Receiver Side Energy Harvesting

Scheduling in Energy Harvesting Networks

Energy Cooperation in Energy Harvesting Networks

Information Theory for Energy Harvesting Communications

Şennur Ulukuş

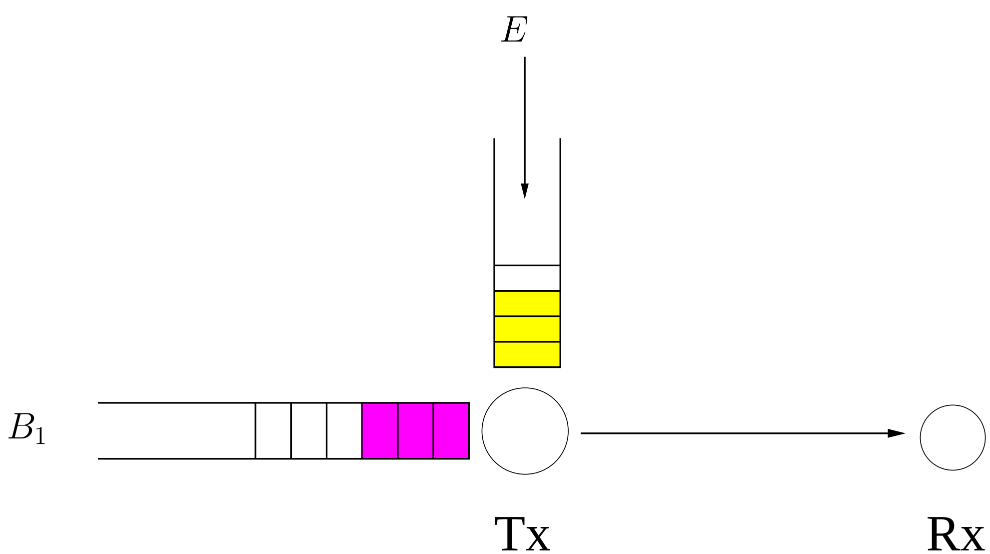
Department of ECE

University of Maryland

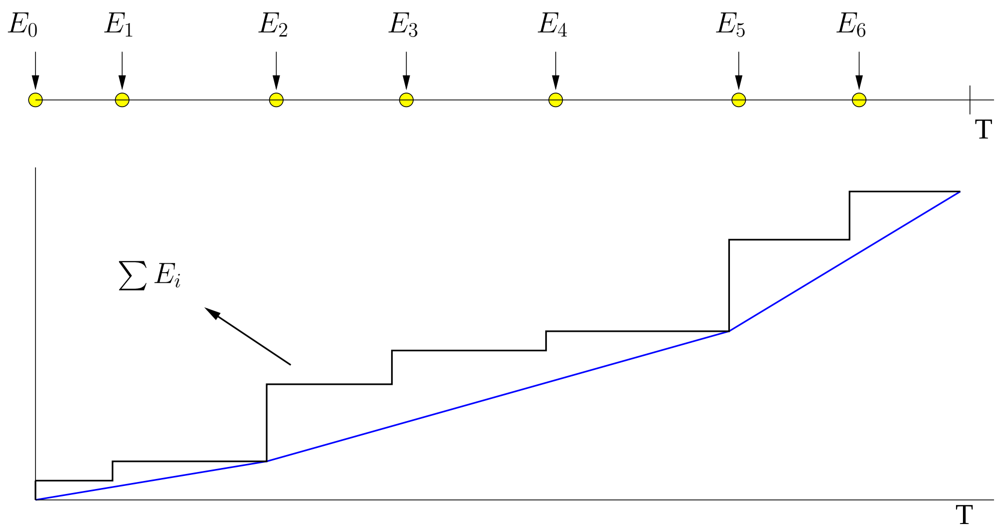
ulukus@umd.edu

So Far, We Learned...

- **Single-user** communication with an energy harvesting transmitter.
- Energy arrives (is harvested) **during the communication session**.
- A non-trivial shift from the conventional battery powered systems.
- Transmission policy is **adapted to energy arrivals.**
- Energy causality constraint and battery capacity limit.
- Objective: Maximize average throughput.

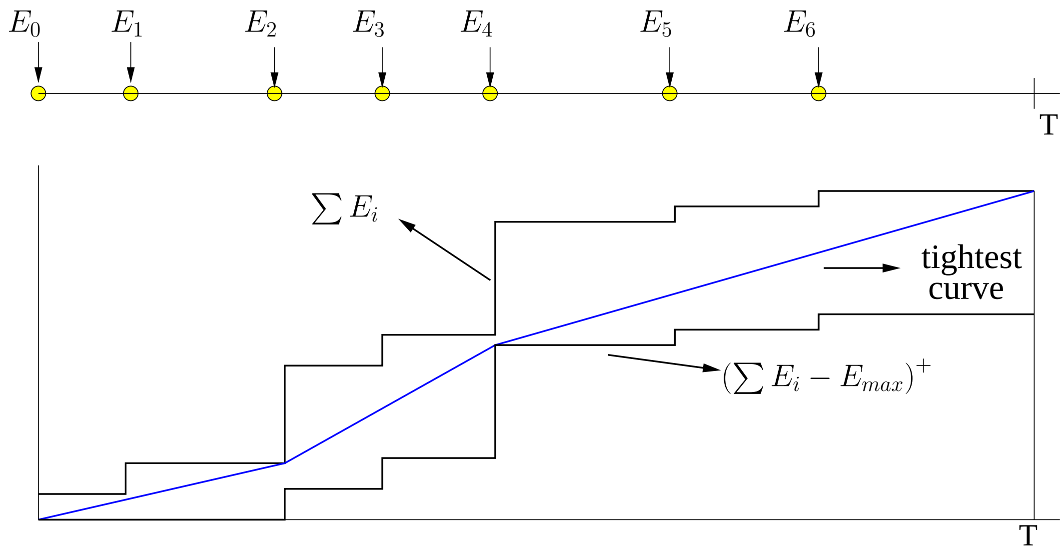


The Optimal Policy for $E_{max} = \infty$



- Upper staircase is the cumulative energy arrivals
- Feasible energy consumption lies below the staircase
- Transmit power remains constant in each epoch
- The tightest curve under the cumulative energy arrival staircase

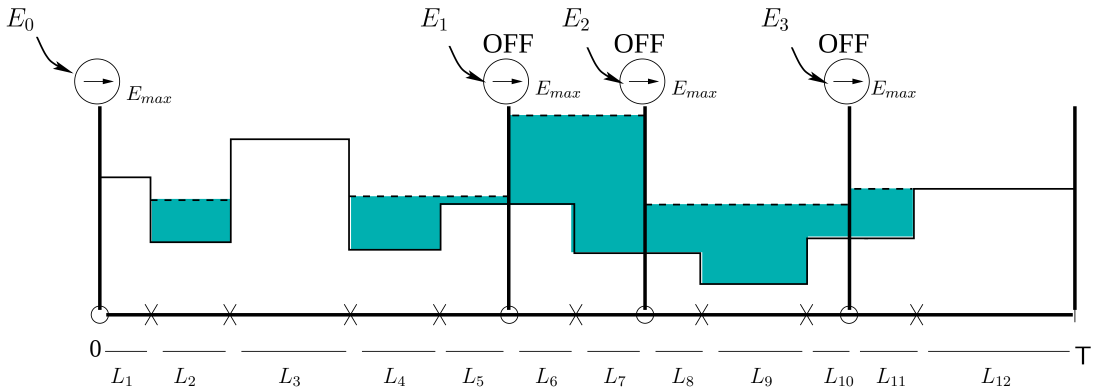
The Optimal Policy for $E_{max} < \infty$



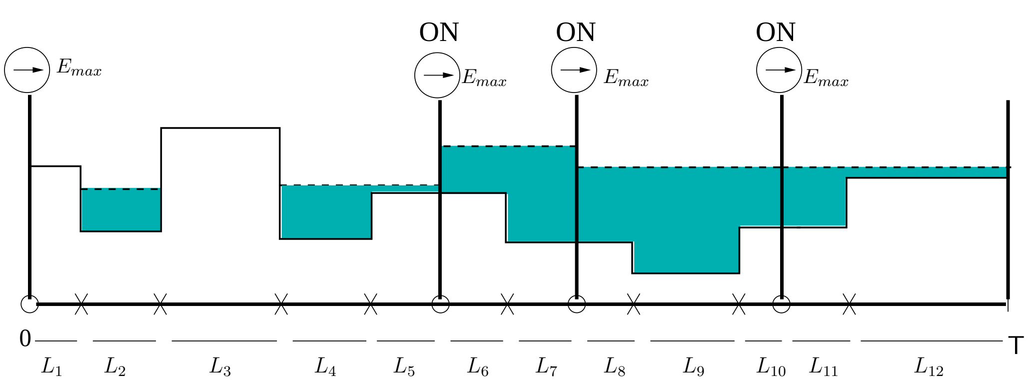
- Upper staircase: energy arrivals
- Lower staircase: finite battery constraint (no overflows)
- Any feasible energy consumption curve must lie **in between**
- Power remains constant in each epoch
- The tightest curve in the feasibility tunnel

Single-User Optimal Policy for Fading Channel

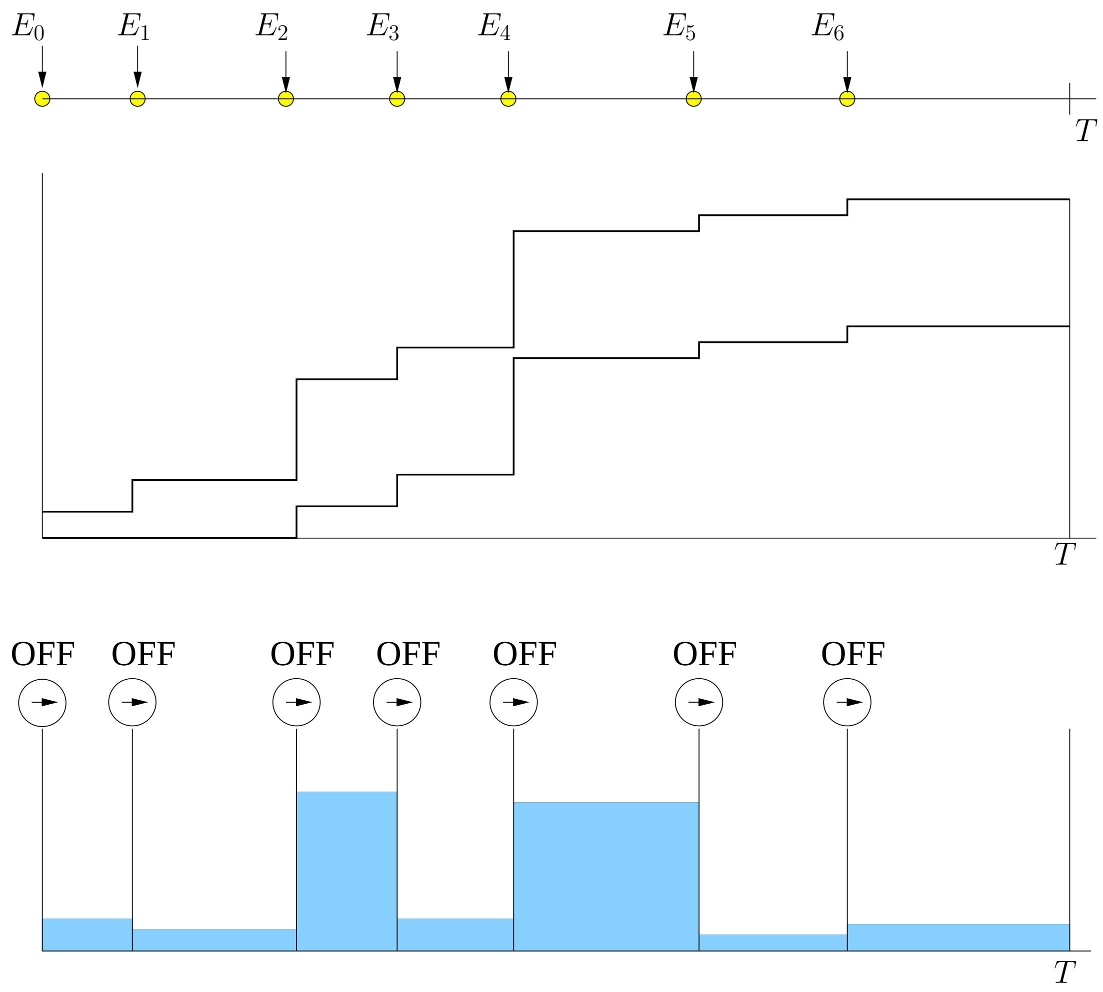
- **Directional water-filling algorithm.**
- First: fill each incoming energy till next energy arrival.



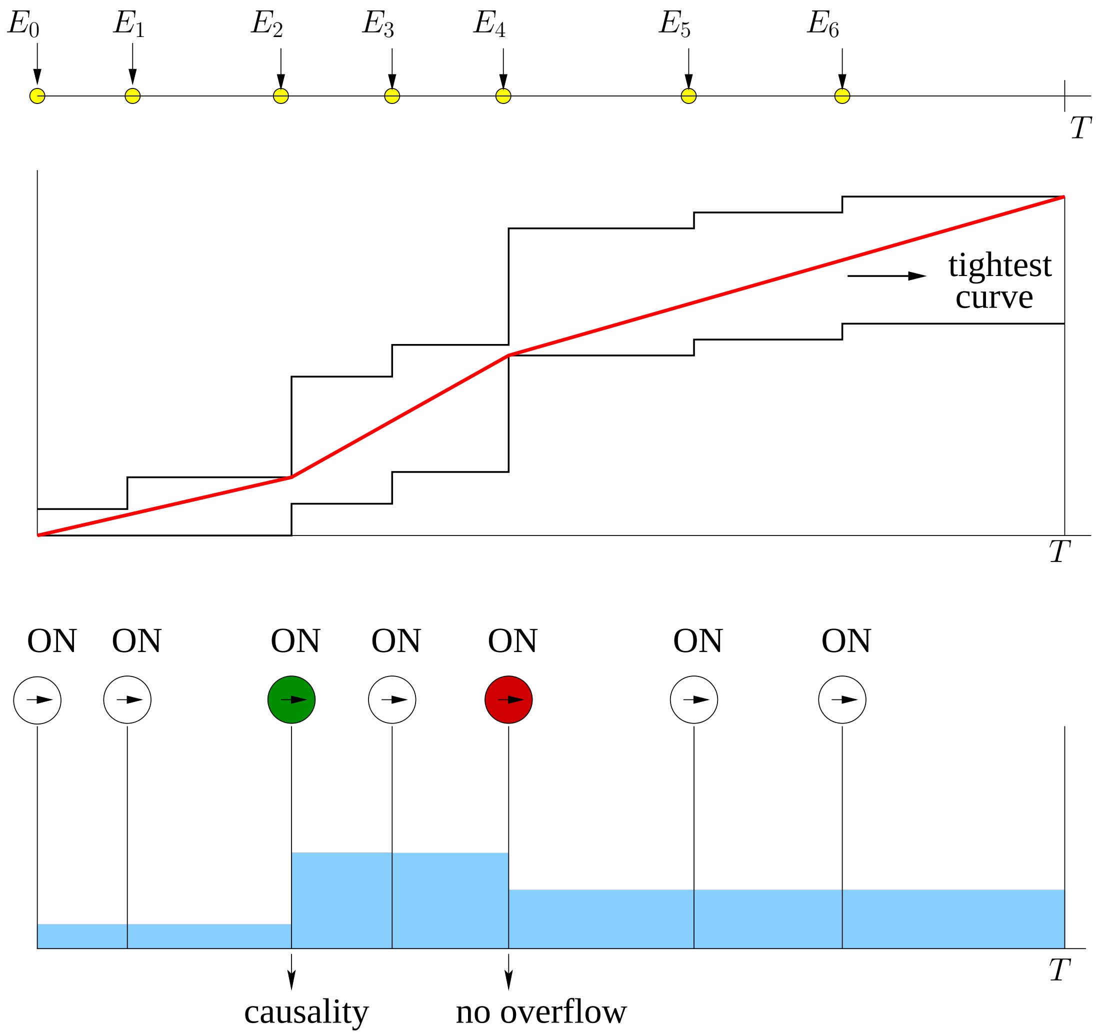
- Second: Allow transfer of energy one by one to the right only.
- Equalize the water level if the water level is higher in the left.



Equivalence of Feasibility Tunnel and Directional Water-filling



Equivalence of Feasibility Tunnel and Directional Water-filling



Single-User Channel with Data Arrivals

- Data is not available before communication; arrives during transmission with amounts $\{B_i\}$.
- Data causality: Source cannot send data packets before receiving them.
- Throughput maximization problem:

$$\begin{aligned}
\max_{\mathbf{p}} \quad & \sum_{i=1}^{N} \frac{1}{2}\log(1+p_i) \triangleq \sum_{i=1}^{N} g(p_i) \\
\text{s.t.} \quad & \sum_{i=1}^{k} p_i \leq \sum_{i=1}^{k} E_i, \quad \forall k \\
& \sum_{i=1}^{k} g(p_i) \leq \sum_{i=1}^{k} B_i, \quad \forall k
\end{aligned}$$

- Either data or energy are bottlenecks.

Single-User Channel with Data Arrivals

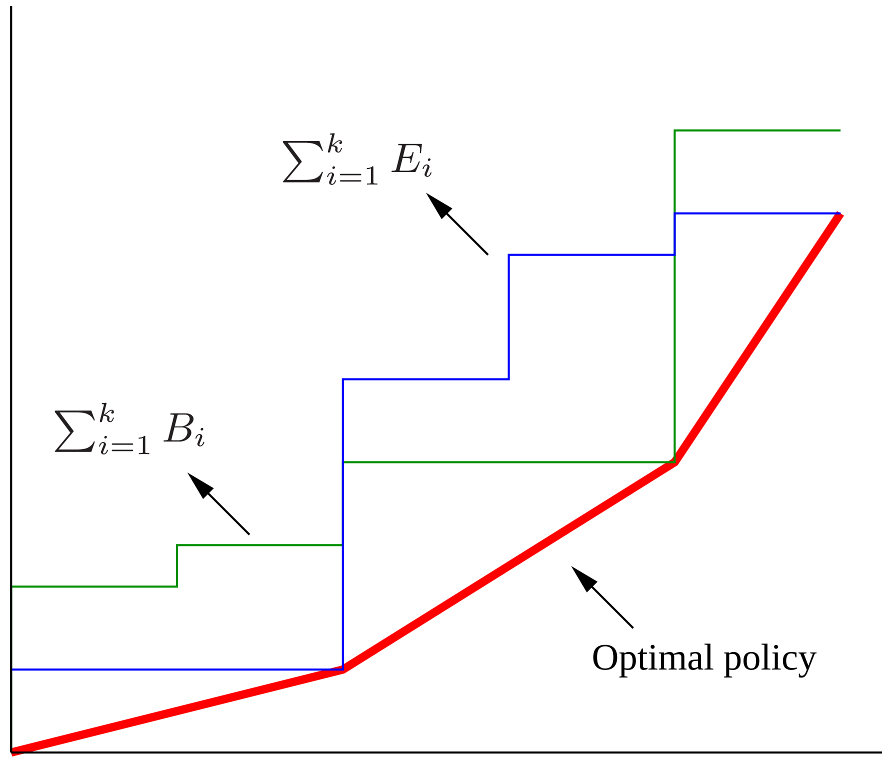
- Data is not available before communication; arrives during transmission with amounts $\{B_i\}$.
- Data causality: Source cannot send data packets before receiving them.
- Throughput maximization problem:

$$\begin{aligned} \max_{\mathbf{r}} \quad & \sum_{i=1}^{N} r_i \\ \text{s.t.} \quad & \sum_{i=1}^{k} 2^{2r_i} - 1 \leq \sum_{i=1}^{k} E_i, \quad \forall k \\ & \sum_{i=1}^{k} r_i \leq \sum_{i=1}^{k} B_i, \quad \forall k \end{aligned}$$

- Either data or energy are bottlenecks.
- Solution given by tightest curve under both cumulative energy and data arrivals:

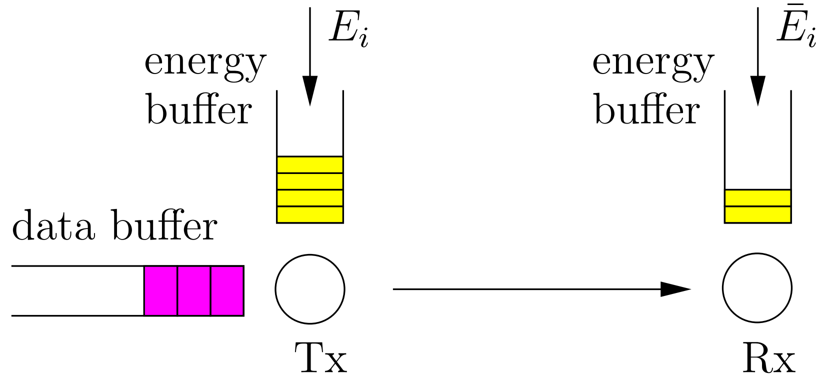
$$r_n = \min\left\{ \frac{1}{2}\log\left(\frac{\sum_{j=1}^{i_n} E_j - \sum_{j=1}^{i_{n-1}} 2^{2r_j} - 1}{i_n - i_{n-1}} \right), \frac{\sum_{j=1}^{i_n} B_j - \sum_{j=1}^{i_{n-1}} r_j}{i_n - i_{n-1}} \right\}$$

Single-User Channel with Data Arrivals: Example



- Optimal policy: Tightest curve under both cumulative energy and data arrivals.

Receiver-Side Energy Harvesting



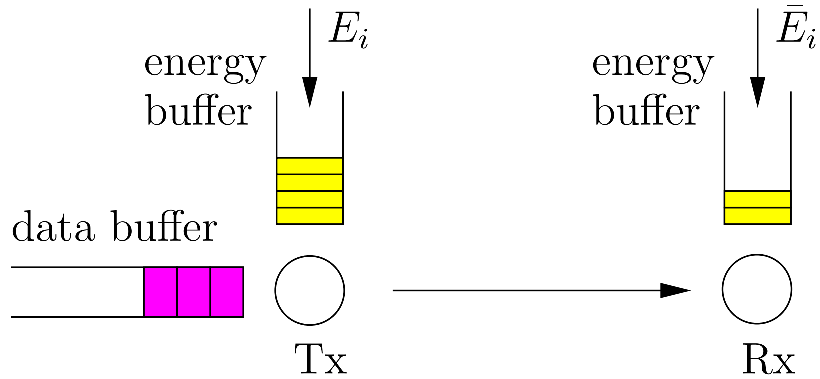
- Receiver spends power mainly in decoding.

- **Decoding causality** constraints:
  - Receiver cannot spend energy in decoding before harvesting it.

- Transmitters should make sure receivers have enough energy to decode.

- Decoding power $\phi(r)$ is convex and increasing. Examples:
  - linear $\phi(r) = ar + b$
  - exponential $\phi(r) = c2^{dr} + e$, specifically, $\phi(r) = 2^{2r} - 1 = g^{-1}$

Receiver-Side Energy Harvesting



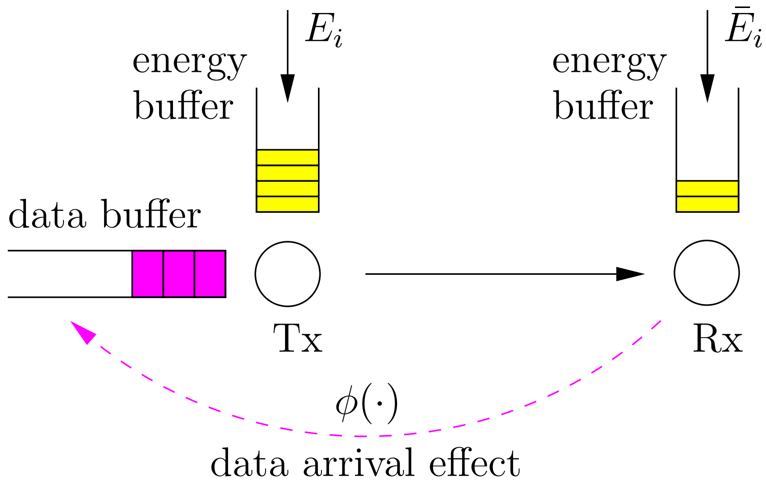
- Throughput maximization problem:

$$\max_{\mathbf{p}} \quad \sum_{i=1}^{N} g(p_i)$$

$$\text{s.t.} \quad \sum_{i=1}^{k} p_i \leq \sum_{i=1}^{k} E_i, \quad \forall k$$

$$\sum_{i=1}^{k} \phi(g(p_i)) \leq \sum_{i=1}^{k} \bar{E}_i, \quad \forall k$$

Receiver-Side Energy Harvesting: Data Arrival Interpretation



- **Decoding causality** constraints:

$$\sum_{i=1}^{k} \phi(g(p_i)) \leq \sum_{i=1}^{k} \bar{E}_i, \quad \forall k$$

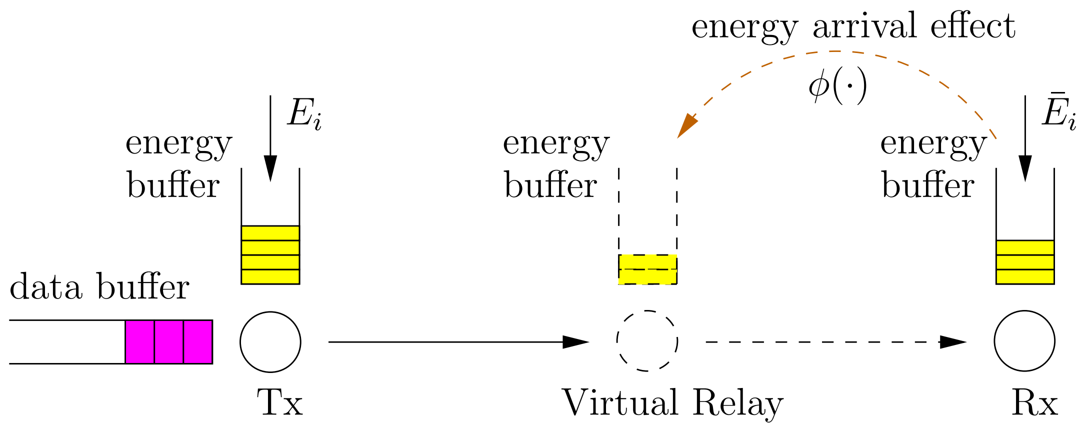
- Interpretation: Gate keeper effect; **generalized data arrival** effect

$$\sum_{i=1}^{k} \phi(r_i) \leq \sum_{i=1}^{k} \bar{E}_i, \quad \forall k$$

- Consider $\phi(r) = r$

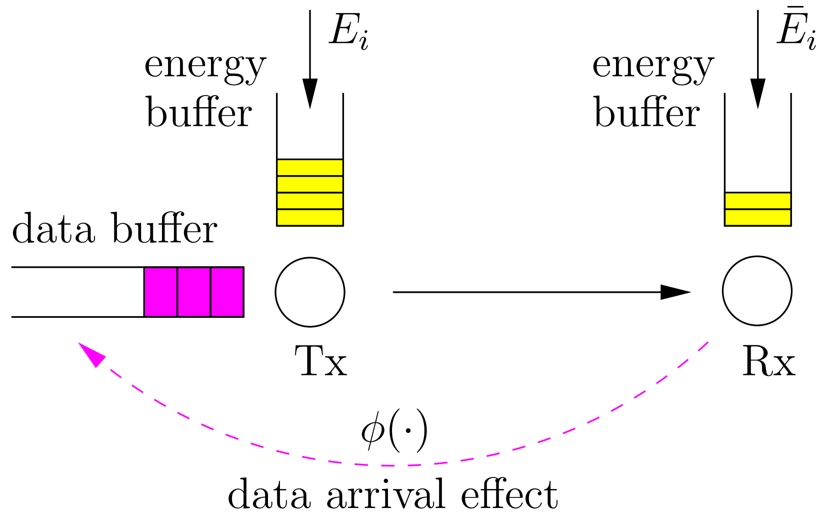
$$\sum_{i=1}^{k} r_i \leq \sum_{i=1}^{k} \bar{E}_i \triangleq \sum_{i=1}^{k} B_i, \quad \forall k$$

Receiver-Side Energy Harvesting: Virtual Relay Interpretation



- Two-hop setting with a virtual relay.
- Relay passes data only if it has energy to forward.
- Relay has no data buffer; rate in equals rate out.
- $\{\bar{E}_i\}$ and ϕ control the amount of energy the relay has to forward data.

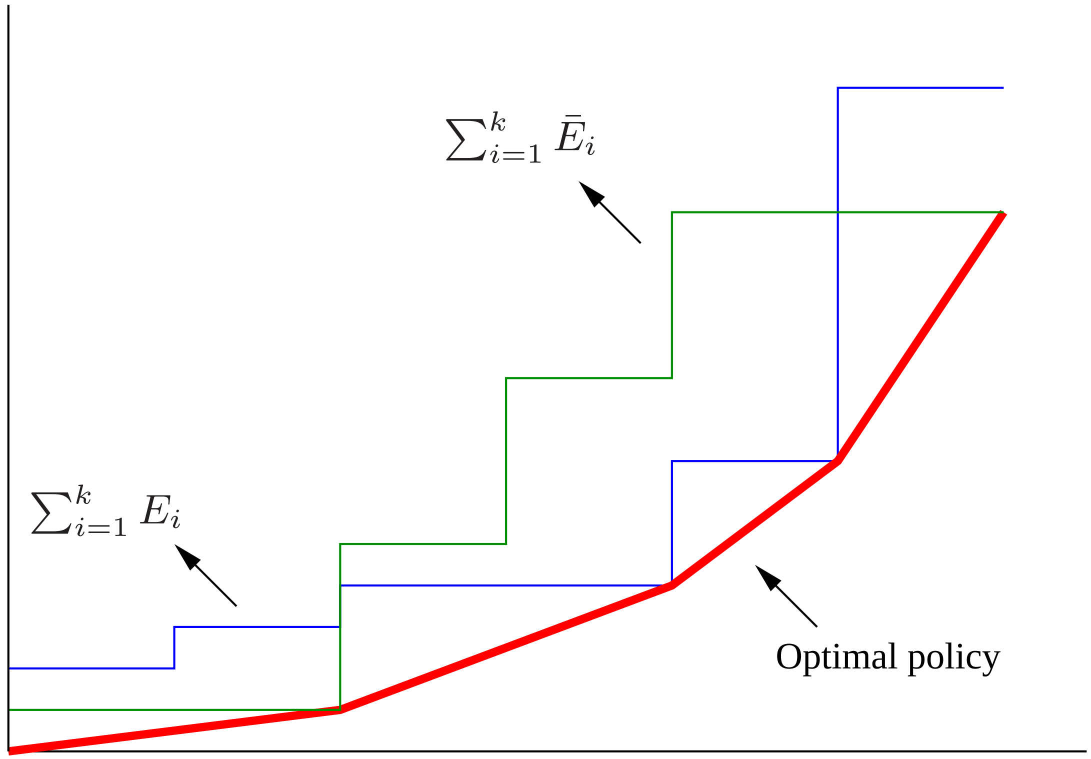
Receiver-Side Energy Harvesting: Solution



- Either transmitter's or receiver's energies are bottlenecks.
- Decoding costs interpreted as **generalized data arrivals.**
- Define $\psi \triangleq \phi^{-1}$, and $f \triangleq g^{-1}$.
- Find tightest curve under both cumulative transmitter's energy and generalized data arrivals:

$$r_n = \min\left\{ g\left(\frac{\sum_{j=1}^{i_n} E_j - \sum_{j=1}^{i_{n-1}} f(r_j)}{i_n - i_{n-1}}\right), \psi\left(\frac{\sum_{j=1}^{i_n} \bar{E}_j - \sum_{j=1}^{i_{n-1}} \phi(r_j)}{i_n - i_{n-1}}\right)\right\}$$

Receiver-Side Energy Harvesting: Example

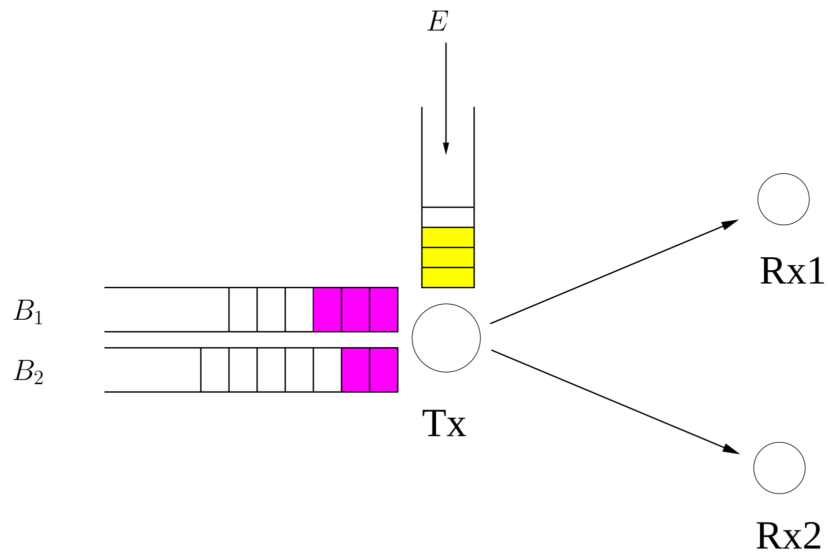


- $\phi(r) = r$.
- Optimal policy: Tightest curve under both cumulative transmitter's and receiver's energies.

Scheduling in Multi-user Energy Harvesting Systems

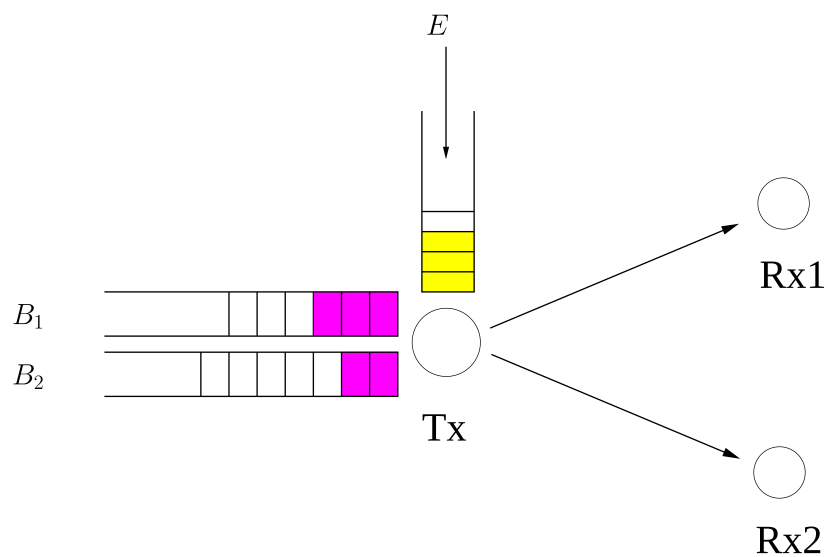
- Broadcasting with an energy harvesting transmitter
  - An energy harvesting transmitter sends messages to two users
  - E.g., a wireless access device sending different messages to users
- Multiple access with energy harvesting transmitters
  - Energy harvesting transmitters communicating with a single receiver
  - E.g., multiple sensors sending data to a center
- Interference channel with energy harvesting transmitters
  - Tx-Rx pairs communicate simultaneously where Txs are energy harvesting.
  - E.g., multiple sensors sending data to different centers.
- Two-hop communication with energy harvesting nodes
  - Source and relay nodes send messages using harvested energy.
  - E.g., end-to-end data delivery in sensor networks.

Broadcasting with an Energy Harvesting Transmitter



- Energy arrives (is harvested) **during the communication session**.
- Assume battery has infinite storage capacity: $E_{max} = \infty$
- Broadcasting data to two users by **adapting to energy arrivals**
- Objective: maximize the data departure region

Broadcast Channel Model



- AWGN broadcast channel:

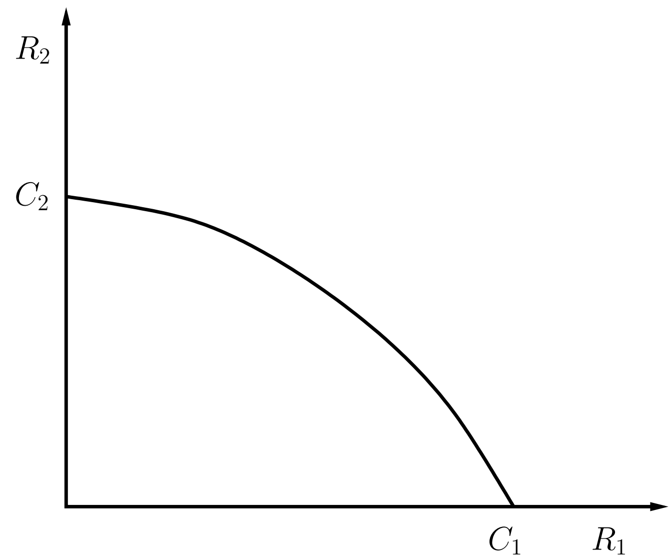
$$Y_1 = X + N_1, \quad Y_2 = X + N_2$$

  where $N_1 \sim \mathcal{N}(0,1)$, $N_2 \sim \mathcal{N}(0,\sigma^2)$

- $\sigma^2 > 1$: 2nd user is degraded

- We call 1st user stronger and 2nd user weaker

Broadcast Channel Model



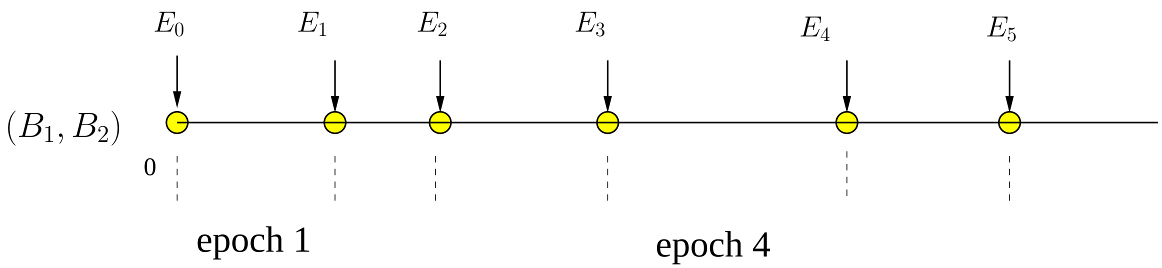
$$r_1 \le \frac{1}{2}\log_2(1+\alpha P)$$
$$r_2 \le \frac{1}{2}\log_2\left(1+\frac{(1-\alpha)P}{\alpha P+\sigma^2}\right)$$

- We work in the (r_1, r_2) domain:

$$P = 2^{2(r_1+r_2)} + (\sigma^2 - 1)2^{2r_2} - \sigma^2 \triangleq F(r_1, r_2)$$

- $F(r_1, r_2)$ is the minimum power required to send at rates (r_1, r_2)

Energy Model

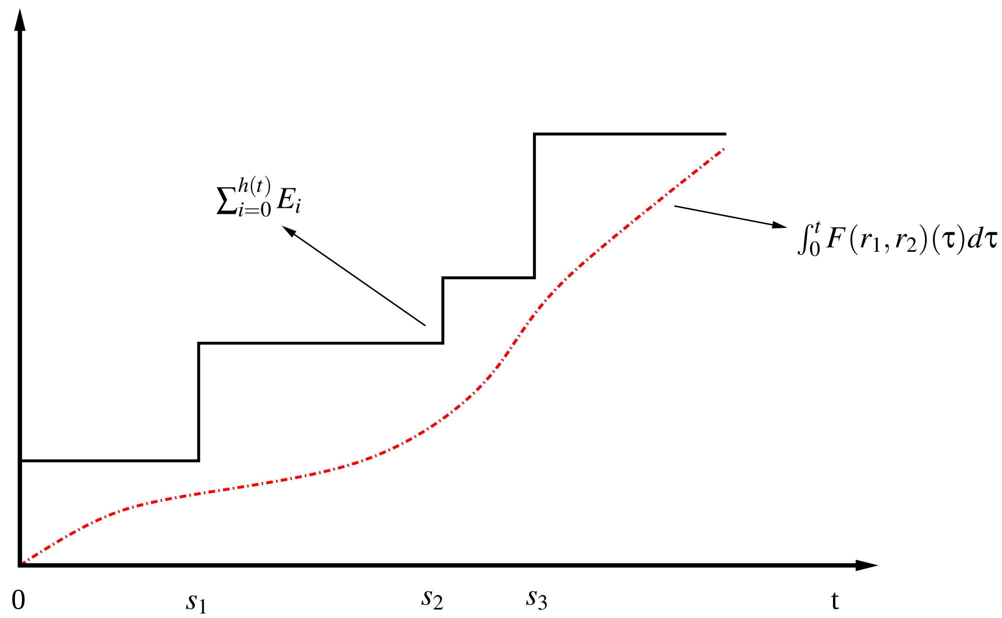


- Energy is *harvested* **during the course of communication**.
- We will consider offline policies.
- Energy causality constraints: energy that has not arrived cannot be used

$$\int_0^{t_i^e} F(r_1, r_2)(\tau)d\tau \leq \sum_{j=0}^{i-1} E_j, \quad \forall i$$

Constraints on the Power Policy

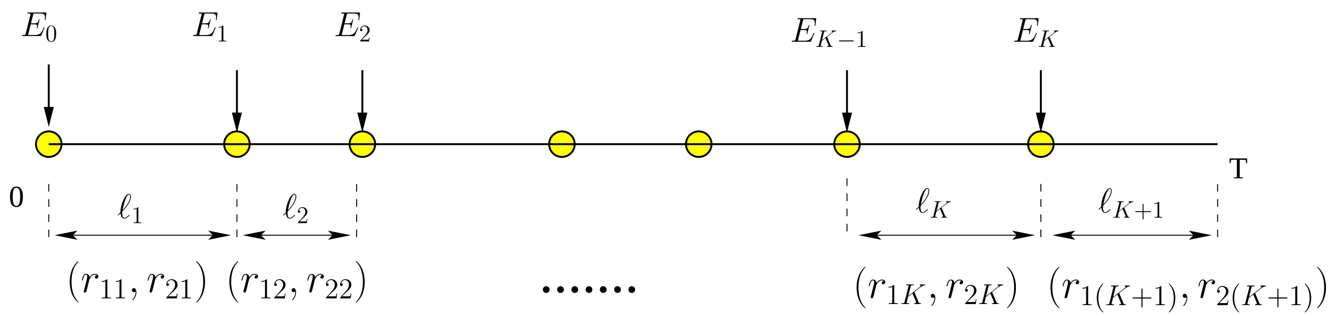
- Energy arrivals known deterministically a priori



- Upper staircase: energy arrivals
- Any feasible energy consumption curve must lie **below the upper staircase**

Find the Maximum Departure Region

- The maximum departure region $\mathcal{D}(T)$: union of (B_1, B_2) pairs achievable by some rate allocation policy that satisfies the energy causality constraint.



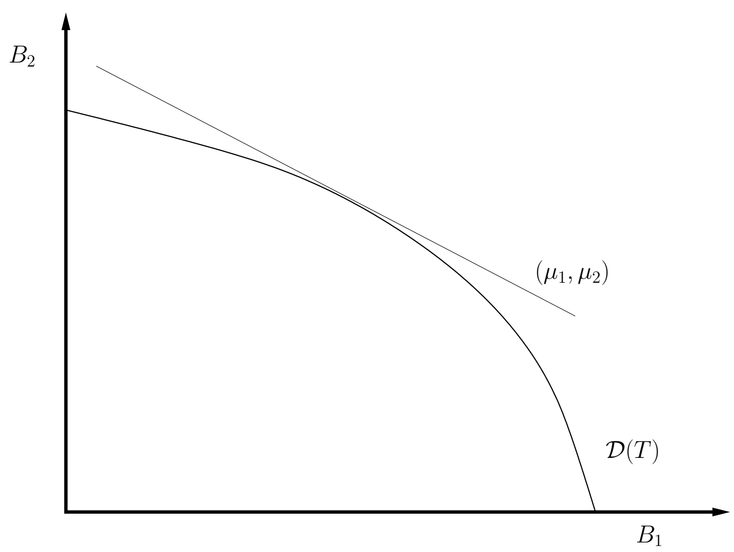
- Transmission rates, and power, remain constant between energy harvests.
- Denote the rates that go to users as (r_{1i}, r_{2i}) over epoch i.
- The **power** at epoch i: $F(r_{1i}, r_{2i})$
- The **energy spent** during epoch i: $F(r_{1i}, r_{2i})\ell_i$
- The energy causality constraint reduces to constraints on (r_{1i}, r_{2i}):

$$\sum_{i=1}^{k} F(r_{1i}, r_{2i})\ell_i \leq \sum_{i=0}^{k-1} E_i, \qquad k = 1, \ldots, K+1$$

Finding the Maximum Departure Region

- $\mathcal{D}(T)$ is a strictly convex region.
- Characterize $\mathcal{D}(T)$ by solving optimization problems for all $\mu_1, \mu_2 \geq 0$:

$$\begin{aligned}\max_{\mathbf{r}_1, \mathbf{r}_2} \quad & \mu_1 \sum_{i=1}^{K+1} r_{1i}\ell_i + \mu_2 \sum_{i=1}^{K+1} r_{2i}\ell_i \\ \text{s.t.} \quad & \sum_{i=1}^{k} F(r_{1i}, r_{2i})\ell_i \leq \sum_{i=0}^{k-1} E_i, \qquad k = 1, \ldots, K+1\end{aligned}$$



Finding the Maximum Departure Region

- The Lagrangian function

$$\mathcal{L} = \mu_1 \sum_{i=1}^{K+1} r_{1i}\ell_i + \mu_2 \sum_{i=1}^{K+1} r_{2i}\ell_i - \sum_{k=1}^{K+1} \lambda_k \left(\sum_{i=1}^{k} F(r_{1i}, r_{2i})\ell_i - \sum_{i=0}^{k-1} E_i \right)$$
$$+ \sum_{i=1}^{K+1} \gamma_{1i} r_{1i} + \sum_{i=1}^{K+1} \gamma_{2i} r_{2i}$$

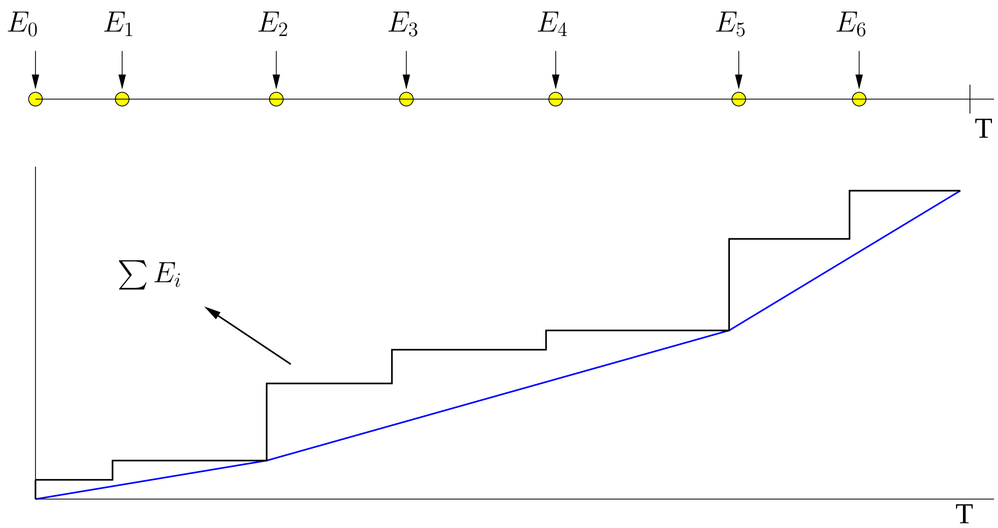
- Total power in terms of Lagrange multipliers

$$P_i = \max \left\{ \frac{\mu_1 + \gamma_{1i}}{\sum_{k=i}^{K+1} \lambda_k} - 1, \frac{\mu_2 + \gamma_{2i}}{\sum_{k=i}^{K+1} \lambda_k} - \sigma^2 \right\}$$

- Structural properties of the optimal policy:
  - Optimal total transmit power, $\{F(r_{1i}^*, r_{2i}^*)\}_{i=1}^{K+1}$, is independent of μ_1, μ_2.
  - In particular, it is the same as the optimal single-user transmit power.

Single User Optimal Policy

- Single user optimal policy is found by calculating the tightest curve below the energy arrival curve:



- Slope of the curve is the allocated power
- Power is monotonically increasing

Structure of the Optimal Policy

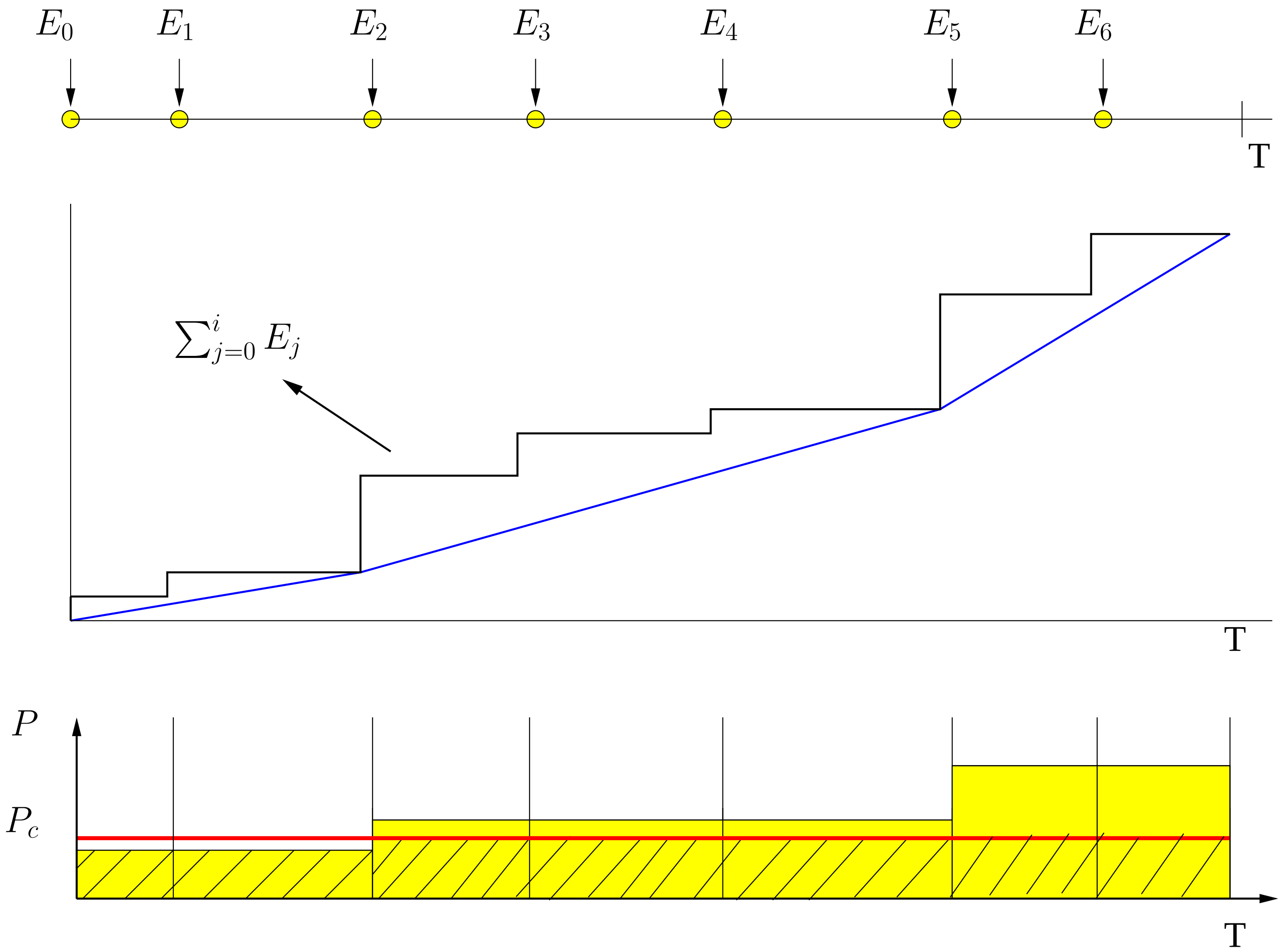
- Total transmit power is the same as the single-user case.
- The power shares follow a cut-off structure:
- Cut-off level P_c

$$P_c = \frac{\mu - 1}{\sigma^2 - \mu}$$

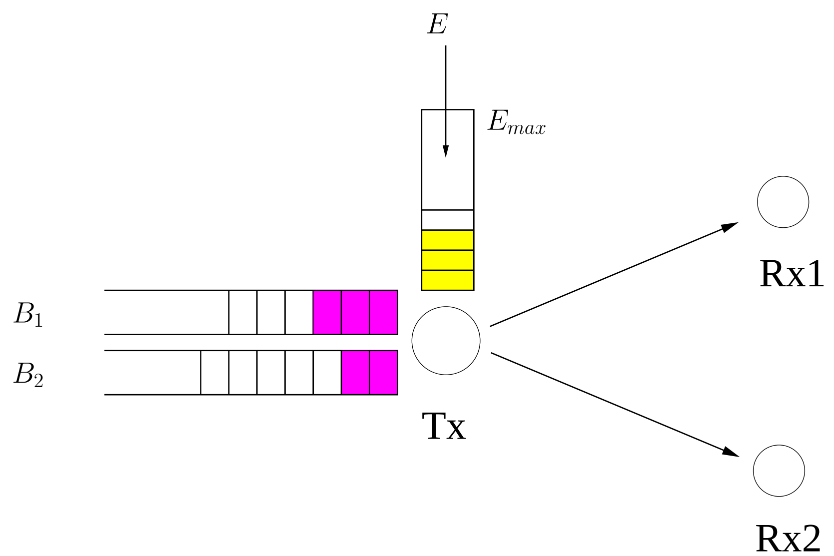
where $\mu = \frac{\mu_2}{\mu_1}$ and $1 < \mu < \sigma^2$.

- If below P_c, then, only transmit to the stronger user.
- Otherwise, stronger user's power share is P_c.
- Extreme cases:
  - If $\mu \leq 1$, only the stronger user's data is transmitted
  - If $\mu \geq \sigma^2$, only the weaker user's data is transmitted

The Structure of the Optimal Policy



Broadcast Channel with Finite E_{max}

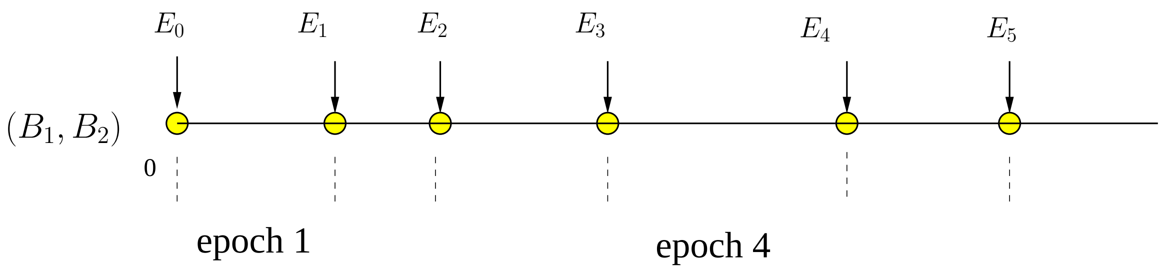


- (B_1, B_2) bits to be sent and battery capacity $E_{max} < \infty$
- AWGN broadcast channel:

$$Y_1 = X + N_1, \quad Y_2 = X + N_2$$

- $N_1 \sim \mathcal{N}(0,1)$ and $N_2 \sim \mathcal{N}(0,\sigma^2)$ with $\sigma^2 > 1$
- 1st user stronger and 2nd user weaker

Broadcast Channel with Finite E_{max}



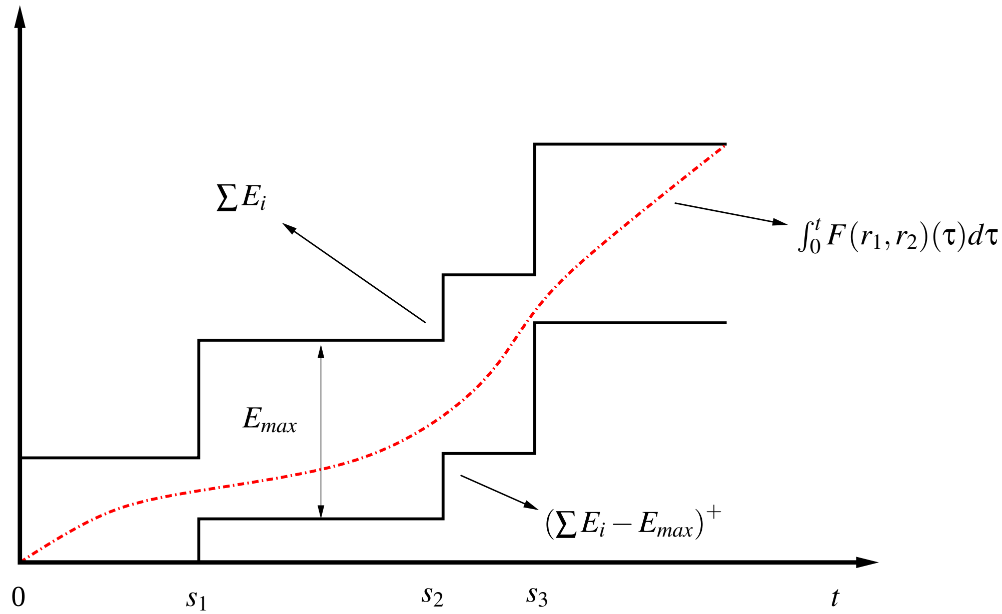
- Incoming energies are smaller than E_{max}: $E_i \leq E_{max}$
- Energy causality constraints: energy that has not arrived cannot be used

$$\int_0^{t_i^e} F(r_1, r_2)(u) du \leq \sum_{j=0}^{i-1} E_j, \quad \forall i$$

- No-energy-overflow condition: energy overflow (wasting) is suboptimal

$$\sum_{j=0}^{h(t)} E_j - \int_0^t F(r_1, r_2)(u) du \leq E_{max}, \quad \forall t$$

Constraints on the Power Policy



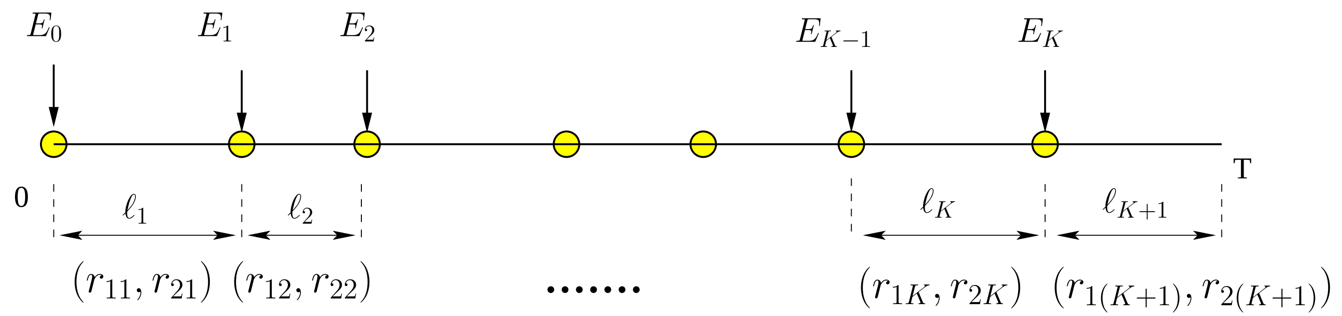
- Energy causality constraints: energy that has not arrived cannot be used

$$\int_0^{t_i^e} F(r_1,r_2)(u)du \le \sum_{j=0}^{i-1} E_j, \quad \forall i$$

- No-energy-overflow condition: energy overflow (wasting) is suboptimal

$$\sum_{j=0}^{h(t)} E_j - \int_0^t F(r_1,r_2)(u)du \le E_{max}, \quad \forall t$$

Find the Maximum Departure Region



- The transmission rates, and hence the transmission power, remain constant between energy harvests in any optimal policy

- The energy causality constraint reduces to constraints on (r_{1i}, r_{2i}):

$$\sum_{i=1}^{k} F(r_{1i}, r_{2i})\ell_i \leq \sum_{i=0}^{k-1} E_i, \qquad k = 1, \ldots, K+1$$

- The no-energy-overflow condition:

$$\sum_{i=0}^{k} E_i - \sum_{i=1}^{k} F(r_{1i}, r_{2i})\ell_i \leq E_{max}, \qquad k = 1, \ldots, K$$

Finding the Maximum Departure Region

- $\mathcal{D}(T)$ is a strictly convex region.
- Characterize $\mathcal{D}(T)$ by solving optimization problems for all $\mu_1, \mu_2 \geq 0$:

$$\begin{aligned}\max_{\mathbf{r}_1,\mathbf{r}_2} \quad & \mu_1 \sum_{i=1}^{K+1} r_{1i}\ell_i + \mu_2 \sum_{i=1}^{K+1} r_{2i}\ell_i \\ \text{s.t.} \quad & \sum_{i=1}^{k} F(r_{1i}, r_{2i})\ell_i \leq \sum_{i=0}^{k-1} E_i, \ 1 \leq k \leq K+1 \\ & \sum_{i=0}^{k} E_i - \sum_{i=1}^{k} F(r_{1i}, r_{2i})\ell_i \leq E_{max}, \ 1 \leq k \leq K\end{aligned}$$

Finding the Maximum Departure Region

- The Lagrangian function

$$\mathcal{L} = \mu_1 \sum_{i=1}^{K+1} r_{1i}\ell_i + \mu_2 \sum_{i=1}^{K+1} r_{2i}\ell_i - \sum_{k=1}^{K+1} \lambda_k \left(\sum_{i=1}^{k} F(r_{1i}, r_{2i})\ell_i - \sum_{i=0}^{k-1} E_i \right)$$
$$- \sum_{k=1}^{K} \eta_k \left(\sum_{i=0}^{k} E_i - \sum_{i=1}^{k} F(r_{1i}, r_{2i})\ell_i - E_{max} \right) + \sum_{i=1}^{K+1} \gamma_{1i} r_{1i} + \sum_{i=1}^{K+1} \gamma_{2i} r_{2i}$$

- Total power in terms of Lagrange multipliers

$$P_i = \max \left\{ \frac{\mu_1}{\left(\sum_{k=i}^{K+1} \lambda_k - \sum_{k=i}^{K} \eta_k\right)} - 1, \frac{\mu_2}{\left(\sum_{k=i}^{K+1} \lambda_k - \sum_{k=i}^{K} \eta_k\right)} - \sigma^2 \right\}$$

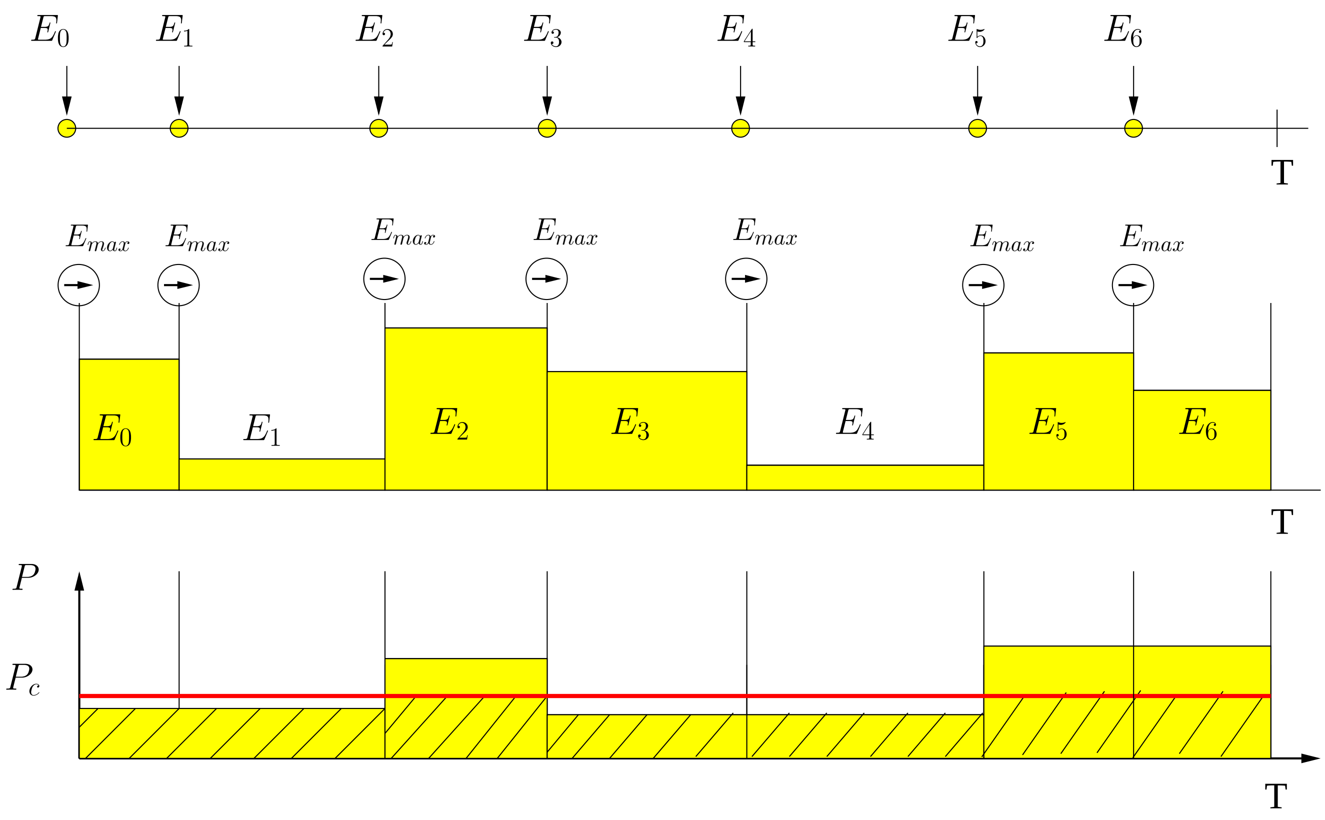
Structure of the Optimal Policy

- Optimal total transmit power, $\{F(r_{1i}^*, r_{2i}^*)\}_{i=1}^{K+1}$, is independent of μ_1, μ_2.
- In particular, it is the same as the optimal single-user transmit power.
- The power shares follow a cut-off structure:
- Cut-off level P_c

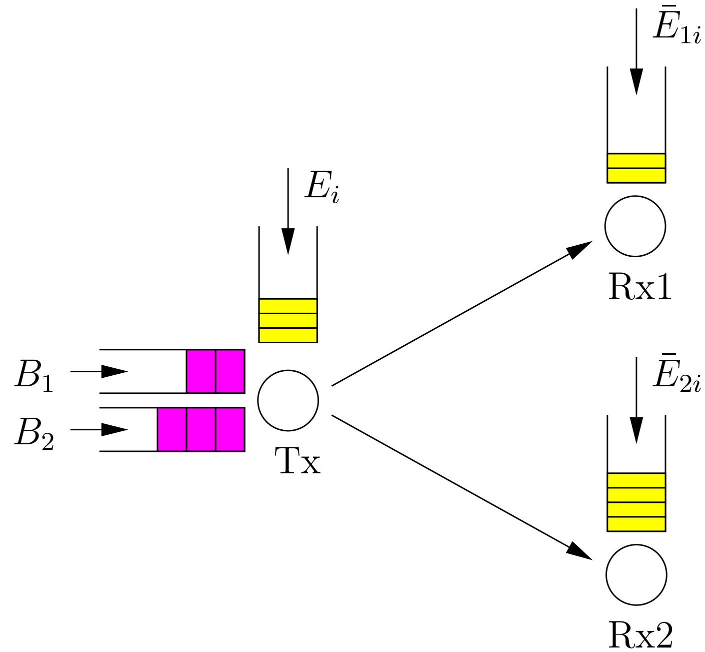
$$P_c = \frac{\mu - 1}{\sigma^2 - \mu}$$

  where $\mu = \frac{\mu_2}{\mu_1}$ and $1 < \mu < \sigma^2$.
- If below P_c, then, only the stronger user
- Otherwise, stronger user's power share is P_c.
- Extreme cases:
  - If $\mu \leq 1$, only the stronger user's data is transmitted
  - If $\mu \geq \sigma^2$, only the weaker user's data is transmitted

The Structure of the Optimal Policy



Broadcast Channel with Energy Harvesting Transmitter and Receivers



- Transmitter uses superposition coding.
- Weak user only decodes its message:
  - Decoding power is a function of its own rate: $\phi(r_2)$.
- Strong user decodes both messages:
  - Decoding power is a function of sum rate: $\phi(r_1 + r_2)$.

Broadcast Channel with Energy Harvesting Transmitter and Receivers

- Characterizing the maximum departure region $\mathcal{D}(N)$:

$$
\begin{aligned}
\max_{\mathbf{r_1},\mathbf{r_2}} \quad & \mu_1 \sum_{i=1}^{N} r_{1i} + \mu_2 \sum_{i=1}^{N} r_{2i} \\
\text{s.t.} \quad & \sum_{i=1}^{k} F\left(r_{1i}, r_{2i}\right) \leq \sum_{i=1}^{k} E_i, \quad \forall k \\
& \sum_{i=1}^{k} \phi(r_{1i} + r_{2i}) \leq \sum_{i=1}^{k} \bar{E}_{1i}, \quad \forall k \\
& \sum_{i=1}^{k} \phi(r_{2i}) \leq \sum_{i=1}^{k} \bar{E}_{2i}, \quad \forall k
\end{aligned}
$$

- Consider exponential decoding power function: $\phi(r) = 2^{2r} - 1$.

Problem Formulation

- Change of variables: $p_{ti} \triangleq 2^{2(r_{1i}+r_{2i})}$, and $p_{2i} \triangleq 2^{2r_{2i}} - 1$.
- Problem in terms of powers:

$$
\begin{aligned}
\max_{\mathbf{p}_t, \mathbf{p}_2} \quad & \mu_1 \sum_{i=1}^{N} g(p_{ti}) + (\mu_2 - \mu_1) \sum_{i=1}^{N} g(p_{2i}) \\
\text{s.t.} \quad & \sum_{i=1}^{k} (\sigma^2 - 1) p_{2i} + p_{ti} \leq \sum_{i=1}^{k} E_i, \quad \forall k \\
& \sum_{i=1}^{k} p_{ti} \leq \sum_{i=1}^{k} \bar{E}_{1i}, \quad \forall k \\
& \sum_{i=1}^{k} p_{2i} \leq \sum_{i=1}^{k} \bar{E}_{2i}, \quad \forall k \\
& p_{ti} \geq p_{2i}, \quad \forall i
\end{aligned}
$$

Problem Decomposition

- Problem decomposition: Inner problem for fixed $\mathbf{p}_2$

$$\max_{\mathbf{p}_t, \mathbf{p}_2} \quad \mu_1 \sum_{i=1}^{N} g\left(p_{ti}\right) + \left(\mu_2 - \mu_1\right) \sum_{i=1}^{N} g\left(p_{2i}\right)$$

$$\text{s.t.} \quad \sum_{i=1}^{k} p_{ti} \leq \sum_{i=1}^{k} E_i - \left(\sigma^2 - 1\right) p_{2i}, \quad \forall k$$

$$\sum_{i=1}^{k} p_{ti} \leq \sum_{i=1}^{k} \bar{E}_{1i}, \quad \forall k$$

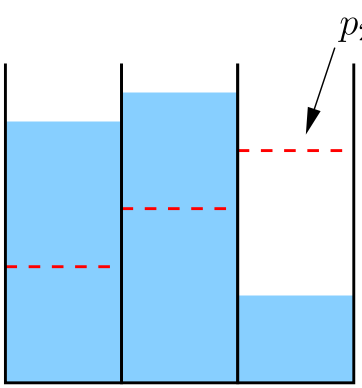
$$\sum_{i=1}^{k} p_{2i} \leq \sum_{i=1}^{k} \bar{E}_{2i}, \quad \forall k$$

$$p_{ti} \geq p_{2i}, \quad \forall i$$

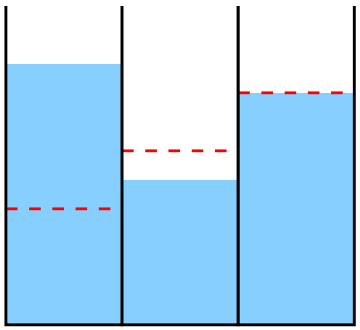
Problem Decomposition: Inner Problem

- Inner problem: For fixed $\mathbf{p}_2$, solve the following problem with minimum power constraints:

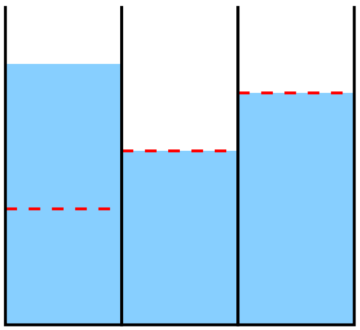
$$
\begin{aligned}
H(\mathbf{p}_2) \triangleq \max_{\mathbf{p}_t} \quad & \sum_{i=1}^{N} g(p_{ti}) \\
\text{s.t.} \quad & \sum_{i=1}^{k} p_{ti} \leq \sum_{i=1}^{k} V_i, \quad \forall k \\
& p_{ti} \geq p_{2i}, \quad \forall i
\end{aligned}
$$



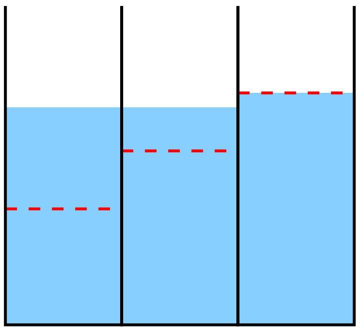
1: Initialization



2: Filling last bin first



3: Filling middle bin



4: Directional water-filling from first bin

Problem Decomposition

- Problem decomposition: Inner problem for fixed $\mathbf{p}_2$

$$
\begin{aligned}
\max_{\mathbf{p}_t, \mathbf{p}_2} \quad & \mu_1 \sum_{i=1}^{N} g\left(p_{ti}\right) + \left(\mu_2 - \mu_1\right) \sum_{i=1}^{N} g\left(p_{2i}\right) \\
\text{s.t.} \quad & \sum_{i=1}^{k} p_{ti} \leq \sum_{i=1}^{k} E_i - \left(\sigma^2 - 1\right) p_{2i}, \quad \forall k \\
& \sum_{i=1}^{k} p_{ti} \leq \sum_{i=1}^{k} \bar{E}_{1i}, \quad \forall k \\
& \sum_{i=1}^{k} p_{2i} \leq \sum_{i=1}^{k} \bar{E}_{2i}, \quad \forall k \\
& p_{ti} \geq p_{2i}, \quad \forall i
\end{aligned}
$$

Problem Decomposition: Outer Problem

- Problem decomposition: Outer problem in terms of $\mathbf{p}_2$

$$\begin{aligned} \max_{\mathbf{p}_2} \quad & \mu_1 H(\mathbf{p}_2) + (\mu_2 - \mu_1) \sum_{i=1}^{N} g(p_{2i}) \\ \text{s.t.} \quad & \sum_{i=1}^{k} p_{2i} \leq \sum_{i=1}^{k} \bar{E}_{2i}, \quad \forall k \end{aligned}$$

Problem Decomposition: Outer Problem

- Problem decomposition: Outer problem in terms of $\mathbf{p}_2$

$$\begin{aligned} \max_{\mathbf{p}_2} \quad & \mu_1 H(\mathbf{p}_2) + (\mu_2 - \mu_1) \sum_{i=1}^{N} g(p_{2i}) \\ \text{s.t.} \quad & \sum_{i=1}^{k} p_{2i} \leq \sum_{i=1}^{k} \bar{E}_{2i}, \quad \forall k \end{aligned}$$

- $H(\mathbf{p}_2)$ is a decreasing concave function in $\mathbf{p}_2$.
- For a fixed *increasing* $\mathbf{p}_2$, the solution $\mathbf{p}_t$ is also *increasing*.
- Convex problem. Possibly not all energies will be used.
- Iterate between inner and outer problems until convergence.

Conclusions for the Broadcasting Scenario

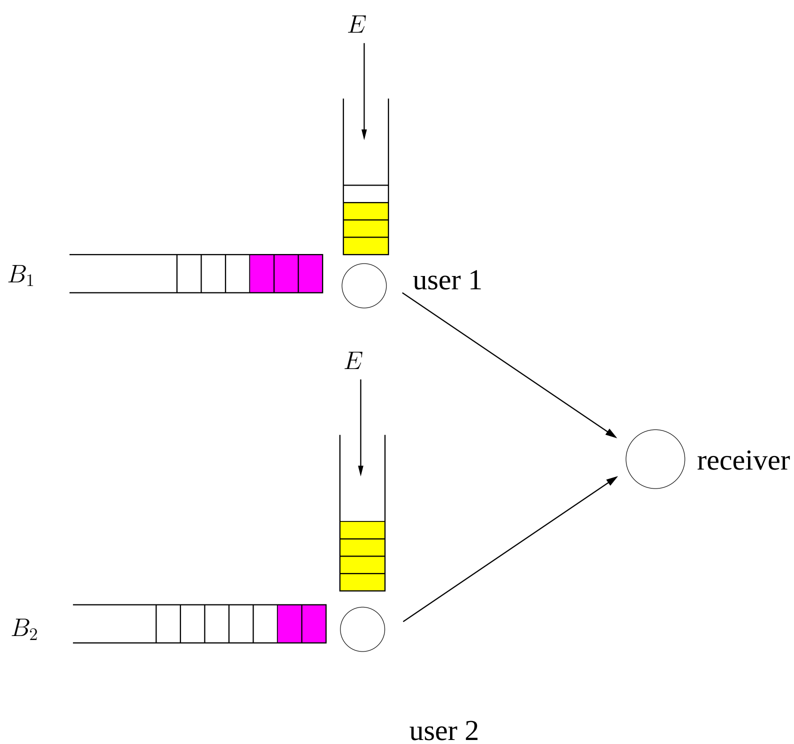
- Energy harvesting transmitter with infinite and finite capacity battery
- Maximize the departure region.
- Obtain the structure such as
  - the monotonicity of the transmit power
  - the cut-off power property
- Energy harvesting transmitter **and receivers:**
  - Exponential decoding costs.
  - Superposition coding: Strong user's decoding cost higher than weak user's.
  - Maximum departure region found by inner/outer problem decomposition.

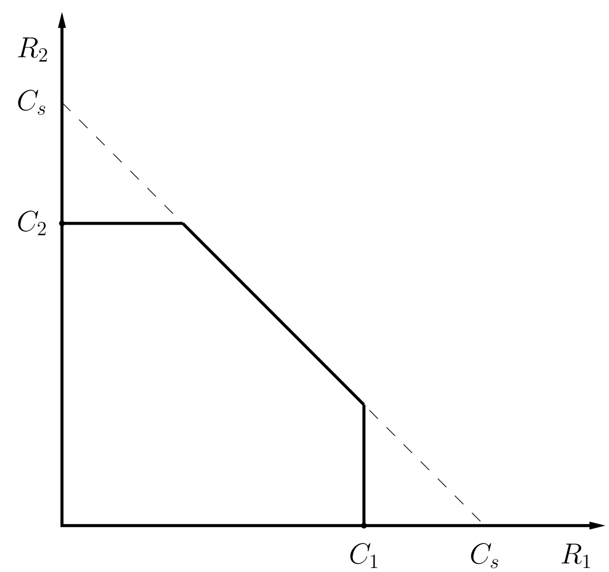
Optimal Packet Scheduling: Multiple Access Channel

- AWGN MAC channel $Y = X_1 + X_2 + Z$, $Z \sim N(0,1)$.
- The capacity region is a pentagon denoted as $\mathcal{C}(P_1, P_2)$:

$$R_1 \leq g(P_1), \quad R_2 \leq g(P_2), \quad R_1 + R_2 \leq g(P_1 + P_2)$$

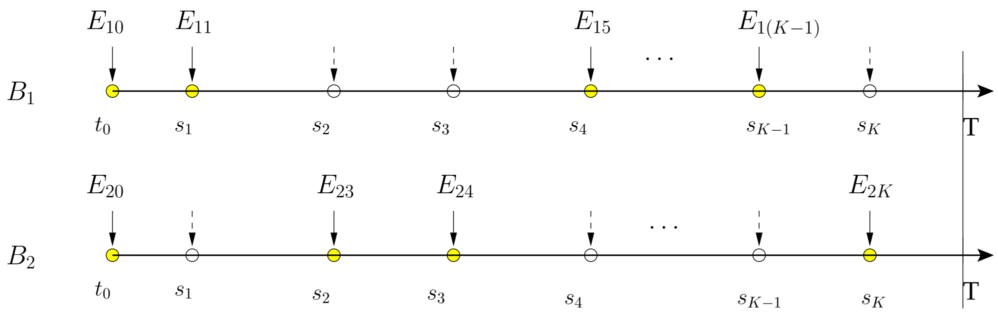
where $g(p) = \frac{1}{2}\log(1+p)$.

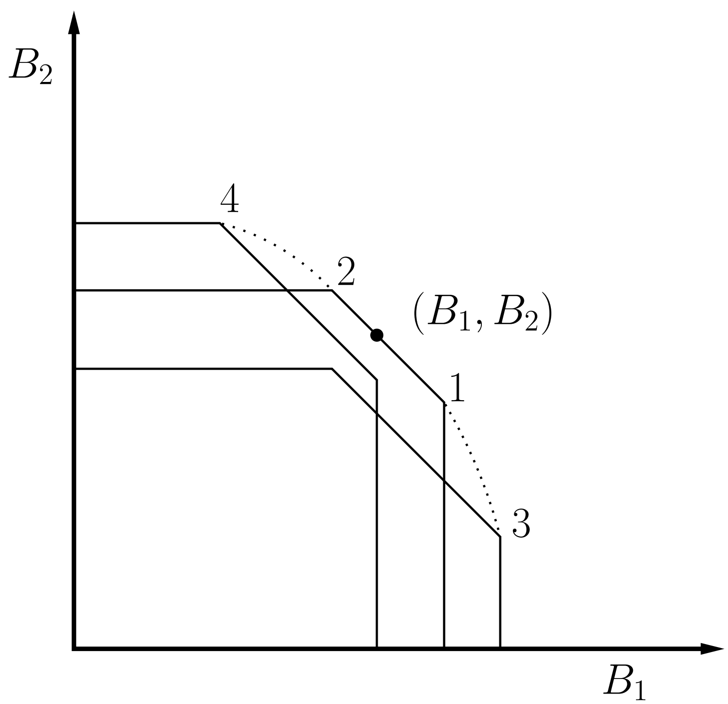




Problem Formulation

- Maximize departure region $\mathcal{D}(T)$ by time T.





Characterizing $\mathcal{D}(T)$

- Transmission rate remains constant between energy harvests.
- For any feasible transmit power sequences $\mathbf{p}_1$, $\mathbf{p}_2$ over $[0,T)$, the departure region is a pentagon defined as

$$B_1 \leq \sum_{n=1}^{N} g(p_{1n}) l_n$$

$$B_2 \leq \sum_{n=1}^{N} g(p_{2n}) l_n$$

$$B_1 + B_2 \leq \sum_{n=1}^{N} g(p_{1n} + p_{2n}) l_n$$

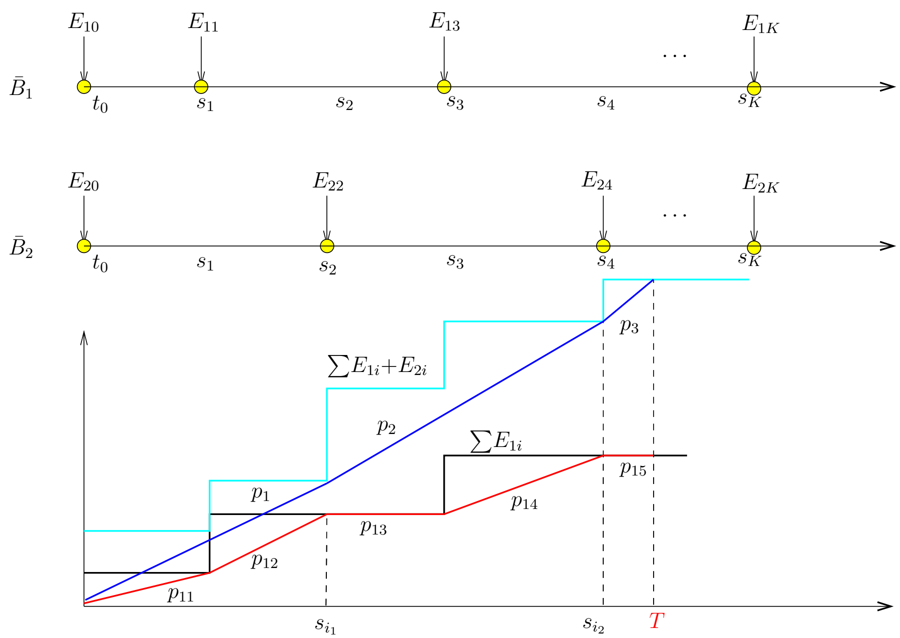
- $\mathcal{D}(T)$ is a union of (B_1, B_2) and convex.
- The boundary points maximize $\mu_1 B_1 + \mu_2 B_2$ for some $\mu_1, \mu_2 \geq 0$.

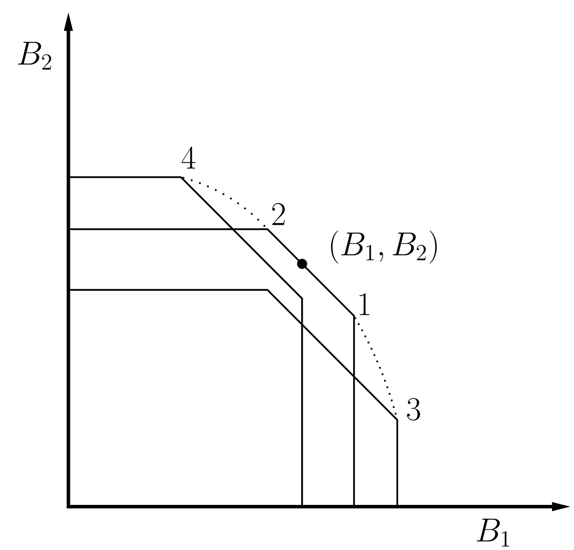
$\mu_1 = \mu_2$

- The problem becomes $\max_{\mathbf{p}_1,\mathbf{p}_2} B_1 + B_2$.
- Sum of powers has same "majorization" property as in single-user.
- Merge energy arrivals of the users, get the optimal sum powers, $p_1, \ldots, p_n$
- Each feasible sequence of p_{1n} and p_{2n} gives a pentagon.
- Union of them is a larger pentagon: dominant faces on the same line.
- Need to identify the boundary of this larger pentagon.

Achieving Corner Points of the Boundary

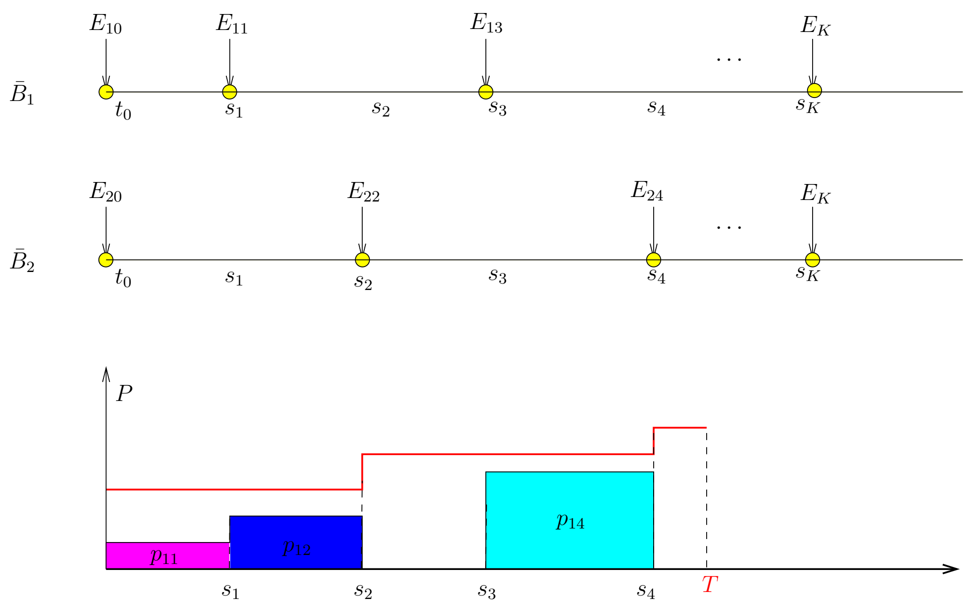
- Maximize B_1 s.t. $B_1 + B_2$ is maximized at the same time $\Rightarrow$ point 1.
  - Equalize the transmit powers of the first user as much as possible
  - Additionally: both users' energy constraints are tight if sum power changes.

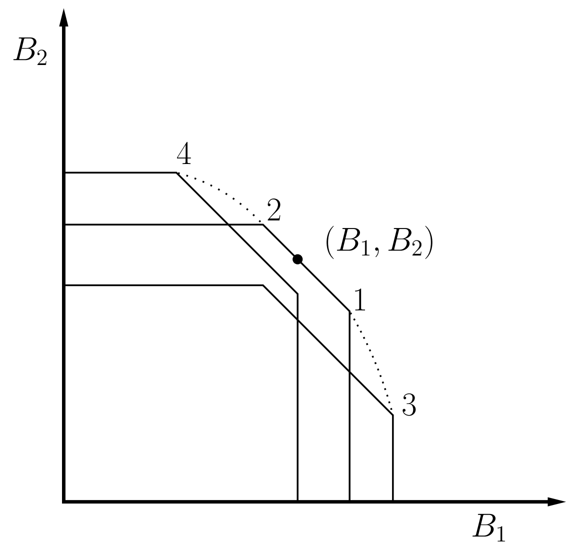




$\mu_1 = 0$ or $\mu_2 = 0$

- Maximize B_1 or B_2 $\Rightarrow$ a single-user scenario.
- Given p^*_{1n}, maximize B_2: backward/directional waterfilling with base level p^*_{1n} $\Rightarrow$ point 3.





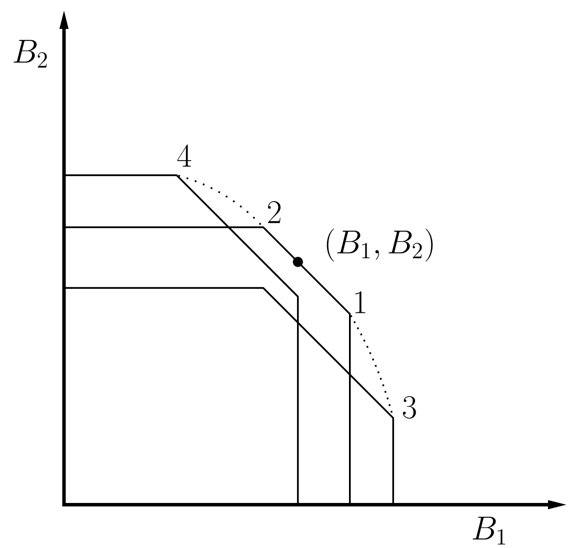
$\mu_1, \mu_2 > 0$

- Each boundary point corresponds to a corner point on some pentagon.
- $\mu_1 > \mu_2 \Rightarrow$ achieving points between point 1 and point 3:

$$\max_{\mathbf{p}_1,\mathbf{p}_2} \quad (\mu_1-\mu_2)\sum_n g(p_{1n})l_n + \mu_2\sum_n g(p_{1n}+p_{2n})l_n$$

$$\text{s.t.} \quad \sum_{n=1}^{j} p_{1n}l_n \leq \sum_{n=0}^{j-1} E_{1n}, \quad \forall j: 0 < j \leq N$$

$$\sum_{n=1}^{j} p_{2n}l_n \leq \sum_{n=0}^{j-1} E_{2n}, \quad \forall j: 0 < j \leq N$$



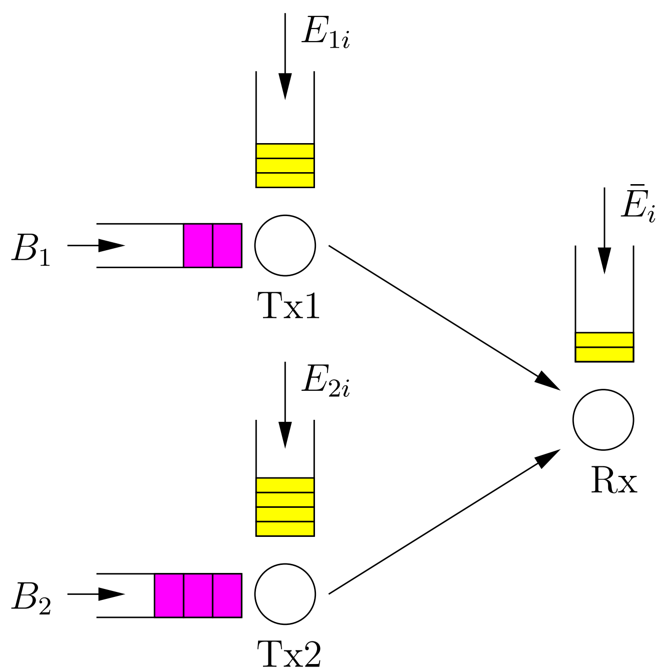
Generalized Iterative Backward Waterfilling

- Solve the problem via generalized iterative backward waterfilling:
- Given $\mathbf{p}_2^*$, solve for $\mathbf{p}_1$:

$$\begin{aligned} \max_{\mathbf{p}_1} \quad & (\mu_1 - \mu_2) \sum_{n=1}^{N} g(p_{1n}) l_n + \mu_2 \sum_{n=1}^{N} g(p_{1n} + p_{2n}^*) l_n \\ \text{s.t.} \quad & \sum_{n=1}^{j} p_{1n} l_n \leq \sum_{n=0}^{j-1} E_{1n}, \quad 0 < j \leq N \end{aligned}$$

- Once $\mathbf{p}_1^*$ is obtained, we do a backward waterfilling for the second user.
- We perform the optimization for both users in an alternating way.
- The iterative algorithm converges to the global optimal solution.

Multiple Access Channel with Energy Harvesting Transmitters and Receiver



- Decoding power is a function of the two incoming rates r_1, r_2.
- Structure of the function depends on the decoding scheme:
  - Simultaneous decoding; successive cancellation decoding.

Multiple Access Channel: Simultaneous Decoding

- Decoding power is a function of the sum rate: $\phi(r_1 + r_2)$.
- A policy $\{p_{1i}, p_{2i}\}$ is feasible if

$$\sum_{i=1}^{k} p_{1i} \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k$$

$$\sum_{i=1}^{k} p_{2i} \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k$$

$$\sum_{i=1}^{k} \phi\left(g\left(p_{1i} + p_{2i}\right)\right) \leq \sum_{i=1}^{k} \bar{E}_i, \quad \forall k$$

- Consider exponential decoding function $\phi(r) = 2^{2r} - 1 = g^{-1}$. The last inequality becomes

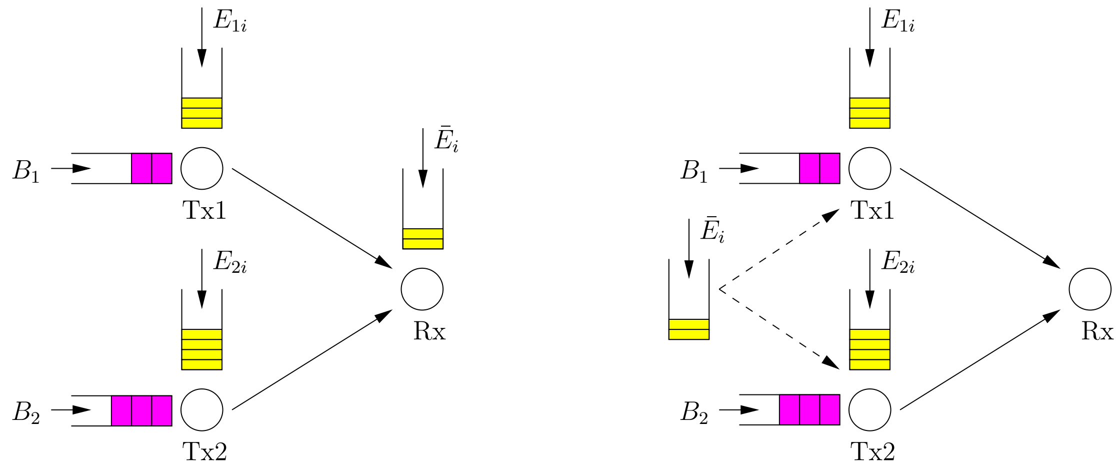
$$\sum_{i=1}^{k} p_{1i} + p_{2i} \leq \sum_{i=1}^{k} \bar{E}_i, \quad \forall k$$

Multiple Access Channel: Simultaneous Decoding

- Characterizing the maximum departure region $\mathcal{D}(N)$:

$$
\begin{aligned}
\max_{\mathbf{p}_1,\mathbf{p}_2} \quad & (\mu_1-\mu_2)\sum_{i=1}^{N} g(p_{1i}) + \mu_2 \sum_{i=1}^{N} g(p_{1i}+p_{2i}) \\
\text{s.t.} \quad & \sum_{i=1}^{k} p_{1i} \le \sum_{i=1}^{k} E_{1i}, \quad \forall k \\
& \sum_{i=1}^{k} p_{2i} \le \sum_{i=1}^{k} E_{2i}, \quad \forall k \\
& \sum_{i=1}^{k} p_{1i}+p_{2i} \le \sum_{i=1}^{k} \bar{E}_{i}, \quad \forall k
\end{aligned}
$$

Multiple Access Channel: Simultaneous Decoding



- Receiver-side constraints become joint transmitter-side constraints.

$$\max_{\mathbf{p}_1,\mathbf{p}_2} \quad (\mu_1-\mu_2)\sum_{i=1}^{N} g(p_{1i}) + \mu_2 \sum_{i=1}^{N} g(p_{1i}+p_{2i})$$

$$\text{s.t.} \quad \sum_{i=1}^{k} p_{1i} \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k$$

$$\sum_{i=1}^{k} p_{2i} \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k$$

$$\sum_{i=1}^{k} p_{1i}+p_{2i} \leq \sum_{i=1}^{k} \bar{E}_i, \quad \forall k$$

Problem Decomposition

- Problem decomposition: Inner problem for fixed $\mathbf{p}_1$

$$
\begin{aligned}
\max_{\mathbf{p}_1,\mathbf{p}_2} \quad & (\mu_1-\mu_2)\sum_{i=1}^{N} g(p_{1i}) + \mu_2 \sum_{i=1}^{N} g(p_{1i}+p_{2i}) \\
\text{s.t.} \quad & \sum_{i=1}^{k} p_{1i} \le \sum_{i=1}^{k} E_{1i}, \quad \forall k \\
& \sum_{i=1}^{k} p_{2i} \le \sum_{i=1}^{k} E_{2i}, \quad \forall k \\
& \sum_{i=1}^{k} p_{2i} \le \sum_{i=1}^{k} \bar{E}_i - p_{1i}, \quad \forall k
\end{aligned}
$$

Problem Decomposition: Inner Problem

- Inner problem: Fix a feasible $\mathbf{p}_1$; solve the following fading problem:

$$
\begin{aligned}
G(\mathbf{p}_1) \triangleq \max_{\mathbf{p}_2} \quad & \sum_{i=1}^{N} g(p_{1i} + p_{2i}) \\
\text{s.t.} \quad & \sum_{i=1}^{k} p_{2i} \leq Q_i, \quad \forall k
\end{aligned}
$$

- Directional water filling over the inverse of the fading levels: $\{1 + p_{1i}\}$.

Problem Decomposition

- Problem decomposition: Inner problem for fixed $\mathbf{p}_1$

$$
\begin{aligned}
\max_{\mathbf{p}_1, \mathbf{p}_2} \quad & (\mu_1 - \mu_2) \sum_{i=1}^{N} g(p_{1i}) + \mu_2 \sum_{i=1}^{N} g(p_{1i} + p_{2i}) \\
\text{s.t.} \quad & \sum_{i=1}^{k} p_{1i} \le \sum_{i=1}^{k} E_{1i}, \quad \forall k \\
& \sum_{i=1}^{k} p_{2i} \le \sum_{i=1}^{k} E_{2i}, \quad \forall k \\
& \sum_{i=1}^{k} p_{2i} \le \sum_{i=1}^{k} \bar{E}_i - p_{1i}, \quad \forall k
\end{aligned}
$$

Problem Decomposition: Outer Problem

- Problem decomposition: Outer problem in terms of $\mathbf{p}_1$

$$
\begin{aligned}
\max_{\mathbf{p}_1} \quad & (\mu_1 - \mu_2) \sum_{i=1}^{N} g(p_{1i}) + \mu_2 G(\mathbf{p}_1) \\
\text{s.t.} \quad & \sum_{i=1}^{k} p_{1i} \le \sum_{i=1}^{k} E_{1i}, \quad \forall k
\end{aligned}
$$

Problem Decomposition: Outer Problem

- Problem decomposition: Outer problem in terms of $\mathbf{p}_1$

$$\max_{\mathbf{p}_1} \quad (\mu_1 - \mu_2)\sum_{i=1}^{N} g(p_{1i}) + \mu_2 G(\mathbf{p}_1)$$

$$\text{s.t.} \quad \sum_{i=1}^{k} p_{1i} \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k$$

- $G(\mathbf{p}_1)$ is a decreasing concave function in $\mathbf{p}_1$.
- Convex problem. Possibly not all energies will be used.
- Iterate between inner and outer problems until convergence.

Multiple Access Channel: Successive Cancellation Decoding

- Rates achieved by decoding corner points, and time sharing if necessary.
- For $\mu_1 > \mu_2$, we always hit a lower corner point at each time slot:

$$r_2 = g\left(\frac{p_2}{1+p_1}\right), \quad r_1 = g(p_1)$$

- Receiver decodes sequentially:
  - First decodes second user's message by treating first user's signal as noise.
  - Then subtracts second user's signal and decodes first user's message interference free.
- Decoding power is spent **sequentially:** $\phi(r_2) + \phi(r_1)$.

Multiple Access Channel: Successive Cancellation Decoding

- A policy $\{p_{1i}, p_{2i}\}$ is feasible if

$$\sum_{i=1}^{k} p_{1i} \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k$$

$$\sum_{i=1}^{k} p_{2i} \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k$$

$$\sum_{i=1}^{k} \phi\left(g\left(p_{1i}\right)\right) + \phi\left(g\left(\frac{p_{2i}}{1+p_{1i}}\right)\right) \leq \sum_{i=1}^{k} \bar{E}_{i}, \quad \forall k$$

- Departure region is non-convex. Time sharing may be necessary.

- By convexity of ϕ, successive decoding is **more energy saving** than simultaneous decoding:

$$\phi\left(g\left(p_1\right)\right) + \phi\left(g\left(\frac{p_2}{1+p_1}\right)\right) \leq \phi\left(g\left(p_1+p_2\right)\right)$$

- Consider exponential decoding function $\phi(r) = 2^{2r} - 1 = g^{-1}$.

Problem Formulation

- Characterizing the maximum departure region $\mathcal{D}(N)$:

$$\max_{\mathbf{p}_1,\mathbf{p}_2} \quad \mu_1 \sum_{i=1}^{N} g(p_{1i}) + \mu_2 \sum_{i=1}^{N} g\left(\frac{p_{2i}}{1+p_{1i}}\right)$$

$$\text{s.t.} \quad \sum_{i=1}^{k} p_{1i} \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k$$

$$\sum_{i=1}^{k} p_{2i} \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k$$

$$\sum_{i=1}^{k} p_{1i} + \frac{p_{2i}}{1+p_{1i}} \leq \sum_{i=1}^{k} \bar{E}_i, \quad \forall k$$

Problem Formulation

- Problem in terms of rates:

$$\begin{aligned}
\max_{\mathbf{r}_1,\mathbf{r}_2} \quad & \mu_1 \sum_{i=1}^{N} r_{1i} + \mu_2 \sum_{i=1}^{N} r_{2i} \\
\text{s.t.} \quad & \sum_{i=1}^{k} 2^{2r_{1i}} - 1 \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k \\
& \sum_{i=1}^{k} 2^{2r_{1i}} \left(2^{2r_{2i}} - 1\right) \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k \\
& \sum_{i=1}^{k} 2^{2r_{1i}} + 2^{2r_{2i}} - 2 \leq \sum_{i=1}^{k} \bar{E}_i, \quad \forall k
\end{aligned}$$

Problem Formulation

- Problem in terms of rates:

$$\max_{\mathbf{r}_1,\mathbf{r}_2} \quad \mu_1 \sum_{i=1}^{N} r_{1i} + \mu_2 \sum_{i=1}^{N} r_{2i}$$

$$\text{s.t.} \quad \sum_{i=1}^{k} 2^{2r_{1i}} - 1 \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k$$

$$\sum_{i=1}^{k} 2^{2r_{1i}} \left(2^{2r_{2i}} - 1\right) \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k$$

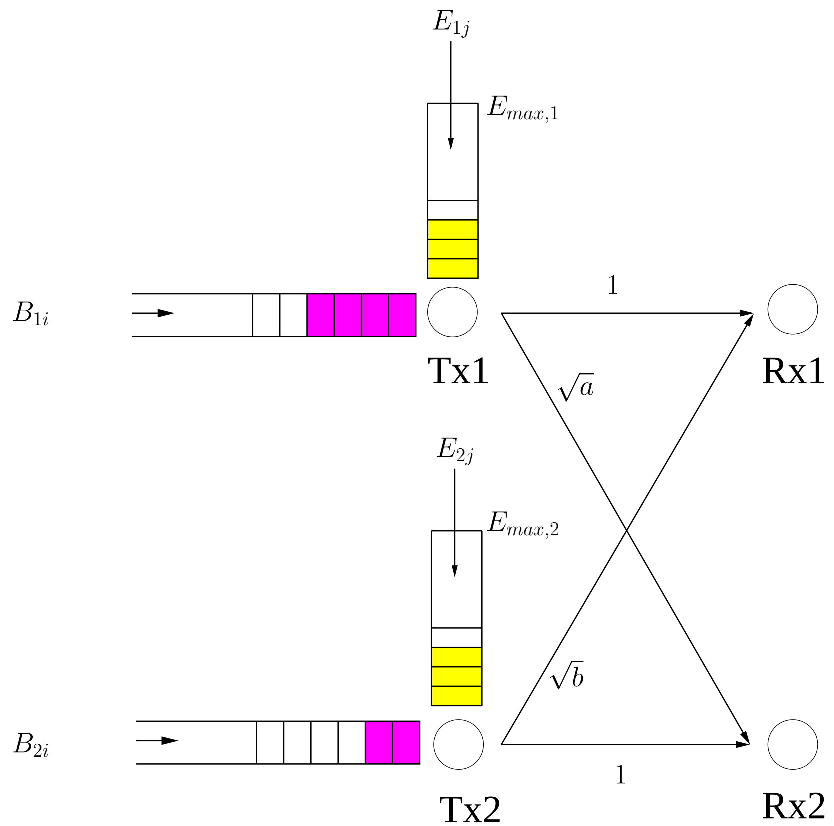
$$\sum_{i=1}^{k} 2^{2r_{1i}} + 2^{2r_{2i}} - 2 \leq \sum_{i=1}^{k} \bar{E}_i, \quad \forall k$$

- Non-convex problem.
- Signomial program:
  - Local optimal (KKT) points can be found by majorization maximization arguments.

Conclusions for the Multiple Access Scenario

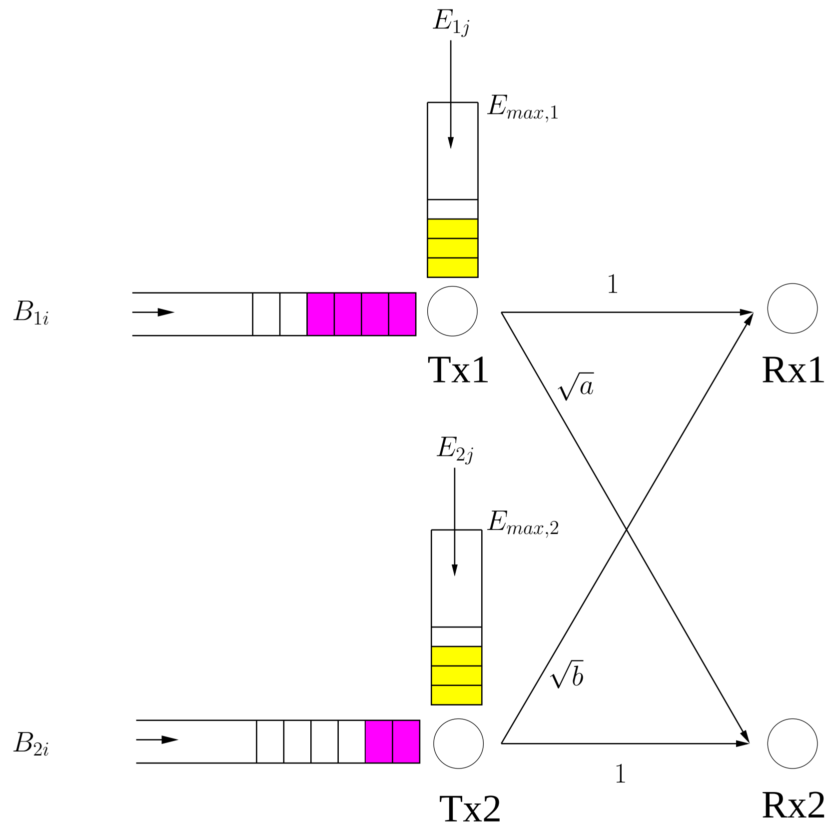
- Energy harvesting transmitters sending messages to a single access point.
- The problem: maximization of the departure region.
- Obtain the structure using generalized iterative waterfilling.
- Energy harvesting transmitters **and receiver:**
  - Decoding power is function of both rates r_1, r_2.
  - Structure of the decoding function depends on the decoding scheme:
    - Simultaneous decoding: $\phi(r_1 + r_2)$.
    - Successive cancellation decoding: $\phi(r_2) + \phi(r_1)$.

Interference Channel with an Energy Harvesting Transmitter



- Two transmitter-receiver pairs communicate in the same medium simultaneously.
- Energy arrives (is harvested) **during the communication session**.
- Batteries have finite storage capacities: $E_{max} < \infty$
- Objective: Maximize sum-rate of the users by **adapting to energy arrivals**

Interference Channel Model



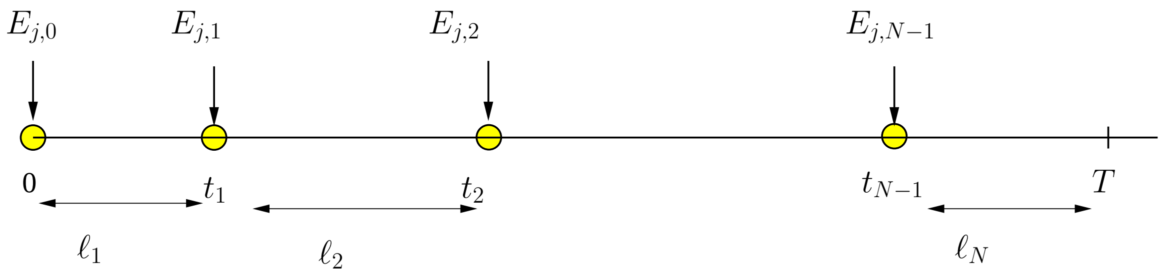
- AWGN interference channel:

$$Y_1 = X_1 + \sqrt{a}X_2 + N_1, \quad Y_2 = X_2 + \sqrt{b}X_1 + N_2$$

  where $N_1 \sim \mathcal{N}(0,1)$, $N_2 \sim \mathcal{N}(0,1)$

- Sum-rate under $E[X_1^2] \leq p_1$ and $E[X_2^2] \leq p_2$ denoted as $r(p_1, p_2)$.

Energy Model



- Energy is *harvested* **during the course of communication**.
- We will consider offline policies.
- We have a slotted system with slot duration τ.
- Energy causality constraints in the Txs: energy that has not arrived cannot be used

$$\sum_{i=1}^{n} p_{j,i}\ell_i \leq \sum_{i=0}^{n-1} E_{j,i}, \quad n = 1,\ldots,N \text{ and } j = 1,2$$

- Battery limit constraints in the Txs: energy overflows are suboptimal:

$$\sum_{i=1}^{n}(p_{j,i}\ell_i - E_{j,i}) + E_{j,max} - E_{j,i+1} \geq 0, \qquad n = 1,\ldots,N-1, \text{ and } j = 1,2$$

Sum-Rate Optimal Policy

- Sum-rate optimal policy is found by solving the following problem:

$$\begin{aligned} \max_{\mathbf{p}_1,\mathbf{p}_2} \quad & \sum_{i=1}^{K+1} r(p_{1i}, p_{2i})\ell_i \\ \text{s.t.} \quad & \sum_{i=1}^{n} p_{j,i}\ell_i \leq \sum_{i=1}^{n} E_{j,i}, \ 1 \leq n \leq N \\ & \sum_{i=1}^{n} E_{j,i} - \sum_{i=1}^{n} p_{j,i}\ell_i \leq E_{max}, \ 1 \leq n \leq N \end{aligned}$$

A General Iterative Solution

- For any achievable $r(p_1, p_2)$, there exists another achievable scheme with
  - $\hat{r}(p_1, p_2) > r(p_1, p_2)$
  - $\hat{r}(p_1, p_2)$ is jointly concave
- Solve the problem iteratively.
- Given $\mathbf{p}_2^*$, solve for $\mathbf{p}_1$:

$$\begin{aligned} \max_{\mathbf{p}_1} \quad & \sum_{i=1}^{K+1} r(p_{1i}, p_{2i}^*)\ell_i \\ \text{s.t.} \quad & \sum_{i=1}^{n} p_{1,i}\ell_i \leq \sum_{i=1}^{n} E_{1,i}, \ 1 \leq n \leq N \\ & \sum_{i=1}^{n} E_{1,i} - \sum_{i=1}^{n} p_{1,i}\ell_i \leq E_{max}, \ 1 \leq n \leq N \end{aligned}$$

- Once this solution is found, we fix it and solve for $\mathbf{p}_2^*$.
- The iterative algorithm converges to the global optimal solution.

Asymmetric Interference with $ab > 1$

- Let $a \geq 1$ and $b \leq 1$ with $ab > 1$.

$$r(p_1, p_2) = \frac{1}{2}\log\left(1 + \frac{p_1}{1+ap_2}\right) + \frac{1}{2}\log(1+p_2)$$

- For fixed p_2, user 1 observes a fading level of $\frac{1}{1+ap_2}$.
- Use directional waterfilling for user 1's problem.
- For fixed p_1, user 2 has the generalized water level

$$\frac{\partial}{\partial p_2} r(p_1, p_2) = -\frac{ap_1}{2(1+p_1+ap_2)(1+ap_2)} + \frac{1}{2(1+ap_2)}$$

- Use generalized directional waterfilling for user 2's problem.

Asymmetric Interference with $ab < 1$

- Let $a \leq 1$ and $b \geq 1$ with $ab < 1$.

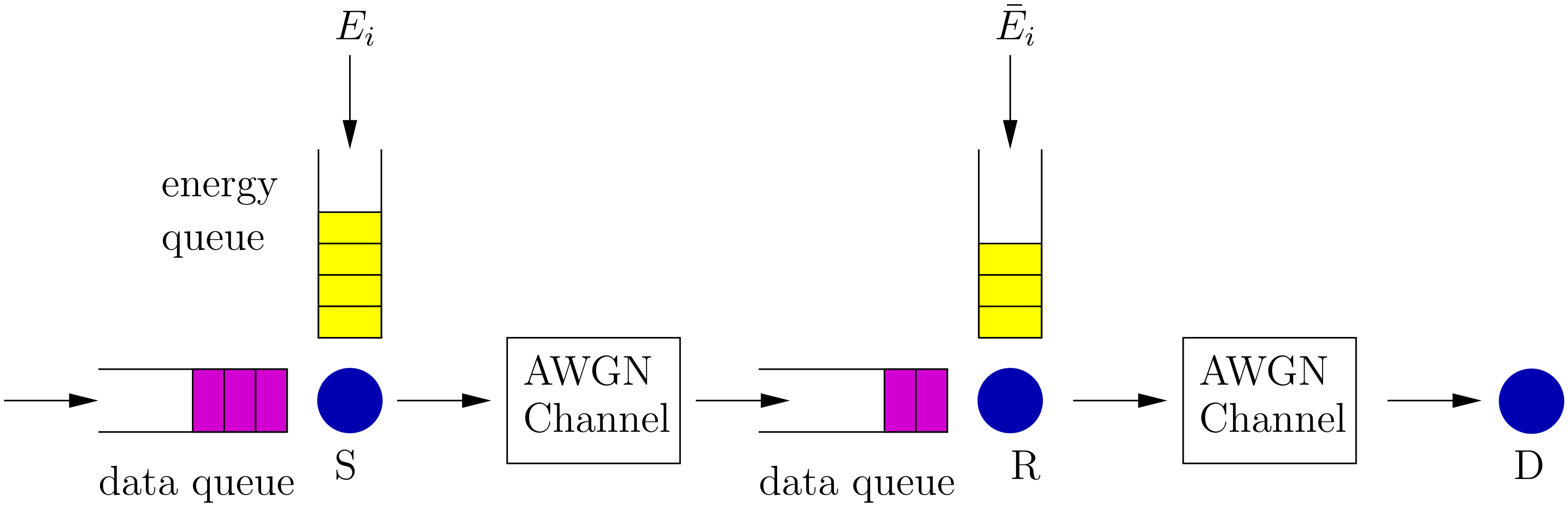
$$r(p_1, p_2) = \min\{\frac{1}{2}\log\left(1 + \frac{p_1}{1 + ap_2}\right) + \frac{1}{2}\log(1 + p_2), \frac{1}{2}\log(1 + bp_1 + p_2)\}$$

- Define $p_c = \frac{b-1}{1-ab}$.
- User 1 observes fading level $\frac{1}{1+ap_2}$ if $p_2 < p_c$ and $\frac{b}{1+p_2}$ otherwise.
- Use directional waterfilling for user 1's problem.
- Similarly, a generalized waterfilling algorithm solves user 2's problem.

Conclusions for the Interference Channel Scenario

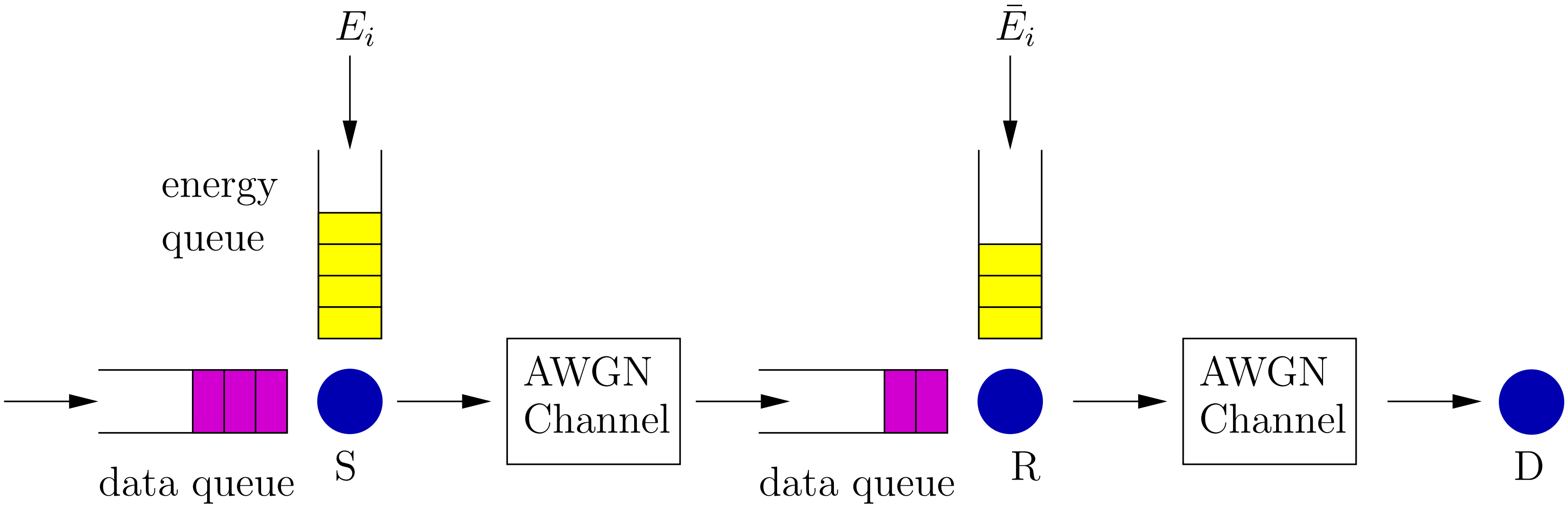
- Energy harvesting transmitter-receiver pairs with finite capacity batteries.
- Maximize the sum-rate of the communication.
- Sum-rate is a jointly concave function of powers.
- Iterative generalized directional water-filling algorithm.
- Specific cases such as asymmetric interference with $ab < 1$ and $ab > 1$.
- Extension to bit arrivals is available.

Two-Hop Communication with Energy Harvesting Nodes



- Source (S) sends messages to the destination (D) via a relay (R).
- Source and relay uses energy harvested from the environment.
- Source adapts its transmission to the energy profiles of both nodes.
- Relay adapts its transmission to the data stream from the source and its energy profile.
- Objective: maximize end-to-end throughput

Channel Model



- Channel between S and R is AWGN with gain h:

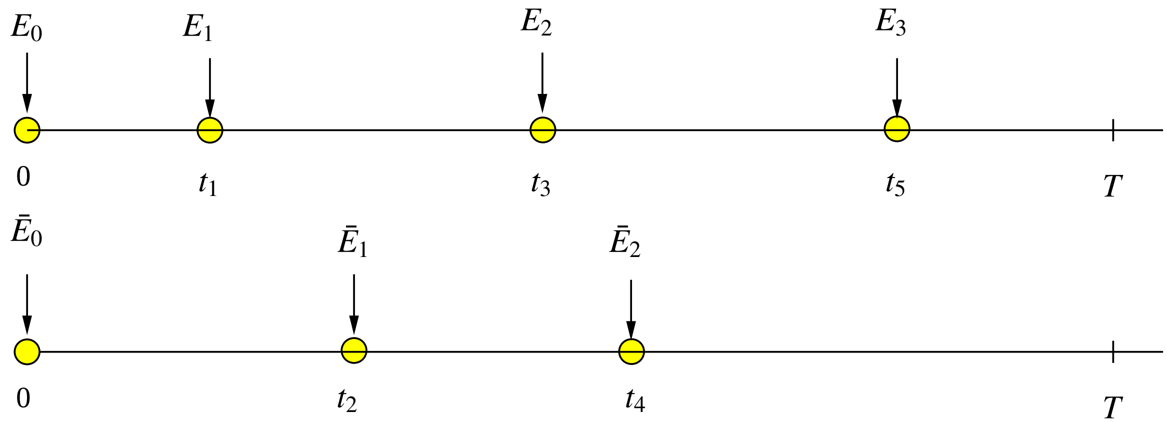
$$r(p) = \frac{1}{2}\log(1 + hp)$$

- Channel between R and D is AWGN with gain $\bar{h}$:

$$\bar{r}(\bar{p}) = \frac{1}{2}\log\left(1 + \bar{h}\bar{p}\right)$$

- Relay operates in full duplex mode.

Energy Model



- Energy is *harvested* **during the communication**. We consider offline policies.
- Energy causality constraints in the nodes: energy that has not arrived cannot be used

$$\sum_{i=1}^{k} p_i \ell_i \leq \sum_{i=0}^{k-1} E_i, \quad \forall k$$

$$\sum_{i=1}^{k} \bar{p}_i \ell_i \leq \sum_{i=0}^{k-1} \bar{E}_i, \quad \forall k$$

- Data causality constraints in the relay: data that has not arrived cannot be forwarded.

$$\sum_{i=1}^{k} \bar{r}(\bar{p}_i) \leq \sum_{i=1}^{k} r(p_i), \quad \forall k$$

Finding Optimal Policies of the Nodes

- Maximize end-to-end throughput

$$
\begin{aligned}
\max \quad & \sum_{i=1}^{N} \bar{r}(\bar{p}_i)\ell_i \\
\text{s.t.} \quad & \sum_{i=1}^{k} p_i \ell_i \leq \sum_{i=0}^{k-1} E_i, \quad \forall k \\
& \sum_{i=1}^{k} \bar{p}_i \ell_i \leq \sum_{i=0}^{k-1} \bar{E}_i, \quad \forall k \\
& \sum_{i=1}^{k} \bar{r}(\bar{p}_i) \leq \sum_{i=1}^{k} r(p_i), \quad \forall k
\end{aligned}
$$

- Optimal policies are not unique.

- There is a separation-based optimal policy.

Separation-Based Optimal Policy

- Source maximizes its throughput **without regard to relay energy profile:**

$$\max \quad \sum_{i=1}^{N} r(p_i)$$

$$\sum_{i=1}^{k} p_i \leq \sum_{i=0}^{k-1} E_i, \quad \forall k$$

- Relay maximizes its throughput **according to the optimal source data stream:**

$$\max \quad \sum_{i=1}^{N} \bar{r}(\bar{p}_i)$$

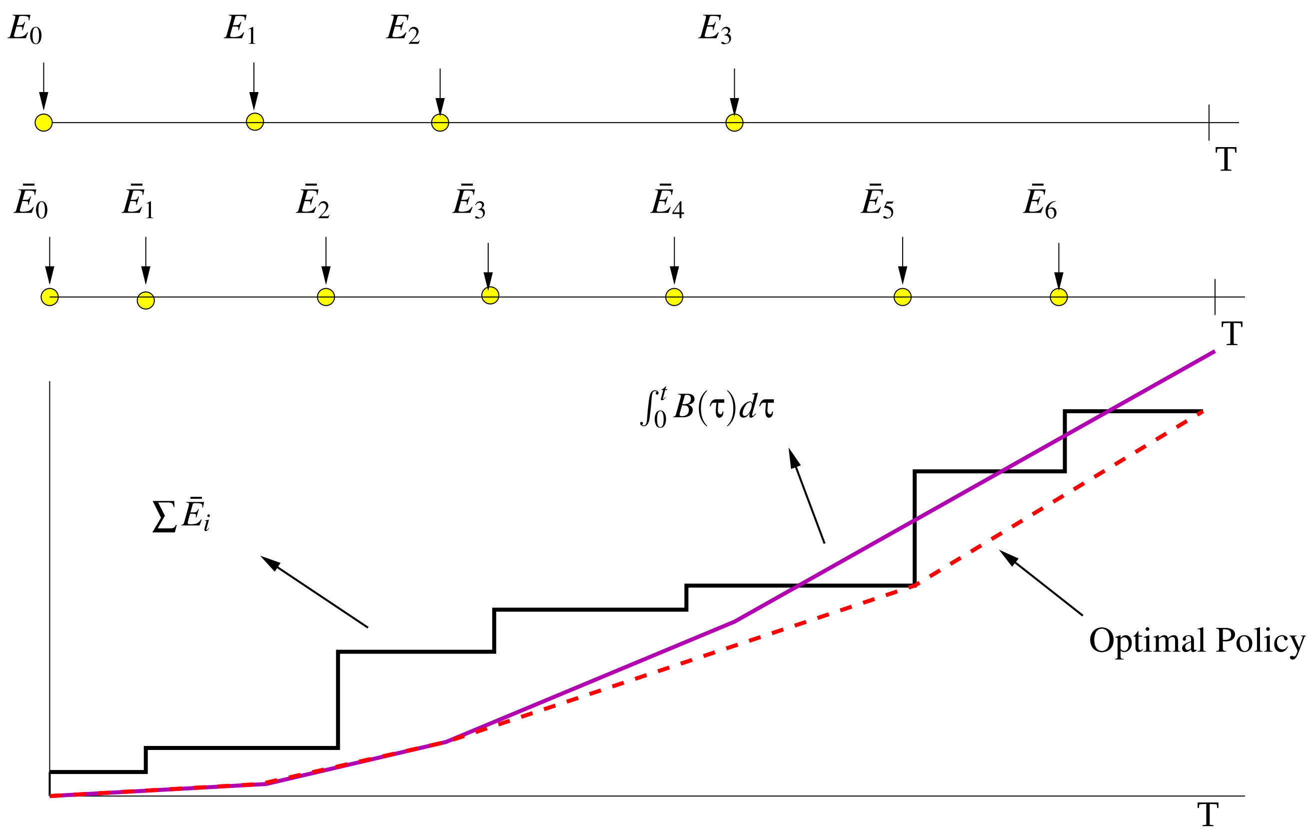
$$\sum_{i=1}^{k} \bar{p}_i \leq \sum_{i=0}^{k-1} \bar{E}_i, \quad \forall k$$

$$\sum_{i=1}^{k} \bar{r}(\bar{p}_i) \leq \sum_{i=1}^{k} r(p_i), \quad \forall k$$

- Both problems are single-user throughput maximization problems.

- This policy is not energy minimal.

Structure of Separation-Based Optimal Policy

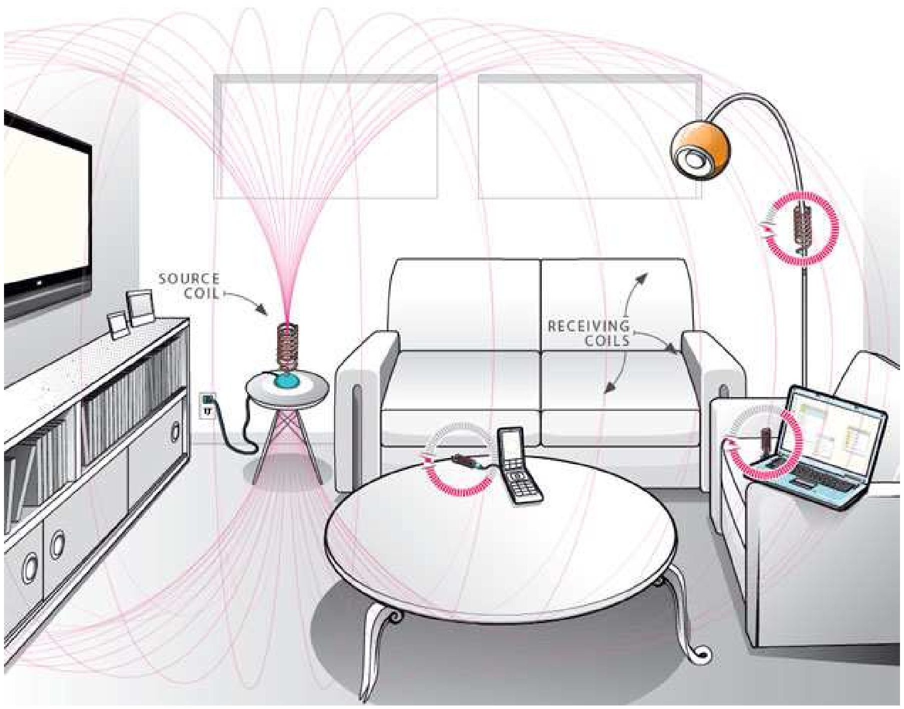


Conclusions for the Two-Hop Communication Scenario

- Energy harvesting source and relay with infinite capacity batteries.
- Maximize the end-to-end throughput.
- Optimal policy is not unique.
- An optimal policy is obtained based on a separation principle:
  - Both source and relay perform single-user optimizations.
  - It is not *energy minimal*.
- There is no simple extension for the finite battery case.

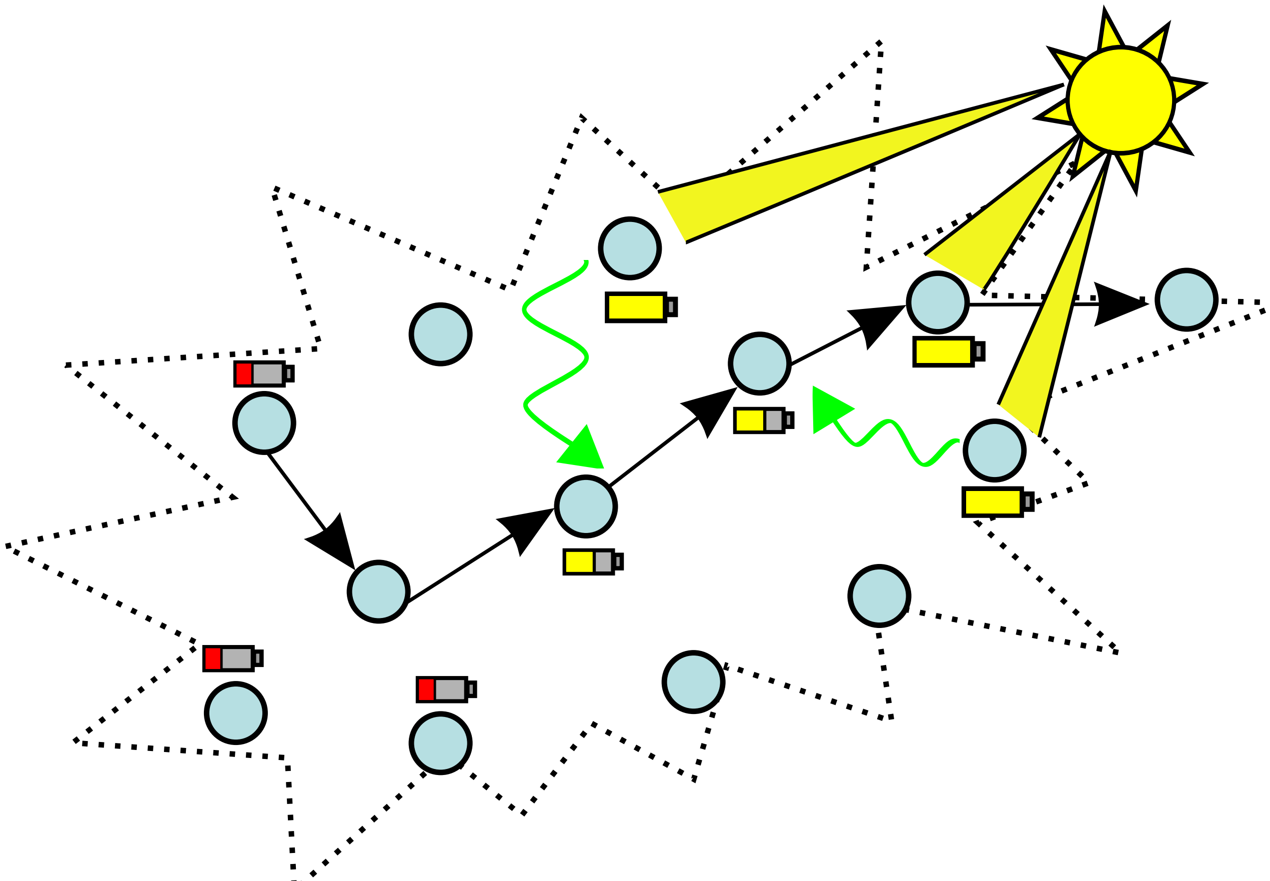
Wireless Energy Transfer

- Newly emerging technologies have enabled us to perform wireless energy transfer efficiently.
- Inductive coupling can be used to wirelessly transfer energy.



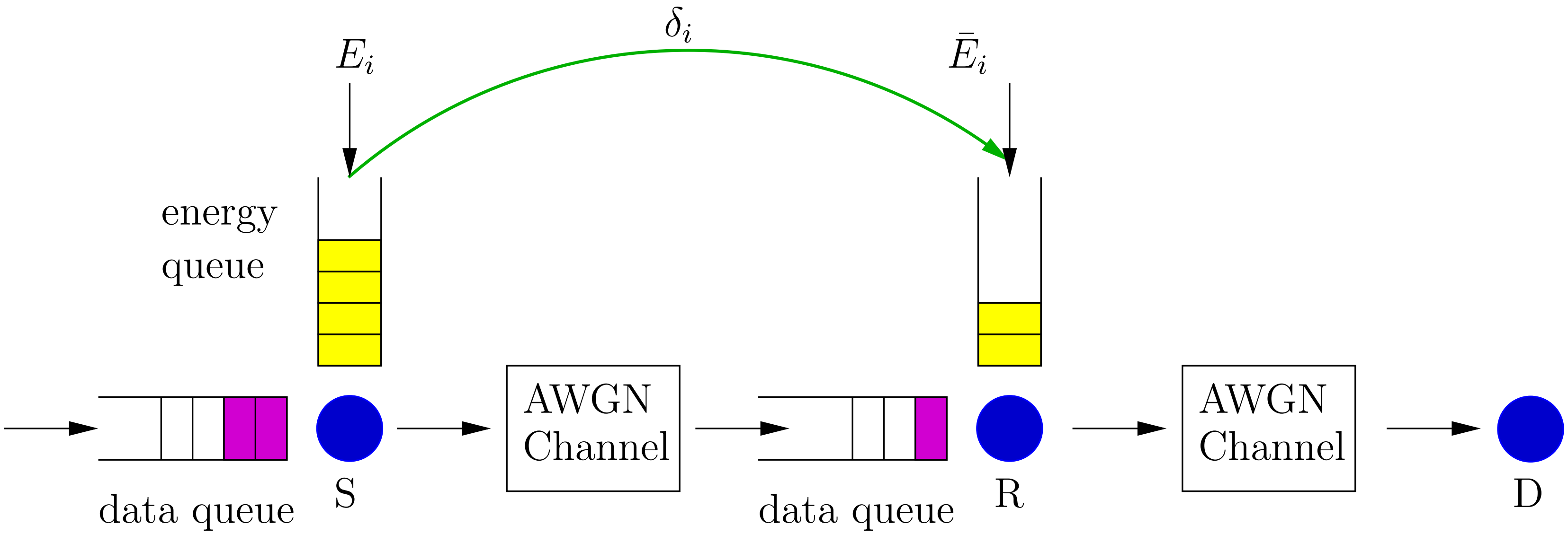
Energy Cooperation in Multi-user Energy Harvesting Communications

- Wireless energy transfer is a **new cooperation paradigm.**



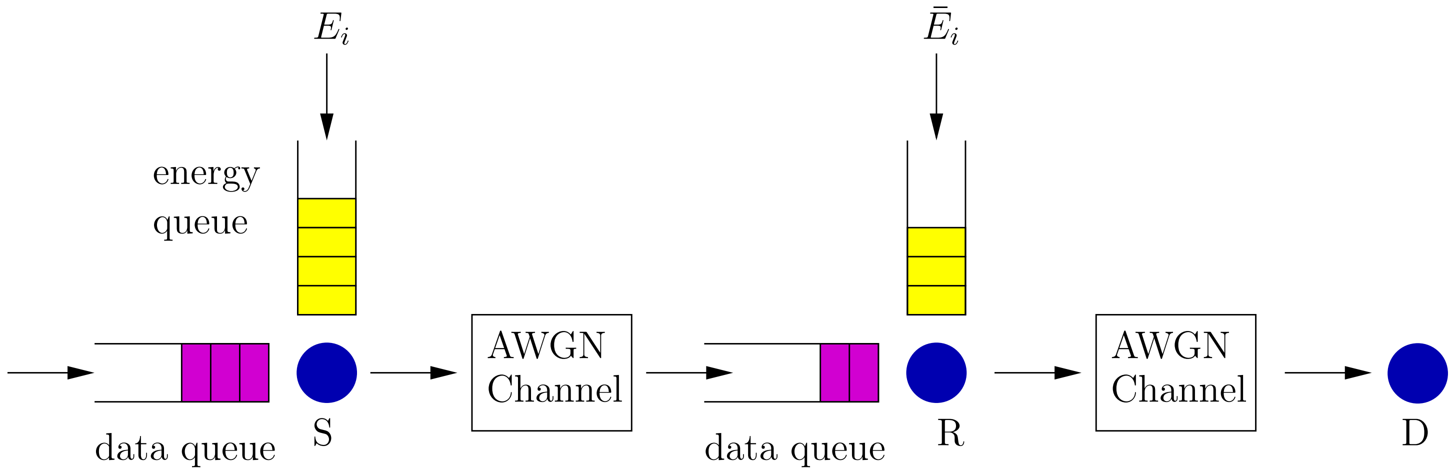
- **Energy cooperation:** Nodes share their energy as well as their information.

Gaussian Two-Hop Relay Channel with Energy Cooperation



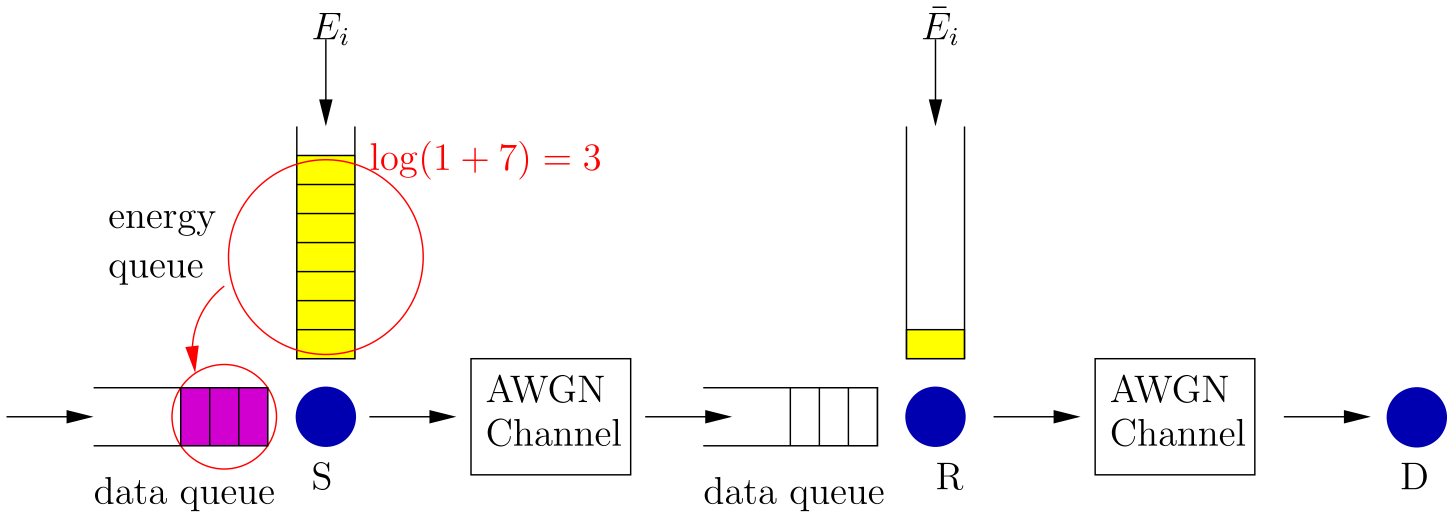
- Energy harvesting source and relay with deterministic energy arrivals E_i, $\bar{E}_i$.
- Wireless energy transfer unit that allows the source to transfer some of its energy to the relay (with $0 \leq \alpha \leq 1$ efficiency).
- Unlimited data and energy buffers at the source and the relay.
- New energy arrivals at every slot i, $1 \leq i \leq T$.
- The source transfers δ_i energy to the relay at slot i.
- Relay receives $\alpha\delta_i$ of this transferred energy at the next slot.

Two Hop Relay Channel without Energy Cooperation



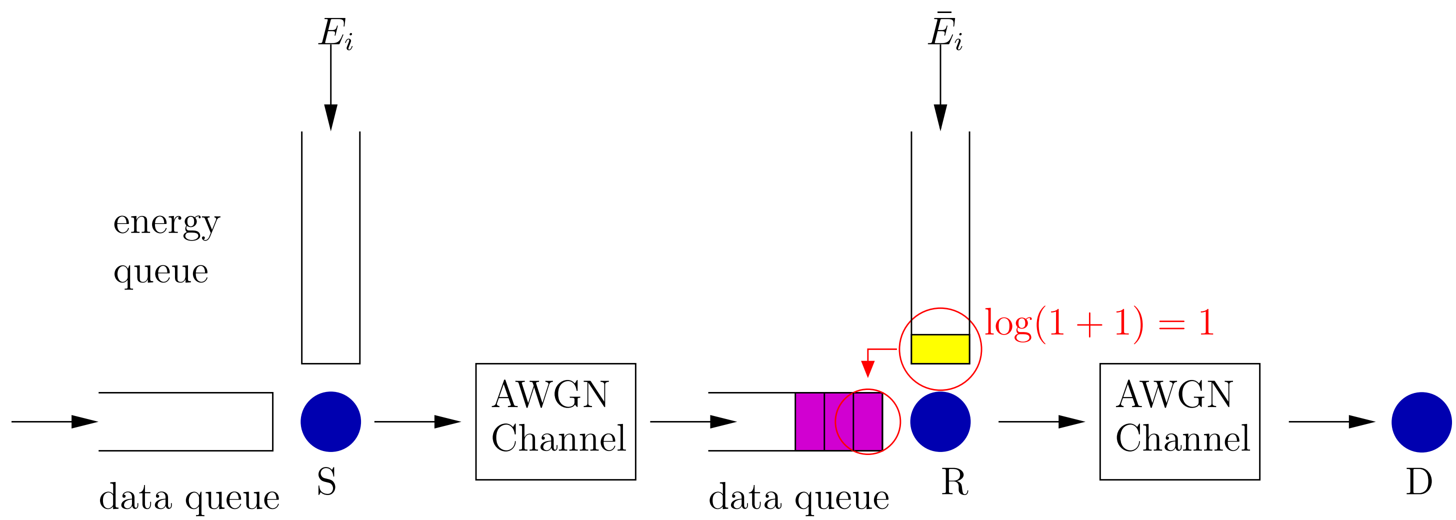
- Optimal source/relay profile is a separable policy.
- Source performs single user throughput maximization with respect to its own energy arrivals.
- Relay forwards as many of the received bits as possible, satisfying data causality and energy causality.

Two Hop Relay Channel without Energy Cooperation



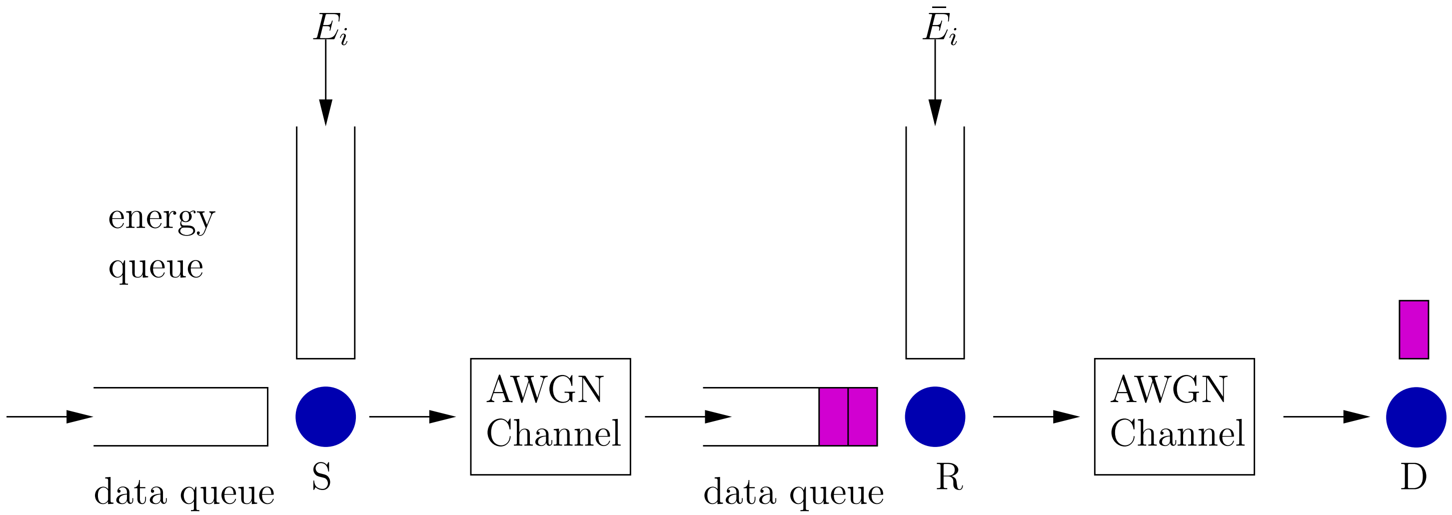
- Separable policy, source maximizes its own throughput.

Two Hop Relay Channel without Energy Cooperation



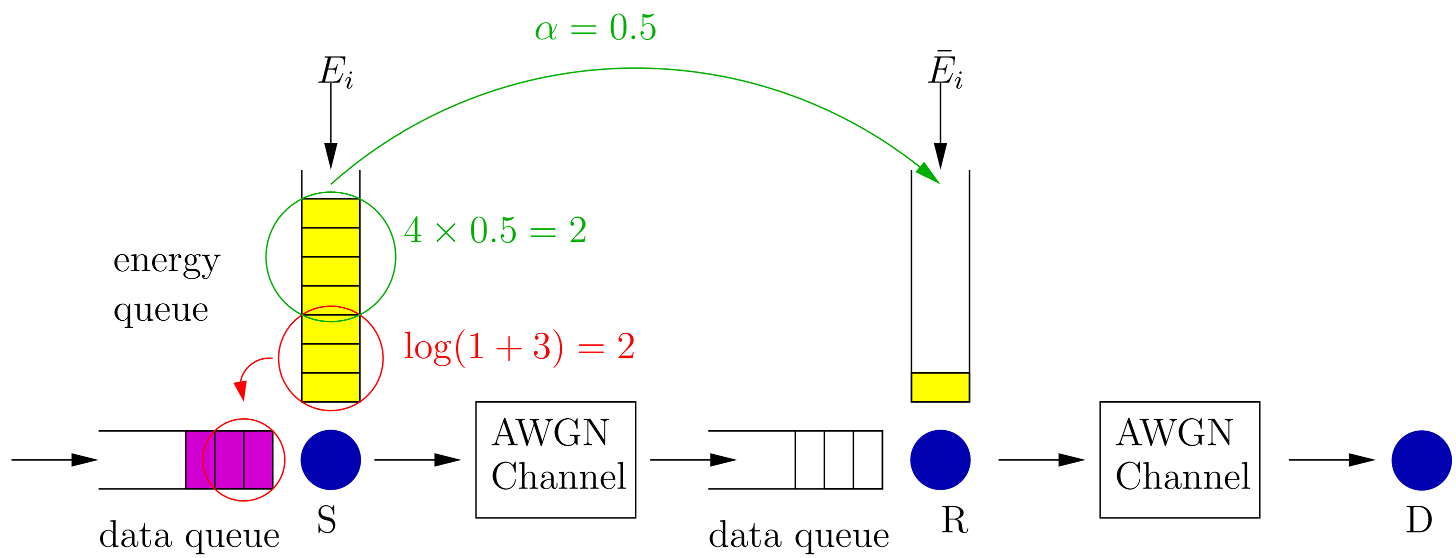
- Separable policy, source maximizes its own throughput.
- Relay tries to send as much as it can.

Two Hop Relay Channel without Energy Cooperation



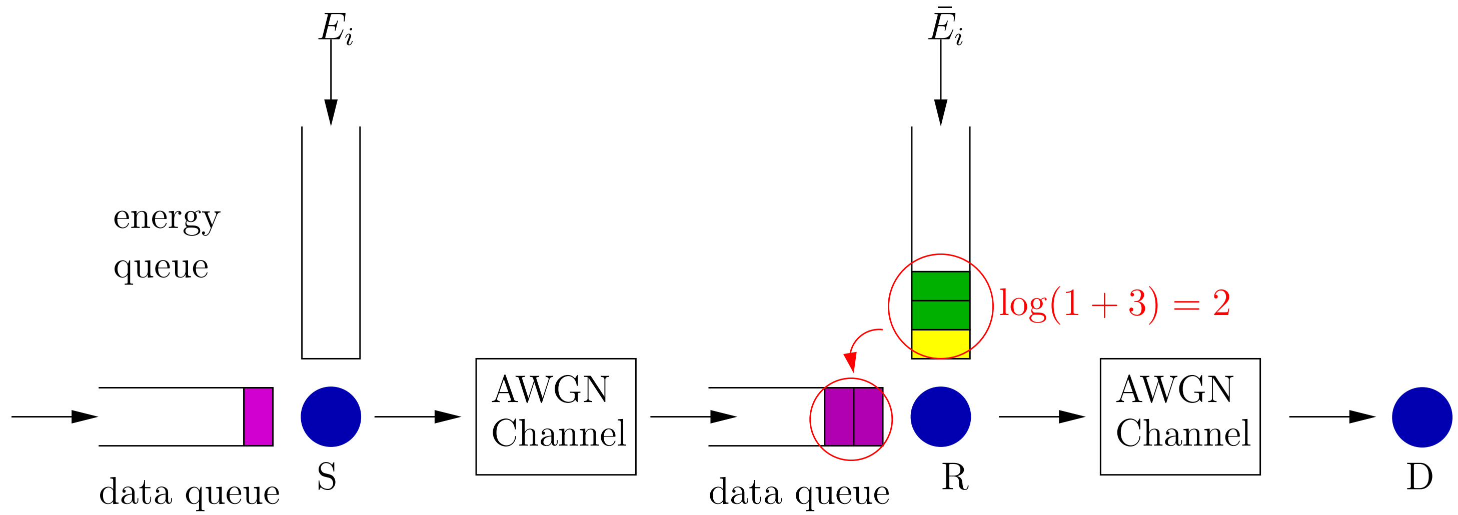
- Separable policy, source maximizes its own throughput.
- Relay tries to send as much as it can.
- 1 bit sent to destination, 2 bits remaining at the relay.
- End-to-end throughput is 1 bit.

Two Hop Relay Channel with Energy Cooperation



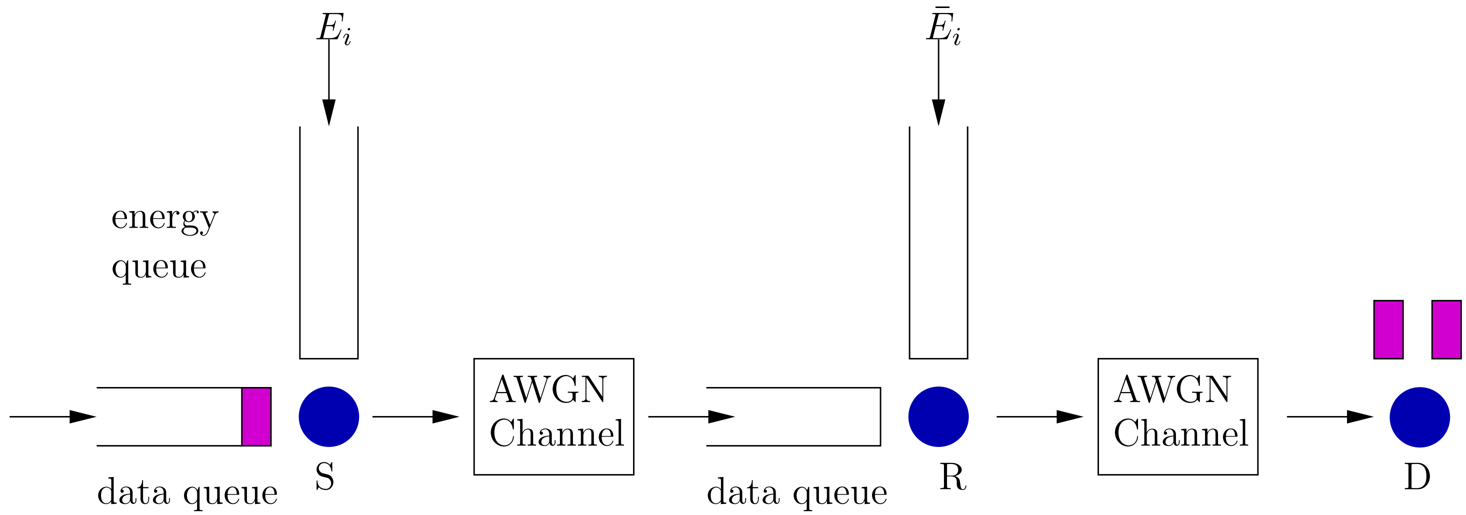
- Source sends less data, but some energy to assist the relay.

Two Hop Relay Channel with Energy Cooperation



- Source sends less data, but some energy to assist the relay.
- Relay uses this extra energy to forward more data.

Two Hop Relay Channel with Energy Cooperation



- Source sends less data, but some energy to assist the relay.
- Relay uses this extra energy to forward more data.
- 2 bits sent to destination, 0 bits remaining at the relay.
- End-to-end throughput is 2 bits.

End-to-end Throughput Maximization

- Maximize end-to-end throughput

$$\max \sum_{i=1}^{T} \frac{1}{2} \log\left(1+\bar{P}_i\right)$$

$$\text{s.t.} \sum_{i=1}^{k} P_i \leq \sum_{i=1}^{k} (E_i - \delta_i), \quad \forall k$$

$$\sum_{i=1}^{k} \bar{P}_i \leq \sum_{i=1}^{k} (\bar{E}_i + \alpha\delta_i), \quad \forall k$$

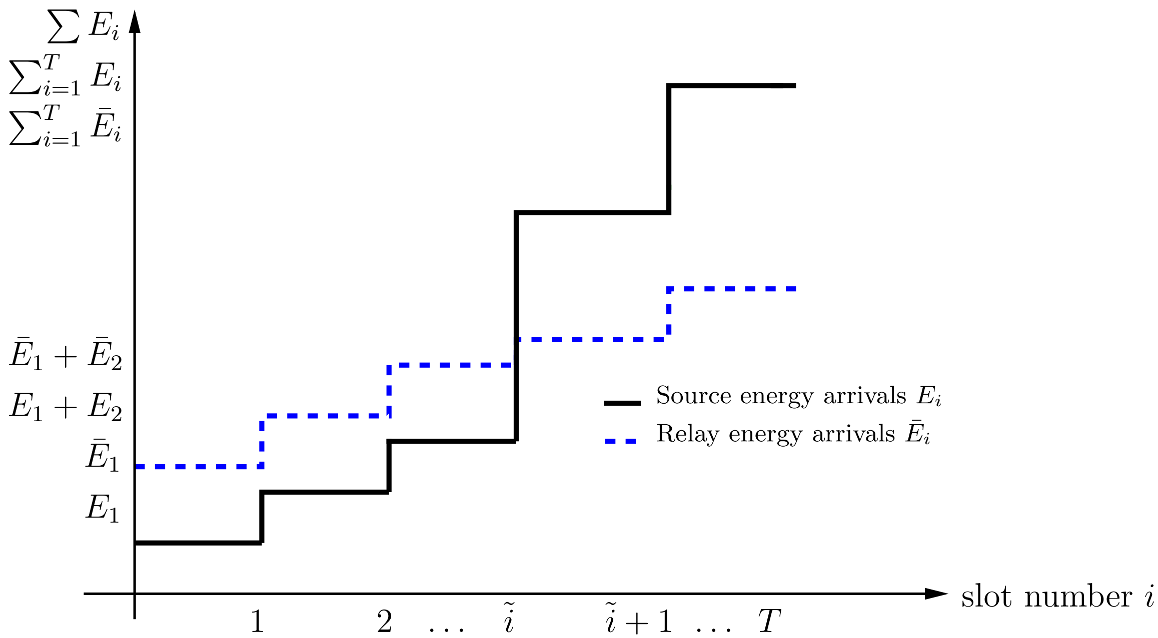
$$\sum_{i=1}^{k} \frac{1}{2} \log\left(1+\bar{P}_i\right) \leq \sum_{i=1}^{k} \frac{1}{2} \log\left(1+P_i\right), \quad \forall k$$

  subject to:

  - Data causality at the relay node
  - Energy causality at both nodes
  - (Possibly) non-zero energy transfers

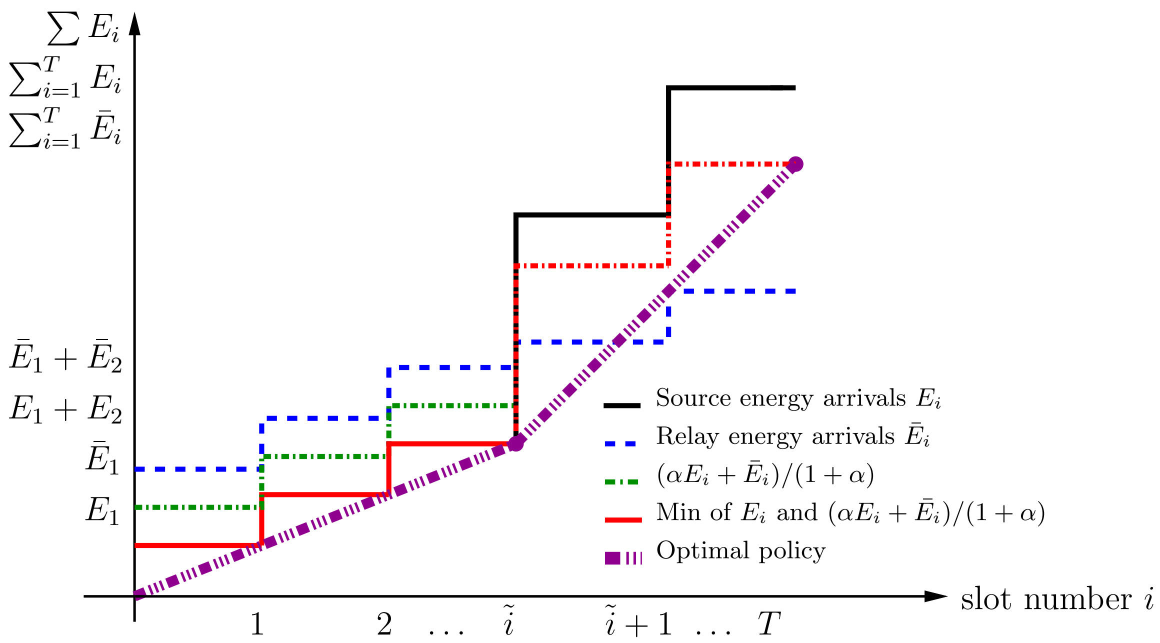
- Solution could be identified only in special cases.

Case I: Higher Initial Relay Energy



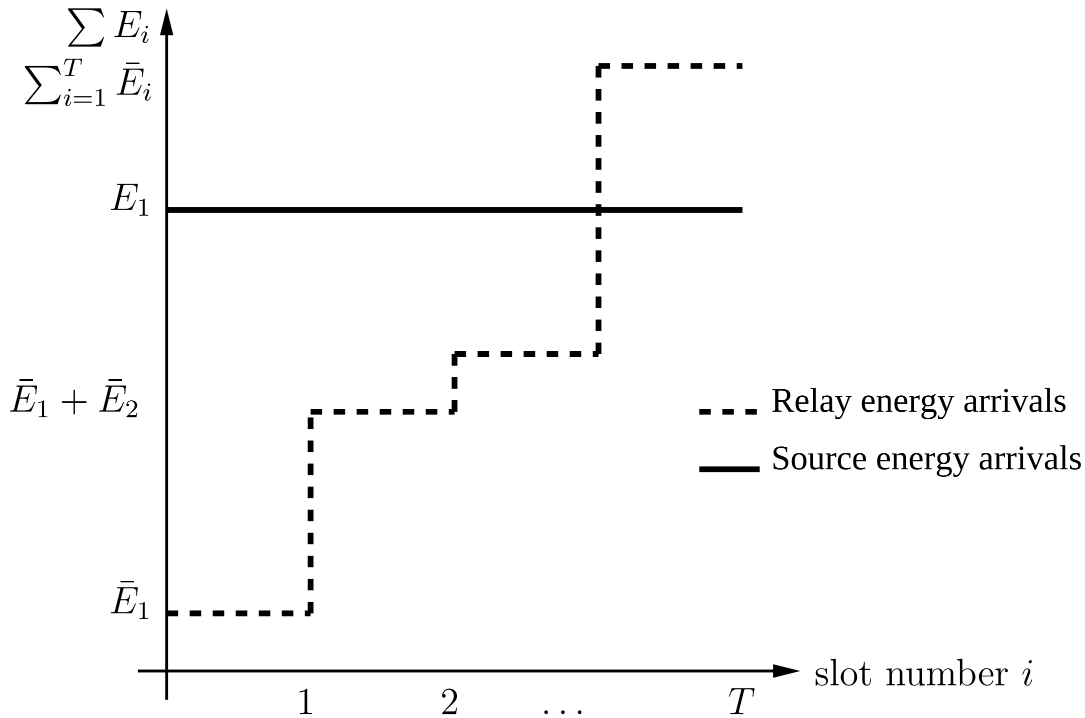
- Higher initial relay energy and single intersection with source energy curve
- Covers the case when the source is energy harvesting and all relay energy is available initially

Case I: Higher Initial Relay Energy



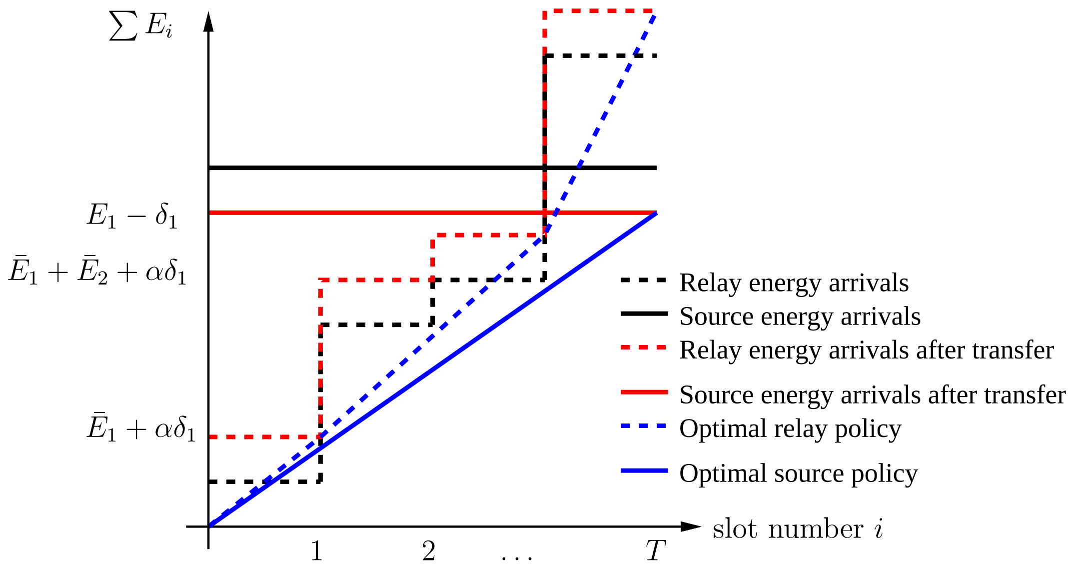
- Since source energy is low initially, no energy transfer until the intersection
- Form a new energy profile $\min\left(\frac{\alpha E_i+\bar{E}_i}{\alpha+1}, E_i\right)$ and maximize throughput
- Source and relay powers are matched to ensure relay data queue is empty.

<u>Case II:</u> Non energy harvesting source and energy harvesting relay



- All source energy is available initially
- Relay is energy harvesting

Case II: Non energy harvesting source and energy harvesting relay



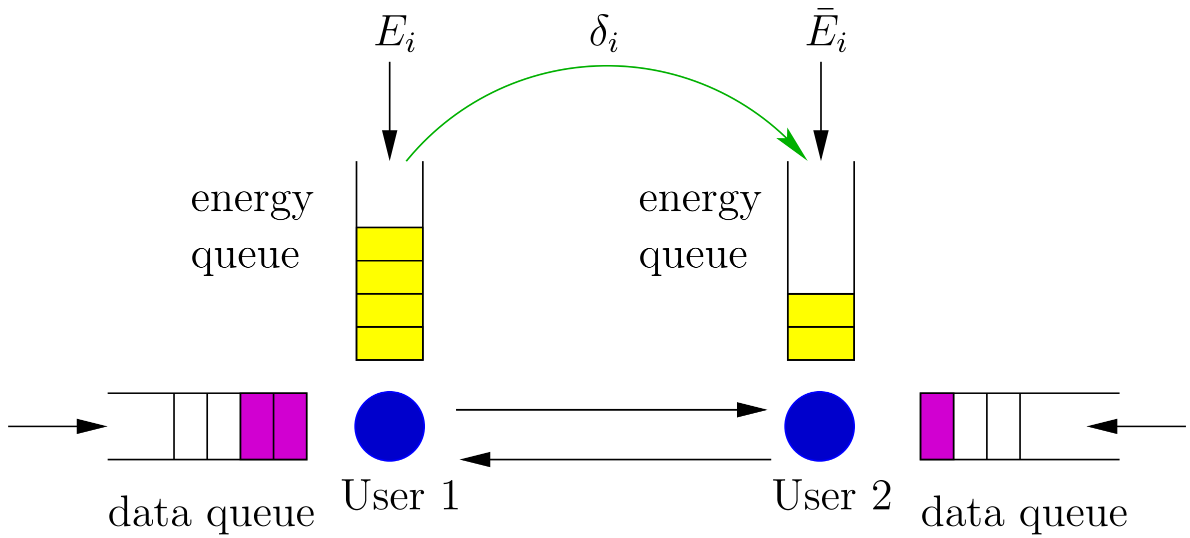
- Transferring energy at a slot can only increase relay powers after that slot.
- Since source is not energy harvesting, energy transfer at first slot is optimal.

$$f(\bar{E}_1 + \delta_1^*, \bar{E}_2, \ldots, \bar{E}_T) = \frac{T}{2} \log\left(1 + \frac{E_1 - \delta_1^*}{T}\right)$$

- $f(\bar{E}_1, \ldots, \bar{E}_T)$ is the maximum number of bits for arrivals $(\bar{E}_1, \ldots, \bar{E}_T)$.

Gaussian Two Way Channel with Energy Cooperation

- Energy harvesting users with deterministic energy arrivals E_i, $\bar{E}_i$
- One-way wireless energy transfer with efficiency $0 < \alpha < 1$.



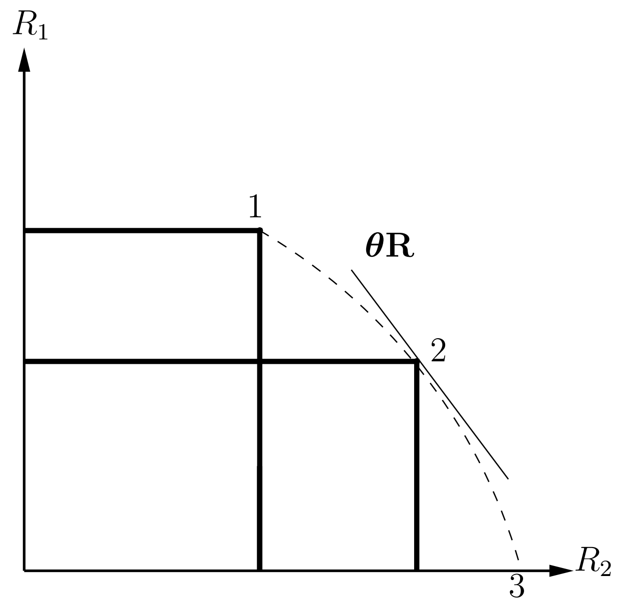
- Physical layer is a Gaussian two-way channel:

$$Y_1 = X_1 + X_2 + N_1$$

$$Y_2 = X_1 + X_2 + N_2$$

N_1, N_2 are Gaussian noises with zero mean and unit power.

Capacity Region



- Convex region, boundary is characterized by solving

$$\begin{array}{ll} \max\limits_{\bar{P}_i, P_i, \delta_i} & \sum_{i=1}^{T} \theta_1 \frac{1}{2} \log(1+P_i) + \theta_2 \frac{1}{2} \log(1+\bar{P}_i) \\ \text{s.t.} & (\boldsymbol{\delta}, \mathbf{P}, \bar{\mathbf{P}}) \in \mathcal{F} \end{array}$$

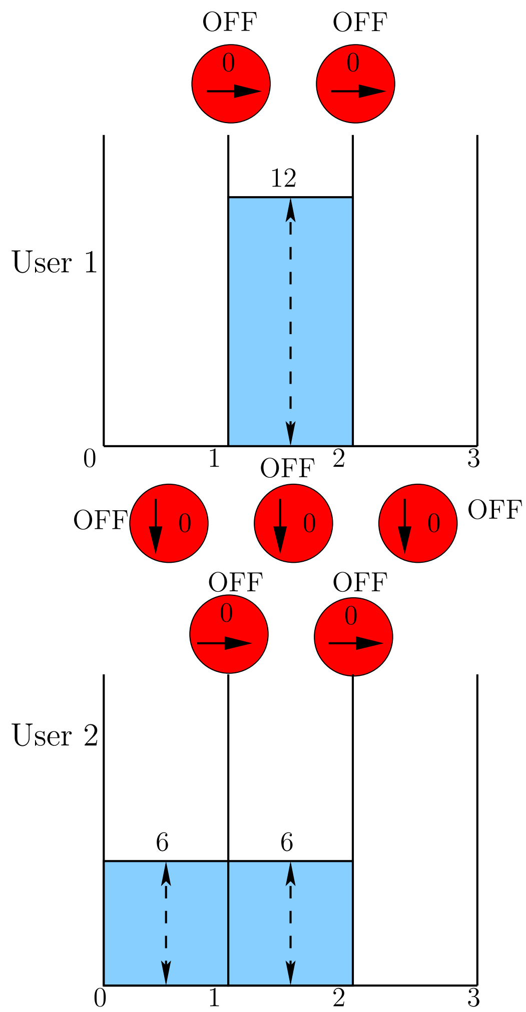
- Point 1 is achieved by $\boldsymbol{\delta} = 0$: **no energy transfer.**
- Point 3 is achieved by $\boldsymbol{\delta} = \mathbf{E}$: **full energy transfer.**

Water-filling Approach

- **Generalized two-dimensional directional water-filling algorithm.**
- Transfer energy from one user to another while maintaining optimal allocation in time.
- Spread the energy as much as possible in time and user dimensions.
- Now we give a numerical example for $\theta_1 = \theta_2$ and $\alpha = 1$.

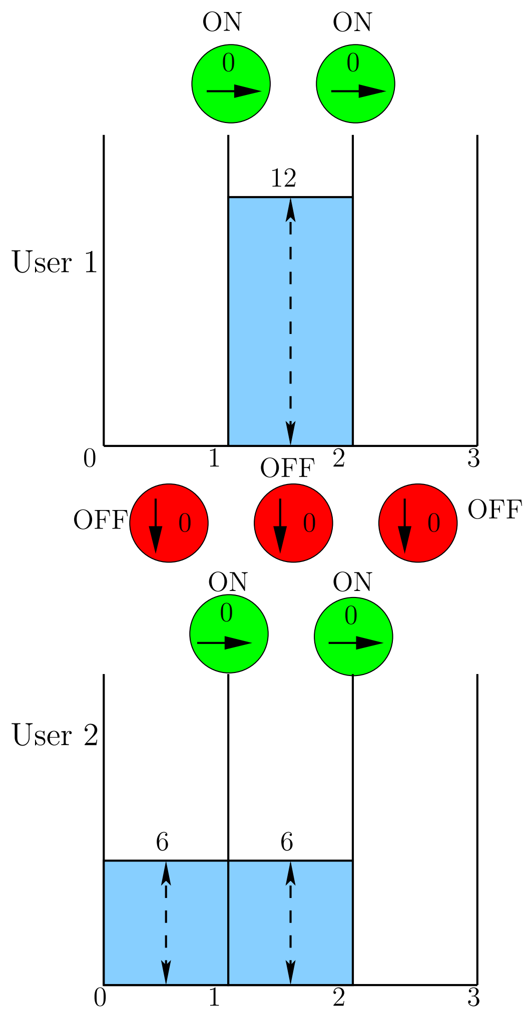
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



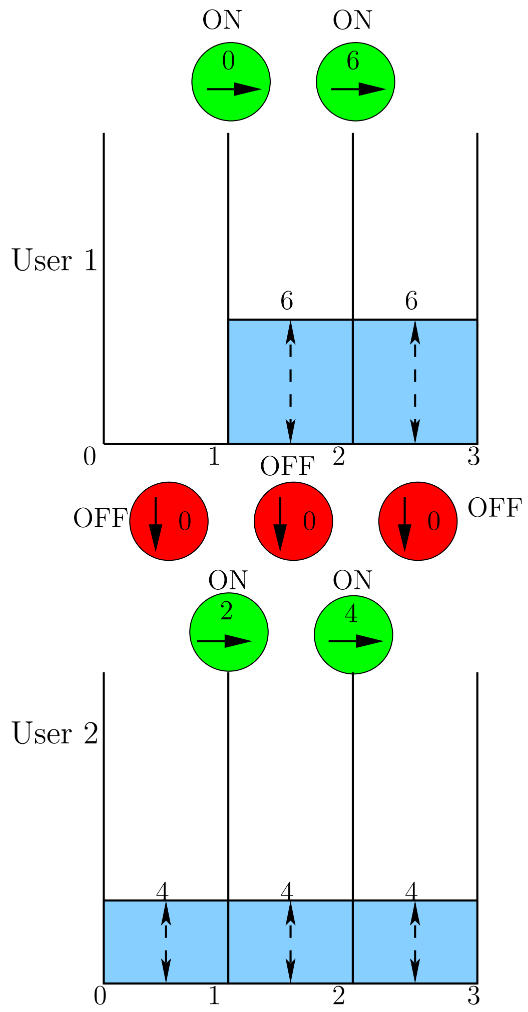
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



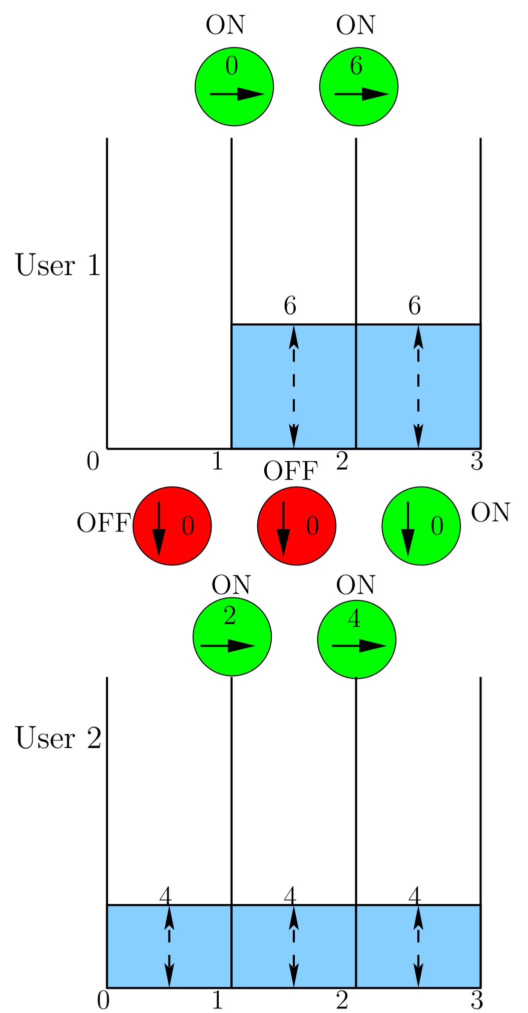
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



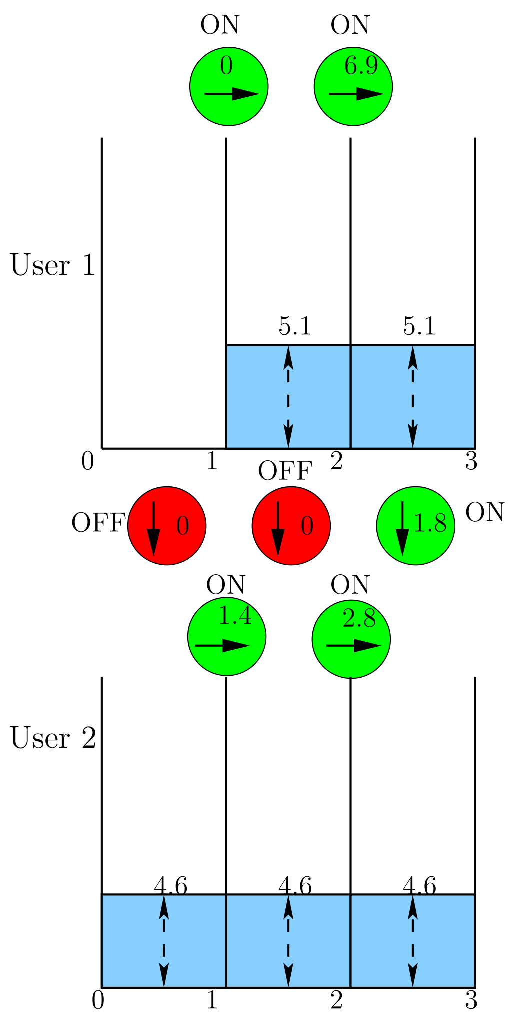
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



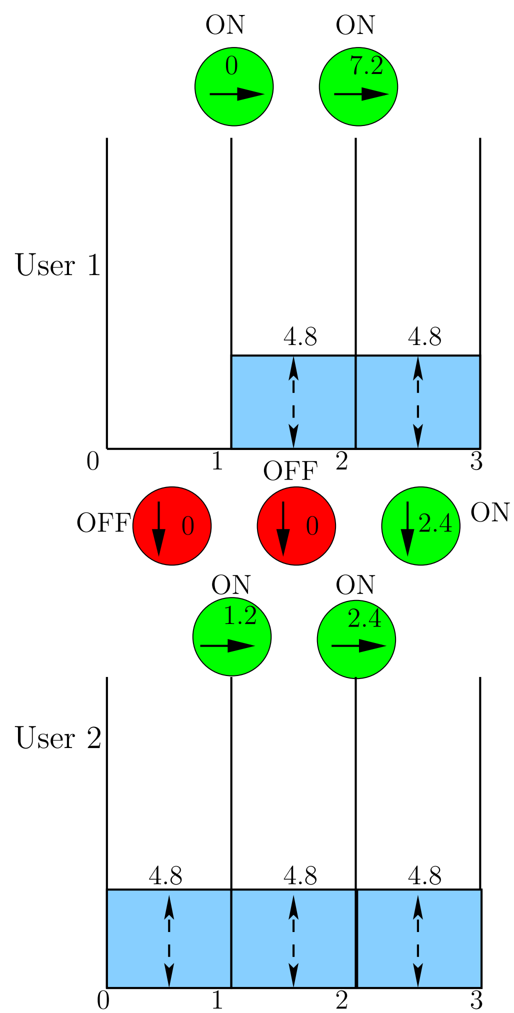
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



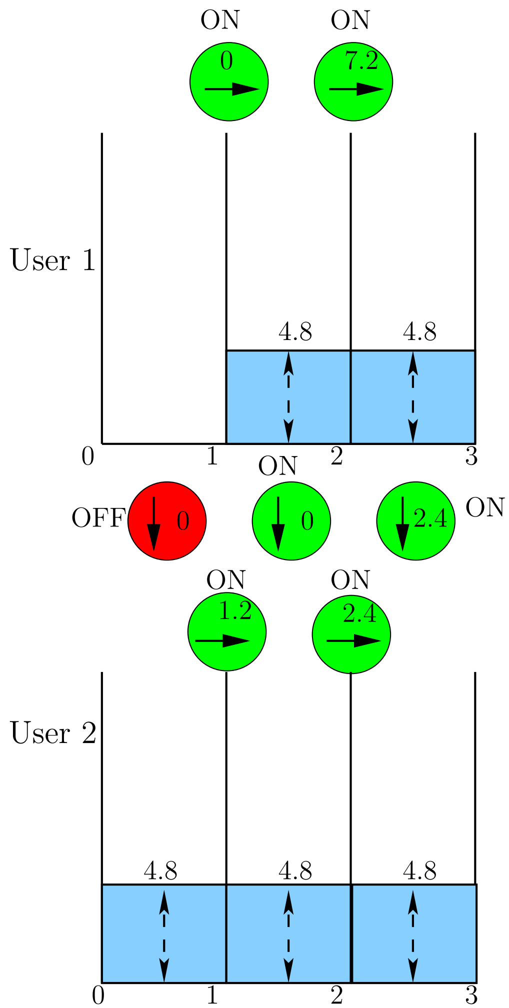
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



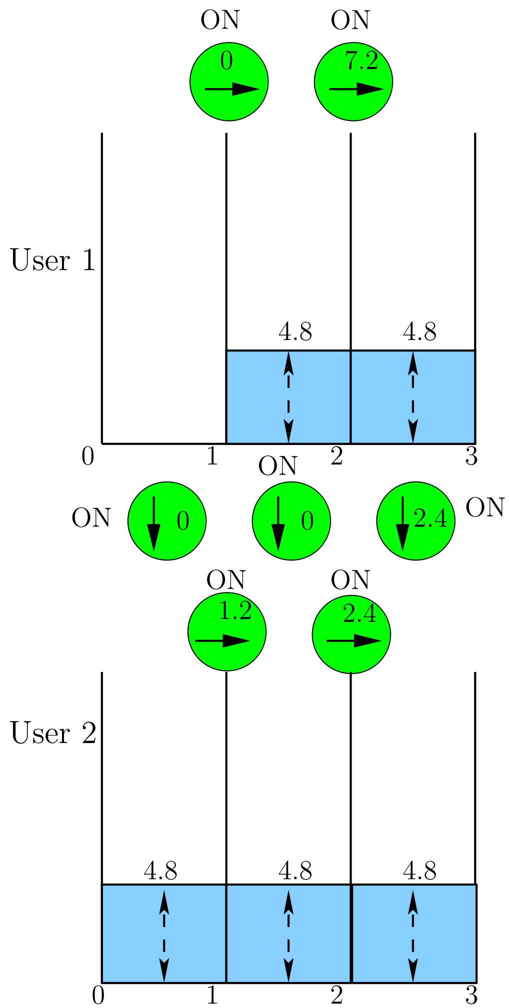
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



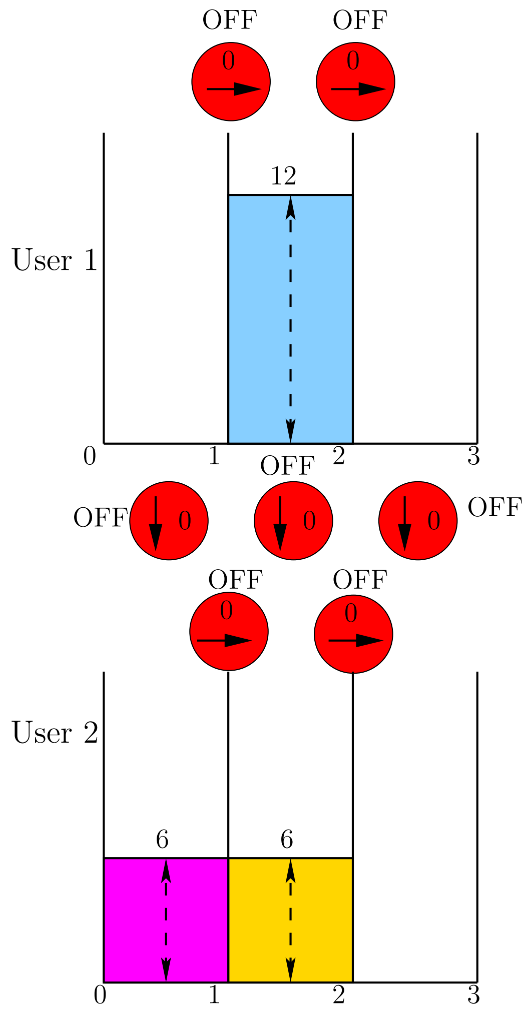
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



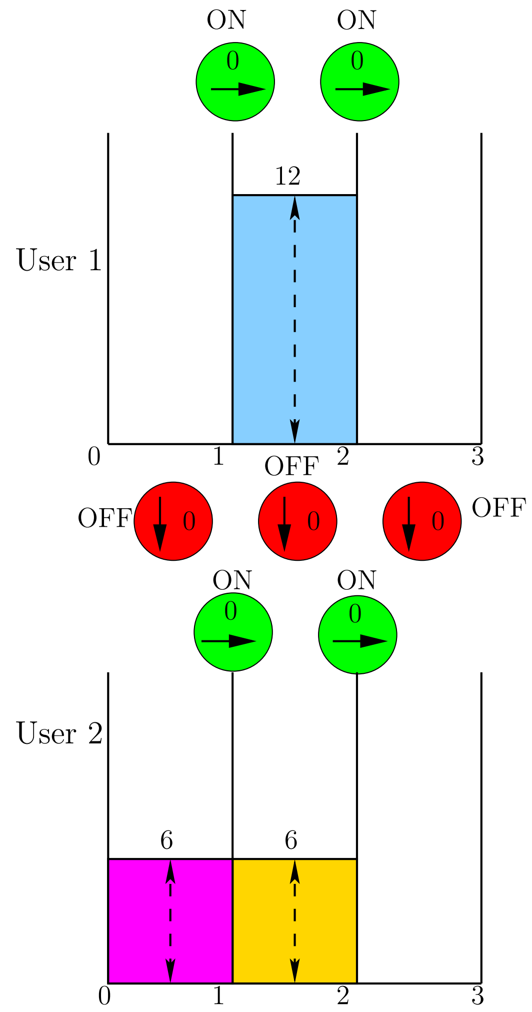
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



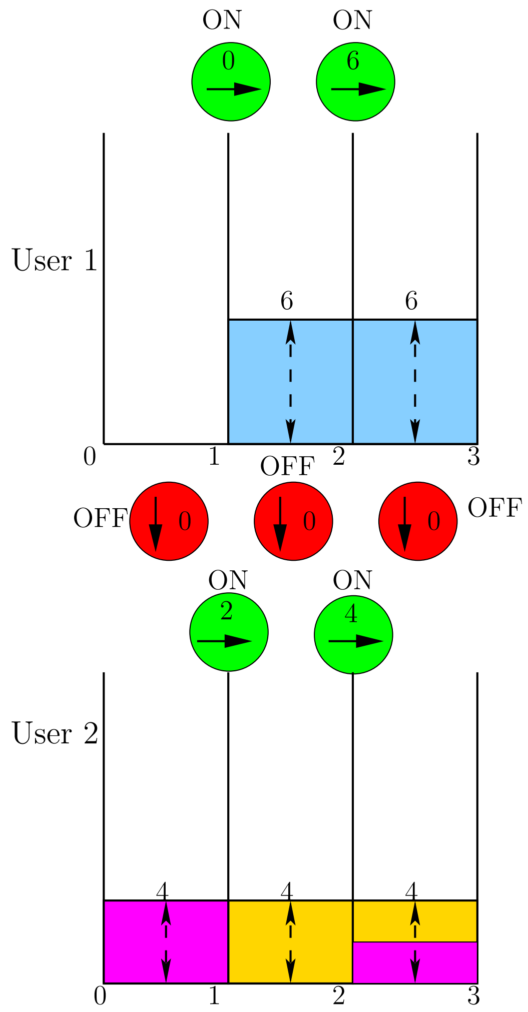
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



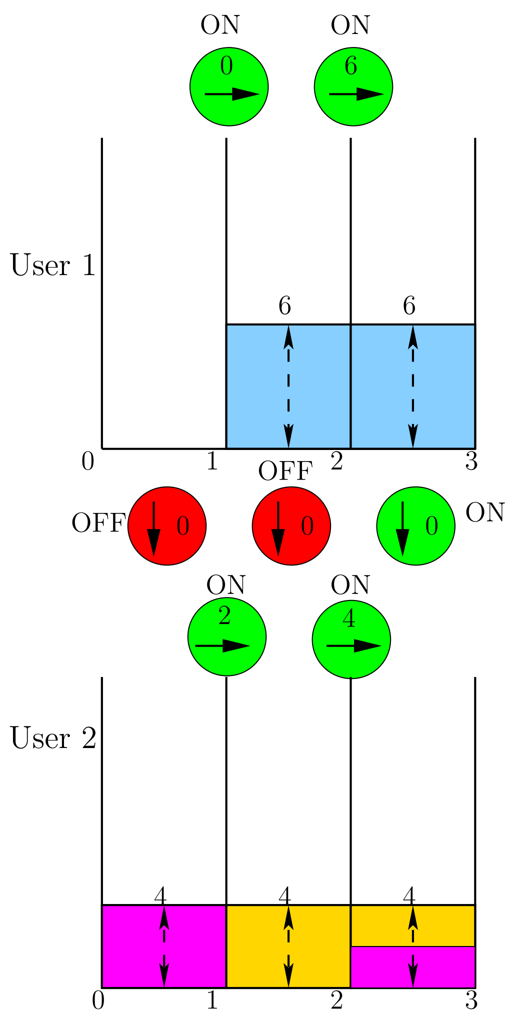
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



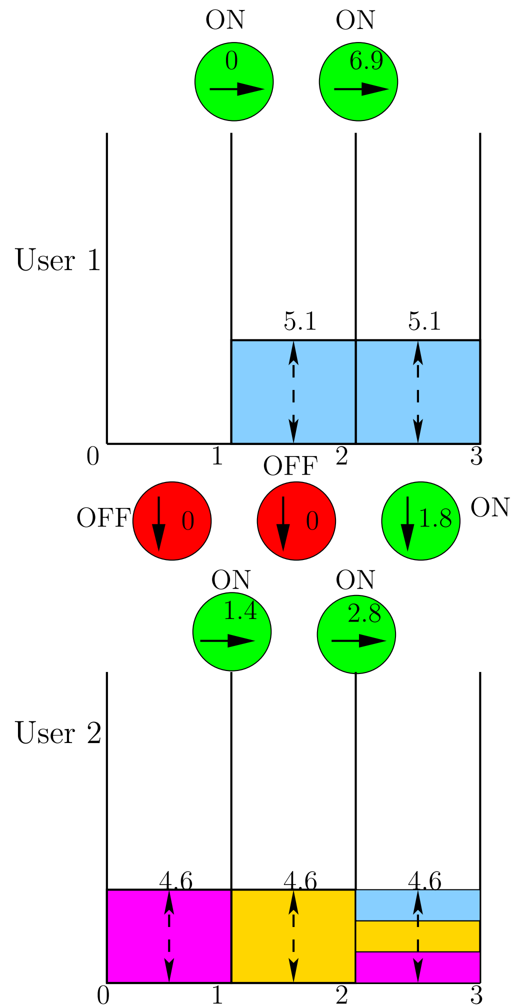
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



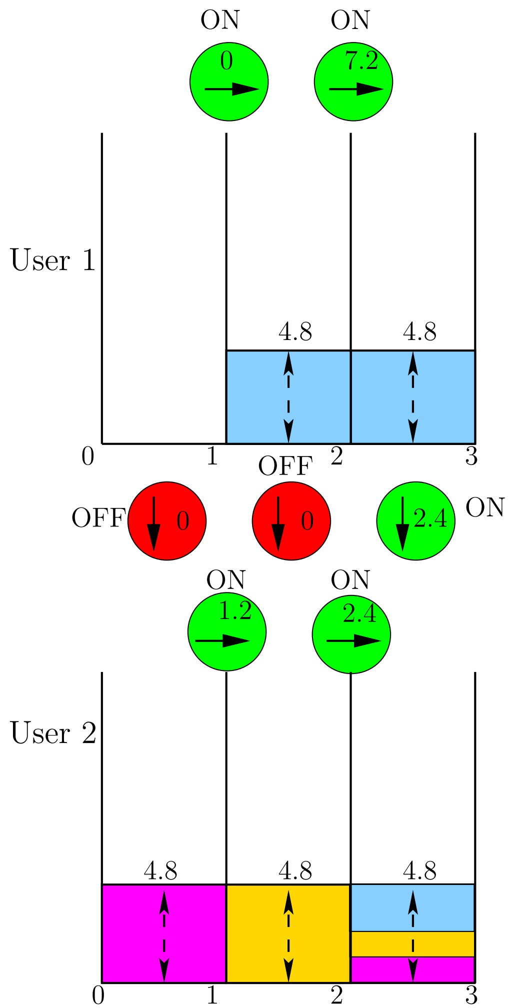
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



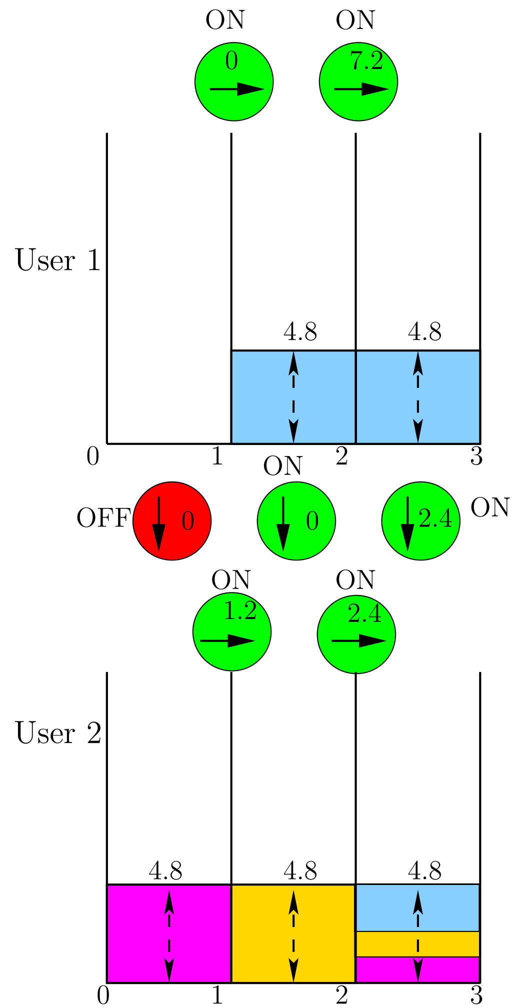
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



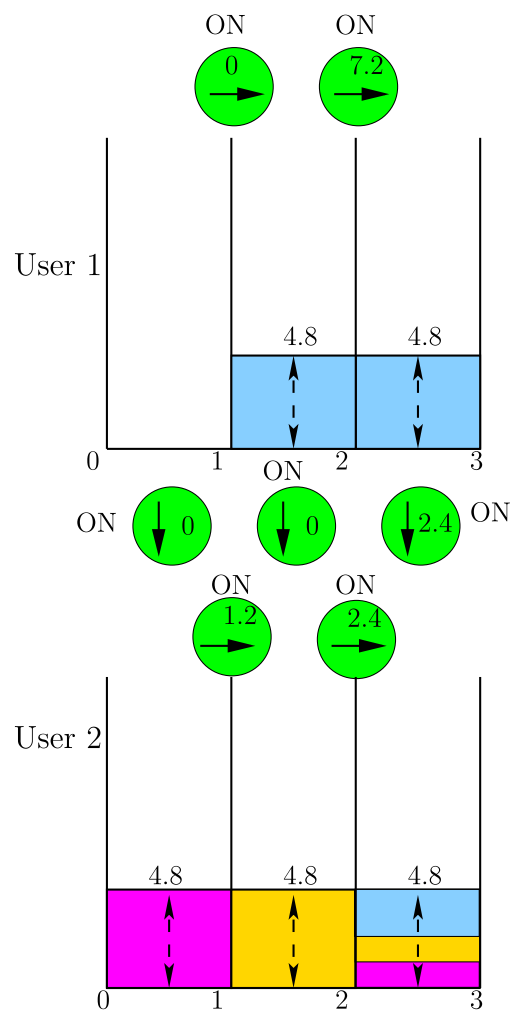
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ



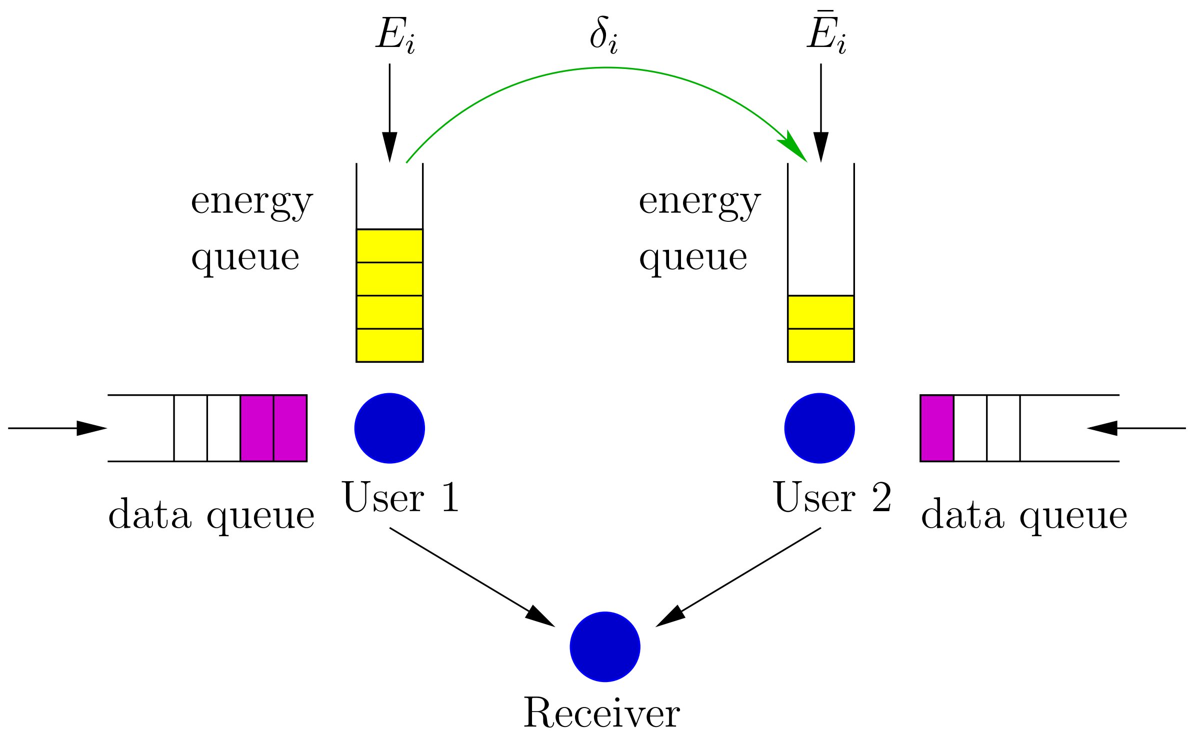
Numerical Example

$\mathbf{E} = [0, 12, 0]$ mJ $\bar{\mathbf{E}} = [6, 6, 0]$ mJ

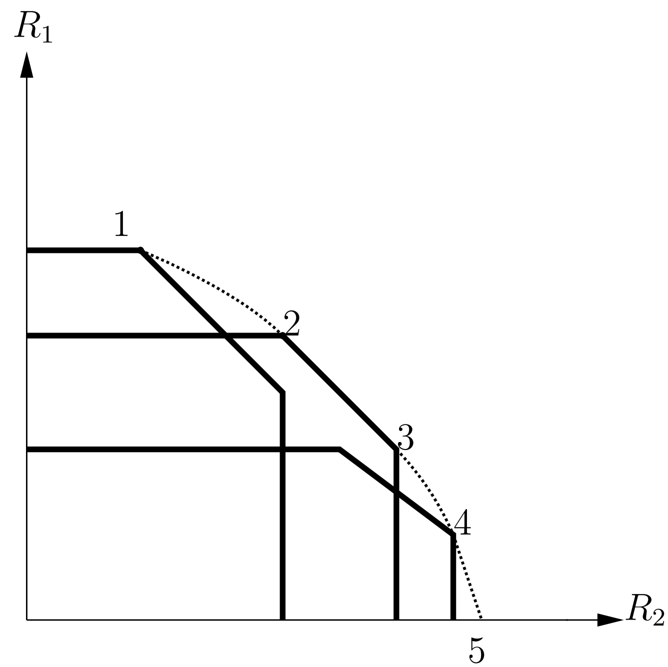


Two User Gaussian MAC with Energy Cooperation

- Energy harvesting users with deterministic energy arrivals E_i, $\bar{E}_i$
- One-way wireless energy transfer with efficiency $0 < \alpha < 1$.

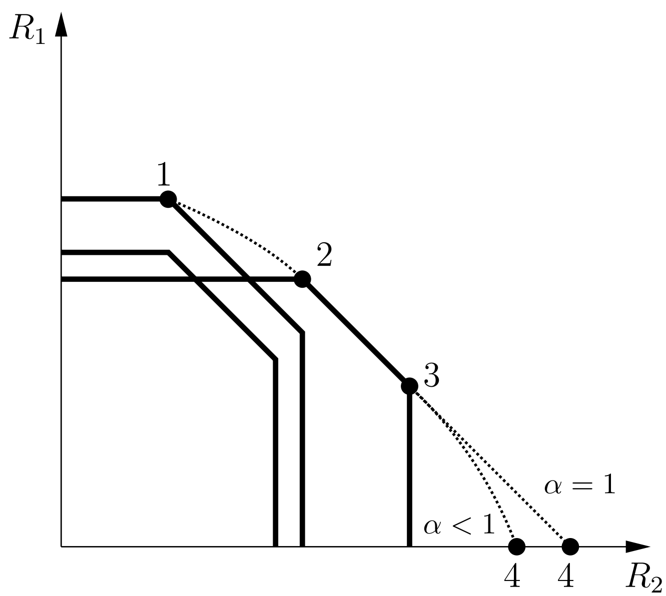


Capacity Region



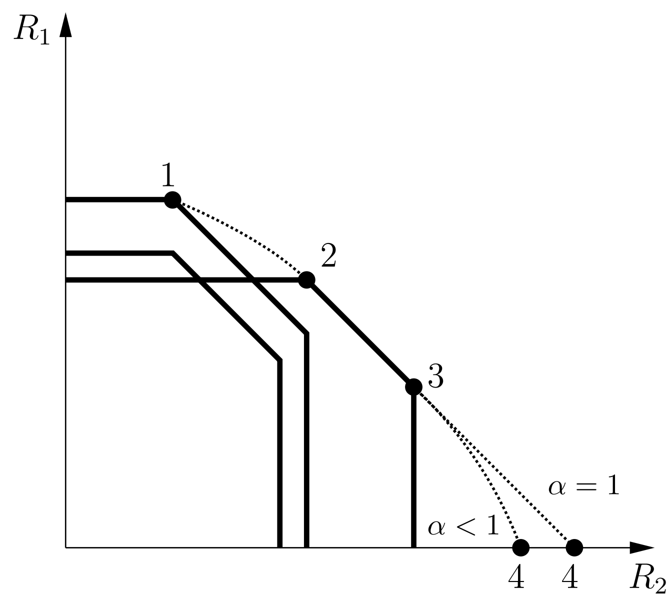
- Convex region, boundary is characterized as $\max_{\mathbf{R} \in \mathcal{C}^M} \boldsymbol{\theta}\mathbf{R}$, $\boldsymbol{\theta} \geq 0$
- We investigate $\theta_1 \geq \theta_2$ and $\theta_1 < \theta_2$ separately.

The Case $\theta_1 \geq \theta_2$



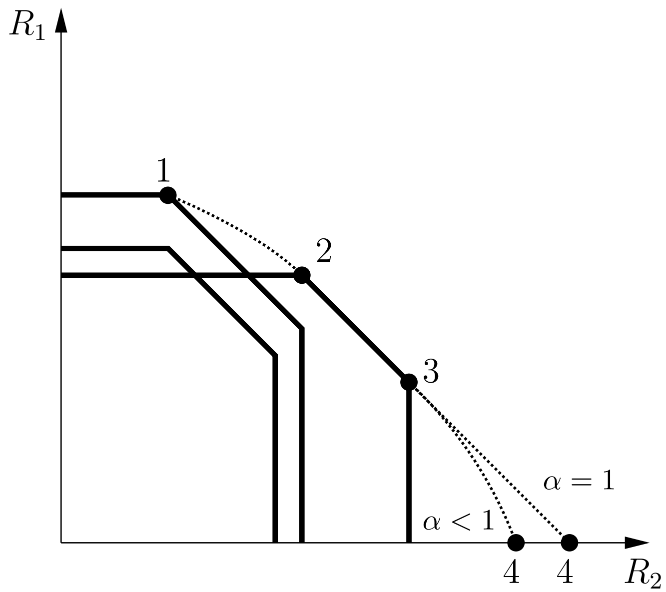
- **In the optimal solution, no energy is transferred.**
- Solution is found by generalized backward directional water-filling algorithm.

The Case $\theta_1 < \theta_2$, $\alpha < 1$



- Point 4 is achieved by **full energy transfer.**
- Energy transfer is necessary to achieve points between 3 and 4.

The Case $\theta_1 < \theta_2$, $\alpha = 1$



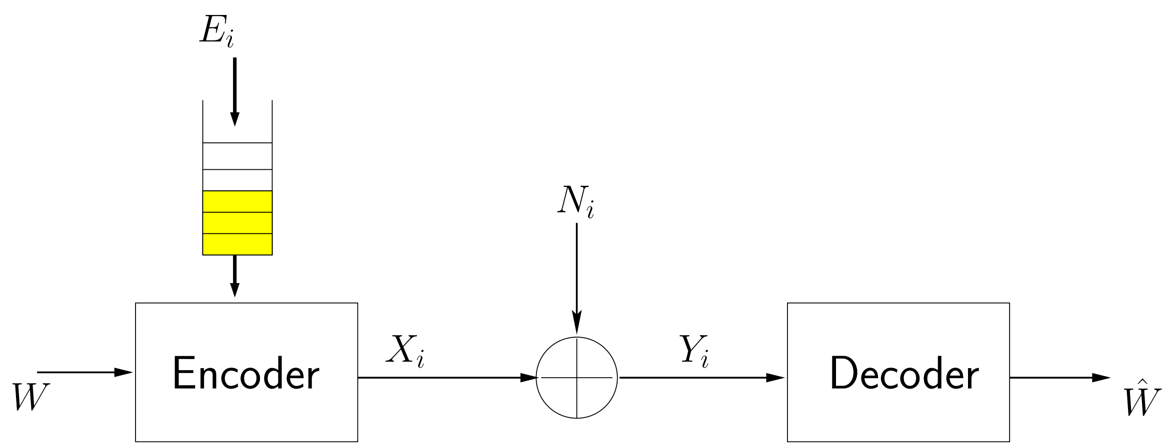
- When $\alpha = 1$, boundary points between 3 and 4 are linear.
- $2, 3$ and 4 are all sum rate optimal.

Conclusions for Energy Cooperation Scenarios

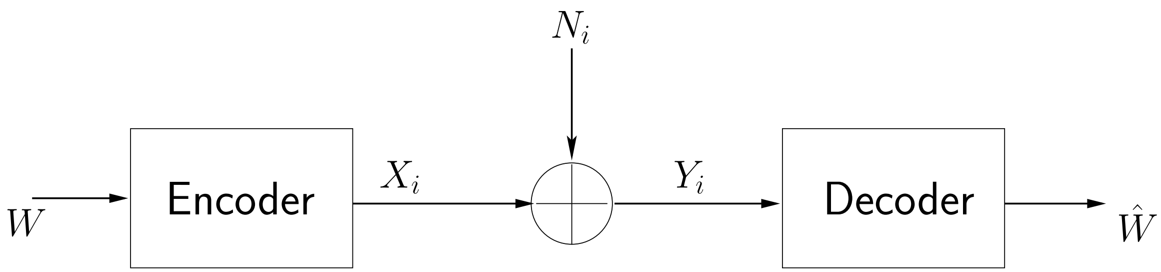
- Energy harvesting users with infinite capacity batteries.
- Energy transfer capability in an orthogonal channel in one way.
- Energy transfer provides a new degree of freedom to smooth out the energy profiles.
- Optimal policies identified for Gaussian two-hop relay, two-way and MAC channels.
- End-to-end throughput maximization for the two-hop relay channel.
- Capacity regions for two-way and MAC channels.

Information Theory of Single-User Energy Harvesting Communication

- Energy is not available up front, arrives randomly in time.
- Energy can be saved in the battery for future use.
- Transmission is interrupted if battery energy is run out.
- What is the highest achievable rate?



Classical AWGN Channel



- AWGN channel:

$$Y = X + N$$

- Average power constraint:

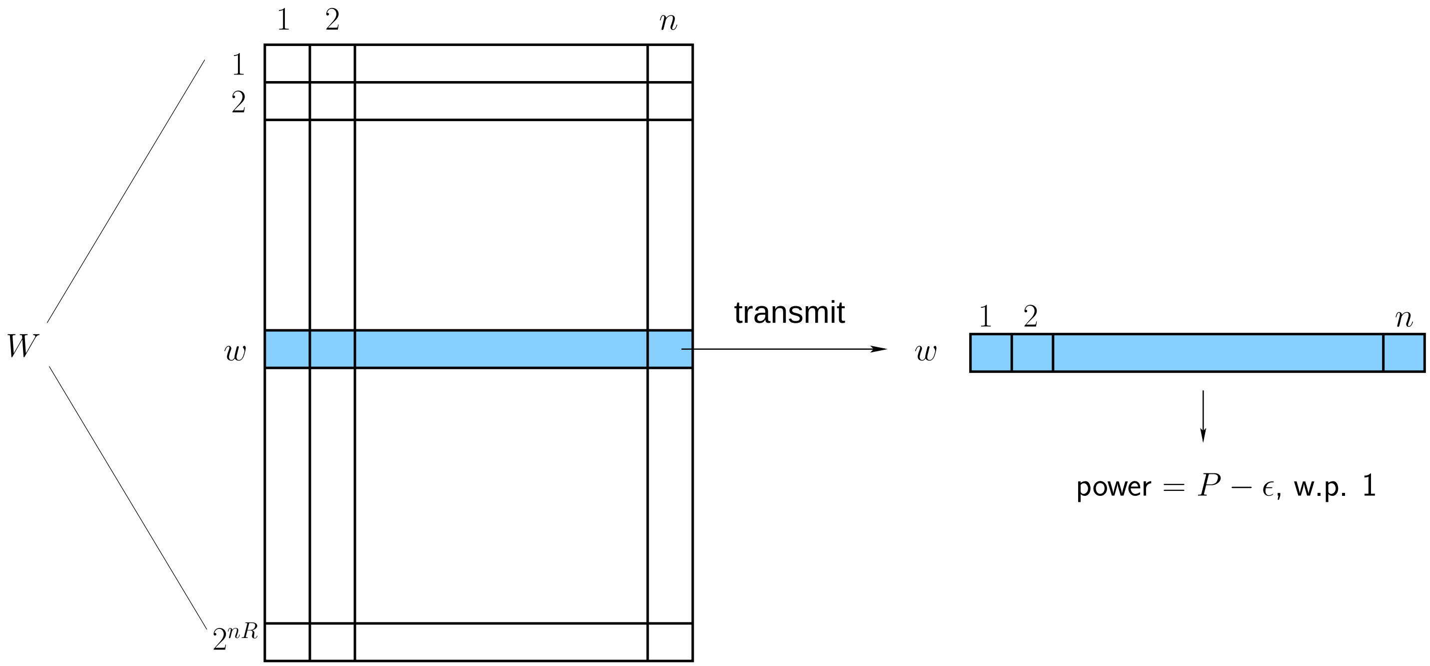
$$\frac{1}{n}\sum_{i=1}^{n} X_i^2 \leq P$$

- AWGN capacity formula with an average power constraint P:

$$C = \frac{1}{2}\log_2(1+P)$$

Achievability in the Classical AWGN Channel

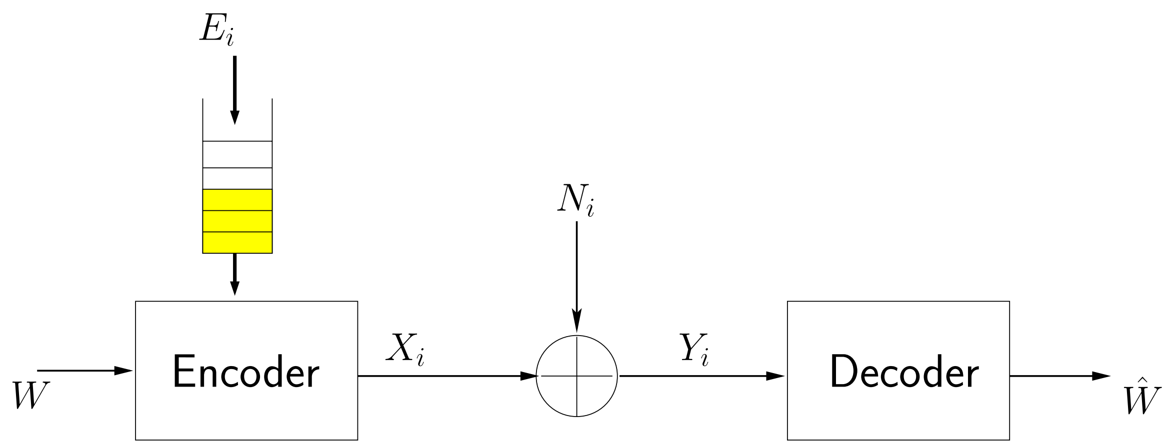
- Generate codebook with i.i.d. Gaussians with zero-mean, variance $P - \varepsilon$.



- By SLLN, codewords so generated obey the power constraint w.p. 1,

$$\frac{1}{n}\sum_{i=1}^{n} X_i^2 \to P - \varepsilon, \quad \text{w.p. } 1$$

Energy Harvesting AWGN Channel Model ($E_{max} = \infty$)



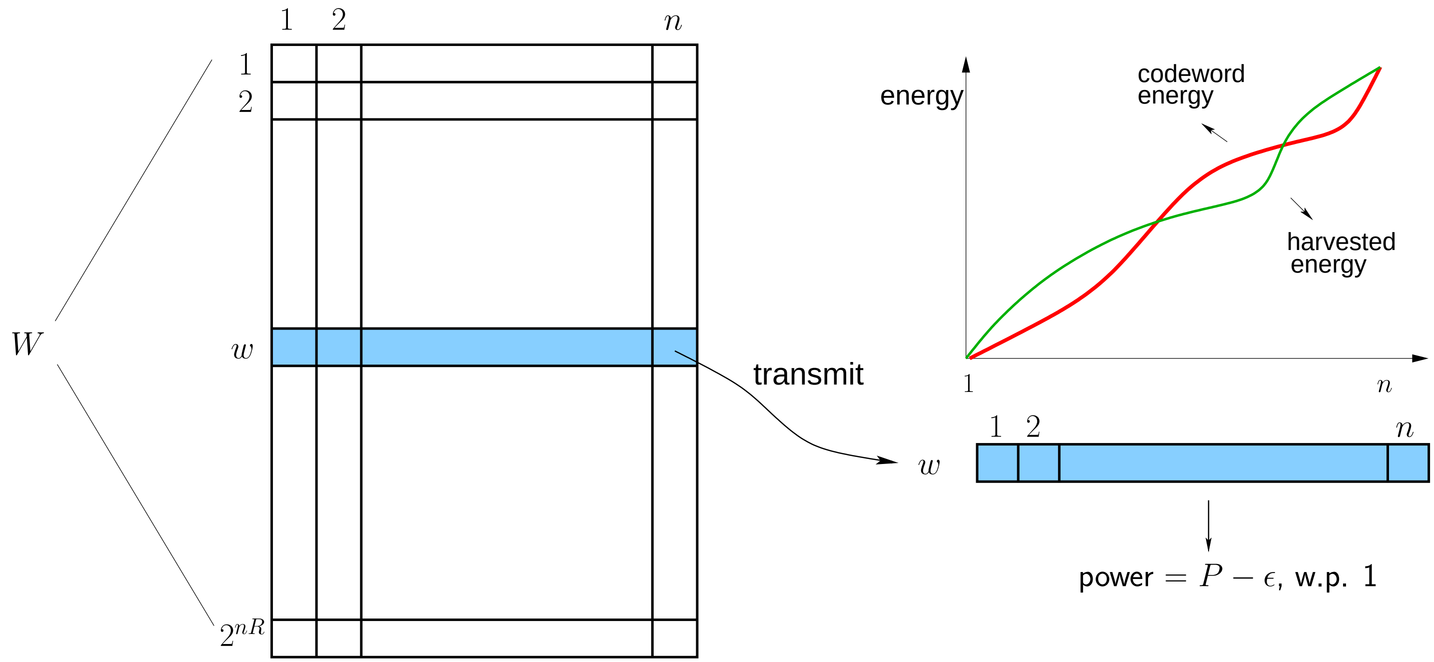
- Code symbols are constrained to the battery energy at each channel use:

$$\sum_{i=1}^{k} X_i^2 \leq \sum_{i=1}^{k} E_i, \qquad k = 1, 2, \ldots, n$$

- Energy harvesting: n constraints.
- Average power constraint: a single constraint, $k = n$.
- $E[E_i] = P$: average recharge rate.
- Battery storage capacity is infinite.

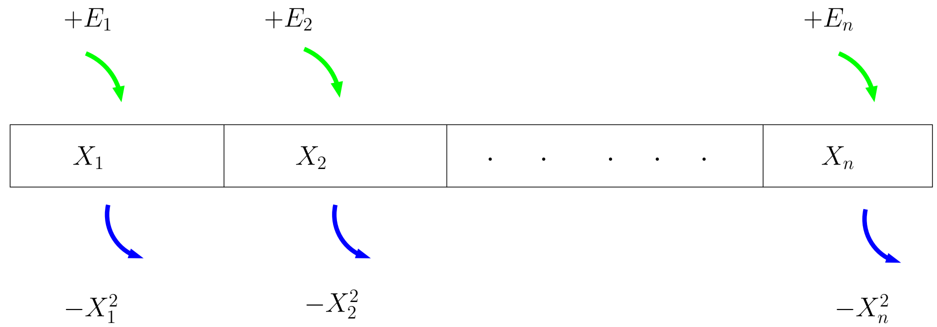
Achievability in the Energy Harvesting AWGN Channel: Major Concerns

- If we generate an i.i.d. Gaussian codebook with zero-mean, variance $P - \varepsilon$.



- How do we design the codebook such that:
  - all codewords are energy-feasible for all channel uses.
- Do we need energy arrival state information:
  - causally, non-causally or not at all, at the transmitter and/or receiver.

The Capacity with Energy Harvesting



- Upper bound: Average power constrained AWGN capacity:

$$C \leq \frac{1}{2} \log(1+P)$$

- This is an upper bound because:
  - Average power constraint imposes a single constraint:

$$\frac{1}{n} \sum_{i=1}^{n} X_i^2 \leq \frac{1}{n} \sum_{i=1}^{n} E_i \to P \quad \text{(by SLLN)}$$

  - While energy harvesting imposes n constraints:

$$\sum_{i=1}^{n} X_i^2 \leq \sum_{i=1}^{n} E_i, \qquad k = 1, \ldots, n$$

- Our contribution: This bound can be achieved.

Achieving the Capacity

- Probability of error: decoding error and violation of energy constraints

- **A first approach:**
  Design a codebook that obeys all n energy constraints.

- **An alternative approach:**
  Design a simple codebook and show the insignificance of energy shortages.

- We will follow the second approach.

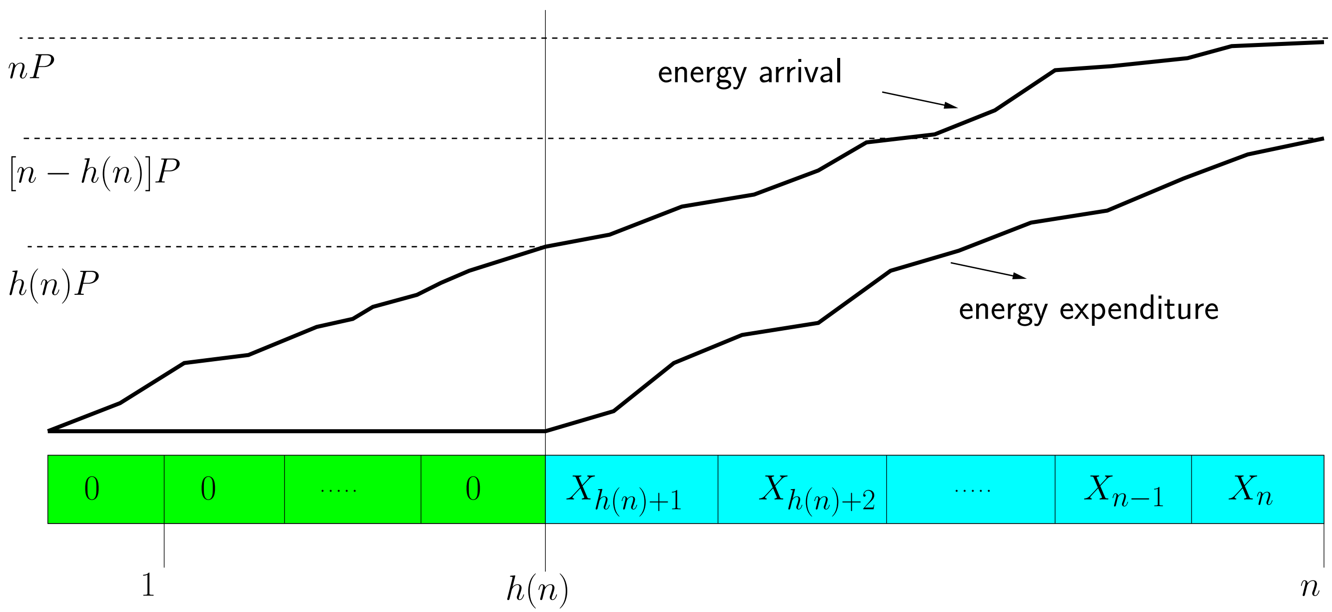
- Two achievable schemes:

  1) Save-and-Transmit Scheme

  2) Best-Effort-Transmit Scheme

Save-and-Transmit Scheme

- Save energy in the first $h(n)$ channel uses, do not transmit.
- In the remaining $n-h(n)$ channel uses, send i.i.d. Gaussian signals.
- Saving period of $h(n)$ channel uses makes the remaining symbols feasible.
- Choose $h(n) \in o(n)$ so that saving incurs no loss in rate, i.e., $h(n)/n \to 0$.
- Rates $< \frac{1}{2}\log(1+P)$ are achievable.



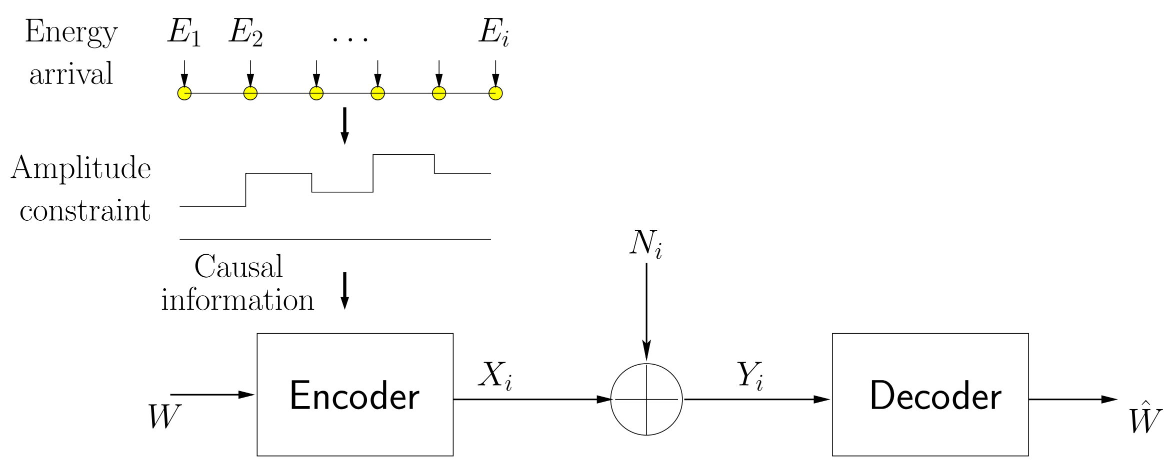
Best-Effort-Transmit Scheme

- X_i: i.i.d. Gaussian.
- $S(i)$: battery energy in the ith channel use.
- If $S(i) \geq X_i^2$, put X_i otherwise put 0 to the channel.
- Mismatch between the codewords and the transmitted symbols.
- Battery energy updates:

$$S(i+1) = S(i) + E_i - X_i^2 \mathbf{1}(S(i) \geq X_i^2)$$

- Since $E[X_i^2] = P - \varepsilon$, only finitely many symbols are infeasible.
- Finitely many mismatches. Inconsequential for joint typical decoding.
- Rates $< \frac{1}{2}\log(1+P)$ are achievable.

Energy Harvesting AWGN Channel Model ($E_{max} = 0$)



- At the ith channel use, i.i.d. E_i energy arrives

$$|X_i| \leq \sqrt{E_i}$$

- Alphabet $\mathcal{E}$ of energies is finite. For simplicity, binary: $\mathcal{E} = \{E_1, E_2\}$
- The transmitter knows energy arrivals **causally**.
- The receiver **does not know** energy arrivals.

The Channel Model

- **A state dependent channel with side information at the transmitter.**
- At realization E of the energy arrivals, the channel is

$$p(y|x,E) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-x)^2}{2}}, \qquad |x| \leq \sqrt{E}$$

- **Combination of**
  - **Smith's** static amplitude constrained AWGN channel
  - **Shannon's** channel with side information at the transmitter

Smith's Amplitude Constrained AWGN Channel

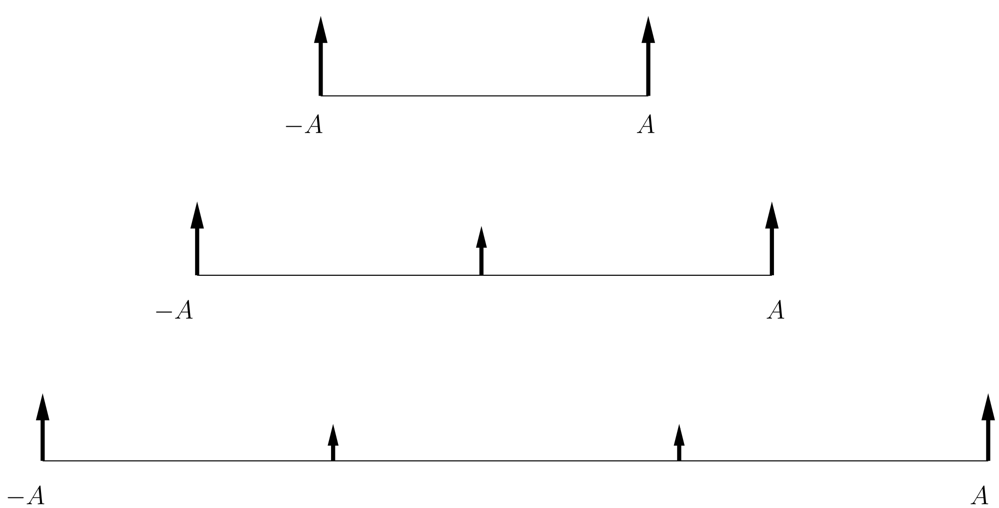
- In 1971, Smith studied **static** amplitude constraints:

$$p(y|x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-x)^2}{2}}, \qquad |x| \leq A$$

- At each channel use, channel symbol is amplitude constrained to A.

$$C_{Sm}(A) = \max_{|X| \leq A} I(X;Y)$$

- This is a convex functional optimization problem.
- The capacity achieving input distribution is **discrete.**



Shannon's Channels with Side Information at the Transmitter

- The state-dependent channel $p(y|x,s)$, $s \in \mathcal{S}$
- i.i.d. states with $P(s = s_i) = p_{s_i}$.
- s is available causally at the transmitter, not available at the receiver.
- Shannon proved in 1958 that

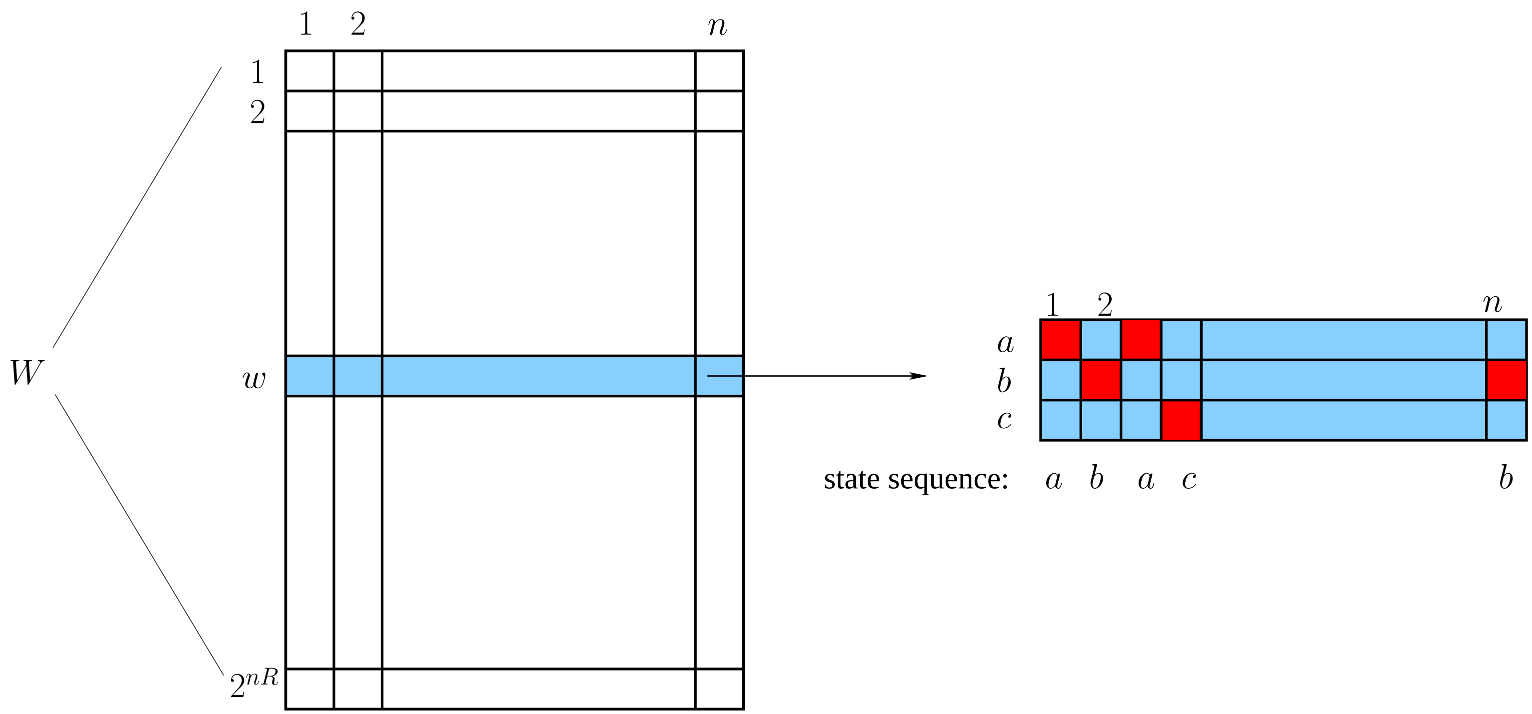
$$C_{Sh} = \max_{p(T)} I(T;Y)$$

- T is the extended input $T = [T_1, \ldots, T_{|\mathcal{S}|}]$ with

$$p(y|t = (t_1, \ldots, t_{|\mathcal{S}|})) = \sum_{i=1}^{|\mathcal{S}|} p_{s_i} p(y|t_i, s_i)$$

Shannon's Channels with Side Information at the Transmitter

- Shannon strategy: codewords are $|\mathcal{S}| \times n$ matrices.



Capacity of AWGN Channel with Time-Varying Amplitude Constraints

- Applying Shannon's result,

$$C_{Sh} = \max_{p(T)} I(T;Y)$$

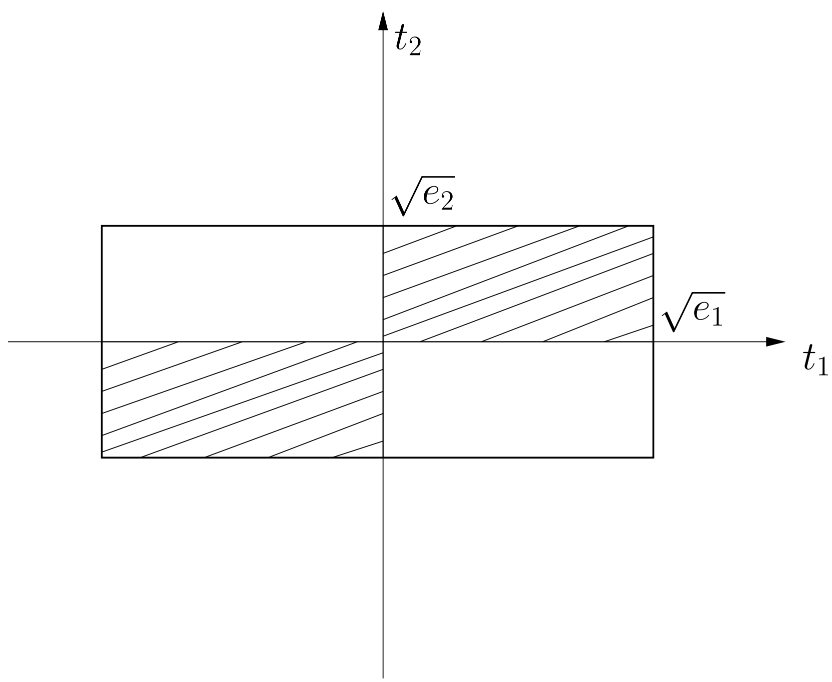
- $T = [T_1, T_2]$

$$p(y|t_1,t_2) = \underbrace{\frac{p_1}{\sqrt{2\pi}} e^{-\frac{(y-t_1)^2}{2}}}_{|t_1| \leq \sqrt{E_1}} + \underbrace{\frac{p_2}{\sqrt{2\pi}} e^{-\frac{(y-t_2)^2}{2}}}_{|t_2| \leq \sqrt{E_2}}$$

- If E is observed, the channel symbol needs to satisfy $|X| \leq \sqrt{E}$.
- The capacity achieving distribution is **discrete.**
- $[T_1, T_2]$ takes values from a finite set in $\mathbb{R}^2$.

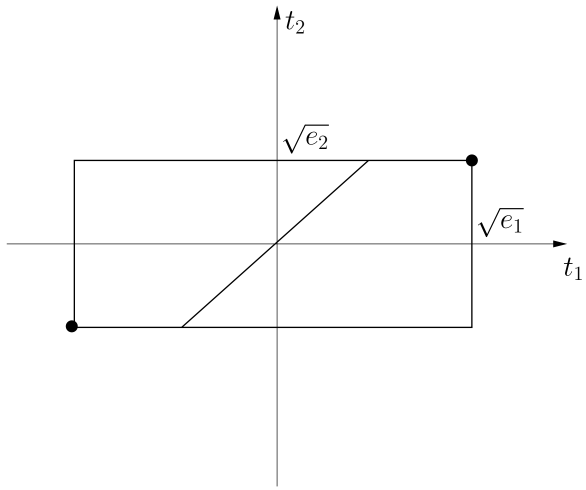
Structure of the Optimal Mass Points

- Symmetric with respect to $(0,0)$
- Constrained to the shaded area
- $a_1 = \sqrt{E_1}$ and $a_2 = \sqrt{E_2}$

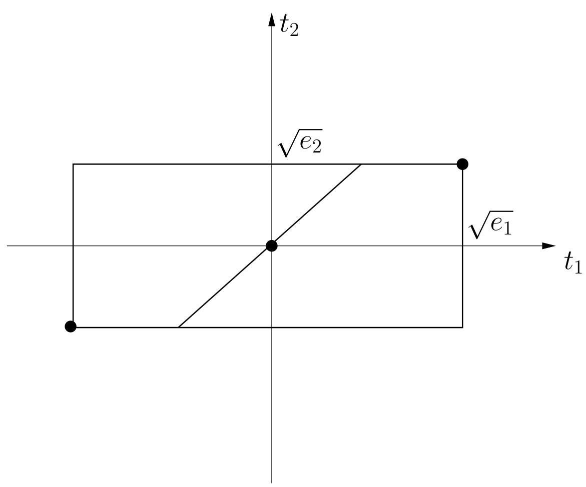


Structure of the Optimal Mass Points

- If a_1 and a_2 are sufficiently small: binary

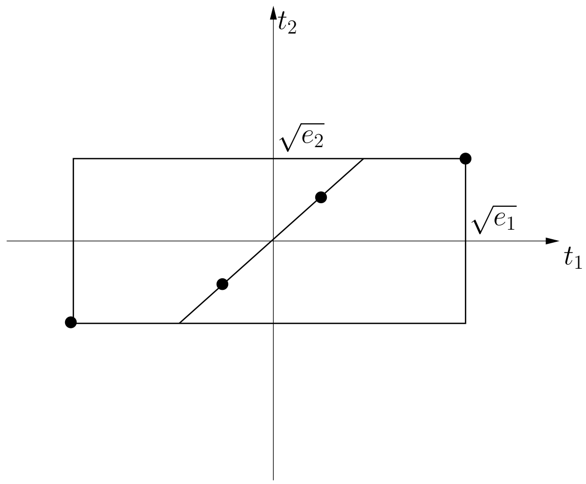


- If a_1 and a_2 are increased: ternary

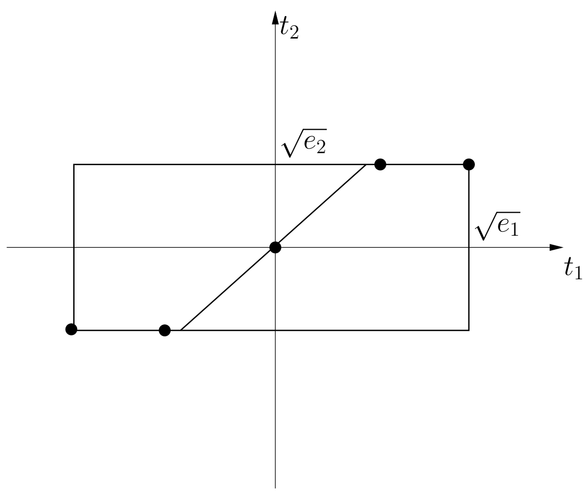


Structure of the Optimal Mass Points

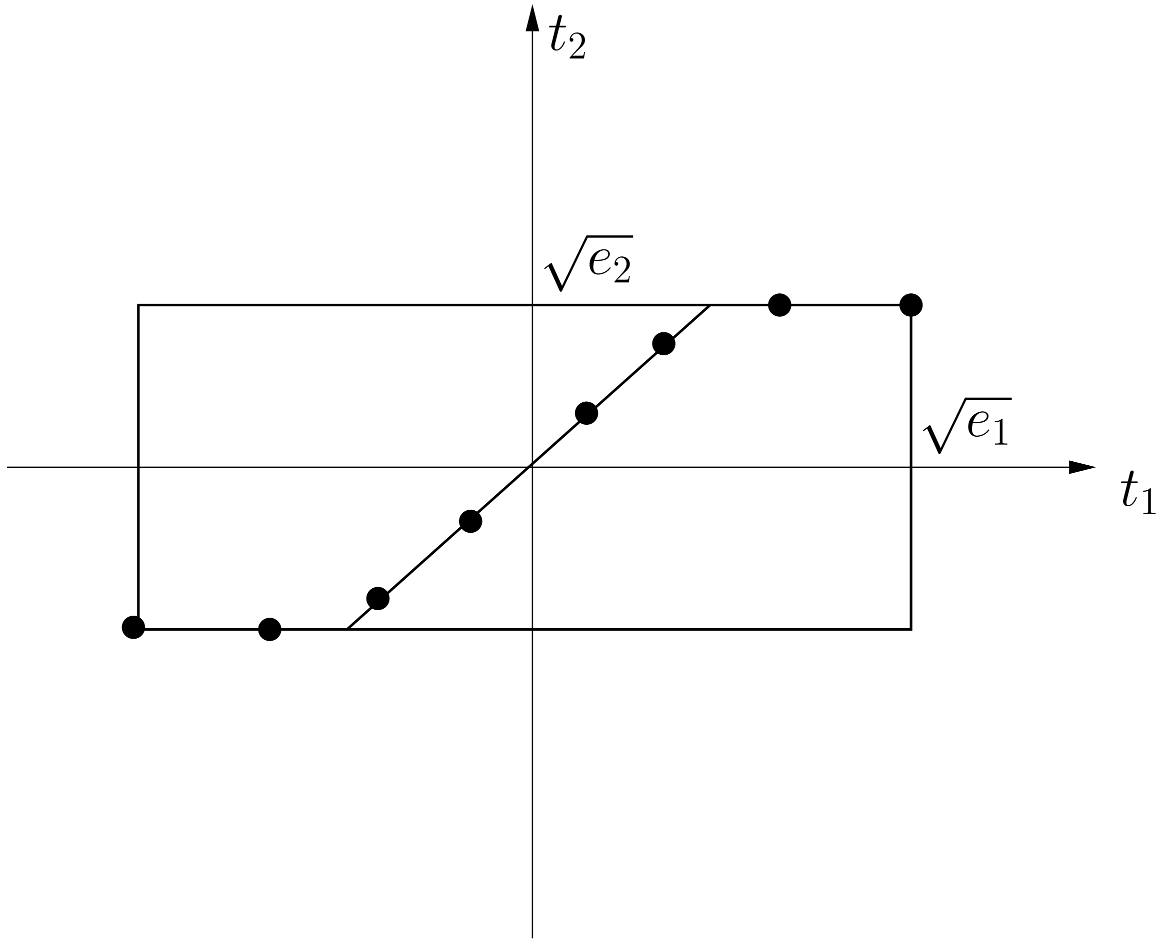
- If a_1 and a_2 are increased: quaternary



- If a_1 and a_2 are increased: quintuple

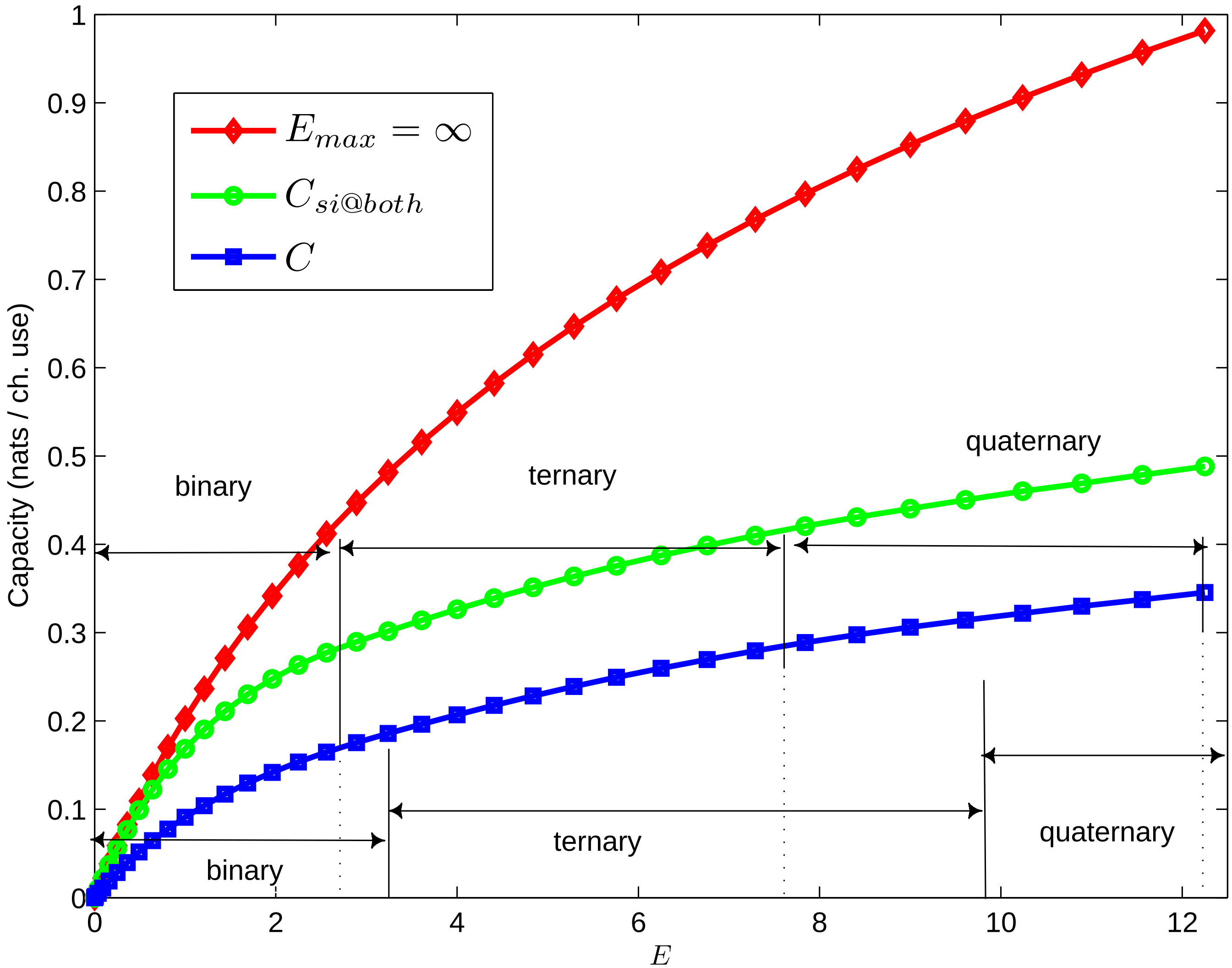


Experimental Observation of the Optimal Mass Points



- Experiments are based on verification of the necessary optimality conditions.

AWGN Channel with On-Off Energy Arrivals, $p_{on} = 0.5$

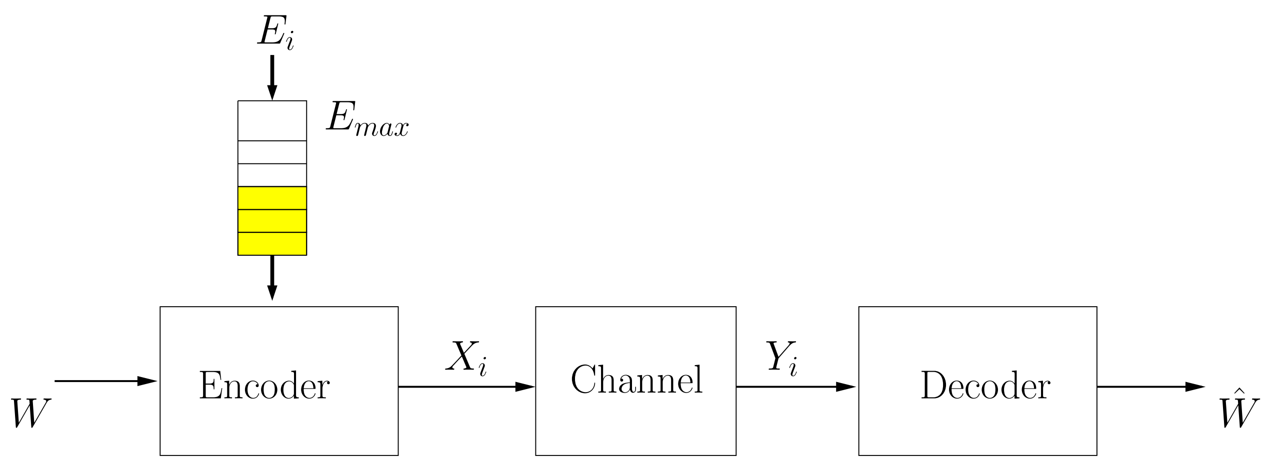


Energy Harvesting Channel with Finite Energy Storage

- Channel input: $X_i \in \{0, 1, \ldots, K\}$
- Each symbol k has k-unit energy cost.
- The transmitter has E_{max} unit battery.
- At channel use i, the symbol energy of X_i must be smaller than the energy in the battery S_i.
- A state-dependent channel with state S_i:

$$S_{i+1} = \min\{S_i - X_i + E_i, E_{max}\}$$

- State has memory and input dependence.

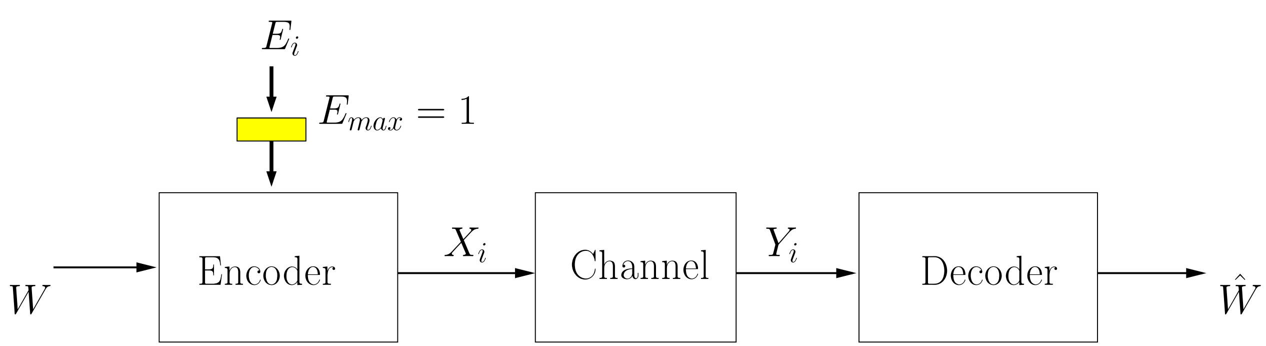


Energy Harvesting Channel with Finite Energy Storage

- Channel input: $X_i \in \{0, 1, \ldots, K\}$
- Each symbol k has k-unit energy cost.
- The transmitter has E_{max} unit battery.
- At channel use i, the symbol energy of X_i must be smaller than the energy in the battery S_i.
- A state-dependent channel with state S_i:

$$S_{i+1} = \min\{S_i - X_i + E_i, E_{max}\}$$

- State has memory and input dependence.

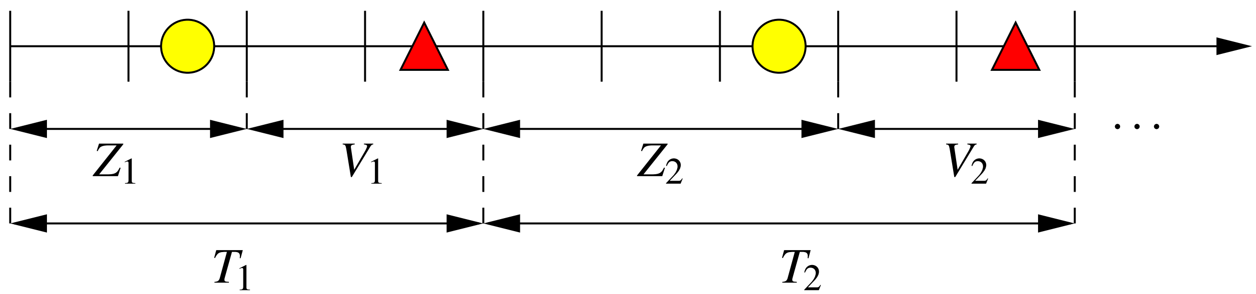


Binary Energy Harvesting Channel with Unit Storage

- A noiseless binary channel: $X_i \in \{0,1\}$
- The transmitter has one unit battery: $E_{max} = 1$.
- Encoding/decoding can be equivalently done in terms of time intervals between 1s.
- An additive noise timing-channel:

$$T_n = V_n + Z_n$$

where V_n is waiting time, Z_n is additive noise, T_n is the length of the interval between two 1s.



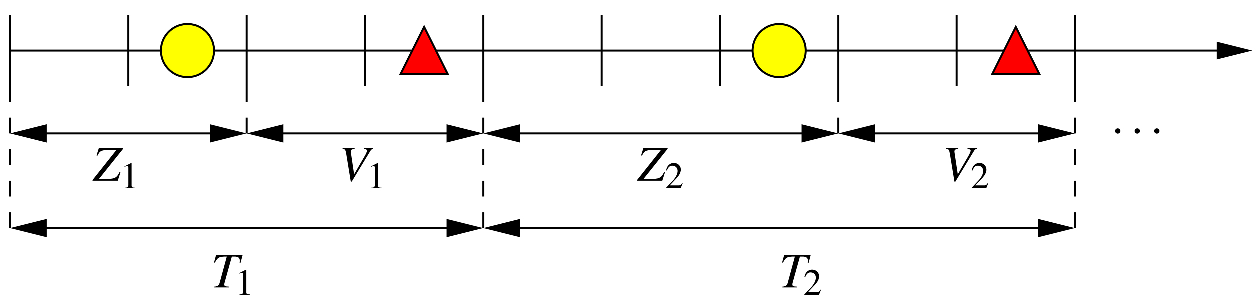
Binary Energy Harvesting Channel with Unit Storage

- An additive noise timing-channel:

$$T_n = V_n + Z_n$$

- Z_n: i.i.d. geometric noise
- Transmitter causally knows Z_n before deciding on V_n.
- The additive timing channel is state dependent where the state is the noise.
- Capacity is found by using Shannon strategy in the timing channel:

$$C = \max_{p(u), f(u,z)} \frac{I(U;Z)}{E[T]}$$



Conclusions

- Capacity of energy harvesting AWGN channel under two extremes.

- When battery capacity is $E_{max} = \infty$:
  - Transmitter/receiver do not need energy arrival information.
  - Equal to the AWGN channel capacity with average power $E[E_i] = P$.
  - Save-and-Transmit Scheme and Best-Effort-Transmit Scheme

- When battery capacity is $E_{max} = 0$:
  - Transmitter has causal energy information, receiver has no information.
  - Smith's static amplitude constraints and Shannon's causal side information
  - Discrete signaling is optimal.

- Open problem: When battery capacity E_{max} is finite.
  - When $E_{max} = 1$, capacity found through a corresponding timing channel.

Acknowledgements

- NSF CNS 0964632, NSF CCF 1422111
- Slides: Omur Ozel, Berk Gurakan, and Ahmed Arafa

References

[1] J. Yang and S. Ulukus, Optimal Packet Scheduling in an Energy Harvesting Communication System, *IEEE Transactions on Communications*, January 2012.

[2] K. Tutuncuoglu and A. Yener, Optimum Transmission Policies for Battery Limited Energy Harvesting Nodes, *IEEE Transactions on Wireless Communications*, February 2012.

[3] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus and A. Yener, Transmission with Energy Harvesting Nodes in Fading Wireless Channels: Optimal Policies, *IEEE Journal on Selected Areas in Communications*, September 2011.

[4] A. Arafa and S. Ulukus, Single-User and Multiple Access Channels with Energy Harvesting Transmitters and Receivers, *IEEE GlobalSIP*, December 2014.

[5] J. Yang, O. Ozel and S. Ulukus, Broadcasting with an Energy Harvesting Rechargeable Transmitter, IEEE Transactions on Wireless Communications, February, 2012.

[6] J. Yang and S. Ulukus, Optimal Packet Scheduling in a Multiple Access Channel with Energy Harvesting Transmitters, JCN Special Issue on Energy Harvesting in Wireless Networks, April 2012.

[7] O. Ozel, J. Yang and S. Ulukus, Optimal Broadcast Scheduling for an Energy Harvesting Rechargeable Transmitter with a Finite Capacity Battery, IEEE Transactions on Wireless Communications, June 2012.

References

[8] K. Tutuncuoglu and A. Yener, Sum-Rate Optimal Power Policies for Energy Harvesting Transmitters in an Interference Channel, JCN Special Issue on Energy Harvesting in Wireless Networks, April 2012.

[9] C. Huang, R. Zhang and S. Cui, Throughput maximization for the Gaussian Relay Channel with Energy Harvesting Constraints, *IEEE Journal on Selected Areas in Communications*, August 2013.

[10] D. Gunduz and B. Devillers, Two-Hop Communication with Energy Harvesting, IEEE CAMSAP, San Juan, PR, December 2011.

[11] B. Gurakan, O. Ozel, J. Yang and S. Ulukus, Energy Cooperation in Energy Harvesting Communications, *IEEE Transactions on Communications*, December 2013.

[12] O. Ozel and S. Ulukus, Achieving AWGN Capacity under Stochastic Energy Harvesting, IEEE Transactions on Information Theory, October 2012.

[13] O. Ozel and S. Ulukus, AWGN Channel under Time-Varying Amplitude Constraints with Causal Information at the Transmitter, Asilomar Conference, Pacific Grove, CA, November 2011.

[14] K. Tutuncuoglu, O. Ozel, A. Yener and S. Ulukus, Binary Energy Harvesting Channel with Finite Energy Storage, IEEE ISIT, Istanbul, Turkey, July 2013.