Afternoon Session
Scheduling in Multi-User Energy Harvesting Networks
and
Information-Theoretic Treatment of Single-User Energy Harvesting Communication

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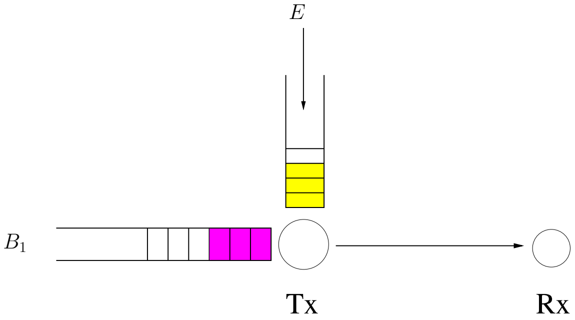
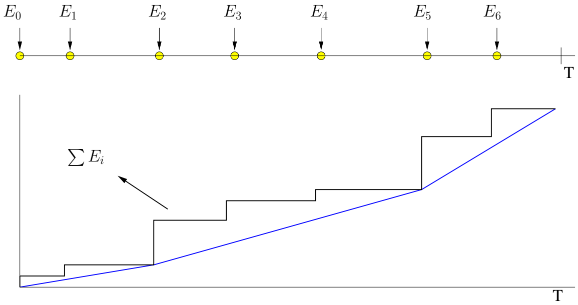
So Far, We Learned...

- **Single-user** communication with an energy harvesting transmitter.
- Energy arrives (is harvested) **during the communication session**
- Transmission policy is **adapted to energy arrivals**
- Two dual objectives:
  - minimize transmission completion time
  - maximize average throughput

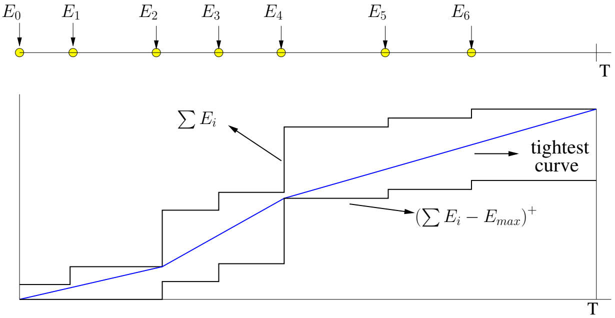


The Optimal Policy for $E_{max} = \infty$



- Upper staircase is the cumulative energy arrivals
- Feasible energy consumption lies below the staircase
- Transmit power remains constant in each epoch
- The tightest curve under the cumulative energy arrival staircase

The Optimal Policy for $E_{max} < \infty$

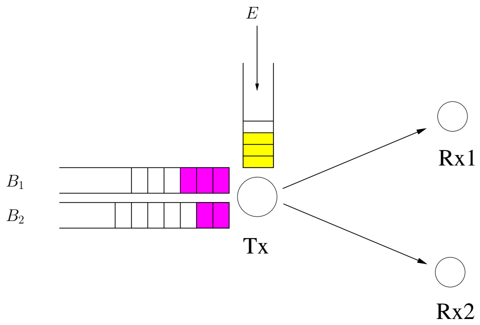


- Upper staircase: energy arrivals
- Lower staircase: finite battery constraint (no overflows)
- Any feasible energy consumption curve must lie **in between**
- Power remains constant in each epoch
- The tightest curve in the feasibility tunnel

Scheduling in Multi-user Energy Harvesting Systems

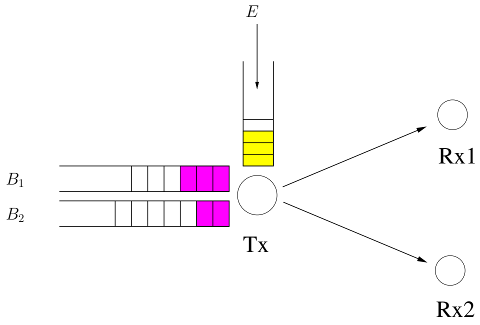
- Extend the system model to a multi-user setting
- Broadcasting with an energy harvesting transmitter
  - An energy harvesting transmitter sends messages to two users
  - E.g., a wireless access device sending different messages to users
- Multiple access with energy harvesting transmitters
  - Energy harvesting transmitters communicating with a single receiver
  - E.g., multiple sensors sending data to a center

Broadcasting with an Energy Harvesting Transmitter



- Energy arrives (is harvested) **during the communication session**.
- Assume battery has infinite storage capacity: $E_{max} = \infty$
- Broadcasting data to two users by **adapting to energy arrivals**
- Objective: minimize the transmission completion time

Broadcast Channel Model

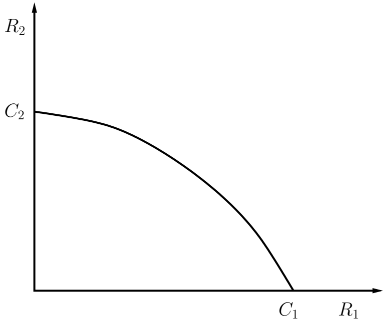


- AWGN broadcast channel:

$$Y_1 = X + N_1, \quad Y_2 = X + N_2$$

  where $N_1 \sim \mathcal{N}(0,1)$, $N_2 \sim \mathcal{N}(0,\sigma^2)$
- $\sigma^2 > 1$: 2nd user is degraded
- We call 1st user stronger and 2nd user weaker

Broadcast Channel Model



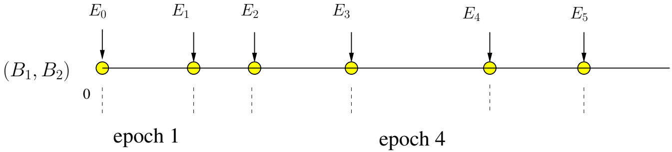
$$r_1 \leq \frac{1}{2}\log_2\left(1+\alpha P\right)$$
$$r_2 \leq \frac{1}{2}\log_2\left(1+\frac{(1-\alpha)P}{\alpha P+\sigma^2}\right)$$

- We work in the (r_1, r_2) domain:

$$P = 2^{2(r_1+r_2)} + (\sigma^2 - 1)2^{2r_2} - \sigma^2 \triangleq g(r_1, r_2)$$

- $g(r_1, r_2)$ is the minimum power required to send at rates (r_1, r_2)

Energy Model

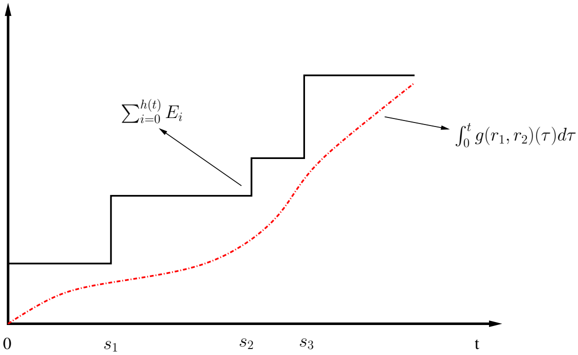


- Energy is *harvested* **during the course of communication**.
- We will consider offline policies.
- Energy causality constraints: energy that has not arrived cannot be used

$$\int_0^{t_i^e} g(r_1, r_2)(\tau) d\tau \leq \sum_{j=0}^{i-1} E_j, \quad \forall i$$

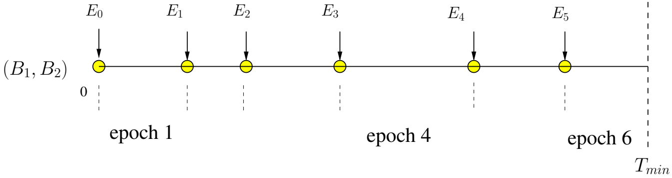
Constraints on the Power Policy

- Energy arrivals known deterministically a priori



- Upper staircase: energy arrivals
- Any feasible energy consumption curve must lie **below the upper staircase**

Problem Formulation

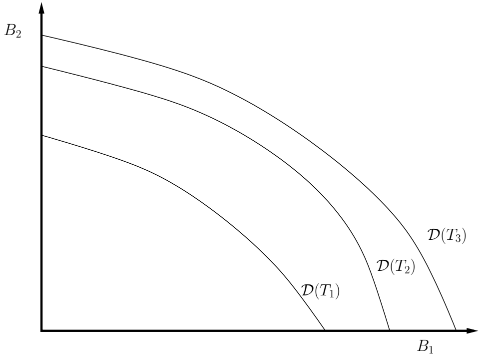


- Minimize transmission completion time of (B_1, B_2) bits.
- By adapting the transmission to the energy arrivals.
- Subject to energy causality constraints

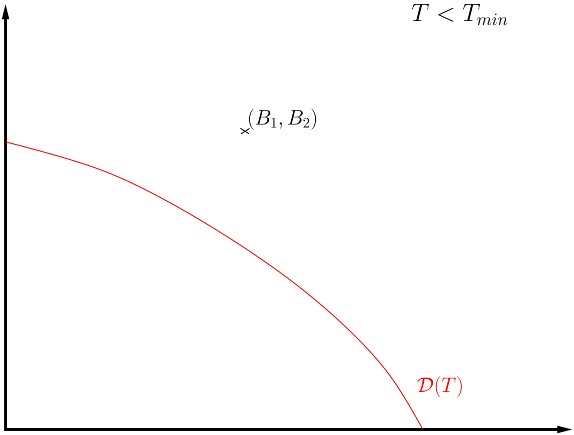
$$\begin{aligned} \min_{\mathbf{r}_1, \mathbf{r}_2} \quad & T \\ \text{s.t.} \quad & \int_0^t g(r_1, r_2)(\tau) d\tau \leq \sum_{n=0}^{h(t)} E_n, \quad \forall t \geq 0 \\ & \int_0^T r_1(\tau) d\tau = B_1 \\ & \int_0^T r_2(\tau) d\tau = B_2 \end{aligned}$$

Dual Problem: Finding the Maximum Departure Region

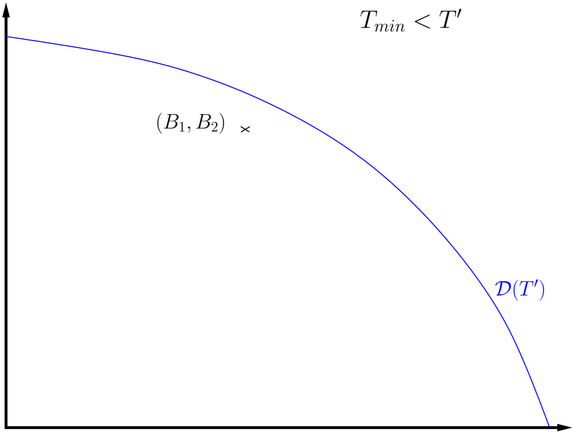
- The maximum departure region $\mathcal{D}(T)$: union of (B_1, B_2) pairs achievable by some rate allocation policy that satisfies the energy causality constraint.
- $\mathcal{D}(T)$ monotonically increases with T. For example, when $T_1 < T_2 < T_3$:



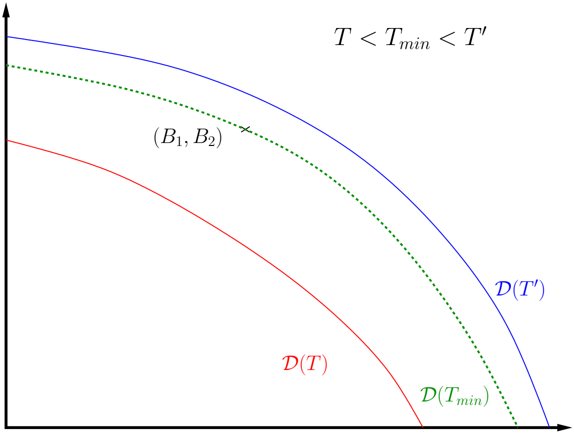
Dual Problem: Finding Maximum Departure Region



Dual Problem: Finding Maximum Departure Region

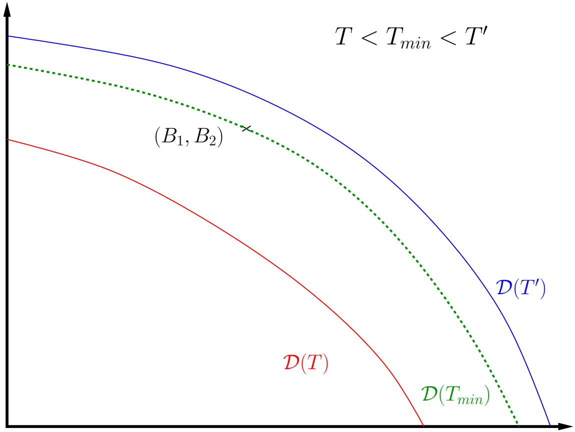


Dual Problem: Finding Maximum Departure Region



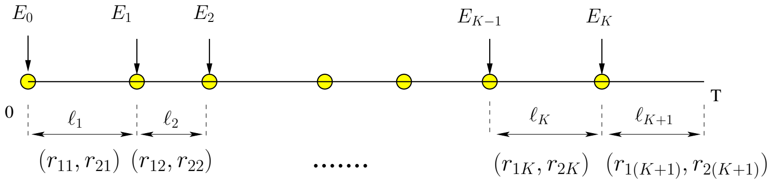
Dual Problem: Finding Maximum Departure Region

- These problems are dual because: if (B_1, B_2) bits can be transmitted in $T_{\min}$ then (B_1, B_2) must be in $D(T_{\min})$.



Dual Problem: Maximum Departure Region

- Find $\mathcal{D}(T)$ for a given T.



- Transmission rates, and power, remain constant between energy harvests.
- Denote the rates that go to users as (r_{1i}, r_{2i}) over epoch i.
- The **power** at epoch i: $g(r_{1i}, r_{2i})$
- The **energy spent** during epoch i: $g(r_{1i}, r_{2i})\ell_i$
- The energy causality constraint reduces to constraints on (r_{1i}, r_{2i}):

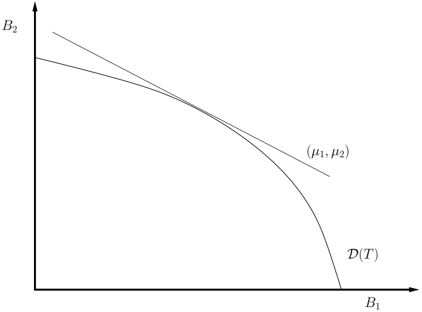
$$\sum_{i=1}^{k} g(r_{1i}, r_{2i})\ell_i \leq \sum_{i=0}^{k-1} E_i, \qquad k = 1, \ldots, K+1$$

Dual Problem: Maximum Departure Region

- $\mathcal{D}(T)$ is a strictly convex region.
- Characterize $\mathcal{D}(T)$ by solving optimization problems for all $\mu_1, \mu_2 \geq 0$:

$$\max_{\mathbf{r}_1, \mathbf{r}_2} \quad \mu_1 \sum_{i=1}^{K+1} r_{1i}\ell_i + \mu_2 \sum_{i=1}^{K+1} r_{2i}\ell_i$$

$$\text{s.t.} \quad \sum_{i=1}^{k} g(r_{1i}, r_{2i})\ell_i \leq \sum_{i=0}^{k-1} E_i, \qquad k = 1, \ldots, K+1$$



Dual Problem: Finding the Maximum Departure Region

- The Lagrangian function

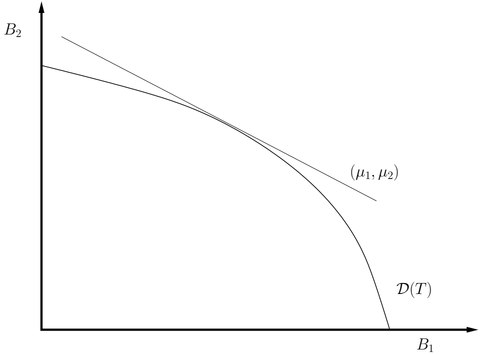
$$\mathcal{L} = \mu_1 \sum_{i=1}^{K+1} r_{1i}\ell_i + \mu_2 \sum_{i=1}^{K+1} r_{2i}\ell_i - \sum_{k=1}^{K+1} \lambda_k \left(\sum_{i=1}^{k} g(r_{1i}, r_{2i})\ell_i - \sum_{i=0}^{k-1} E_i \right) + \sum_{i=1}^{K+1} \gamma_{1i} r_{1i} + \sum_{i=1}^{K+1} \gamma_{2i} r_{2i}$$

- Total power in terms of Lagrange multipliers

$$P_i = \max \left\{ \frac{\mu_1 + \gamma_{1i}}{\sum_{k=i}^{K+1} \lambda_k} - 1, \frac{\mu_2 + \gamma_{2i}}{\sum_{k=i}^{K+1} \lambda_k} - \sigma^2 \right\}$$

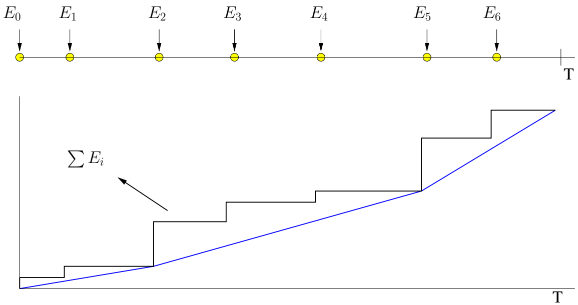
A Structural Property of the Optimal Policy

- Optimal total transmit power, $\{g(r_{1i}^*, r_{2i}^*)\}_{i=1}^{K+1}$, is independent of μ_1, μ_2.
- In particular, it is the same as the optimal single-user transmit power.



Single User Optimal Policy

- Single user optimal policy is found by calculating the tightest curve below the energy arrival curve:



- Slope of the curve is the allocated power
- Power is monotonically increasing

Full Structure of an Optimal Policy

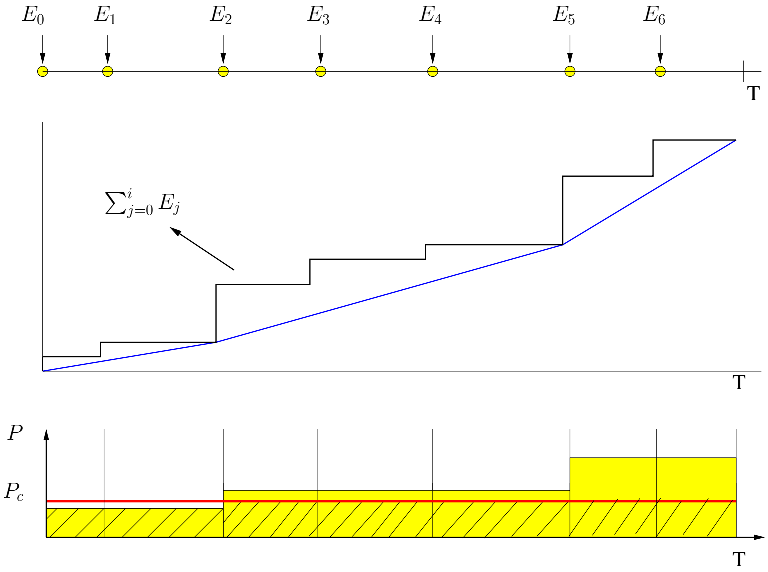
- Total transmit power is the same as the single-user case.
- The power shares follow a cut-off structure:
- Cut-off level P_c

$$P_c = \frac{\mu - 1}{\sigma^2 - \mu}$$

where $\mu = \frac{\mu_2}{\mu_1}$ and $1 < \mu < \sigma^2$.

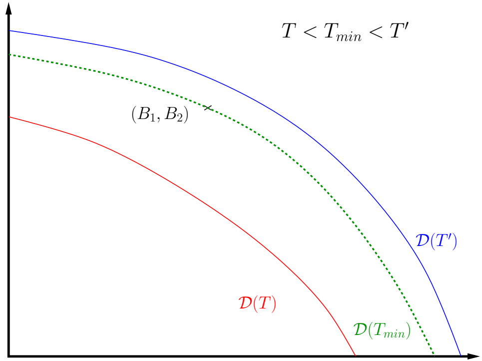
- If below P_c, then, only transmit to the stronger user
- Otherwise, stronger user's power share is P_c.
- Extreme cases:
  - If $\mu \leq 1$, only the stronger user's data is transmitted
  - If $\mu \geq \sigma^2$, only the weaker user's data is transmitted

The Structure of an Optimal Policy



Back to the Transmission Completion Time Minimization Problem

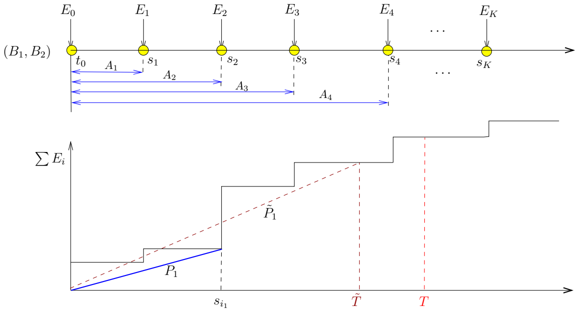
- (B_1, B_2) and $\{E_i\}$ are given
- Find the minimum time to transmit (B_1, B_2) subject to energy causality.
- (B_1, B_2) point must lie on the boundary of $\mathcal{D}(T_{min})$:



- Use derived structure of the optimal policy
- Transmissions for strong and weak users must end at the same time.

Algorithm to Find the Optimal Policy

- Find P_1: the power level allocated at the first epoch



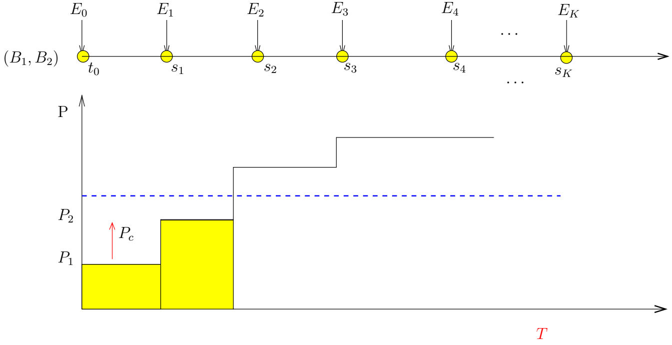
- Set $P_c = P_1$ and calculate

$$T = \frac{B_1}{\frac{1}{2}\log(1 + P_c)}$$

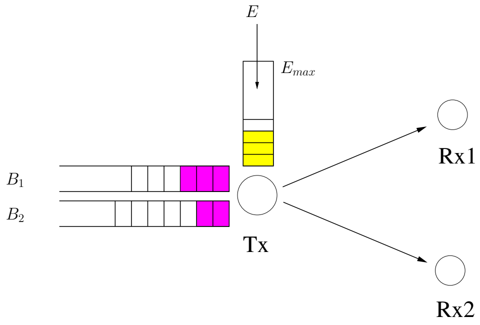
- Calculate $D_2(T, P_c)$: bits sent for weaker user by T treating P_c as noise.

Algorithm to Find the Optimal Policy

- If $D_2(T, P_c) > B_2$, decrease P_c.
- Otherwise find P_2: the next allocated power level. Repeat the procedure
- Once $D_2(T, P_c) = B_2$, stop.



Broadcast Channel with Finite E_{max}

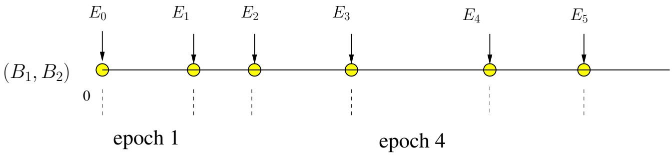


- (B_1, B_2) bits to be sent and battery capacity $E_{max} < \infty$
- AWGN broadcast channel:

$$Y_1 = X + N_1, \quad Y_2 = X + N_2$$

- $N_1 \sim \mathcal{N}(0, 1)$ and $N_2 \sim \mathcal{N}(0, \sigma^2)$ with $\sigma^2 > 1$
- 1st user stronger and 2nd user weaker

Broadcast Channel with Finite E_{max}



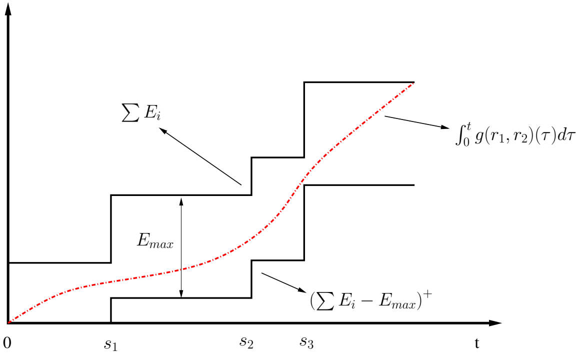
- Incoming energies are smaller than E_{max}: $E_i \leq E_{max}$
- Energy causality constraints: energy that has not arrived cannot be used

$$\int_0^{t_i^e} g(r_1, r_2)(u)du \leq \sum_{j=0}^{i-1} E_j, \quad \forall i$$

- No-energy-overflow condition: energy overflow (wasting) is suboptimal

$$\sum_{j=0}^{h(t)} E_j - \int_0^{t} g(r_1, r_2)(u)du \leq E_{max}, \quad \forall t$$

Constraints on the Power Policy



- Energy causality constraints: energy that has not arrived cannot be used

$$\int_0^{t_i^e} g(r_1, r_2)(u)du \leq \sum_{j=0}^{i-1} E_j, \quad \forall i$$

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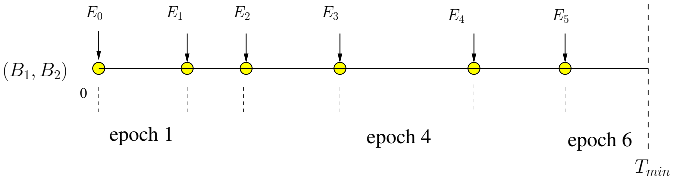
$$\sum_{j=0}^{h(t)} E_j - \int_0^t g(r_1, r_2)(u)du \leq E_{max}, \quad \forall t$$

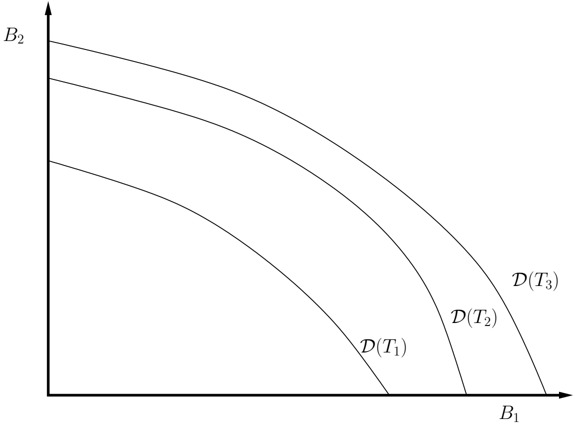
Problem Formulation



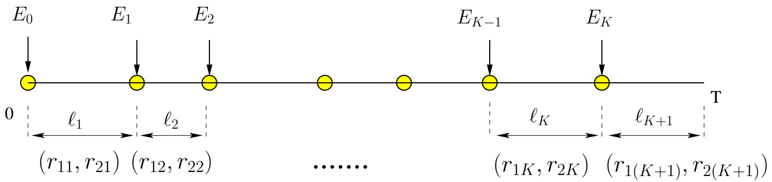
- Minimize transmission completion time of (B_1, B_2) bits.
- By adapting the transmission to the energy arrivals.
- Subject to energy causality and finite battery constraints

Dual Problem: Finding the Maximum Departure Region

- $\mathcal{D}(T)$: union of (B_1, B_2) pairs achievable by some rate allocation policy that satisfies the energy causality and no-energy-overflow constraints.



Dual Problem: Maximum Departure Region



- The transmission rates, and hence the transmission power, remain constant between energy harvests in any optimal policy
- The energy causality constraint reduces to constraints on (r_{1i}, r_{2i}):

$$\sum_{i=1}^{k} g(r_{1i}, r_{2i})\ell_i \leq \sum_{i=0}^{k-1} E_i, \qquad k = 1, \ldots, K+1$$

- The no-energy-overflow condition:

$$\sum_{i=0}^{k} E_i - \sum_{i=1}^{k} g(r_{1i}, r_{2i})\ell_i \leq E_{max}, \qquad k = 1, \ldots, K$$

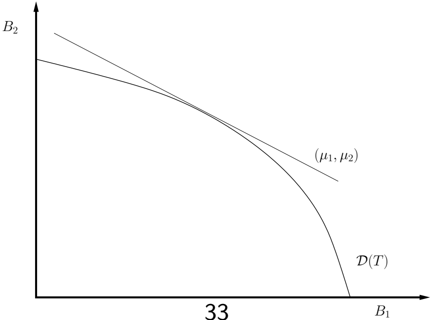
Dual Problem: Maximum Departure Region

- $\mathcal{D}(T)$ is a strictly convex region.
- Characterize $\mathcal{D}(T)$ by solving optimization problems for all $\mu_1, \mu_2 \geq 0$:

$$\max_{\mathbf{r}_1, \mathbf{r}_2} \quad \mu_1 \sum_{i=1}^{K+1} r_{1i}\ell_i + \mu_2 \sum_{i=1}^{K+1} r_{2i}\ell_i$$

$$\text{s.t.} \quad \sum_{i=1}^{k} g(r_{1i}, r_{2i})\ell_i \leq \sum_{i=0}^{k-1} E_i, \ 1 \leq k \leq K+1$$

$$\sum_{i=0}^{k} E_i - \sum_{i=1}^{k} g(r_{1i}, r_{2i})\ell_i \leq E_{max}, \ 1 \leq k \leq K$$



Dual Problem: Finding the Maximum Departure Region

- The Lagrangian function

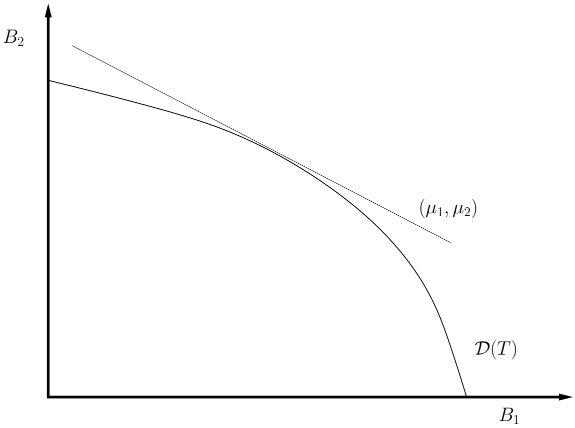
$$\mathcal{L} = \mu_1 \sum_{i=1}^{K+1} r_{1i}\ell_i + \mu_2 \sum_{i=1}^{K+1} r_{2i}\ell_i - \sum_{k=1}^{K+1} \lambda_k \left(\sum_{i=1}^{k} g(r_{1i}, r_{2i})\ell_i - \sum_{i=0}^{k-1} E_i \right) - \sum_{k=1}^{K} \eta_k \left(\sum_{i=0}^{k} E_i - \sum_{i=1}^{k} g(r_{1i}, r_{2i})\ell_i - E_{max} \right) + \sum_{i=1}^{K+1} \gamma_{1i} r_{1i} + \sum_{i=1}^{K+1} \gamma_{2i} r_{2i}$$

- Total power in terms of Lagrange multipliers

$$P_i = \max \left\{ \frac{\mu_1}{\left(\sum_{k=i}^{K+1} \lambda_k - \sum_{k=i}^{K} \eta_k \right)} - 1, \frac{\mu_2}{\left(\sum_{k=i}^{K+1} \lambda_k - \sum_{k=i}^{K} \eta_k \right)} - \sigma^2 \right\}$$

A Structural Property of the Optimal Policy

- Optimal total transmit power, $\{g(r_{1i}^*, r_{2i}^*)\}_{i=1}^{K+1}$, is independent of μ_1, μ_2.
- In particular, it is the same as the optimal single-user transmit power.



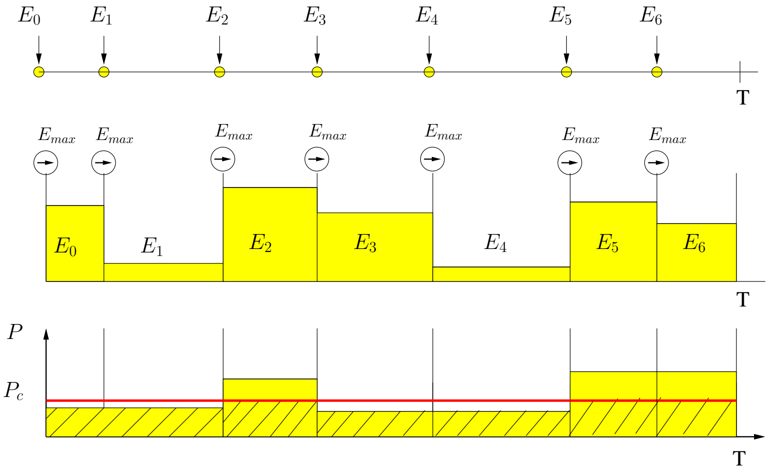
Full Structure of an Optimal Policy

- Total transmit power is the same as the single-user case.
- The power shares follow a cut-off structure:
- Cut-off level P_c

$$P_c = \frac{\mu - 1}{\sigma^2 - \mu}$$

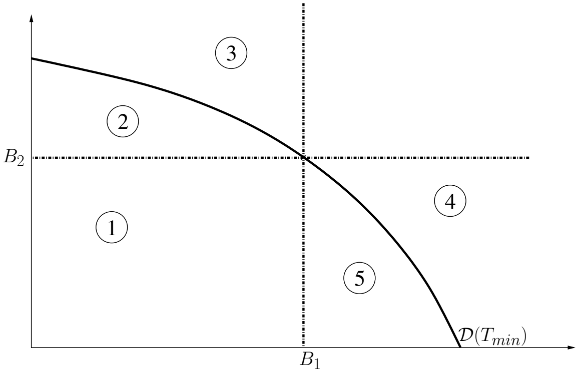
  where $\mu = \frac{\mu_2}{\mu_1}$ and $1 < \mu < \sigma^2$.
- If below P_c, then, only the stronger user
- Otherwise, stronger user's power share is P_c.
- Extreme cases:
  - If $\mu \leq 1$, only the stronger user's data is transmitted
  - If $\mu \geq \sigma^2$, only the weaker user's data is transmitted
- **Powers are not monotonically increasing due to finite E_{max}.**
- **Need to devise a new algorithm.**

The Structure of an Optimal Policy



Back to the Transmission Completion Time Minimization Problem

- (B_1, B_2) and $\{E_i\}$ are given
- Find the minimum time to transmit (B_1, B_2) subject to
  - energy causality
  - no-energy-overflow
- We divide the positive quadrant in 5 regions as follows

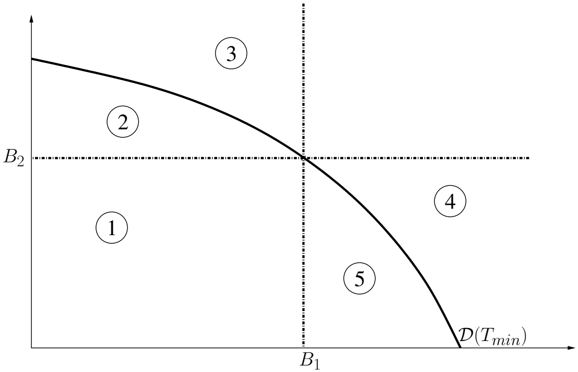


Algorithm to Find the Optimal Policy

- Start with an arbitrary P_c and calculate

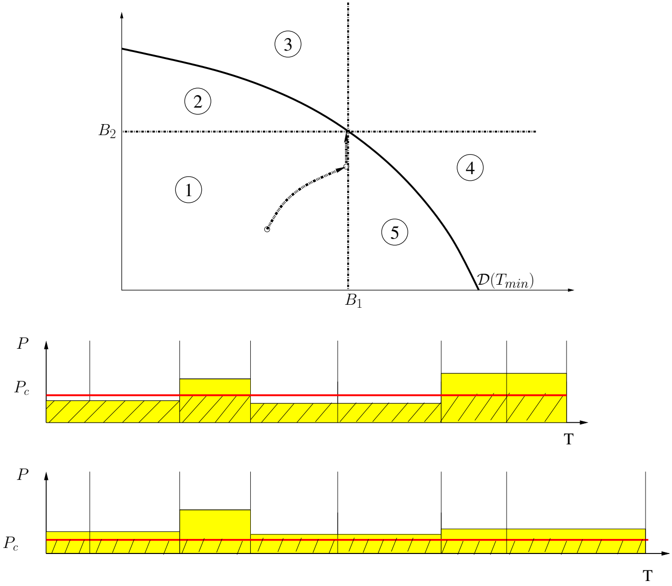
$$T = \frac{B_1}{\frac{1}{2}\log(1 + P_c)}$$

- Assume, WLOG, we start in ①. Decrease P_c and recalculate T
- There are two possible cases.



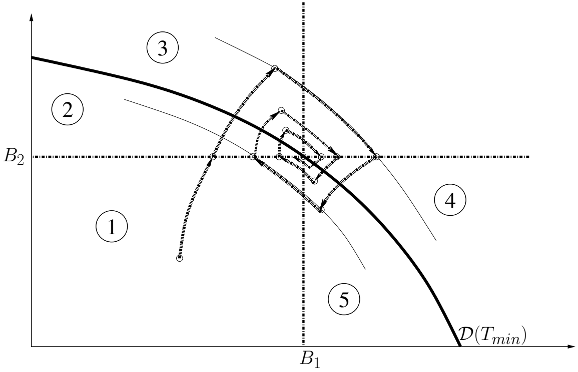
Algorithm to Find the Optimal Policy

- In case B_1 is achieved, iterations on P_c is sufficient.



Algorithm to Find the Optimal Policy

- Otherwise, iterate P_c and T separately.
- Suitable step size updates exist due to continuity.



Conclusions for the Broadcasting Scenario

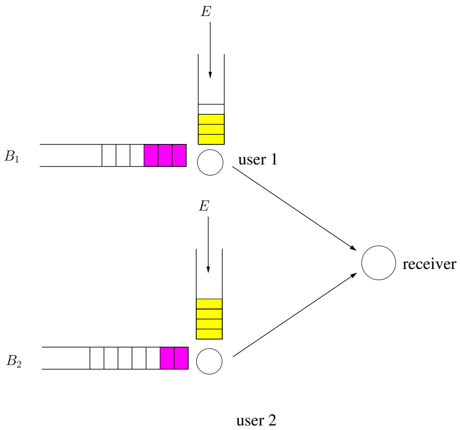
- Energy harvesting transmitter with infinite and finite capacity battery
- Transmission completion time minimization in a broadcast setting
- The dual problem: maximization of the departure region.
- Obtain the structure such as
  - the monotonicity of the transmit power
  - the cut-off power property
- Use structural properties to devise an algorithm

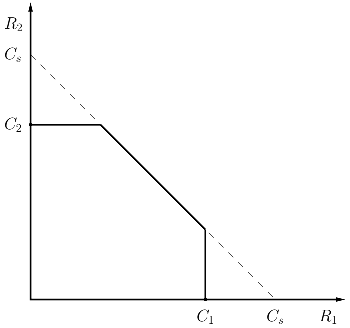
Optimal Packet Scheduling: Multiple Access Channel

- AWGN MAC channel $Y = X_1 + X_2 + Z$, $Z \sim N(0, 1)$.
- The capacity region is a pentagon denoted as $\mathcal{C}(P_1, P_2)$:

$$R_1 \leq f(P_1), \quad R_2 \leq f(P_2), \quad R_1 + R_2 \leq f(P_1 + P_2)$$

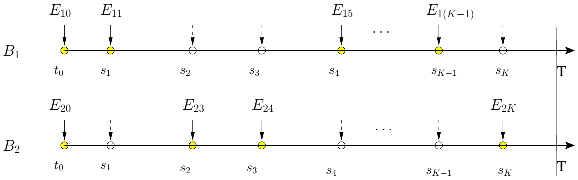
where $f(p) = \frac{1}{2}\log(1 + p)$.



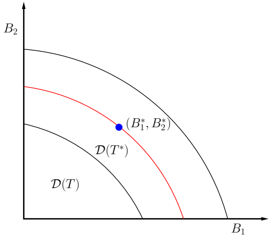


Problem Formulation

- Given (B_1, B_2), minimize transmission completion time, T.



- Start with the dual problem:



Characterizing $\mathcal{D}(T)$

- Transmission rate remains constant between energy harvests.
- For any feasible transmit power sequences $\mathbf{p}_1$, $\mathbf{p}_2$ over $[0, T)$, the departure region is a pentagon defined as

$$B_1 \leq \sum_{n=1}^{N} f(p_{1n}) l_n$$

$$B_2 \leq \sum_{n=1}^{N} f(p_{2n}) l_n$$

$$B_1 + B_2 \leq \sum_{n=1}^{N} f(p_{1n} + g_{2n}) l_n$$

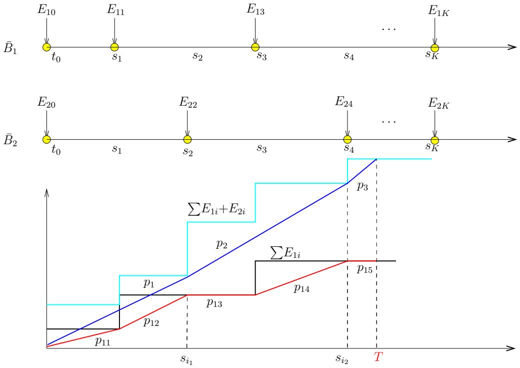
- $\mathcal{D}(T)$ is a union of (B_1, B_2) and convex.
- For any $T' > T$, $\mathcal{D}(T)$ is strictly inside $\mathcal{D}(T')$.
- The boundary points maximize $\mu_1 B_1 + \mu_2 B_2$ for some $\mu_1, \mu_2 \geq 0$.

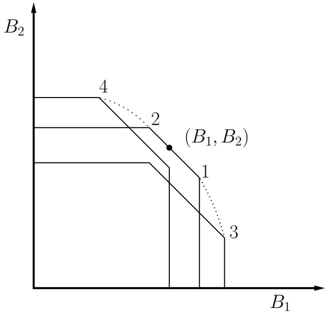
$\mu_1 = \mu_2$

- The problem becomes $\max_{\mathbf{p}_1, \mathbf{p}_2} B_1 + B_2$.
- Sum of powers has same "majorization" property as in single-user.
- Merge energy arrivals of the users, get the optimal sum powers, $p_1, \ldots, p_n$
- Each feasible sequence of p_{1n} and p_{2n} gives a pentagon.
- Union of them is a larger pentagon: dominant faces on the same line.
- Need to identify the boundary of this larger pentagon.

Achieving Corner Points of the Boundary

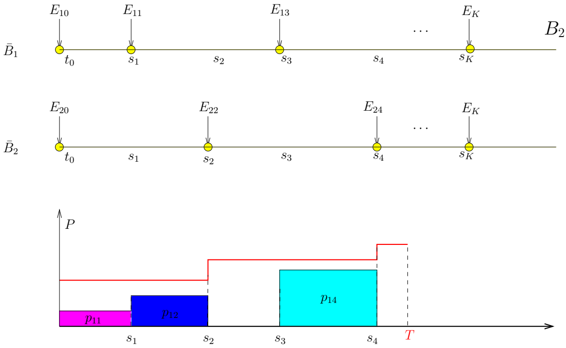
- Maximize B_1 s.t. $B_1 + B_2$ is maximized at the same time $\Rightarrow$ point 1.
  - Equalize the transmit powers of the first user as much as possible
  - Additionally: both users' energy constraints are tight if sum power changes.

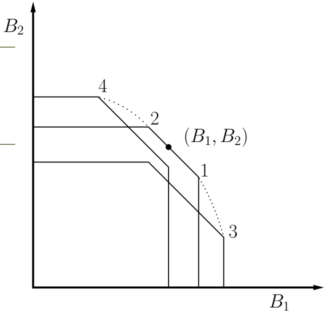




$\mu_1 = 0$ or $\mu_2 = 0$

- Maximize B_1 or B_2 $\Rightarrow$ a single-user scenario.
- Given p_{1n}^*, maximize B_2: backward/directional waterfilling with base level p_{1n}^* $\Rightarrow$ point 3.





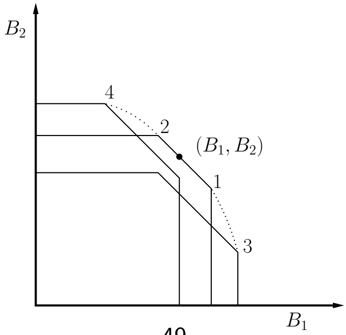
$\mu_1, \mu_2 > 0$

- Each boundary point corresponds to a corner point on some pentagon.
- $\mu_1 > \mu_2 \Rightarrow$ achieving points between point 1 and point 3:

$$\max_{\mathbf{p}_1, \mathbf{p}_2} \quad (\mu_1 - \mu_2) \sum_n f(p_{1n}) l_n + \mu_2 \sum_n f(p_{1n} + p_{2n}) l_n$$

$$\text{s.t.} \quad \sum_{n=1}^{j} p_{1n} l_n \leq \sum_{n=0}^{j-1} E_{1n}, \quad \forall j : 0 < j \leq N$$

$$\sum_{n=1}^{j} p_{2n} l_n \leq \sum_{n=0}^{j-1} E_{2n}, \quad \forall j : 0 < j \leq N$$



Generalized Iterative Backward Waterfilling

- Solve the problem via generalized iterative backward waterfilling:
- Given $\mathbf{p}_2^*$, solve for $\mathbf{p}_1$:

$$\begin{aligned}\max_{\mathbf{p}_1} \quad & (\mu_1 - \mu_2) \sum_{n=1}^{N} f(p_{1n}) l_n + \mu_2 \sum_{n=1}^{N} f(p_{1n} + p_{2n}^*) l_n \\ \text{s.t.} \quad & \sum_{n=1}^{j} p_{1n} l_n \leq \sum_{n=0}^{j-1} E_{1n}, \quad 0 < j \leq N\end{aligned}$$

- Once $\mathbf{p}_1^*$ is obtained, we do a backward waterfilling for the second user.
- We perform the optimization for both users in an alternating way.
- The iterative algorithm converges to the global optimal solution.

Minimizing T for a Given (B_1, B_2)

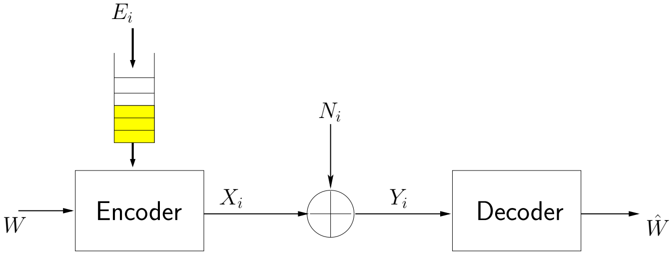
- Need to obtain optimal power policy and rate policy at the same time.
- First calculate $\mathcal{D}(t)$ for $t = s_1, s_2, \ldots, s_K$.
- Locate (B_1, B_2) on the maximum departure region.
- If (B_1, B_2) is outside $\mathcal{D}(s_i)$ but inside $\mathcal{D}(s_{i+1})$ for some s_i, then, $s_i < T < s_{i+1}$.
- Solve this optimization problem in two steps.
  1. Find a power policy to minimize T s.t (B_1, B_2) is within $\mathcal{D}(T)$, convex optimization.
  2. Find a feasible rate allocation within the capacity regions, linear programming.
- Complexity is reduced: the number of unknown variables is about half.

Conclusions for the Multiple Access Scenario

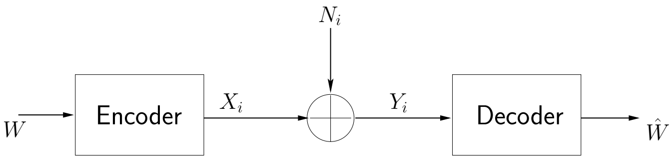
- Energy harvesting transmitters sending messages to a single access point.
- Transmission completion time minimization in a multiple access scenario.
- The dual problem: maximization of the departure region.
- Obtain the structure using generalized iterative waterfilling.

Information Theoretic Analysis of Single-User Energy Harvesting Communication

- Energy is not available up front, arrives randomly in time.
- Energy can be saved in the battery for future use.
- Transmission is interrupted if battery energy is run out.
- What is the highest achievable rate?



Classical AWGN Channel



- AWGN channel:

$$Y = X + N$$

- Average power constraint:

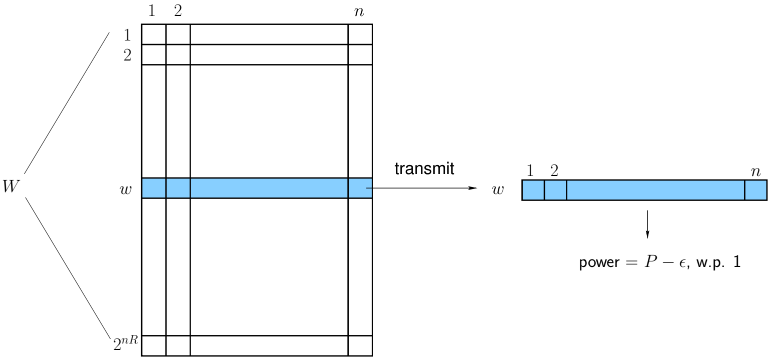
$$\frac{1}{n}\sum_{i=1}^{n} X_i^2 \leq P$$

- AWGN capacity formula with an average power constraint P:

$$C = \frac{1}{2}\log_2(1+P)$$

Achievability in the Classical AWGN Channel

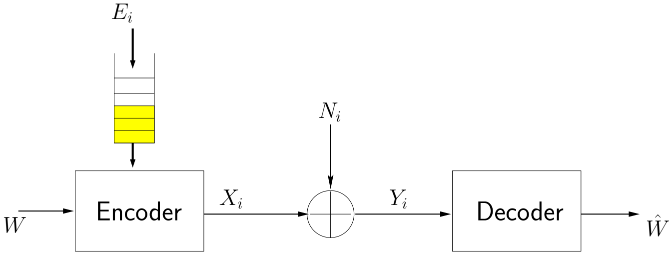
- Generate codebook with i.i.d. Gaussians with zero-mean, variance $P - \epsilon$.



- By SLLN, codewords so generated obey the power constraint w.p. 1,

$$\frac{1}{n}\sum_{i=1}^{n} X_i^2 \rightarrow P - \epsilon, \quad \text{w.p. } 1$$

Energy Harvesting AWGN Channel Model



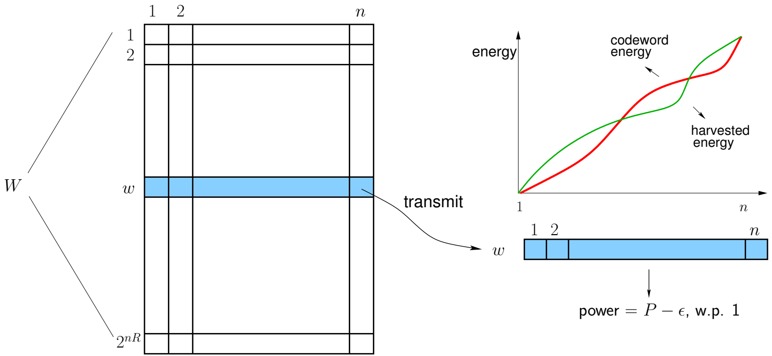
- Code symbols are constrained to the battery energy at each channel use:

$$\sum_{i=1}^{k} X_i^2 \leq \sum_{i=1}^{k} E_i, \qquad k = 1, 2, \dots, n$$

- Energy harvesting: n constraints.
- Average power constraint: a single constraint, $k = n$.
- $E[E_i] = P$: average recharge rate.
- Battery storage capacity is infinite.

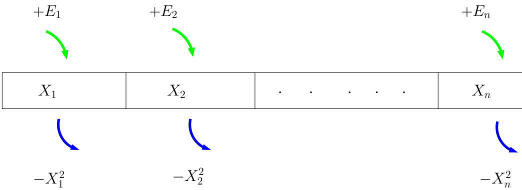
Achievability in the Energy Harvesting AWGN Channel: Major Concerns

- If we generate an i.i.d. Gaussian codebook with zero-mean, variance $P - \epsilon$.



- How do we design the codebook such that:
  - all codewords are energy-feasible for all channel uses.
- Do we need energy arrival state information:
  - causally, non-causally or not at all, at the transmitter and/or receiver.

The Capacity with Energy Harvesting



- Upper bound: Average power constrained AWGN capacity:

$$C \leq \frac{1}{2} \log (1 + P)$$

- This is an upper bound because:
  - Average power constraint imposes a single constraint:

$$\frac{1}{n} \sum_{i=1}^{n} X_i^2 \leq \frac{1}{n} \sum_{i=1}^{n} E_i \to P \quad \text{(by SLLN)}$$

  - While energy harvesting imposes n constraints:

$$\sum_{i=1}^{n} X_i^2 \leq \sum_{i=1}^{n} E_i, \qquad k = 1, \ldots, n$$

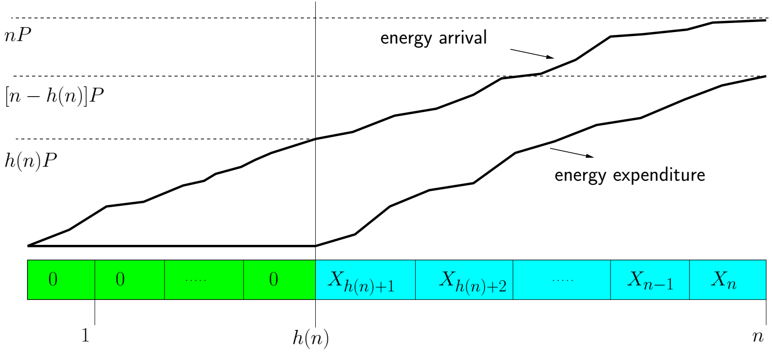
- Our contribution: This bound can be achieved.

Achieving the Capacity

- Probability of error $P_e = \Pr(E_1 \cup E_2)$:
  - E_1: decoding error
  - E_2: violation of energy constraints
- A first approach: Design a codebook that obeys all n energy constraints.
- **An alternative approach:**
  Design a simple codebook and show the insignificance of energy shortages.
- We will follow the second approach.
- Two achievable schemes:
  1) Save-and-Transmit scheme
  2) Best-Effort-Transmit scheme

Save-and-Transmit Scheme

- Save energy in the first $h(n)$ channel uses, do not transmit.
- In the remaining $n - h(n)$ channel uses, send i.i.d. Gaussian signals.
- Saving period of $h(n)$ channel uses makes the remaining symbols feasible.
- Choose $h(n) \in o(n)$ so that saving incurs no loss in rate, i.e., $h(n)/n \to 0$.



Save-and-Transmit Scheme

- When $E[X_i^2] = P - \epsilon$,
  - $h(n) \in o(n)$ guarantees no loss in rate.
  - $h(n) \to \infty$ guarantees sufficient energy storage.
  - An $h(n)$ that works is $h(n) = \log(n)$.
- When $E[X_i^2] = P$,
  - Additionally, we need $E[e^{E_i^\gamma}] < \infty$ for some $0 < \gamma < 1$.
  - Then, we need $h(n) > n^{1/\alpha}(\log(n))^{1/\gamma}$, for some $1 < \alpha \leq 2$.
  - An $h(n)$ that works is $h(n) = \sqrt{n}(\log(n))^2$.
- Hence, for $E[X_i^2] \leq P$, there exists an $h(n)$ such that achievable rate:

$$
\begin{aligned}
\frac{1}{n} I(X^n; Y^n) &= \frac{1}{n} \sum_{j=h(n)}^{n} I(X_j; Y_j) \\
&= \frac{n - h(n)}{2n} \log(1 + P) \\
&\to \frac{1}{2} \log(1 + P)
\end{aligned}
$$

Best-Effort-Transmit Scheme

- X_i: i.i.d. Gaussian.
- $S(i)$: battery energy in the ith channel use.
- If $S(i) \geq X_i^2$, put X_i otherwise put 0 to the channel.
- Mismatch between the codewords and the transmitted symbols.
- Battery energy updates:

$$S(i+1) = S(i) + E_i - X_i^2 \mathbf{1}(S(i) \geq X_i^2)$$

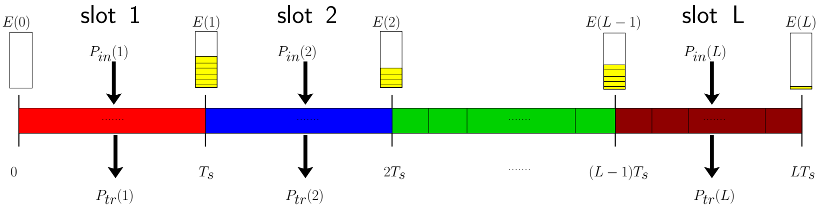
- Since $E[X_i^2] = P - \epsilon$, only finitely many symbols are infeasible.
- Finitely many mismatches. Inconsequential for joint typical decoding.
- Rates $< \frac{1}{2}\log(1+P)$ are achievable.

Conclusions So Far

- AWGN capacity with i.i.d. recharge process is equal to the capacity with average power constrained to average recharge rate.
- Two-achievable schemes:
  - Save-and-Transmit scheme
  - Best-Effort-Transmit scheme
- **Next:**
  - Energy arrival rate changes in large time slots.

The Large Time Scale Case

- Average recharge rate changes in large time slots.
- We consider L time slots.



Optimizing the Average Throughput

- We optimize average throughput over L slots subject to energy causality:

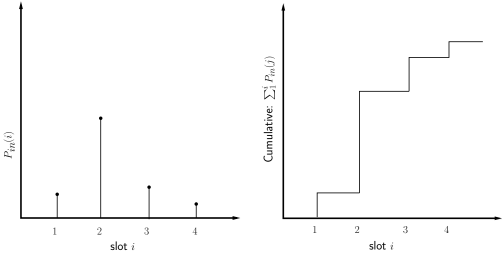
$$\max \quad \frac{1}{L}\sum_{i=1}^{L}\frac{1}{2}\log\left(1+P_{tr}(i)\right)$$

$$\text{s.t} \quad \sum_{i=1}^{\ell}P_{tr}(i) \leq \sum_{i=1}^{\ell}P_{in}(i), \qquad \ell = 1,2,\ldots,L$$

- Objective function is Schur-concave.
- The solution: **most majorized** feasible power vector.

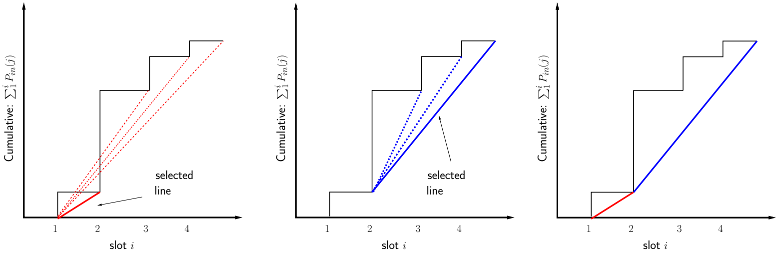
Optimum Power Control Algorithm

- Make the transmit power as constant as possible.
- Select the feasible line with the minimum slope.



Deciding P_1

Deciding P_2-P_4



Numerical Example

- Given the input power sequence: $P_{in}(1), P_{in}(2), \ldots, P_{in}(L)$.
- Use the developed optimum power control algorithm.
- Lower bound: no power control.

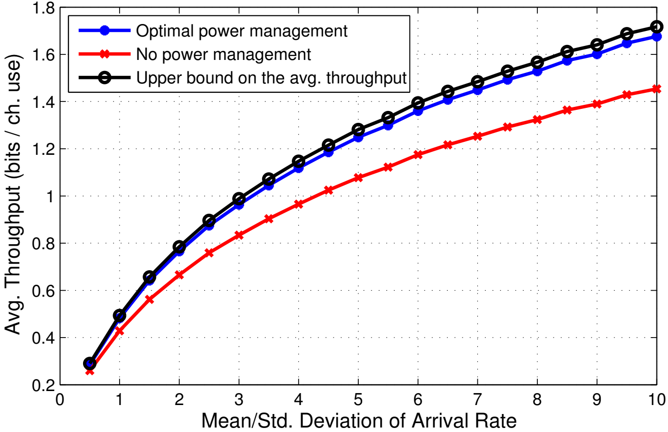
$$T_{lb} = \frac{1}{L} \sum_{i=1}^{L} \frac{1}{2} \log\left(1 + P_{in}(i)\right)$$

- Upper bound: all power is available at time zero.

$$T_{ub} = \frac{1}{2} \log\left(1 + \frac{1}{L} \sum_{i=1}^{L} P_{in}(i)\right)$$

Numerical Example: $L = 20$ Slots

- $\{P_i\}_{i=1}^{L}$ are i.i.d. exponential random variables.



Conclusions

- AWGN capacity with i.i.d. recharge process is equal to the capacity with average power constrained to average recharge rate.
- Two-achievable schemes:
  - Save-and-Transmit scheme
  - Best-Effort-Transmit scheme
- Optimal power control in a large scale time constrained system.
  - Optimal power vector: most majorized feasible vector subject to causality.

References

(1) J. Yang, O. Ozel and S. Ulukus, Optimal Packet Scheduling in a Broadcast Channel with an Energy Harvesting Transmitter, IEEE International Conference on Communications (ICC), Kyoto, Japan, June 2011

(2) J. Yang and S. Ulukus, Optimal Packet Scheduling in a Multiple Access Channel with Rechargeable Nodes, IEEE International Conference on Communications(ICC), Kyoto, Japan, June 2011.

(3) O. Ozel, J. Yang and S. Ulukus, Broadcasting with a Battery Limited Energy Harvesting Rechargeable Transmitter, 9th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt), Princeton, NJ, May 2011.

(4) O. Ozel and S. Ulukus, Information-Theoretic Analysis of an Energy Harvesting Communication System, International Workshop on Green Wireless (W-GREEN) at IEEE Personal, Indoor and Mobile Radio Conference, Istanbul, Turkey, September 2010.