Morning Session
Capacity-based Power Control

Şennur Ulukuş
Department of Electrical and Computer Engineering
University of Maryland

So Far, We Learned...

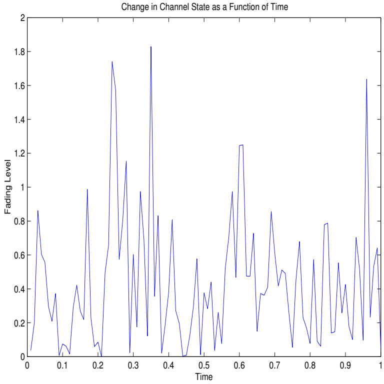
- Power control with SIR-based QoS guarantees
  - Suitable for delay-intolerant services, e.g., voice
  - Satisfy all SIR constraints with minimum transmit power
  - Leading to energy-efficient communications
- Power control for capacity
  - Suitable for delay-tolerant services, e.g., data
  - Maximize rate with a given average transmit power
  - Equivalently, support a given rate with minimum power
  - Leading to energy-efficient communications

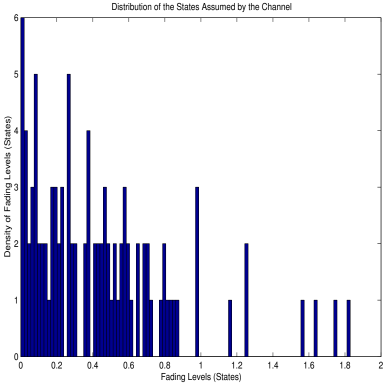
Capacity-Based Power Control

- Fading: random fluctuations in channel gains.
- Perfect CSI at both the transmitter and the receiver
- Maximize ergodic capacity subject to average power constraints
- Main operational difference:
  - QoS based power control: compensate for channel fading
  - Capacity-based power control: exploit the channel fading

Channel Fading

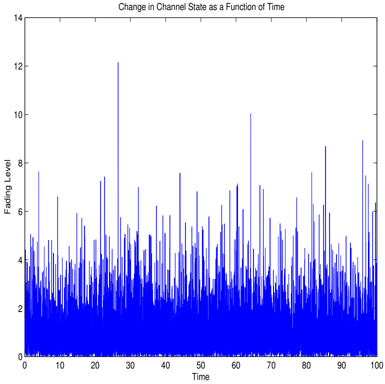
$$r = \sqrt{h}x + n$$

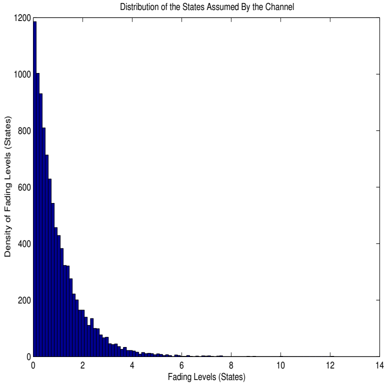




Channel Fading

$$r = \sqrt{h}x + n$$





Single-User Fading Channel (Goldsmith-Varaiya'94)

- Channel capacity for single user

$$
\begin{aligned}
C &= \frac{1}{2}\log\left(1 + SNR\right) \\
&= \frac{1}{2}\log\left(1 + \frac{p}{\sigma^2}\right)
\end{aligned}
$$

- In the presence of fading, the capacity for a fixed channel state h,

$$
C(h) = \frac{1}{2}\log\left(1 + \frac{p(h)h}{\sigma^2}\right)
$$

- Ergodic (expected) capacity under an average power constraint

$$
\begin{aligned}
\max_{p(h)} \quad & \mathbb{E}_h\left[\frac{1}{2}\log\left(1 + \frac{p(h)h}{\sigma^2}\right)\right] \\
\text{s.t.} \quad & \mathbb{E}_h\left[p(h)\right] \leq P \\
& p(h) \geq 0, \qquad \forall h
\end{aligned}
$$

Optimal Power Allocation using Waterfilling

- The Lagrangian function

$$\mathbb{E}_h\left[\log\left(1+\frac{p(h)h}{\sigma^2}\right)\right]-\lambda\left(\mathbb{E}_h\left[p(h)\right]-P\right)+\int\mu(h)f(h)dh$$

- Optimality conditions

$$\frac{h}{p^*(h)h+\sigma^2}+\frac{\mu(h)}{f(h)}=\lambda$$

  where $f(h)$ is the PDF of h.
- The complementary slackness conditions $\mu(h)p^*(h)=0$ for all h.
- If $p^*(h)>0$, we get

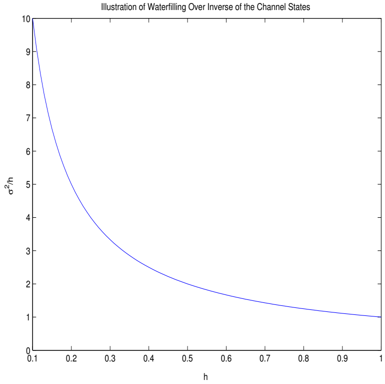
$$p^*(h)=\frac{1}{\lambda}-\frac{\sigma^2}{h}$$

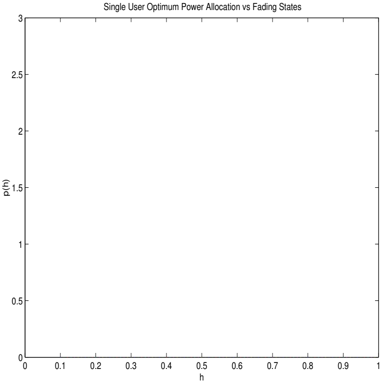
- Otherwise, $p^*(h)=0$.

Optimal Power Allocation using Waterfilling

- Optimal power allocation: waterfilling of power over time

$$p(h) = \left(\frac{1}{\lambda} - \frac{\sigma^2}{h}\right)^+$$

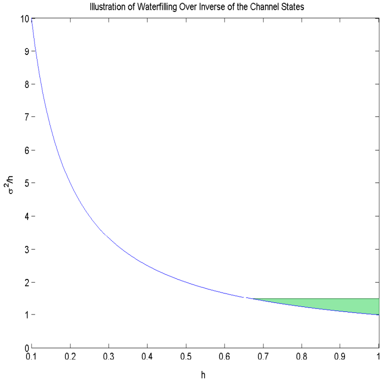


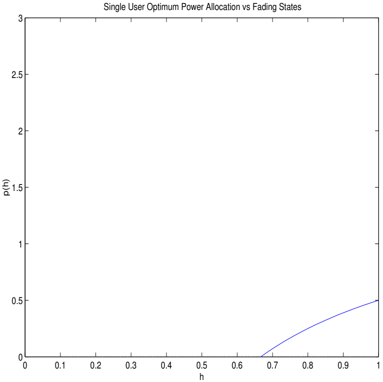


Optimal Power Allocation using Waterfilling

- Optimal power allocation: waterfilling of power over time

$$p(h) = \left(\frac{1}{\lambda} - \frac{\sigma^2}{h}\right)^+$$

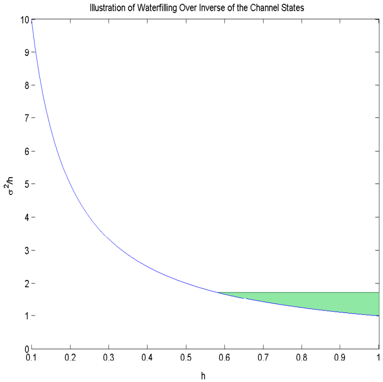


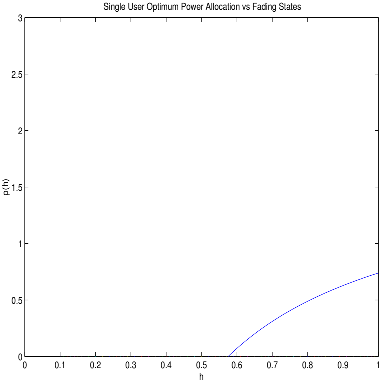


Optimal Power Allocation using Waterfilling

- Optimal power allocation: waterfilling of power over time

$$p(h) = \left(\frac{1}{\lambda} - \frac{\sigma^2}{h}\right)^+$$

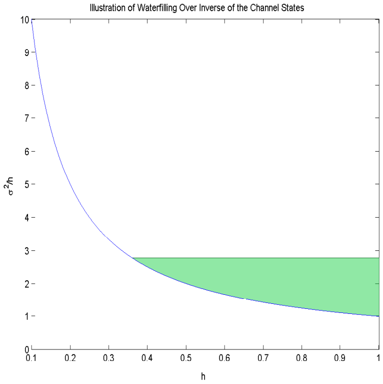


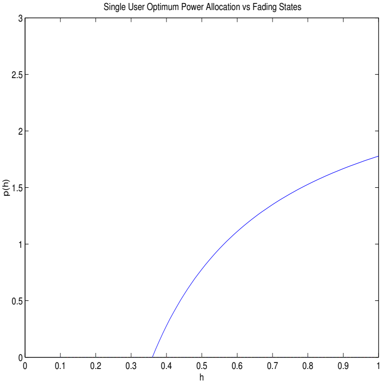


Optimal Power Allocation using Waterfilling

- Optimal power allocation: waterfilling of power over time

$$p(h) = \left(\frac{1}{\lambda} - \frac{\sigma^2}{h}\right)^+$$

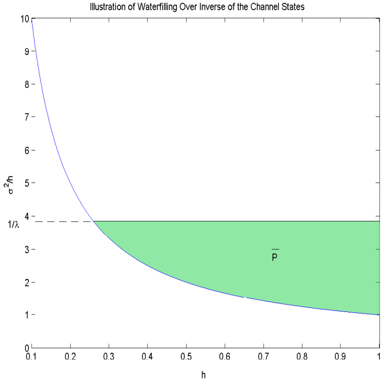


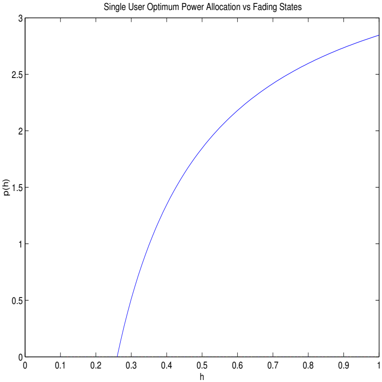


Optimal Power Allocation using Waterfilling

- Optimal power allocation: waterfilling of power over time

$$p(h) = \left(\frac{1}{\lambda} - \frac{\sigma^2}{h}\right)^+$$





Differences Between QoS-Based and Capacity-Based Power Control

- Single-user system

$$y = \sqrt{h}x + n$$

- SIR-based

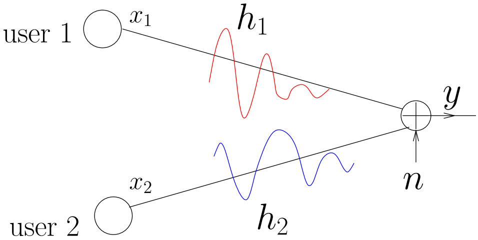
$$\frac{p(h)h}{\sigma^2} \geq \gamma \qquad \Leftrightarrow \qquad p(h) = \frac{\gamma\sigma^2}{h}$$

- Channel inversion; more power if bad channel, less if good channel
- Compensate for channel fading via power control
- Capacity-based

$$\max \mathbb{E}_h \left[\frac{1}{2} \log \left(1 + \frac{p(h)h}{\sigma^2} \right) \right] \qquad \Rightarrow \qquad p(h) = \left(\frac{1}{\lambda} - \frac{\sigma^2}{h} \right)^+$$

- Waterfilling; more power if good channel, less if bad channel
- Exploit variations, opportunistic transmission

Fading Gaussian Multiple Access Channel



- Channel model

$$y = \sqrt{h_1} x_1 + \sqrt{h_2} x_2 + n$$

- Simultaneously achievable ergodic rates for both users (R_1, R_2)
- Channel state vector $\mathbf{h} = (h_1, h_2)$
- Adapt powers as functions of $\mathbf{h}$: $p_1(\mathbf{h})$ and $p_2(\mathbf{h})$

Fading Gaussian Multiple Access Channel (Tse-Hanly'98)

- Union of pentagons

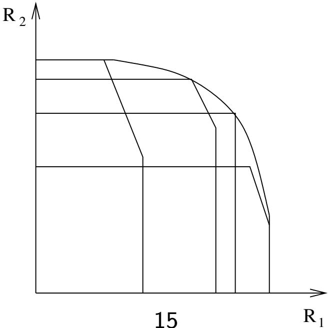
$$R_1 < E\left[\frac{1}{2}\log\left(1+\frac{p_1(\mathbf{h})h_1}{\sigma^2}\right)\right] \quad (\triangleq C_1)$$

$$R_2 < E\left[\frac{1}{2}\log\left(1+\frac{p_2(\mathbf{h})h_2}{\sigma^2}\right)\right] \quad (\triangleq C_2)$$

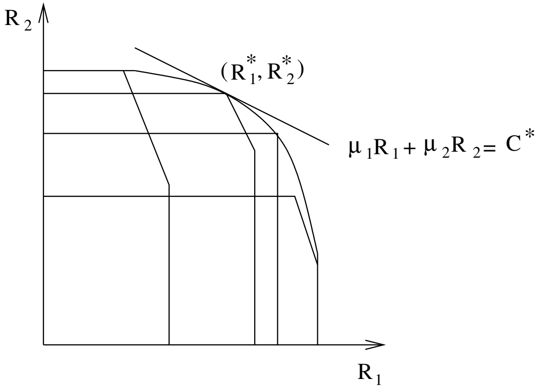
$$R_1 + R_2 < E\left[\frac{1}{2}\log\left(1+\frac{p_1(\mathbf{h})h_1 + p_2(\mathbf{h})h_2}{\sigma^2}\right)\right] \quad (\triangleq C_s)$$

over all feasible power distributions

$$E\left[p_1(\mathbf{h})\right] \le P_1, \quad E\left[p_2(\mathbf{h})\right] \le P_2, \quad p_1(\mathbf{h}) \ge 0, \quad p_2(\mathbf{h}) \ge 0$$



Determining the Boundary of the Capacity Region



- Capacity region is a convex region.
- To determine the boundary, maximize $\mu_1 R_1 + \mu_2 R_2$ for all $\mu_1, \mu_2 \geq 0$.
- Any (R_1^*, R_2^*) on the boundary is a corner of one of the pentagons.
- If $\mu_2 > \mu_1$ then the upper corner; if $\mu_1 > \mu_2$ then the lower corner.

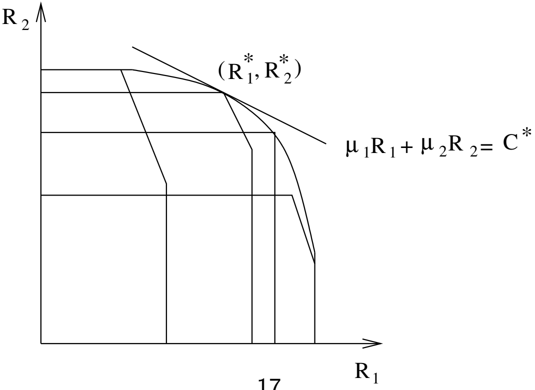
Achieving Arbitrary Rate Tuples on the Boundary

- For given μ_i, maximize $C_\mu \triangleq \mu_1 R_1 + \mu_2 R_2$ s.t. $E_{\mathbf{h}}[p_i(\mathbf{h})] \leq P_i$, $\mathbf{R} \in \mathcal{C}$.
- Wlog, $\mu_2 > \mu_1$. Given power policy, the optimum $\mathbf{R}$ is the upper corner.
- The coordinates of the upper corner are:

$$R_2 = C_2, \qquad R_1 = C_s - C_2$$

and

$$\begin{aligned} C_\mu &= \mu_1(C_s - C_2) + \mu_2 C_2 \\ &= (\mu_2 - \mu_1)C_2 + \mu_1 C_s \end{aligned}$$

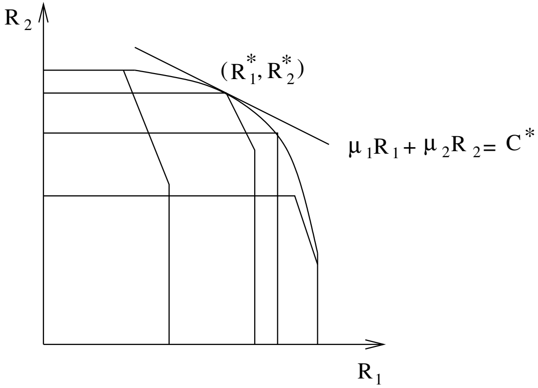


Achieving Arbitrary Rate Tuples on the Boundary

- Therefore, the optimum power allocation policy is the solution of:

$$\begin{aligned} \max_{\mathbf{p}(\mathbf{h})} \quad & \mathbb{E}_{\mathbf{h}}\left[(\mu_2-\mu_1)\log\left(1+\frac{p_2(\mathbf{h})h_2}{\sigma^2}\right)+\mu_1\log\left(1+\frac{p_1(\mathbf{h})h_1+p_2(\mathbf{h})h_2}{\sigma^2}\right)\right] \\ \text{s.t.} \quad & \mathbb{E}_{\mathbf{h}}[p_i(\mathbf{h})] \leq P_i, \qquad i=1,2 \\ & p_i(\mathbf{h}) \geq 0, \quad \forall\ \mathbf{h}, \quad i=1,2 \end{aligned}$$

- Objective function is concave, and constraint set is convex in powers.



Optimality Conditions

- $\mathbf{p}^*(\mathbf{h})$ achieves the global maximum of C_μ iff it satisfies the KKTs,

$$\frac{\mu_1 h_1}{h_1 p_1(\mathbf{h}) + h_2 p_2(\mathbf{h}) + \sigma^2} \leq \lambda_1, \quad \forall\, \mathbf{h}$$

$$\frac{\mu_1 h_2}{h_1 p_1(\mathbf{h}) + h_2 p_2(\mathbf{h}) + \sigma^2} + \frac{(\mu_2 - \mu_1) h_2}{h_2 p_2(\mathbf{h}) + \sigma^2} \leq \lambda_2, \quad \forall\, \mathbf{h}$$

  with equality at $\mathbf{h}$, if $p_1(\mathbf{h}) > 0$ and $p_2(\mathbf{h}) > 0$, respectively.
- Solution based on utilities in Tse-Hanly'98.
- Solve KKTs iteratively: generalized iterative waterfilling (Kaya-Ulukus)

Generalized Iterative Waterfilling

- Given $p_2(\mathbf{h})$, find $p_1(\mathbf{h})$ and λ_1 such that

$$\frac{\mu_1 h_1}{h_1 p_1(\mathbf{h}) + h_2 p_2(\mathbf{h}) + \sigma^2} \leq \lambda_1, \quad \forall\ \mathbf{h}$$

  with equality at $\mathbf{h}$, if $p_1(\mathbf{h}) > 0$.
- Waterfilling for user 1 given the power of user 2 contributing to noise

$$p_1(\mathbf{h}) = \left(\frac{1}{\lambda_1} - \frac{\sigma^2 + h_2 p_2(\mathbf{h})}{h_1}\right)^+$$

- Given $p_1(\mathbf{h})$, find $p_2(\mathbf{h})$ and λ_2.

$$\frac{\mu_1 h_2}{h_1 p_1(\mathbf{h}) + h_2 p_2(\mathbf{h}) + \sigma^2} + \frac{(\mu_2 - \mu_1) h_2}{h_2 p_2(\mathbf{h}) + \sigma^2} \leq \lambda_2, \quad \forall\ \mathbf{h}$$

  with equality at $\mathbf{h}$, if $p_2(\mathbf{h}) > 0$.
- A second order equation to solve.

Optimal Power Allocation via Generalized Iterative Waterfilling

- KKT conditions for the K-user case

$$\sum_{i=1}^{k} \frac{(\mu_i - \mu_{i-1})\, h_k}{\sum_{j=1}^{i-1} p_j(\mathbf{h}) h_j + p_k(\mathbf{h}) h_k + \sigma^2} \le \lambda_k, \quad \forall\, \mathbf{h}, k = 1, \cdots, K$$

  with equality at $\mathbf{h}$, if $p_k(\mathbf{h}) > 0$.

- **Generalized iterative waterfilling**
  - Given $p_j(\mathbf{h})$, $j < k$, find $p_k(\mathbf{h})$ and λ_k such that

$$\sum_{i=1}^{k} \frac{(\mu_i - \mu_{i-1})\, h_k}{\sum_{j=1}^{i-1} p_j(\mathbf{h}) h_j + p_k(\mathbf{h}) h_k + \sigma^2} \le \lambda_k, \quad \forall\, \mathbf{h}, k = 1, \cdots, K$$

    with equality at $\mathbf{h}$, if $p_k(\mathbf{h}) > 0$.

- One-user-at-a-time algorithm, converges to the optimum.
- **A Gauss-Seidel type of iteration.**

Special Case: Sum Capacity (Knopp-Humblet'95)

- Ergodic sum capacity ($\mu_1 = \mu_2 = 1$)

$$\max_{\mathbf{p}(\mathbf{h})} \quad \mathbb{E}_{\mathbf{h}} \left[\log \left(1 + \frac{p_1(\mathbf{h}) h_1 + p_2(\mathbf{h}) h_2}{\sigma^2} \right) \right]$$
$$\text{s.t.} \quad \mathbb{E}_{\mathbf{h}} \left[p_i(\mathbf{h}) \right] \leq P_i, \qquad p_i(\mathbf{h}) \geq 0, \qquad i = 1, 2$$

- KKT conditions

$$\frac{h_1}{p_1(\mathbf{h}) h_1 + p_2(\mathbf{h}) h_2 + \sigma^2} \leq \lambda_1, \qquad \forall \, \mathbf{h}$$
$$\frac{h_2}{p_1(\mathbf{h}) h_1 + p_2(\mathbf{h}) h_2 + \sigma^2} \leq \lambda_2, \qquad \forall \, \mathbf{h},$$

with equality at $\mathbf{h}$, if $p_1(\mathbf{h}) > 0$ and $p_2(\mathbf{h}) > 0$, respectively.

- For both users to transmit simultaneously at channel state $\mathbf{h} = (h_1, h_2)$,

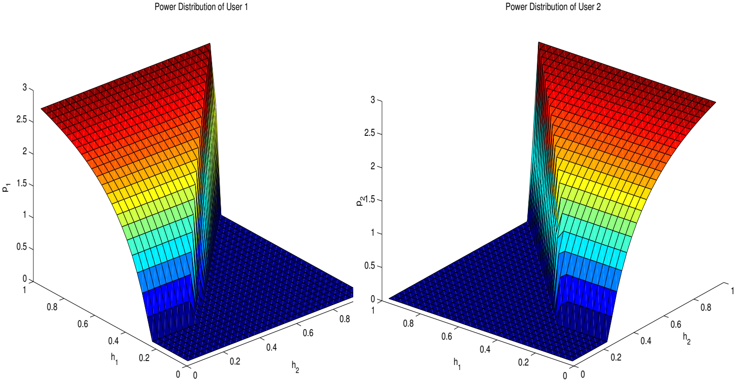
$$\frac{h_1}{h_2} = \frac{\lambda_1}{\lambda_2}$$

- For continuous channel gains, this is a zero-probability event.
- Only the strongest (after some scaling) user transmits at any given time.

Closed-Form Solution

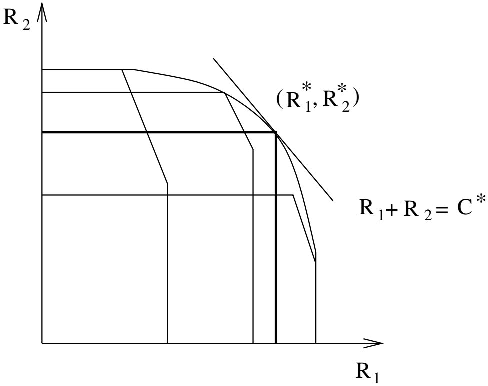
- Single-user waterfilling over favorable channel states

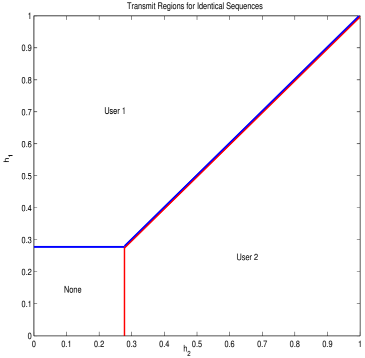
$$p_k(\mathbf{h}) = \begin{cases} \left(\frac{1}{\lambda_k} - \frac{\sigma^2}{h_k}\right)^+, & \text{if} \quad h_k/\lambda_k > h_j/\lambda_j, \quad \forall j \neq k \\ 0, & \text{otherwise} \end{cases}$$



Issue of Simultaneous Transmissions

- Sum capacity achieving pentagon: a rectangle (orthogonal transmissions)





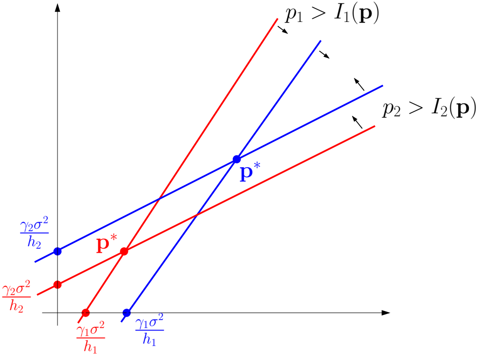
- This result is specific to **sum capacity** and **scalar channel**.

Differences Between QoS-Based and Capacity-Based Power Control in MAC

- SIR-based

$$\frac{p_1 h_1}{p_2 h_2 + \sigma^2} \geq \gamma_1 \quad \Longleftrightarrow \quad p_1 \geq \frac{\gamma_1}{h_1}\left(p_2 h_2 + \sigma^2\right) \triangleq I_1(\mathbf{p})$$

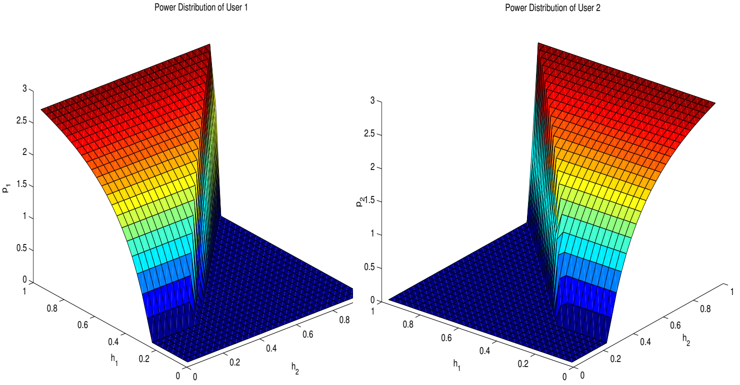
$$\frac{p_2 h_2}{p_1 h_1 + \sigma^2} \geq \gamma_2 \quad \Longleftrightarrow \quad p_2 \geq \frac{\gamma_2}{h_2}\left(p_1 h_1 + \sigma^2\right) \triangleq I_2(\mathbf{p})$$



- Both users transmit simultaneously
- More power if bad channels, less if good channels.

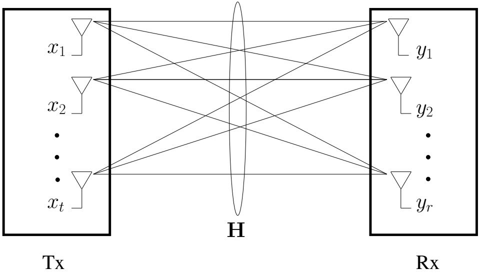
Differences Between QoS-Based and Capacity-Based Power Control in MAC

- Sum-capacity-based



- User with the stronger channel transmits
- Stronger user transmits with more power at better channels
- Multi-user diversity, multi-user opportunistic transmission

Single-User MIMO Channel (Telatar'99)



- Channel gain matrix $\mathbf{H}$: $r \times t$ matrix
- Transmitted signal t-dim. vector $\mathbf{x}$ and received vector r-dim. vector $\mathbf{y}$

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

- $\mathbf{n}$ is i.i.d. zero-mean Gaussian noise vector with equal variance
- $\mathbf{H}$ is deterministic or random and known perfectly at the receiver
- Average power constraint

$$\mathbb{E}[\mathbf{x}^T\mathbf{x}] \le P$$

Single-User MIMO Channel (Telatar'99)

- Use singular value decomposition to express $\mathbf{H}$ as

$$\mathbf{H} = UDV^T$$

- U and V are unitary matrices, D is a diagonal matrix of singular values
- Diagonal entries of D are square roots of the eigenvalues of $\mathbf{H}\mathbf{H}^T$
- Columns of U are the normalized eigenvectors of $\mathbf{H}\mathbf{H}^T$
- Columns of V are the normalized eigenvectors of $\mathbf{H}^T\mathbf{H}$

Single-User MIMO Channel (Telatar'99)

- Obtain an equivalent channel

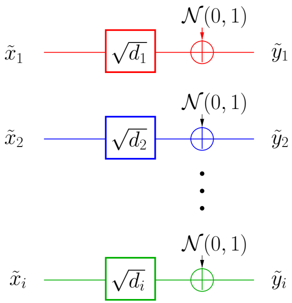
$$\mathbf{y} = UDV^T\mathbf{x} + \mathbf{n}$$

- Let $\tilde{\mathbf{x}} = V^T\mathbf{x}$, $\tilde{\mathbf{y}} = U^T\mathbf{y}$ and $\tilde{\mathbf{n}} = U^T\mathbf{n}$

$$\tilde{\mathbf{y}} = D\tilde{\mathbf{x}} + \tilde{\mathbf{n}}$$

- $\tilde{\mathbf{n}}$ is also i.i.d. zero-mean Gaussian
- Equivalent channel: $\min\{r, t\}$ parallel channels with (squared) gains d_i

$$\tilde{y}_i = \sqrt{d_i}\tilde{x}_i + \tilde{n}_i$$



Single-User MIMO Channel (Telatar'99)

- Independent signalling is optimal over parallel channels
- Let $P_i \triangleq \mathbb{E}[\tilde{x}_i^2]$, power over the ith parallel channel.
- MIMO capacity

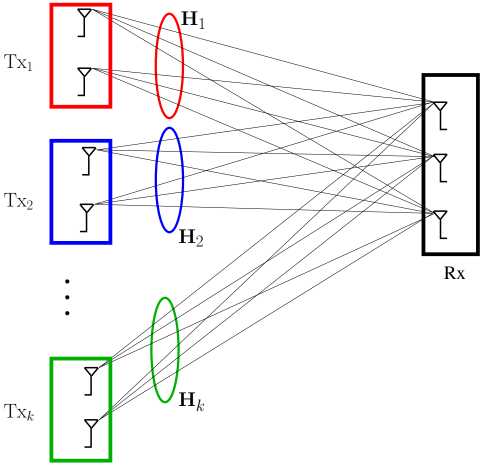
$$\max \sum_{i=1}^{\min\{r,t\}} \frac{1}{2} \log\left(1 + d_i P_i\right)$$

$$\text{s.t.} \sum_{i=1}^{\min\{r,t\}} P_i \leq P$$

- Optimal power allocation: waterfilling

$$P_i = \left(\frac{1}{\lambda} - \frac{1}{d_i}\right)^+$$

MIMO Multiple Access Channel



- The received vector at the receiver,

$$\mathbf{y} = \mathbf{H}_1\mathbf{x}_1 + \mathbf{H}_2\mathbf{x}_2 + \mathbf{n}$$

- $\mathbf{H}_1$ and $\mathbf{H}_2$ are $r \times t$ matrices
- Additive noise $\mathbf{n}$ is i.i.d. Gaussian with covariance $\mathbf{I}$

Capacity Region of MIMO MAC Channel (Yu-Rhee-Boyd-Cioffi'01)

- $\mathbf{Q}_1 = \mathbb{E}[\mathbf{x}_1\mathbf{x}_1^T]$ and $\mathbf{Q}_2 = \mathbb{E}[\mathbf{x}_2\mathbf{x}_2^T]$ are covariances
- Transmit power constraints

$$tr(\mathbf{Q}_1) \leq P_1, \qquad tr(\mathbf{Q}_2) \leq P_2$$

- For fixed $\mathbf{Q}_1$, $\mathbf{Q}_2$, define $\mathcal{B}(\mathbf{Q}_1, \mathbf{Q}_2)$

$$\begin{aligned} R_1 &\leq \frac{1}{2}\log\left|\mathbf{H}_1\mathbf{Q}_1\mathbf{H}_1^T + \mathbf{I}\right| \\ R_2 &\leq \frac{1}{2}\log\left|\mathbf{H}_2\mathbf{Q}_2\mathbf{H}_2^T + \mathbf{I}\right| \\ R_1 + R_2 &\leq \frac{1}{2}\log\left|\mathbf{H}_1\mathbf{Q}_1\mathbf{H}_1^T + \mathbf{H}_2\mathbf{Q}_2\mathbf{H}_2^T + \mathbf{I}\right| \end{aligned}$$

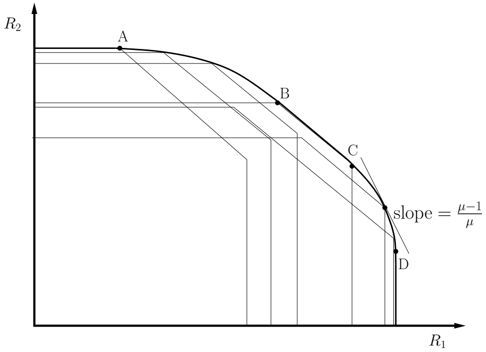
- The capacity region is

$$\mathcal{C} = \bigcup_{tr(\mathbf{Q}_i) \leq P_i} \mathcal{B}(\mathbf{Q}_1, \mathbf{Q}_2)$$

Boundary of the Capacity Region of MIMO MAC (Yu-Rhee-Boyd-Cioffi'01)

- Solve the following wlog for $\mu_1 \leq \mu_2$

$$\begin{aligned} \max_{\mathbf{Q}_1,\mathbf{Q}_2} \quad & \mu_1 \log \left| \mathbf{H}_1\mathbf{Q}_1\mathbf{H}_1^T + \mathbf{H}_2\mathbf{Q}_2\mathbf{H}_2^T + \mathbf{I} \right| + (\mu_2 - \mu_1) \log \left| \mathbf{H}_2\mathbf{Q}_2\mathbf{H}_2^T + \mathbf{I} \right| \\ \text{s.t.} \quad & tr\left(\mathbf{Q}_1\right) \leq P_1, \quad tr\left(\mathbf{Q}_2\right) \leq P_2 \end{aligned}$$



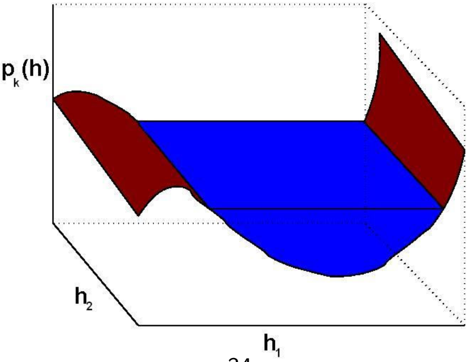
Sum Capacity of the MIMO MAC Channel (Yu-Rhee-Boyd-Cioffi'01)

- Maximize sum rate

$$\max_{\mathbf{Q}_1,\mathbf{Q}_2} \quad \log\left|\mathbf{H}_1\mathbf{Q}_1\mathbf{H}_1^T + \mathbf{H}_2\mathbf{Q}_2\mathbf{H}_2^T + \mathbf{I}\right|$$

$$\text{s.t.} \quad tr\left(\mathbf{Q}_1\right) \le P_1, tr\left(\mathbf{Q}_2\right) \le P_2$$

- Sum rate optimal allocation $\mathbf{Q}_1^*$ and $\mathbf{Q}_2^*$
- Necessary condition:
  - $\mathbf{Q}_1^*$ is the single user waterfilling over the colored noise $\mathbf{H}_2\mathbf{Q}_2\mathbf{H}_2^T + \mathbf{I}$.
  - $\mathbf{Q}_2^*$ is the single user waterfilling over the colored noise $\mathbf{H}_1\mathbf{Q}_1\mathbf{H}_1^T + \mathbf{I}$.
- Multi-dimensional water-filling:



Iterative Waterfilling (Yu-Rhee-Boyd-Cioffi'01)

- Perform single-user optimizations: Iterative waterfilling

$$\log \left| \mathbf{H}_1 \mathbf{Q}_1 \mathbf{H}_1^T + \underbrace{\mathbf{H}_2 \mathbf{Q}_2 \mathbf{H}_2^T + \mathbf{I}}_{\text{effective colored noise}} \right|$$

- Given $\mathbf{Q}_2$, user 1 waterfills over the effective colored noise and updates $\mathbf{Q}_1$

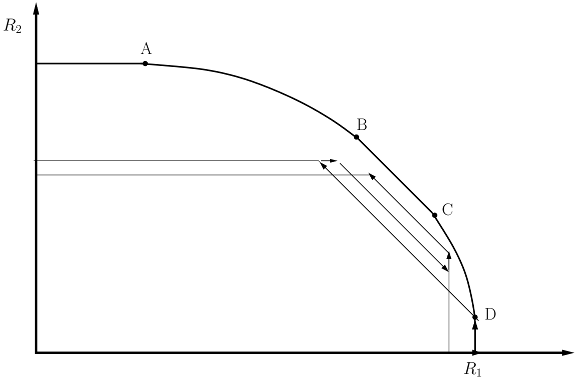
$$\mathbf{Q}_1 = \arg\max_{\mathbf{Q}_1} \log \left| \mathbf{H}_1 \mathbf{Q}_1 \mathbf{H}_1^T + \mathbf{H}_2 \mathbf{Q}_2 \mathbf{H}_2^T + \mathbf{I} \right|$$

- Given $\mathbf{Q}_1$, user 2 waterfills over the effective colored noise and updates $\mathbf{Q}_2$

$$\mathbf{Q}_2 = \arg\max_{\mathbf{Q}_2} \log \left| \mathbf{H}_2 \mathbf{Q}_2 \mathbf{H}_2^T + \mathbf{H}_1 \mathbf{Q}_1 \mathbf{H}_1^T + \mathbf{I} \right|$$

Iterative Waterfilling (Yu-Rhee-Boyd-Cioffi'01)

- An illustration of the trajectory followed during the iterative waterfilling



Conclusions So Far

- Power adaptation for maximizing the capacity in single-user and MAC fading channels, and MIMO and MIMO MAC channels
- Common tool: waterfilling
- Fading channel is equivalent to parallel channels over channel states.
- MIMO channel is equivalent to $\min\{t, r\}$ parallel channels.
- Fading scalar and MIMO multiple access channels
- Sum-rate optimal operating points are reached by iterative waterfilling.
- Any arbitrary point is reached by generalized iterative waterfilling.

Power Adaptation for Energy Minimal Transmission (Uysal-Biyikoglu, Prabhakar, El Gamal'02)

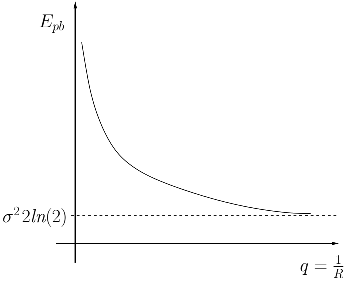
- Rate-power relation in the AWGN channel

$$R = \frac{1}{2}\log\left(1 + \frac{P}{\sigma^2}\right)$$

- Energy-per-bit (E_{pb}) in an AWGN channel

$$E_{pb} = \frac{\sigma^2(2^{2R} - 1)}{R}$$

- E_{pb} monotonically decreases as $\frac{1}{R} \to \infty$

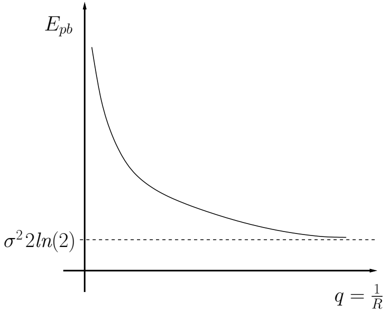


Power Adaptation for Energy Minimal Transmission

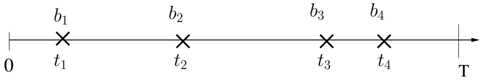
- Serving bits with slower rates is more energy-efficient.
- q: Number of transmissions to send a bit
- Codebook of fixed but sufficiently large block length.
- Code rate is adapted by changing its average power.

$$q \approx \frac{1}{R}$$

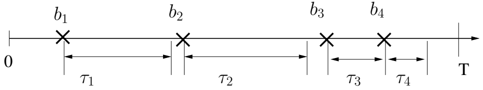
- Given B bits at the transmitter, decreasing transmit power yields
  - longer transmission durations
  - smaller transmission energy
- This is true for any system with a concave rate-power relation.



Energy Minimal Packet Scheduling



- Packets arriving at different times, deadline constrained by T
- Scheduling packets: τ_i is the time allocated for packet i



- Deadline constraint

$$\sum_i \tau_i \leq T$$

- Energy of a schedule $\boldsymbol{\tau}$: $\omega(\boldsymbol{\tau})$
- Find the energy minimal schedule $\boldsymbol{\tau}^*$ to send all packets by T
- Adapt the transmission times of the packets.
- Equivalently, adapt the rate and hence the power.

Offline Packet Scheduling

- Offline schedule: bit arrival times are known a priori
- Bits cannot be served before they arrive at the data buffer: causality
- A necessary condition for optimality in $b_i = b$ case:

$$\tau_i^* \geq \tau_{i+1}^* \quad \text{with} \quad \sum_i \tau_i^* = T$$

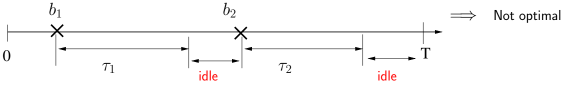
- The necessity is due to the convexity of $\omega(\boldsymbol{\tau})$.
- Assume $\tau_i < \tau_{i+1}$ for some i.
- This is a contradiction since $\sigma_i = \sigma_{i+1} = \frac{\tau_i + \tau_{i+1}}{2}$ and $\sigma_j = \tau_j$ elsewhere

$$\begin{aligned}\omega(\boldsymbol{\sigma}) - \omega(\boldsymbol{\sigma}) &= \omega(\tau_i) + \omega(\tau_{i+1}) - \omega(\sigma_i) - \omega(\sigma_{i+1}) \\ &= \omega(\tau_i) + \omega(\tau_{i+1}) - 2\omega(\frac{\tau_i + \tau_{i+1}}{2}) < 0\end{aligned}$$

- **Equate τ_i subject to feasibility constraints.**

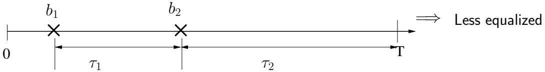
Optimal Offline Packet Scheduling

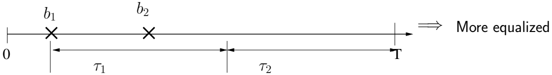
- A schedule with idle intervals is suboptimal.



- **Equate τ_i subject to feasibility constraints.**
- Split the transmission times to the available deadline as much as possible.

$b_1 = b_2 = B$





Optimal Offline Packet Scheduling

- The optimal policy has the structure:

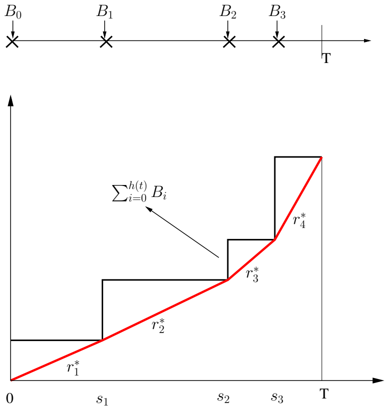
$$\tau_i^* = b_i \max_{k \in \{1,\ldots,M-k_{i-1}\}} \frac{\sum_{j=1}^{k} d_{k_{i-1}+j}}{\sum_{j=0}^{k-1} b_j}$$

$$k_i = k_{i-1} + \arg\max_{k \in \{1,\ldots,M-k_{i-1}\}} \frac{\sum_{j=1}^{k} d_{k_{i-1}+j}}{\sum_{j=0}^{k-1} b_j}$$

- Optimal offline schedule is called lazy scheduling.
- The lazy schedule
  - start slowly and work harder as the deadline approaches
- **Key reason: convexity of energy per bit in transmission time.**

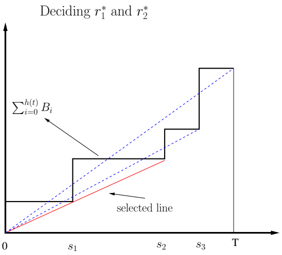
A Calculus Approach for Energy Minimal Packet Scheduling (Zafer-Modiano'09)

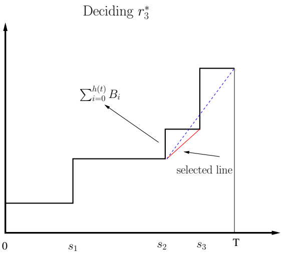
- Uses a geometric framework
- Let $h(t)$ be the last time a packet arrived before t
- $\sum_{i=0}^{h(t)} B_i$ is the cumulative data arrival curve
- A policy is feasible if its cumulative service curve lies below $\sum_{i=0}^{h(t)} B_i$
- The optimal rate policy is the tightest string.

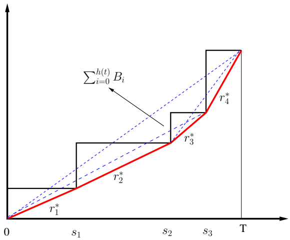


Algorithm to Find the Optimal Policy

- Connect the points at data arrival times to form lines
- Select the line which is feasible and which has minimum slope







Conclusions

- Energy per bit monotonically decreases as $\frac{1}{R}$ increases, i.e., R decreases.
- The slower the transmission, the more energy-efficient it is.
- Scheduling packets that arrive at different times.
- Optimal offline schedule has a "majorization" structure.
- Serve bits with a rate as constant as possible subject to bit feasibility.

References

(1) A. Goldsmith and P. Varaiya, Capacity of fading channels with channel side information , IEEE Trans. on Info. Theory, Nov. 1997

(2) R.Knopp and K. Humblet, Information capacity and power control in single-cell multiuser communications ICC'95, IEEE International Conference on Communications, June 18-22, 1995, Seattle, USA

(3) D.Tse and S. Hanly, Multi-Access Fading Channels: Part I: Polymatroid Structure, Optimal Resource Allocation and Throughput Capacities, IEEE Trans. on Info. Theory, Nov. 1998.

(4) E. Telatar, Capacity of multi-antenna Gaussian channels. European Transactions on Telecommunications, 10(6):585-596, 1999

(5) W.Yu, W. Rhee, S.Boyd and J.Cioffi, Iterative Water-filling for Gaussian Vector Multiple Access Channels, IEEE Trans. on Info. Theory, Jan. 2004.

(6) O. Kaya and S. Ulukus, Achieving the Capacity Region Boundary of Fading CDMA Channels via Generalized Iterative Waterfilling, IEEE Trans. on Wireless Commun., Nov. 2006.

(7) E. Uysal-Biyikoglu, B. Prabhakar and A. El-Gamal, Energy-efficient Packet Transmission over a Wireless Link, IEEE/ACM Trans. on Networking, Aug. 2002.

(8) M. Zafer and E. Modiano, A calculus approach to energy-efficient data transmission with quality-of-service constraints., IEEE/ACM Trans. on Networking, July 2009.