ABSTRACT

Title of thesis: SCHEDULING IN ENERGY HARVESTING SYSTEMS WITH HYBRID ENERGY STORAGE

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In wireless networks, efficient energy storage and utilization plays a vital role, resulting in a prolonged lifetime and enhanced throughput. This factor becomes even more important in systems employing energy harvesting as compared to utility or battery powered networks, where a constant supply of energy is available. Therefore, it is crucial to design schemes that make the best use of available energy resources, keeping in view the practical constraints.

In this work, we consider data transmission with an energy harvesting transmitter which has hybrid energy storage with a perfect super-capacitor (SC) and an inefficient battery. The SC has finite storage space while the battery has unlimited storage space. The transmitter can choose to store the harvested energy in the SC or in the battery, while draining energy from the SC and the battery simultaneously. Under this energy storage setup, we solve throughput optimal energy allocation problem over a point-to-point channel in an offline setting. The hybrid energy storage model with finite and unlimited storage capacities imposes a generalized set of constraints on the transmission policy. We show that the solution is found by a

sequential application of the directional water-filling algorithm.

Next, we consider offline throughput maximization in the presence of an additive time-linear processing cost in the transmitter's circuitry. In this case, the transmitter has to additionally decide on the portions of the processing cost to be drained from the SC and the battery. Despite this additional complexity, we show that the solution is obtained by a sequential application of a directional glue-pouring algorithm, parallel to the costless processing case. Finally, we provide numerical illustrations for optimal policies and performance comparisons with some heuristic online policies.

SCHEDULING IN ENERGY HARVESTING SYSTEMS WITH HYBRID ENERGY STORAGE

by

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Professor Şennur Ulukuş, Chair/Advisor
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To my parents,

Whose love and support means the world to me.

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Chapter 1

Introduction

1.1 Energy Harvesting in Wireless Communication

Energy concerns in wireless communication networks have recently gained a considerable attention from the research community [1]. Energy-limited wireless systems e.g., wireless sensor networks, are equipped with fixed energy supply devices such as batteries which possess limited operation time and energy. For applications where replacing the energy source is cumbersome, unaffordable or some times even impossible, e.g., in unsafe and toxic environments, energy harvesting (EH) appears as a reasonable solution for safe and unlimited energy supply to communication networks. Environmental energy harvesting has been recently considered for improving the sustainable lifetimes of systems e.g., wearable computers and sensor networks etc. Numerous harvesting approaches have been successfully demonstrated including wind, solar, vibrational, biochemical, and motion based, and several others are currently being developed [2, 3]. The amount of energy captured from the environment is highly dependent on the energy source. Power densities of different harvesting technologies are shown in Table 1.1 [2].

In energy harvesting wireless systems, energy acquired for data transmission is incrementally harvested from the environment during the data transmission as the energy producing phenomena is not always present. Moreover, energy is first

Table 1.1: Power Densities of Harvesting Technologies

Harvesting Technology	**Power Density**
Solar Cells (outdoors at noon)	$15mW/cm^3$
Piezoelectric (shoe inserts)	$330\mu W/cm^3$
Vibration (small microwave oven)	$116\mu W/cm^3$
Thermoelectric (10C Gradient)	$40\mu W/cm^3$
Acoustic Noise (100dB)	$960nW/cm^3$

saved in an energy storage unit before it is used for data transmission and unused energy remains in the storage unit for future use. In order to obtain best utility for this unlimited, time-varying and uncontrollable source of energy, energy consumption has to be carefully managed according to the times and amounts of energy harvests and the rate of data transmission must be adapted accordingly. One of the key parameters that determine a system's lifetime and hence performance is the efficiency with which the harvested energy is stored and utilized. This is especially important in a distributed harvesting system, such as a sensor network, where each node may have different environmental harvesting opportunities and hence, instead of just minimizing the total energy consumption, it becomes necessary to adapt the power management scheme to account for these spatio-temporal variations [2]. In this regard, a few energy-aware designs and aspects of energy management in sensor networks are discussed in [4], while a game theoretic approach to energy management in sensor networks is described in [5].

1.2 Energy Storage Technologies

Energy storage design is, possibly, one of the most complex design aspect in an energy harvesting system. Commonly used options for energy storage are batteries and Electric Double Layer Capacitor (EDLC), also known as super-capacitor (SC). Both of these storage choices offer their own benefits and limitations. EDLC uses carbon as the electrodes and stores charge in the electric field at the interface using aqueous or non-aqueous electrolyte. The charging of EDLC is a purely physical phenomena rather than a chemical reaction and hence highly reversible process, which results in high cycle life, long shelf life and a maintenance-free product. Some of the advantages that the EDLC offers are [6]:

- Unlimited charge cycle life
- High power density
- No thermal heat during discharge
- No risk of overcharging
- Unaffected by deep discharges
- Longer lifetime
- Operating temperature range as great as between $-50^{o}C$ to $85^{o}C$

However supercapacitors involve intrinsic leakage due to parasitic paths in the external circuitry [7, 8], which precludes their use for long term energy storage [2]. Batteries, as compared to supercapacitors are a relatively mature technology and have a higher energy density. Different kinds of rechargeable batteries are used in

EH applications including Nickel Cadmium (NiCd), Nickel Metal Hydride (NiMH), Lithium based (Li), and Sealed Lead Acid (SLA). Of these, SLA and NiCD batteries are less used because of relatively low energy density and temporary capacity loss caused by shallow discharge cycles, termed as the memory effect. The choice between NiMH and Li batteries involves several tradeoffs. Li batteries are more efficient than NiMH, have a longer cycle lifetime, and involve a lower rate of self-discharge. However, they are more expensive, even after accounting for their increased cycle life. Li batteries also require a significantly more complicated charging circuit [2].

Although batteries do offer high energy densities but the lack of high power densities sometimes makes them unsuitable for applications where instantaneous power transmissions are required as is typical of sensor networks. On the other hand, supercapacitors suffer from low energy densities and thus cannot work as a standalone unit for energy storage. Tiered energy storage mechanisms using a combination of supercapacitor and batteries have been proposed in the literature for similar applications [9–11] to achieve a better overall performance. This combination has been advocated to inherently offer better performance in comparison to the use of either of them alone. In this thesis, we consider throughput optimal energy allocation for energy harvesting transmitters with such a hybrid energy storage unit.

In data transmission with such a device, aside from determining the transmit power level, the transmitter has to decide the portions of the incoming energy to be saved in the SC and the battery. While it is desirable to save incoming energy in the SC due to its perfect storage efficiency, the storage capacity limitation necessitates careful management of the energy saved in this device. In this regard, the transmitter may wish to save energy in the inefficient battery rather than losing it. Therefore, the extra degree of freedom to choose the portions of incoming energy to save in different storage units significantly complicates the energy management problem.

1.3 Literatue Review and Contribution

We utilize the offline nature of the work where we assume that the transmitter knows exactly the amount and time of energy arrivals in advance. Offline throughput maximization for energy harvesting systems has recently received considerable interest [12–28]. In [12], the transmission completion time minimization problem is solved in energy harvesting systems with an unlimited capacity battery that operates over a static channel. The solution of this problem has later been extended for a finite capacity battery [13], fading channel [14], broadcast channel [15–17], multiple access channel [18], interference channel [19] and relay channel [20, 21]. Offline throughput maximization for energy harvesting systems with leakage in energy storage were studied in [22]. In [23–25], offline optimal performance limits of multi-user wireless systems with energy transfer are studied. Finally, [28] considers offline throughput maximization for energy harvesting devices in the presence of energy storage losses.

As emphasized in [12–28], energy arrivals impose causality constraints on the energy management policy. In addition, battery limitation imposes no-energy-overflow constraints [13, 14, 16]. As the rate-power relation is concave, energy allocation has to be made as constant as possible in time subject to the energy causality and no-energy-overflow constraints. In the presence of hybrid energy storage, the energy causality and no-energy-overflow constraints take a new form since the transmitter has to govern the internal energy dynamics of the storage unit in addition to the power levels drained from these devices. We capture the inefficiency of the battery by a factor η and solve the resulting offline throughput maximization problem.

Although previous work on offline throughput maximization did not address this more realistic energy storage model, a hybrid storage model has appeared in [29]. In this work, the authors analyze a save-then-transmit protocol in energy harvesting wireless systems with a hybrid storage model that operates over fading channels. The optimal save ratio that minimizes outage probability is derived and some useful guidelines are given. Our work is different from [29] in that our objective is throughput maximization and we perform the optimization over a sequence of variables. Moreover, unlike the hybrid storage model in our work, both of the storage devices have unlimited capacities in the model of [29].

A natural way of formulating this problem for the specified model is over the energies drained from the SC and the battery and the portion of the incoming energy to be saved in the SC. Instead, in the spirit of [30], we formulate the problem in terms of energies drained from the SC and the battery and energy transferred from the SC to the battery after initially storing all incoming energy in the SC as much as possible. This formulation reveals many commonalities of this problem with the previous works. This problem relates to sum-throughput maximization in a multiple access channel with energy harvesting transmitters [18] since energies drained from two queues contribute to transmission of a common data. Battery storage loss model is reminiscent of that in [28] where the transmitter is allowed to save the incoming energy in a lossy battery or use it immediately for data transmission. Finally, one-way energy transfer from the SC to the battery relates to the problem considered in [24] where a two-user multiple access channel is considered with energy transfer from one node to the other.

Despite the coupling between the variables that represent energies drained from and transferred within the energy storage unit, we show that the problem can be solved by application of directional water-filling algorithm [14] in multiple stages. In particular, we first forbid energy transfer from the SC to the battery and solve this restricted optimization problem. We show that this problem is solved by optimizing the SC allocation first and then the battery allocation given the SC allocation. Next, we allow energy transfer from the SC to the battery and show that the optimal allocation is obtained by directional water-filling in a setting transformed by the storage efficiency η. As a consequence, we obtain a generalization of the directional water-filling algorithm which yields useful insight on the structure of the optimal offline energy allocation in energy harvesting systems. Byproducts of this analysis are new insights about the optimal policies over the multiple access channel under finite battery constraints.

In the second part of the thesis, we extend the offline throughput maximization problem to the case where a time-linear additive processing cost is present in the data transmission circuitry. It is well-known that circuit power consumption is non-negligible compared to the power spent for data transmission in small scale and short range applications [31]. We note that a considerable portion of energy harvesting communication applications falls into this category, and the effects of circuit power have been investigated in previous works on energy harvesting communications [26, 27,32,33]. Among these works, the framework that is most pertinent to ours has been proposed in [27]. In contrast to [27], in our case, the transmitter has to additionally decide the portions of the energy cost drained from the SC and the battery in the presence of hybrid energy storage. Despite this additional complexity, we show that the solution of the throughput maximization problem with hybrid energy storage is obtained by a sequential application of an extended version of the directional glue pouring algorithm in [27]. To this end, we first construct an equivalent single epoch problem by introducing new time and power variables. In particular, we divide the available time for the SC and the battery and enforce SC and the battery to pay the energy cost in the corresponding time intervals. Moreover, we allow to drain energy from the SC only in its time interval while battery energy can be drained in both intervals. We show that this specific scheme yields a jointly optimal transmission and energy cost drainage scheme. We, then, generalize the single epoch analysis to multiple epochs and obtain an extension of the framework in [27] to the case of hybrid energy storage.

Rest of the thesis is organized as follows. Chapter 2 introduces our hybrid energy storage system model. Mathematical notations and basic constraints on the optimization problem are described. Chapter 3 considers the throughput maximization problem in the given setup. Chapter 4 extends the work of Chapter 3 to include the processing power overhead. In both these chapters, optimal policies are described in terms of transmit powers keeping in view the system constraints. In Chapter 5, we illustrate the optimal policies with and without processing cost in specific numerical studies and provide performance comparisons with heuristic policies in the online regime. Finally, Chapter 6 concludes the thesis.

Chapter 2

System Model

We consider a single-user additive white Gaussian noise (AWGN) channel where the transmitter is equipped with the energy harvesting capability. The transmitter has three buffers: two energy storing buffers and one buffer for storing the data to be transmitted. We assume that the transmitter always has data to transmit, hence an infinite backlog. The two energy buffers represent the hybrid storage system, one for the SC and one for the battery as shown in Fig. 2.1. The SC has a finite storage capacity and can store E_{max} units of energy at maximum. The battery, on the other hand, can store infinite energy but it comes at the cost of storage inefficiency i.e., a certain portion of the stored energy is lost and a fraction is available for use.

The physical channel's input-output model is given by $y = \sqrt{h}x + N$, where x and y represent the input and output of the channel respectively, h is the squared channel gain and N is Gaussian noise with zero-mean and unit-variance. Without loss of generality, we set $h = 1$ throughout the communication. We assume that the transmitter can adjust its power and data rate at will, thus following a continuous time model. The instantaneous data rate is given by

$$r(t) = \frac{1}{2}\log(1 + p(t)) \tag{2.1}$$

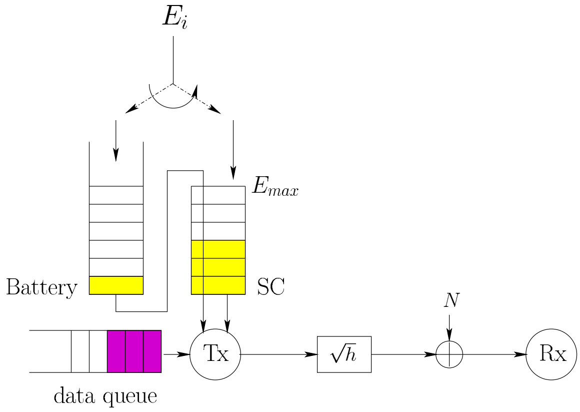

Figure 2.1: System model with hybrid energy storage.

The function $r(.)$ is non-negative, strictly increasing, continuously differentiable and strictly concave, thus for a fixed amount of energy, the number of bits that can be transmitted increases as the transmission duration increases [12]. At time t_i^e, E_i amount of energy arrives. E_0^b and E_0^{sc} amounts of energies are available at the beginning in the battery and in the SC, respectively. In the following, we refer to the time interval between two energy arrivals as an epoch. More specifically, epoch i is the time interval $[t_i^e, t_{i+1}^e)$ and the length of the epoch i is $\ell_i = t_{i+1}^e - t_i^e$.

Whenever energy E_i arrives at time t_i^e, the transmitter stores E_i^{sc} amount in the SC and $E_i^b = E_i - E_i^{sc}$ amount in the battery. Since SC can store at most E_{max} units of energy, E_i^{sc} must be chosen such that no energy unnecessarily overflows. For this reason, $E_i^{sc} \leq E_{max}$ must necessarily be satisfied. The efficiency of the battery is given by the parameter η where $0 \leq \eta < 1$: If E_i^b units of energy is stored in the battery, then ηE_i^b units can be drained and $(1-\eta)E_i^b$ units are lost. Moreover, we assume that the available energy in the battery can be transferred to SC instantaneously[1]. As a consequence, none of the arrived energy overflows; however, there is an energy loss due to inefficiency of the battery.

We assume a simple form of circuitry power consumption i.e., a constant circuit power ϵ is consumed whenever the transmitter is in *active* mode. This constant power represents the energy consumed by the transmitter hardware including power amplifiers, active filters and synthesizers and is assumed to be independent of the level of transmit power [31]. We also assume that the transmitter does not consume any energy while switching between states. Thus when the processing power is non-negligible, the power consumption is $p_i + \epsilon$ in epoch i whenever the transmitter is in *active* state and 0 when transmitter is *inactive*. Due to the presence of ϵ, the nature of the transmitter becomes bursty i.e., to transmit in a certain fraction of the available time and stay *inactive* for rest of the time slot.

A transmit power policy is denoted as $p(t)$ over $[0, T]$. $p(t)$ is constrained by the energy that can be drained from the hybrid storage system:

$$\int_0^{t_i^e} p(u)du \leq \sum_{j=0}^{i-1} E_j^{sc} + \eta E_j^b, \qquad \forall i \tag{2.2}$$

where t_i^e in the upper limit of the integral is considered as $t_i^e - \varepsilon$ for sufficiently small ε.

Moreover, we note that the power policy should cause no energy overflow in the SC. In order to express this constraint, we divide each incremental drained energy $p(u)du$ as a linear combination of the energy drained from the SC, $p^{sc}(u)du$, and the

[1]In real systems, switching time between the battery and the SC is very small compared to epoch lengths of interest [2].

energy drained from the battery, $p^b(u)du$. That is, $p(u)du = p^{sc}(u)du + p^b(u)du$. We are allowed to divide $p(u)du$ into such components since the energy in the battery can be instantaneously transferred to the SC. No-energy-overflow constraint in the SC can now be expressed as follows:

$$\sum_{j=0}^{i} E_j^{sc} - \int_0^{t_i^e} p^{sc}(u)du \leq E_{max}, \qquad \forall i \tag{2.3}$$

We note that the constraints in (2.2) and (2.3) generalize the energy causality and no-energy-overflow constraints in the single-stage energy storage models studied, e.g., in [14].

Chapter 3

Throughput Maximization with Hybrid Energy Storage Model

3.1 Introduction

A key determinant of the performance of energy management policies in energy harvesting systems is the efficiency of energy storage. In order to use the harvested energy for data transmission, energy has to be saved in an energy storage unit, which may foster imperfections such as leakage of the available energy and inefficiency due to other physical reasons. In some energy harvesting systems, energy storage units possess a hybrid storage composed of perfectly efficient and inefficient components with storage capacity limitations [30]. The extra degree of freedom to choose to save energy in different storage units significantly complicates the energy management problems in such systems. While it is desirable to save incoming energy in perfectly efficient storage device, the storage capacity limitation on this device necessitates careful management of the energy saved in this device and save energy in the inefficient one rather than loosing it.

Although the variables in the problem are highly coupled, we show that the problem is solved by a multi-stage algorithm that involves repeated application of the directional water-filling algorithm [14]. The solution generalizes the directional water-filling algorithm for a single stage energy storage and yields valuable insight on the structure of the optimal energy allocation.

3.2 Offline Throughput Maximization

Our objective is to determine the optimal offline schedule for determining the portions of energies that are stored in the SC and the battery and resulting power policy that maximizes throughput by a deadline T. The power policy $p(t)$ has to be constant over each epoch, due to the concavity of the rate-power relation in (2.1). Therefore, the power policy is represented by the sequence $p_i = p_i^{sc} + p_i^b$ where p_i^{sc} and p_i^b are the portions of the power drained from the SC and the battery, respectively, in epoch i. We note that it suffices to assume constant portions p_i^{sc} and p_i^b over epoch i; however, time-varying $f_i^{sc}(t)$ and $f_i^b(t)$ with $p_i = f_i^{sc}(t) + f_i^b(t)$ for all $t \in [t_i^e, t_{i+1}^e)$ and $\int_{t_i^e}^{t_{i+1}^e} f_i^{sc}(t)dt = p_i^{sc}$, $\int_{t_i^e}^{t_{i+1}^e} f_i^b(t)dt = p_i^b$ would have the same performance as well. Moreover, the transmitter decides the portions of the incoming energy E_i^{sc} and E_i^b where $E_i^{sc} + E_i^b = E_i$. Since the battery is inefficient ($0 \leq \eta < 1$), we prefer to initially allocate incoming energy to the SC and the remaining energy to the battery while still allowing to transfer a portion of the energy in SC to the battery. We denote the energy transfer power at epoch i as δ_i with the convention that the transferred energy becomes available for use in epoch $i+1$. The variables in the original problem formulation and its equivalent formulation are depicted in Fig. 3.1.

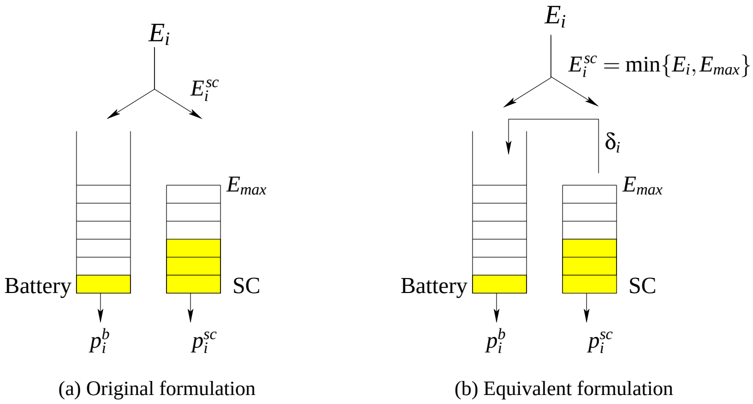

Figure 3.1: The variables in the original problem formulation and its equivalent formulation followed in this thesis.

In view of (2.2)-(2.3), we get the following constraints for all i:

$$\sum_{j=1}^{i} \left(p_j^{sc}\ell_j + \delta_j\ell_j\right) \leq \sum_{j=0}^{i-1} E_j^{sc} \tag{3.1}$$

$$\sum_{j=0}^{i} E_j^{sc} - \sum_{j=1}^{i} \left(p_j^{sc}\ell_j + \delta_j\ell_j\right) \leq E_{max} \tag{3.2}$$

$$\sum_{j=1}^{i} p_j^b\ell_j \leq \sum_{j=0}^{i-1} \left(\eta E_j^b + \eta\delta_j\ell_j\right) \tag{3.3}$$

$$p_i^{sc} \geq 0,\ p_i^b \geq 0,\ \delta_i \geq 0 \tag{3.4}$$

where $E_i^{sc} = \min\{E_i, E_{max}\}$ and $E_i^b = (E_i - E_{max})^+$. We set $\delta_0 = 0$ and $\delta_N = 0$ by convention. We remark that in the system model, energy transfer from SC to the battery is not allowed. However, due to the offline nature, we have the freedom to allocate energy to SC first and then transfer it to the battery. Moreover, one epoch delay in this energy transfer emphasizes the fact that if the energy in the SC in epoch i is transferred to the battery, that energy must be utilized starting from epoch $i+1$ as otherwise such an energy transfer cannot increase the throughput since the battery is inefficient.

Offline throughput maximization problem by deadline T with hybrid energy storage unit is:

$$\begin{aligned} \max_{p_i^{sc}, p_i^b, \delta_i \geq 0} \quad & \sum_{i=1}^{N} \frac{\ell_i}{2} \log\left(1 + p_i^{sc} + p_i^b\right) \\ \text{s.t.} \quad & (3.1) - (3.4) \end{aligned} \tag{3.5}$$

We note that the problem in (3.5) is a convex optimization problem and we can solve it using standard techniques [34]. In fact, the problem in (3.5) is equivalent to sum-throughput maximization in a multiple access channel where battery of one of the users has finite capacity while the other has infinite capacity and one-way energy transfer from the user with finite capacity battery to the other is allowed. A simpler version of this problem is addressed in [24] where both users have unlimited battery. The Lagrangian function for (3.5) is

$$\begin{aligned} \mathcal{L} = &- \sum_{i=1}^{N} \frac{\ell_i}{2} \log\left(1 + p_i^{sc} + p_i^b\right) \\ &+ \sum_{i=1}^{N} \lambda_i \left(\sum_{j=1}^{i} \left(p_j^{sc}\ell_j + \delta_j \ell_j\right) - \sum_{j=0}^{i-1} E_j^{sc} \right) \\ &+ \sum_{i=1}^{N-1} \mu_i \left(\sum_{j=0}^{i} E_j^{sc} - \sum_{j=1}^{i} \left(p_j^{sc}\ell_j + \delta_j \ell_j\right) - E_{max} \right) \\ &+ \sum_{i=1}^{N} \nu_i \left(\sum_{j=1}^{i} p_j^b \ell_j - \sum_{j=0}^{i-1} \left(\eta E_j^b + \eta \delta_j \ell_j\right) \right) \\ &- \sum_{i=0}^{N} \gamma_i \delta_i - \sum_{i=1}^{N} \rho_{1i} p_i^{sc} - \sum_{i=1}^{N} \rho_{2i} p_i^b \end{aligned} \tag{3.6}$$

KKT optimality conditions for (3.5) are:

$$-\frac{1}{1+p_i^{sc}+p_i^b}+\sum_{j=i}^{N}\lambda_j-\sum_{j=i}^{N-1}\mu_j-\rho_{1i}=0, \quad \forall i \tag{3.7}$$

$$-\frac{1}{1+p_i^{sc}+p_i^b}+\sum_{j=i}^{N}\nu_j-\rho_{2i}=0, \quad \forall i \tag{3.8}$$

$$\sum_{j=i}^{N}\lambda_j-\sum_{j=i}^{N-1}\mu_j-\eta\sum_{j=i+1}^{N}\nu_j-\gamma_i=0, \quad \forall i \tag{3.9}$$

and the complementary slackness conditions are:

$$\lambda_i\left(\sum_{j=1}^{i}\left(p_j^{sc}\ell_j+\delta_j\ell_j\right)-\sum_{j=0}^{i-1}E_j^{sc}\right)=0, \qquad \forall i \tag{3.10}$$

$$\mu_i\left(\sum_{j=0}^{i}E_j^{sc}-\sum_{j=1}^{i}\left(p_j^{sc}\ell_j+\delta_j\ell_j\right)-E_{max}\right)=0, \qquad \forall i \tag{3.11}$$

$$\nu_i\left(\sum_{j=1}^{i}p_j^b\ell_j-\sum_{j=0}^{i-1}\left(\eta E_j^b+\eta\delta_j\ell_j\right)\right)=0, \qquad \forall i \tag{3.12}$$

$$\gamma_i\delta_i=\rho_{1i}p_i^{sc}=\rho_{2i}p_i^b=0, \qquad \forall i \tag{3.13}$$

We remark that the optimization problem (3.5) may have many solutions. In order to get a solution, it suffices to find power sequences p_i^{sc}, p_i^b and Lagrange multipliers that are consistent with (3.7)-(3.9) and (3.10)-(3.13). We observe properties of an optimal solution p_i^{sc*}, p_i^{b*} and δ_i^* in the following lemmas. We assume $\eta < 1$ as for $\eta = 1$, there is no cost incurred due to saving energy in the battery and therefore energy can be blindly saved in the SC or the battery, yielding a single energy storage with unlimited space for which the solution is well known [14].

Lemma 3.2.1 *If $p_i^{b*} \neq 0$, $p_i^{sc*}+p_i^{b*}$ does not decrease in the passage from epoch i*

to epoch $i+1$.

Proof: If $p_i^{b*} \neq 0$, then $\rho_{2i} = 0$. By (3.8), we have $p_i^{sc*} + p_i^{b*} = \frac{1}{\sum_{j=i}^{N} \nu_j} - 1$ and $p_{i+1}^{sc*} + p_{i+1}^{b*} = \frac{1}{\sum_{j=i+1}^{N} \nu_j - \rho_{2(i+1)}} - 1$. Since $\nu_i \geq 0$ and $\rho_{i+1} \geq 0$, we conclude the desired result. ■

Lemma 3.2.2 *If $E_{i-1}^b \neq 0$, $p_i^{b*} = 0$ and $p_{i+1}^{b*} \neq 0$, then $p_i^{sc*} + p_i^{b*}$ does not increase in the passage from epoch i to epoch $i+1$. Similarly, if $E_{i-1}^b = 0$, $E_i^b = 0$, $p_i^{b*} = 0$ and $p_{i+1}^{b*} \neq 0$, then $p_i^{sc*} + p_i^{b*}$ does not increase in the passage from epoch i to epoch $i+1$.*

Proof: As $p_i^{b*} = 0$ and $p_{i+1}^{b*} \neq 0$, we have $\rho_{2i} \geq 0$ and $\rho_{2(i+1)} = 0$. Moreover, since $p_i^{b*} = 0$, $\nu_i = 0$ as the constraint $\sum_{j=1}^{i} p_j^b \ell_j \leq \sum_{j=0}^{i-1} \left(\eta E_j^b + \eta \delta_j \ell_j\right)$ cannot be satisfied with equality when $E_{i-1}^b \neq 0$ and $p_i^{b*} = 0$. Similarly, we note that if $E_{i-1}^b = 0$, $E_i^b = 0$, then $\sum_{j=1}^{i} p_j^b \ell_j \leq \sum_{j=0}^{i-1} \left(\eta E_j^b + \eta \delta_j \ell_j\right)$ cannot be satisfied with equality when $p_i^{b*} = 0$ and $p_{i+1}^{b*} \neq 0$. Therefore, $\sum_{j=i}^{N} \nu_j - \rho_{2i} \leq \sum_{j=i+1}^{N} \nu_j - \rho_{2(i+1)}$, which by (3.8) implies the desired result. ■

Lemma 3.2.3 *If $p_i^{sc*}, p_i^{b*} \neq 0$, then $\delta_i^* = 0$.*

Proof: If $p_i^{sc*}, p_i^{b*} \neq 0$, from (3.7) and (3.8), we have $\sum_{j=i}^{N} \lambda_j - \sum_{j=i}^{N-1} \mu_j = \sum_{j=i}^{N} \nu_j$. Combining this with (3.9), we conclude that $\gamma_i = \nu_i + (1-\eta) \sum_{j=i+1}^{N} \nu_j > 0$ as $\eta < 1$. In view of the slackness condition $\gamma_i \delta_i = 0$, we get $\delta_i^* = 0$. ■

Lemma 3.2.4 *If $p_i^{sc*}, p_{i+1}^{sc*}, p_{i+1}^{b*} \neq 0$, $p_i^{sc*} + p_i^{b*} \leq p_{i+1}^{sc*} + p_{i+1}^{b*}$, then $\delta_i^* = 0$.*

Proof: As $p_i^{sc*}, p_{i+1}^{sc*}, p_{i+1}^{b*} \neq 0$, $\rho_{1i} = \rho_{1(i+1)} = \rho_{2(i+1)} = 0$. Therefore, by (3.7) and since $p_i^{sc*} + p_i^{b*} \le p_{i+1}^{sc*} + p_{i+1}^{b*}$, we have $\sum_{j=i}^{N} \lambda_j - \sum_{j=i}^{N-1} \mu_j > \sum_{j=i+1}^{N} \lambda_j - \sum_{j=i+1}^{N-1} \mu_j$. Moreover, since $\rho_{2(i+1)} = 0$, we have $\sum_{j=i+1}^{N} \lambda_j - \sum_{j=i+1}^{N-1} \mu_j = \sum_{j=i+1}^{N} \nu_j$. By (3.9), $\gamma_i > 0$ and due to the slackness condition $\gamma_i \delta_i = 0$, we get $\delta_i^* = 0$. ■

Lemmas 3.2.1-3.2.4 reveal several useful properties of the optimal power sequences p_i^{sc*} and p_i^{b*} and their relation to the transfer power δ_i^*. In view of these lemmas, we adopt the following strategy: Initially, we fix $\delta_i = 0$ and find the optimal policy under this constraint. Note that $\delta_i = 0$ is a good candidate for an optimal selection in view of Lemmas 3.2.3-3.2.4. If the resulting optimal policy is compatible with the KKT conditions, then we stop. Otherwise, we carefully update δ_i so that the KKT conditions are satisfied.

3.2.1 Optimal Policy for Fixed $\delta_i = 0$

For fixed $\delta_i = 0$, the problem becomes maximizing the throughput by the deadline subject to energy causality and finite SC E_{max} constraints only:

$$
\begin{aligned}
\max_{p_i^{sc}, p_i^b \ge 0} \quad & \sum_{i=1}^{N} \frac{\ell_i}{2} \log\left(1 + p_i^{sc} + p_i^b\right) \\
\text{s.t.} \quad & (3.1) - (3.4) \\
& \delta_i = 0, \ \forall i
\end{aligned}
\tag{3.14}
$$

where $E_i^{sc} = \min\{E_i, E_{max}\}$ and $E_i^b = (E_i - E_{max})^+$. We note that (3.14) is equivalent to sum-throughput maximization in a two-user multiple access channel with finite and infinite capacity batteries. A simpler version of this problem where both users have unlimited battery is addressed in [18]. While the problem of sum-throughput maximization has a simple solution when batteries are unlimited by summing the energies of the users and performing single-user throughput maximization [18], the finite battery constraint in (3.14) disables such a simple solution. As in the general problem in [18], the solution of (3.14) is found by iterative directional water-filling where infinitely many iterations are required in general.

Next, we show that due to the problem structure, we can find the solution of (3.14) only in two iterations. Note that the energy arrivals of the storage units are $E_i^{sc} = \min\{E_i, E_{max}\}$ and $E_i^b = (E_i - E_{max})^+$: Energy is first allocated to the SC and the remaining energy is allocated to the battery. This specific way of allocation allows us to find the solution in two iterations. We state this result in the following lemma and provide the proof in Appendix 3.4.1.

Lemma 3.2.5 *For fixed $\delta_i = 0$, let $\hat{p}_i^{sc}$ be the outcome of directional water-filling given $p_i^b = 0$. Let $\hat{p}_i^b$ be the outcome of directional water-filling given $\hat{p}_i^{sc}$. Then, $\hat{p}_i^{sc}$ and $\hat{p}_i^b$ are jointly optimal for (3.14).*

We note that the claim in Lemma 3.2.5 would not be true if E_i^{sc} and E_i^b were allowed to take arbitrary values. Therefore, apart from providing a crucial step towards finding the solution of (3.5), the optimality result stated in Lemma 3.2.5 is an interesting case in the two-user multiple access channel with finite and infinite batteries where the optimal power sequence can be found only in two iterations.

We provide an illustration of the result of two iterations of directional water-filling in Fig. 3.2 where blue and red waters represent energies in the SC and the battery, respectively. In this specific example, $E_i^{sc} = E_{max}$ only over epochs 1 and

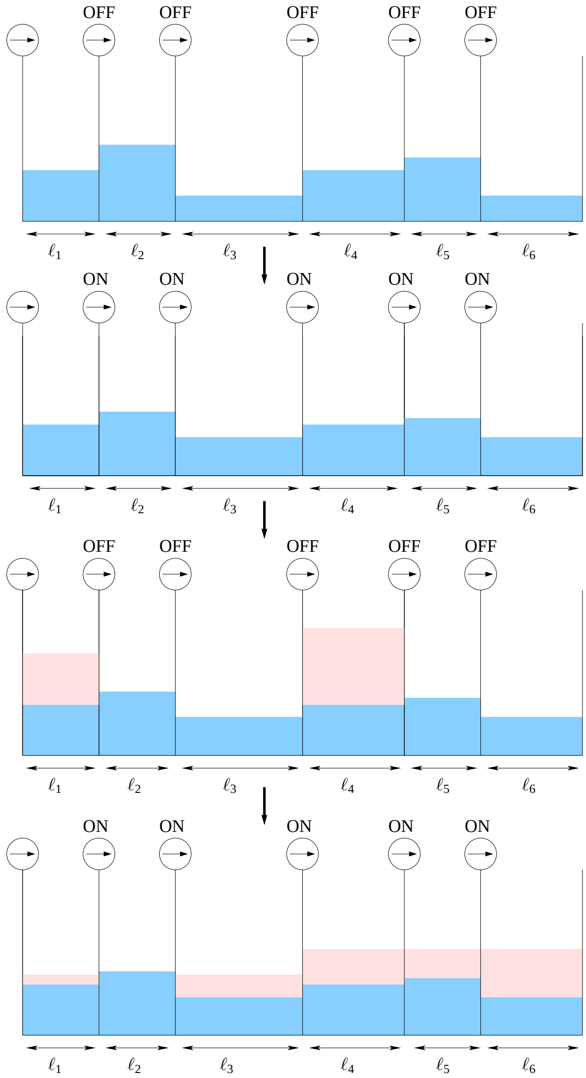

Figure 3.2: An example of optimal power allocation for $\delta_i = 0$.

4. We observe that the red water level is constant over epochs $1-3$ and epochs $4-6$. Moreover, in view of Lemma 3.2.1, whenever p_i^b is non-zero total power level increases. Note that the statement of Lemma 3.2.1, which is originally stated for the solution of (3.5), is also true for the solution of (3.14). This is due to the fact that Lemma 3.2.1 follows from the KKT condition in (3.8) and this condition still holds under the extra constraint $\delta_i = 0$.

3.2.2 Determining Optimal δ_i^*

We note that for $\hat{p}_i^{sc}$ and $\hat{p}_i^b$, there are Lagrange multipliers λ_i, μ_i, ν_i, ρ_{1i} and ρ_{2i} that are compatible with (3.7) and (3.8). Accordingly, $\hat{p}_i^{sc}$ and $\hat{p}_i^b$ are compatible with Lemmas 3.2.1-3.2.2. However, it is not clear if there exist γ_i that are compatible with (3.9). In fact, if $\sum_{j=i}^N \lambda_j - \sum_{j=i+1}^N \mu_j - \eta \sum_{j=i+1}^N \nu_j < 0$, then such γ_i do not exist and if otherwise $\gamma_i = \sum_{j=i}^N \lambda_j - \sum_{j=i+1}^N \mu_j - \eta \sum_{j=i+1}^N \nu_j$. In this section, we propose a method to update the allocations $\hat{p}_i^{sc}$ and $\hat{p}_i^b$ and the Lagrange multipliers $\lambda_i, \mu_i, \rho_{1i}, \rho_{2i}$ that yield δ_i^* and corresponding γ_i so that (3.7)-(3.9) and (3.10)-(3.13) are satisfied. For ease of exposition, we restrict our treatment in this section to the case where $E_1^b > 0$ and $E_i^b = 0$ for $i = 2, \ldots, N$; however, the arguments can be generalized. One can show that in this case, $\nu_N > 0$ and $\nu_i = 0$ for $i = 1, \ldots, N-1$.

Note that if $\hat{p}_i^b \neq 0$ for some i, resulting Lagrange multipliers yield $\gamma_i \geq 0$. In view of the KKT condition (3.9), we transform the directional water-filling setting as in Fig. 3.3: We multiply the water level and the bottom level by $\frac{1}{\eta}$ at epochs where $\hat{p}_i^b > 0$ and leave other epochs unchanged where the bottom level is 1. Moreover, if $\gamma_i \geq 0$, we set $\delta_i^* = 0$ and transform the water level and the bottom level of that epoch. At epochs i with $\gamma_i < 0$, we wish to decrease $\sum_{j=i}^N \nu_j$ and increase $\sum_{j=i}^N \lambda_j - \sum_{j=i}^{N-1} \mu_j$ so that γ_i approaches zero and resulting allocations are compatible with (3.7)-(3.9) and (3.10)-(3.13). We next argue that if energy is transferred from epochs i with $\gamma_i < 0$ in a coordinated fashion, this is possible.

Recall that $\nu_N > 0$ and $\nu_i = 0$ for $i = 1, \ldots, N-1$. We decrease ν_N and increase $\lambda_{\tilde{i}}$, $\mu_{\tilde{i}}$ and $\sum_{j=\tilde{i}}^N \lambda_j - \sum_{j=\tilde{i}}^{N-1} \mu_j$ where $\tilde{i}$ is the epoch index with the lowest

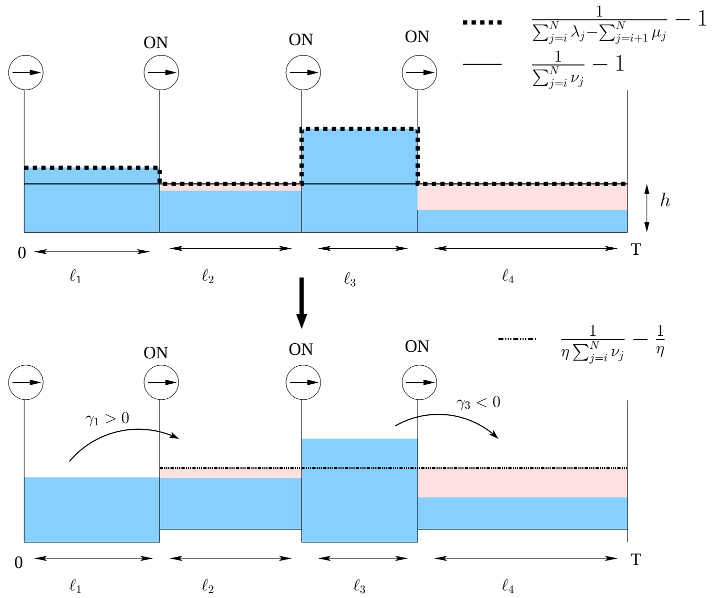

Figure 3.3: Transforming the directional water-filling setting.

$\sum_{j=i}^{N}\lambda_j - \sum_{j=i}^{N-1}\mu_j$. This decreases the power level $p_{\tilde{i}}^{sc}$ and increases the battery power level p_i^b at all epochs. Therefore, a non-zero energy transfer from epoch $\tilde{i}$ occurs. As we decrease ν_N, γ_i also increases. In particular, γ_i may change sign from negative to positive in which case, we make sure that $\delta_i^* = 0$ for that epoch and hence we transform the bottom levels and the water levels for those epochs as in Fig. 3.3. On the other hand, $\sum_{j=\tilde{i}}^{N}\lambda_j - \sum_{j=\tilde{i}}^{N-1}\mu_j$ increases and it may hit the second lowest $\sum_{j=i}^{N}\lambda_j - \sum_{j=i}^{N-1}\mu_j$. In this case, we start to increase λ_i, μ_i and $\sum_{j=i}^{N}\lambda_j - \sum_{j=i}^{N-1}\mu_j$ in both of these epochs.

Note that this procedure corresponds to a coordinated energy transfer: We start energy transfer from the epoch $\tilde{i}$ with the highest power level $\hat{p}_i^{sc}$. In the transformed setting, as we transfer δ_i, $\frac{1}{\eta}\delta_i$ units of water is added to the next epoch as shown in Fig. 3.4. If the power level of epoch $\tilde{i}$ decreases to the level of the second highest power $\hat{p}_i^{sc}$ with $\gamma_i < 0$, then energy is transferred simultaneously from these

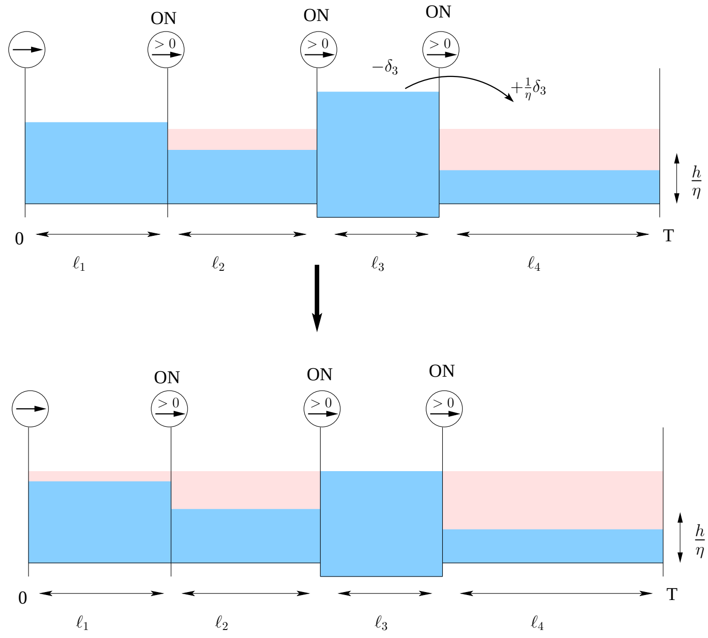

Figure 3.4: The water flow in the transformed directional water-filling setting.

epochs. Causality conditions may forbid decreasing ν_N after some level. This way, all epochs i which have initially $\gamma_i < 0$ are updated so that $\gamma_i \geq 0$ with $\gamma_i = 0$ if $\delta_i > 0$ and (3.7)-(3.9) and (3.10)-(3.13) are satisfied.

Note that when energy is transferred from the SC to the battery in epoch i, this energy spreads over future epochs $i+1, \ldots, N$. Moreover, the energy that was transferred from epochs $1, \ldots, i-1$ in the second directional water-filling of Lemma 3.2.5 given $\hat{p}_i^{sc}$ may flow back to these epochs. We, therefore, measure the transferred energy within the battery at each epoch by means of meters and negate it if energy flows in the opposite direction. This is reminiscent of the meters used for the two-way channel in [24, 25].

3.2.3 Discussion

When $\delta_i = 0$, in general the first directional water-filling yields a non-monotone power sequence $\hat{p}_i^{sc}$ due to finite storage limit E_{max}. The second directional water-filling fills the gaps due to non-monotonicity of $\hat{p}_i^{sc}$ and ameliorates the non-monotonicity of the total power level $\hat{p}_i^{sc}+\hat{p}_i^{b}$. The second stage of the algorithm further smooths out the non-monotonicity of the total power by transferring energy from the SC to the battery in epochs where power is sharply high. Therefore, the cumulative effect of the two-stage algorithm is to collectively transfer energy from the past to the future in both storage devices and make the total power level as constant as possible subject to energy causality and finite SC capacity limit constraints. The extent to which this transfer is continued is determined in a transformed directional water-filling setting where the key parameter is the storage efficiency η.

We remark that for $\eta = 1$, the outcome of the algorithm is the same as the power policy yielded by single-user directional water-filling applied to the energy arrivals E_i with unlimited battery capacity. This is due to the fact that storing energy in the battery or the SC does not cause a performance difference in this case and hence the same performance is achieved if all energy is allocated to the battery only. We also remark that for $\eta = 0$, the algorithm stops after the first directional water-filling since the battery is never used in this case. Therefore, the algorithm reduces to the classical directional water-filling with E_{max} constraint in [14]. Finally, we remark that even when the energy arrivals are always smaller than the SC capacity, i.e., even when $E_i \leq E_{max}$ for all i, the presence of the battery improves the throughput performance as the battery enables smoothing out the variations in the transmit power.

3.3 Conclusion

In this chapter, we analyze data transmission with an energy harvesting transmitter that has a hybrid energy storage unit composed of an inefficient battery and a perfect super-capacitor (SC). We address the offline throughput maximization problem for such an energy harvesting transmitter. In order to optimize performance, internal energy dynamics between the two energy storage units has to be properly adjusted. We utilize the offline nature of the problem and reformulate it in terms of energies drained from the SC and the battery and energy transferred from the SC to the battery. In spite of coupling between the variables in this setting, optimal energy management problem is solved using directional water-filling in multiple stages: First, energy transfer between the two storage elements is fixed to zero and energies drained from the SC and the battery are determined. Then, energy transfer, if necessary, is determined. This solution generalizes the single stage directional water-filling algorithm in [14] and provides valuable insight on how energy is spread in time as equal as possible subject to energy causality and storage limit constraints.

3.4 Appendix

3.4.1 Proof of Lemma 3.2.5

To prove the asserted optimality, it suffices to show that for the power levels $\hat{p}_i^{sc}$ and $\hat{p}_i^b$, there are Lagrange multipliers λ_i, μ_i, ν_i, ρ_{1i}, ρ_{2i} that are consistent with (3.7)-(3.8) and (3.10)-(3.13). Note that we ignored (3.9) as $\delta_i = 0$ fixed. Consider the first directional water-filling that yields $\hat{p}_i^{sc} > 0$ sequence. Let i_n be the epoch indices such that $E_{i_n} = E_{max}$. We remark that the directional water-filling determines the energy allocation between the epochs $i = i_n, i_n + 1, \ldots, i_{n+1} - 1$ independent of the other epochs. For the sequence $\hat{p}_i^{sc}$, $i = i_n, i_n + 1, \ldots, i_{n+1} - 1$, there exist λ_i and μ_i such that

$$\frac{1}{1 + \hat{p}_i^{sc}} = \sum_{k=i}^{i_{n+1}} \lambda_k - \sum_{k=i}^{i_{n+1}-1} \mu_k \tag{3.15}$$

Note that since $\hat{p}_i^{sc} > 0$, $\rho_{1i} = 0$. Therefore, for $i = i_n + 1, \ldots, i_{n+1} - 1$, energy causality and no-energy-overflow conditions cannot be simultaneously active, implying that $\lambda_i \mu_i = 0$. In particular, $\hat{p}_i^{sc}$ increases when $\lambda_i > 0$ and decreases when $\mu_i > 0$.

In the second directional water-filling, $\hat{p}_i^{sc}$ are given and the outcomes are $\hat{p}_i^b$, ν_i and ρ_{2i}. Note that $E_{i_n}^b \geq 0$ and $E_i^b = 0$ for $i = i_n + 1, \ldots, i_{n+1} - 1$. Therefore, the water levels in the second directional water-filling must be constant in between these intervals, i.e., $\nu_i = 0$ for $i = i_n, \ldots, i_{n+1} - 2$ and $\nu_i > 0$ for $i = i_{n+1} - 1$ such that

$$\frac{1}{1+\hat{p}_i^{sc}+\hat{p}_i^b} = \nu_{i_{n+1}} - \rho_{2i} \tag{3.16}$$

for $i = i_n, i_n+1, \ldots, i_{n+1}-1$. Due to the complementary slackness conditions in (3.12), $\rho_{2i} \geq 0$ if $\hat{p}_i^b = 0$ and otherwise $\rho_{2i} = 0$. We note that with the $\hat{p}_i^b$ found from (3.16), Lagrange multipliers λ_i, μ_i in (3.15) do not satisfy (3.7) while they satisfy the corresponding slackness conditions in (3.10)-(3.11). However, current selection of variables satisfy (3.8).

We next argue that λ_i, μ_i can be updated so that (3.7) is satisfied while still satisfying the slackness conditions. In particular, we can combine (3.15) and (3.16) and find $\tilde{\lambda}, \tilde{\mu}$ such that for $i = i_n, i_n+1, \ldots, i_{n+1}-1$:

$$\frac{1}{1+\hat{p}_i^{sc}+\hat{p}_i^b} = \min\left\{\sum_{k=i}^{i_{n+1}} \lambda_k - \sum_{k=i}^{i_{n+1}-1} \mu_k,\ \nu_{i_{n+1}}\right\} \tag{3.17}$$

$$= \sum_{k=i}^{i_{n+1}} \tilde{\lambda}_k - \sum_{k=i}^{i_{n+1}-1} \tilde{\mu}_k \tag{3.18}$$

where, if $\hat{p}_{i-1}^{sc} + \hat{p}_{i-1}^b < \hat{p}_i^{sc} + \hat{p}_i^b$:

$$\tilde{\lambda}_{i-1} = \frac{1}{1+\hat{p}_{i-1}^{sc}+\hat{p}_{i-1}^b} - \frac{1}{1+\hat{p}_i^{sc}+\hat{p}_i^b}, \quad \tilde{\mu}_{i-1} = 0 \tag{3.19}$$

If $\hat{p}_{i-1}^{sc} + \hat{p}_{i-1}^b > \hat{p}_i^{sc} + \hat{p}_i^b$:

$$\tilde{\mu}_{i-1} = \frac{1}{1+\hat{p}_i^{sc}+\hat{p}_i^b} - \frac{1}{1+\hat{p}_{i-1}^{sc}+\hat{p}_{i-1}^b}, \quad \tilde{\lambda}_{i-1} = 0 \tag{3.20}$$

and $\tilde{\lambda}_{i_n} = \frac{1}{1+\hat{p}_i^{sc}+\hat{p}_i^b}$ and $\tilde{\lambda}_i = \tilde{\mu}_i = 0$ otherwise. In view of (3.17), we observe that over the epochs $i = i_n, \ldots, i_{n+1} - 1$, if $\hat{p}_i^{sc} + \hat{p}_i^b < \hat{p}_{i+1}^{sc} + \hat{p}_{i+1}^b$, then $\lambda_i > 0$, $\mu_i = 0$ and hence $\hat{p}_i^{sc} < \hat{p}_{i+1}^{sc}$. Similarly, if $\hat{p}_i^{sc} + \hat{p}_i^b > \hat{p}_{i+1}^{sc} + \hat{p}_{i+1}^b$, then $\mu_i > 0$, $\lambda_i = 0$ and hence $\hat{p}_i^{sc} > \hat{p}_{i+1}^{sc}$. Therefore, $\tilde{\lambda}_i$ and $\tilde{\mu}_i$ have the following property: if $\tilde{\lambda}_i > 0$ then $\lambda_i > 0$ and if $\tilde{\mu}_i > 0$ then $\mu_i > 0$. Hence, $\tilde{\lambda}_i$ and $\tilde{\mu}_i$ satisfy (3.7) as well as (3.10)-(3.11). This proves the existence of Lagrange multipliers that satisfy (3.7)-(3.8) as well as (3.10)-(3.13) and hence the outcomes of two successive directional water-fillings $\hat{p}_i^{sc}, \hat{p}_i^b$ are jointly optimal.

Chapter 4

Hybrid Energy Storage with Non-Ideal Processing Power

4.1 Introduction

Wireless nodes that utilize harvested energy may have to work even when the energy source is not present, thus careful storage and utilization of harvested energy is necessary. In addition, the transmit circuitry consumes power as well in addition to the transmitted power [31]. This circuit power plays a crucial role in energy harvesting systems. This effect appears more prominently in short-range communication systems, where often the circuitry power is comparable to power used for data transmission. In case of systems having a non-negligible processing power cost, bursty transmission has been shown to be optimal [35]. It has been shown that a simple relationship exists between the optimal transmission time and the processing cost whose solution is interpreted as *glue-pouring*.

The presence of processing power complicates the problem of energy management in this setting and the solution to this problem offers useful insights. In this chapter, we consider offline optimization hence the transmitter possesses the knowledge of energy arrivals in advance. This assumption holds true for systems where energy harvests can be predicted based on previous observations and statistical analysis [36]. Being equipped with the knowledge of time and amount of energy arrivals, we find the optimal transmission policies maximizing the data throughput in the presence of processing power overhead.

4.2 Offline Throughput Maximization

In this section, we consider the case in which the transmitter's circuitry causes an additive time-linear processing cost in data transmission. In particular, the processing cost could be viewed as a constant circuit power ϵ whenever it is active. Hence, for a transmit power policy $p(t)$, the total power consumption is $p(t)+\epsilon\mathbf{1}_{p(t)>0}$ where ϵ is in energy units per time units.

The energy causality and no-energy-overflow constraints in (2.2) and (2.3) extend naturally to the case of non-negligible processing power and can be expressed as:

$$\int_0^{t_i^e} (p(u) + \epsilon\mathbf{1}_{p(u)>0})du \le \sum_{j=0}^{i-1} E_j^{sc} + \eta E_j^b, \qquad \forall i \tag{4.1}$$

$$\sum_{j=0}^{i} E_j^{sc} - \int_0^{t_i^e} (p^{sc}(u) + \epsilon^{sc}\mathbf{1}_{p(u)>0})du \le E_{max}, \qquad \forall i \tag{4.2}$$

where ϵ_i^{sc} and ϵ_i^b are the portions of the processing power drained from the SC and the battery, respectively, in epoch i: $\epsilon = \epsilon_i^{sc} + \epsilon_i^b$.

4.2.1 The Case of a Single Epoch

We start our analysis by considering the single epoch case. Assume E^{sc} and E^b units of energy are available before the start of transmission in the SC and the battery, respectively, and let the transmission deadline be set to infinity. We have the following optimization problem:

$$\max_{t,p^{sc}(t),p^{b}(t)} \quad \int_0^t \frac{1}{2} \log\left(1 + p^{sc}(u) + p^{b}(u)\right) du \tag{4.3}$$

where $p^{sc}(u)$ and $p^{b}(u)$ are the powers drained from the SC and the battery during $0 \leq u \leq t$ time interval. The energy constraints for (4.3) are: $\int_0^t (p^{sc}(u) + \epsilon^{sc})du \leq E^{sc}$ and $\int_0^t (p^{b}(u) + \epsilon^{b})du \leq \eta E^{b}$ where $\epsilon^{sc} + \epsilon^{b} = \epsilon$. We remark that the single epoch analysis in [27, 35] does not immediately apply to our problem since our problem involves two power variables and the transmitter incurs a processing cost when either one (or both) of these power variables is non-zero and the processing energy can be drained from two different energy storage devices.

We note that due to the concavity of the log(.) function, $p^{sc}(u) + p^{b}(u)$ must remain constant whenever $p^{sc}(u)+p^{b}(u) > 0$ and such an allocation is always feasible since the energies E^{sc} and E^{b} are assumed to be available before the transmission starts. This, in turn, implies that the transmission duration t is $t = \frac{E^{sc}+\eta E^{b}}{p^{sc}+p^{b}+\epsilon}$ where p^{sc} and p^{b} are constant powers drained from the SC and the battery during $0 \leq u \leq t$ interval. Hence, the objective function in (4.3) is expressed as a single-variable function of $p^{sc} + p^{b}$: $\frac{E^{sc}+\eta E^{b}}{p^{sc}+p^{b}+\epsilon} \log\left(1 + p^{sc} + p^{b}\right)$. Equating its derivative to zero, we obtain the following equation (c.f. [27, 35]):

$$\frac{\log\left(1 + p^{*}\right)}{(p^{*} + \epsilon)} = \frac{1}{1 + p^{*}} \tag{4.4}$$

Let p^{*} be the solution of the equation in (4.4). Then, p^{sc*} and p^{b*} are solutions of (4.3) if $p^{sc*} + p^{b*} = p^*$. Note that p^* is the unique solution of (4.4), which parametrically depends on ϵ and is independent of E^{sc} and E^b [35]. Moreover, we note that the selections of p^{sc*} and p^{b*} are not unique and they determine ϵ^{sc} and ϵ^b. In particular, we have

$$\epsilon^{sc} = \frac{E^{sc}}{E^{sc} + \eta E^b}(p^* + \epsilon) - p^{sc*} \tag{4.5}$$

$$\epsilon^{b} = \frac{\eta E^{b}}{E^{sc} + \eta E^b}(p^* + \epsilon) - p^{b*} \tag{4.6}$$

Now, let us impose a deadline $t \leq T$ to the problem in (4.3). If the deadline T satisfies $T \geq \frac{E^{sc}+\eta E^b}{p^*+\epsilon}$, the solution is the same as the solution with infinite deadline. On the other hand, if $T \leq \frac{E^{sc}+\eta E^b}{p^*+\epsilon}$, then $p^{sc*} + p^{b*} = \frac{E^{sc}+\eta E^b}{T} - \epsilon$ and ϵ^{sc}, ϵ^b are determined as:

$$\epsilon^{sc} = \frac{E^{sc}}{T} - p^{sc*} \tag{4.7}$$

$$\epsilon^{b} = \frac{\eta E^{b}}{T} - p^{b*} \tag{4.8}$$

In the infinite deadline case, one possible selection is $p^{sc*} = \frac{E^{sc}}{E^{sc}+\eta E^b}p^*$ and $p^{b*} = \frac{\eta E^b}{E^{sc}+\eta E^b}p^*$. ϵ^{sc} and ϵ^b are determined according to (4.5)-(4.6). This selection facilitates an alternative view of the problem: If in the first $t^{sc} = \frac{E^{sc}}{p^*+\epsilon}$ time units, $p^{sc} = p^*$, $p^b = 0$ and in the following $t^b = \frac{\eta E^b}{p^*+\epsilon}$ time units, $p^b = p^*$ and $p^{sc} = 0$, then this yields the optimal throughput for (4.3). Moreover, the processing energy is drained from the SC and the battery with power ϵ only when they are active. This selection has the following counterpart if the deadline is finite: When $\frac{E^{sc}}{p^*+\epsilon} \leq T \leq \frac{E^{sc}+\eta E^b}{p^*+\epsilon}$,

$p^{sc} = p^*$ over the first $t^{sc} = \frac{E^{sc}}{p^*+\epsilon}$ time units and p^b is determined by water-filling ηE^b units of energy over $[0, T]$ interval given p^{sc} and no processing cost from the battery in the first t^{sc} units. Secondly, if $T < \frac{E^{sc}}{p^*+\epsilon}$, $p^{sc} = \frac{E^{sc}}{T} - \epsilon$ and $p^b = \frac{\eta E^b}{T}$ over $[0, T]$.

This alternative view of the problem suggests that a solution for (4.3) can be found by solving

$$\max_{t^{sc}, t^b, p^{sc}, p_1^b, p_2^b} \frac{t^{sc}}{2} \log\left(1 + p^{sc} + p_1^b\right) + \frac{t^b}{2} \log\left(1 + p_2^b\right) \tag{4.9}$$

where the energy constraints are $t^{sc}(p^{sc} + \epsilon) \le E^{sc}$ and $t^{sc}p_1^b + t^b(p_2^b + \epsilon) \le \eta E^b$ along with the deadline $t^{sc} + t^b \le T$. Note that the processing energy is drained from the SC in the first t^{sc} units and from the battery in the remaining time units. The problem (4.9) has a unique solution[1] $t^{sc*}, t^{b*}, p^{sc*}, p_1^{b*}, p_2^{b*}$. To see this note that all of the time and energy constraints must be satisfied with equality and whenever $t^{b*} > 0$, we must have $p^{sc*} + p_1^{b*} = p_2^{b*} = \frac{E^{sc} + \eta E^b}{T} - \epsilon$, which along with the time and energy constraints, determine the variables in (4.9) uniquely. Similarly, if $t^{b*} = 0$, then $p^{sc*} = \frac{E^{sc}}{T} - \epsilon$, $p_1^{b*} = \frac{\eta E^b}{T}$ and p_2^{b*} can be selected arbitrarily. Note that using the unique solution $t^{sc*}, t^{b*}, p^{sc*}, p_1^{b*}, p_2^{b*}$ of (4.9), we can get a solution of (4.3) by setting the SC power as $\frac{t^{sc*}}{t^{sc*}+t^{b*}} p^{sc*}$ and the battery power as $\frac{t^{sc*}}{t^{sc*}+t^{b*}} p_1^{b*} + \frac{t^{b*}}{t^{sc*}+t^{b*}} p_2^{b*}$. Moreover, $\frac{t^{sc*}}{t^{sc*}+t^{b*}} \epsilon$ units of processing energy is drained from the SC and the remaining processing energy is drained from the battery. We note that in an optimal solution of (4.9), $p_1^{b*} = 0$ whenever $t_i^{sc*} + t_i^{b*} < T$.

This specific allocation is not necessary for optimality in (4.3) and one may

[1] If $t^{b*} = 0$, p_2^{b*} can be selected arbitrarily; however, this does not violate the uniqueness of the solution.

suggest different optimal allocations. However, we will see in the following section that this allocation enables us to extend the analysis in Chapter 3 and interpret the solutions properly.

4.2.2 The Case of Multiple Epochs

As the rate-power relation is concave and the processing cost is additive and independent of the transmit power level, the transmit power policy $p(t)$ has to be constant during each epoch i as long as $p(t) > 0$. See also [27, 35]. Therefore, we get the following constraints for all $i = 1, \ldots, N$:

$$\sum_{j=1}^{i} \left((p_j^{sc} + \epsilon)t_j^{sc} + \delta_j t_j^{sc}\right) \leq \sum_{j=0}^{i-1} E_j^{sc} \tag{4.10}$$

$$\sum_{j=0}^{i} E_j^{sc} - \sum_{j=1}^{i} \left((p_j^{sc} + \epsilon)t_j^{sc} + \delta_j t_j^{sc}\right) \leq E_{max} \tag{4.11}$$

$$\sum_{j=1}^{i} \left(p_{1j}^{b} t_j^{sc} + (p_{2j}^{b} + \epsilon)t_j^{b}\right) \leq \sum_{j=0}^{i-1} \left(\eta E_j^{b} + \eta \delta_j t_j^{sc}\right) \tag{4.12}$$

$$p_i^{sc} \geq 0, p_{1i}^{b} \geq 0, p_{2i}^{b} \geq 0, \delta_i \geq 0 \tag{4.13}$$

where $E_i^{sc} = \min(E_i, E_{max})$ and $E_i^b = (E_i - E_{max})^+$. We set $\delta_0 = 0$ and $\delta_N = 0$ by convention. $t_i = t_i^{sc} + t_i^b$ is the time portion of epoch i in which the transmitter is active. Thus, $0 \leq t_i \leq \ell_i$. We note that the constraint set in (4.10)-(4.13) is not convex. To circumvent this difficulty, we introduce a change of variables: $\alpha_i \triangleq p_i^{sc} t_i^{sc}$, $\beta_i \triangleq p_{2i}^b t_i^b$, $\theta_i \triangleq p_{1i}^b t_i^{sc}$ and $\gamma_i \triangleq \delta_i t_i^{sc}$. The constraint set in terms of the new variables is:

$$\sum_{j=1}^{i} \left(\alpha_j + \epsilon t_j^{sc} + \gamma_j\right) \leq \sum_{j=0}^{i-1} E_j^{sc} \tag{4.14}$$

$$\sum_{j=0}^{i} E_j^{sc} - \sum_{j=1}^{i} \left(\alpha_j + \epsilon t_j^{sc} + \gamma_j\right) \leq E_{max} \tag{4.15}$$

$$\sum_{j=1}^{i} \left(\theta_i + \beta_i + \epsilon t_j^{b}\right) \leq \sum_{j=0}^{i-1} \left(\eta E_j^{b} + \eta\gamma_j\right) \tag{4.16}$$

$$0 \leq t_i^{sc} + t_i^{b} \leq \ell_i \tag{4.17}$$

$$t_i^{sc}, t_i^{b} \geq 0, \quad \alpha_i \geq 0, \quad \beta_i \geq 0, \quad \theta_i \geq 0 \quad \gamma_i \geq 0 \tag{4.18}$$

Offline throughput maximization problem in the new variable set is:

$$\begin{aligned} \max_{\alpha_i,\beta_i,\theta_i,t_i^{sc},t_i^{b},\gamma_i \geq 0} \quad & \sum_{i=1}^{N} \frac{t_i^{sc}}{2} \log\left(1 + \frac{\alpha_i}{t_i^{sc}} + \frac{\theta_i}{t_i^{sc}}\right) + \frac{t_i^{b}}{2} \log\left(1 + \frac{\beta_i}{t_i^{b}}\right) \\ \text{s.t.} \quad & (4.14) - (4.18) \end{aligned} \tag{4.19}$$

The concavity of the objective function in (4.19) follows from convexity preservation of the perspective operation [34]. Note that the function $\frac{t_i}{2} \log\left(1 + \frac{\alpha_i}{t_i} + \frac{\beta_i}{t_i}\right)$ is the perspective of the strictly concave function $\frac{1}{2} \log\left(1 + \alpha_i + \beta_i\right)$.

The Lagrangian for (4.19) is as follows:

$$
\begin{aligned}
\mathcal{L} = &-\sum_{i=1}^{N}\left[\frac{t_i^{sc}}{2}\log\left(1+\frac{\alpha_i}{t_i^{sc}}+\frac{\theta_i}{t_i^{sc}}\right)+\frac{t_i^{b}}{2}\log\left(1+\frac{\beta_i}{t_i^{b}}\right)\right] \\
&+\sum_{i=1}^{N}\lambda_i\left[\sum_{j=1}^{i}\left(\alpha_j+\epsilon t_j^{sc}+\gamma_j\right)-\sum_{j=0}^{i-1}E_j^{sc}\right] \\
&+\sum_{i=1}^{N-1}\mu_i\left[\sum_{j=0}^{i}E_j^{sc}-\sum_{j=1}^{i}\left(\alpha_j+\epsilon t_j^{sc}+\gamma_j\right)-E_{max}\right] \\
&+\sum_{i=1}^{N}\nu_i\left[\sum_{j=1}^{i}\left(\theta_i+\beta_i+\epsilon t_j^{b}\right)-\sum_{j=0}^{i-1}\left(\eta E_j^{b}+\eta\gamma_j\right)\right] \\
&-\sum_{i=1}^{N}\rho_{1i}\alpha_i-\sum_{i=1}^{N}\rho_{2i}\theta_i-\sum_{i=1}^{N}\rho_{3i}\beta_i \\
&-\sum_{i=0}^{N}\xi_i\gamma_i-\sum_{i=1}^{N}\sigma_{1i}t_i^{sc}-\sum_{i=1}^{N}\sigma_{2i}t_i^{b}+\sum_{i=1}^{N}z_i\left(t_i^{sc}+t_i^{b}-\ell_i\right)
\end{aligned}
\tag{4.20}
$$

where λ_i, μ_i, ν_i, ρ_{1i}, ρ_{2i}, ρ_{3i}, ξ_i, σ_{1i}, σ_{2i} and z_i are the Lagrange multipliers. The KKT optimality conditions for (4.19) are:

$$-\frac{t_i^{sc}}{t_i^{sc}+\alpha_i+\theta_i}+\sum_{j=i}^{N}\lambda_j-\sum_{j=i}^{N-1}\mu_j-\rho_{1i}=0, \quad \forall i \tag{4.21}$$

$$-\frac{t_i^{sc}}{t_i^{sc}+\alpha_i+\theta_i}+\sum_{j=i}^{N}\nu_j-\rho_{2i}=0, \quad \forall i \tag{4.22}$$

$$-\frac{t_i^{b}}{t_i^{b}+\beta_i}+\sum_{j=i}^{N}\nu_j-\rho_{3i}=0, \quad \forall i \tag{4.23}$$

$$\sum_{j=i}^{N}\lambda_j-\sum_{j=i}^{N-1}\mu_j-\eta\sum_{j=i+1}^{N}\nu_j-\xi_i=0, \quad \forall i \tag{4.24}$$

$$\frac{\alpha_i + \theta_i}{t_i^{sc} + \alpha_i + \theta_i} - \log\left[\frac{t_i^{sc} + \alpha_i + \theta_i}{t_i^{sc}}\right] + \epsilon\left[\sum_{j=i}^{N} \lambda_j - \sum_{j=i}^{N-1} \mu_j\right] - \sigma_{1i} + z_i = 0, \quad \forall i \tag{4.25}$$

$$\frac{\beta_i}{t_i^{b} + \beta_i} - \log\left[\frac{t_i^{b} + \beta_i}{t_i^{b}}\right] + \epsilon\sum_{j=i}^{N} \nu_j - \sigma_{2i} + z_i = 0, \quad \forall i \tag{4.26}$$

and the corresponding complementary slackness conditions are:

$$\lambda_i\left[\sum_{j=1}^{i}\left(\alpha_j + \epsilon t_j^{sc} + \gamma_j\right) - \sum_{j=0}^{i-1} E_j^{sc}\right] = 0, \quad \forall i \tag{4.27}$$

$$\mu_i\left[\sum_{j=0}^{i} E_j^{sc} - \sum_{j=1}^{i}\left(\alpha_j + \epsilon t_j^{sc} + \gamma_j\right) - E_{max}\right] = 0, \quad \forall i \tag{4.28}$$

$$\nu_i\left[\sum_{j=1}^{i}\left(\theta_j + \beta_j + \epsilon t_j^{b}\right) - \sum_{j=0}^{i-1}\left(\eta E_j^{b} + \eta\gamma_j\right)\right] = 0, \quad \forall i \tag{4.29}$$

$$\rho_{1i}\alpha_i = \rho_{2i}\theta_i = \rho_{3i}\beta_i = \xi_i\gamma_i = 0, \quad \forall i \tag{4.30}$$

$$\sigma_{1i}t_i^{sc} = \sigma_{2i}t_i^{b} = z_i\left(t_i^{sc} + t_i^{b} - \ell_i\right) = 0, \quad \forall i \tag{4.31}$$

We note that the optimization problem (4.19) may have many solutions. To find a solution, it suffices to find α_i, β_i, γ_i and Lagrange multipliers that are consistent with (4.21)-(4.26) and (4.27)-(4.31). This, in turn, yields optimal transmit power sequences p_i^{sc}, p_{1i}^{b}, p_{2i}^{b} along with time sequences t_i^{sc} and t_i^{b}. Based on our analysis of a single epoch case, we observe properties of an optimal solution for (4.19) in the following lemmas.

Lemma 4.2.1 *If* $0 < t_i^{sc*} < \ell_i$, *then* $\delta_i^* = 0$.

Proof: For the case when $t_i^{b*} = 0$, we have $p_{1i}^{b*} = p_{2i}^{b*} = 0$. Hence, $\sigma_{1i} = 0$, $z_i = 0$ and $\nu_i = 0$. By (4.25), $\frac{\log\left(1+p_i^{sc*}\right)}{p_i^{sc*}+\epsilon} = \frac{1}{1+p_i^{sc*}}$ and therefore $p_i^{sc*} = p^*$. By (4.21) and (4.24), we get $\xi_i > 0$ and hence $\gamma_i = 0$ and $\delta_i^* = 0$.

When $t_i^{b*} > 0$, we have $p_{2i}^{b*} > 0$. From the slackness conditions in (4.30)-(4.31), $\sigma_{1i} = \sigma_{2i} = \rho_{1i} = \rho_{2i} = 0$. By (4.21), $\sum_{j=i}^{N} \lambda_j - \sum_{j=i}^{N-1} \mu_j = \frac{1}{1+p_i^{sc*}}$. By (4.22), $\sum_{j=i}^{N} \nu_j = \frac{1}{1+p_{2i}^{b*}} = \frac{1}{1+p_i^{sc*}}$. Using this in (4.24), we have $\xi_i = (1-\eta)\left(\frac{1}{1+p_i^{sc*}}\right) + \eta\nu_i$. By (4.29), $\nu_i \geq 0$ and hence $\xi_i > 0$ and together with the slackness condition $\xi_i\gamma_i = 0$, we get $\delta_i^* = 0$. ■

Lemma 4.2.2 *If $t_i^{sc*} + t_i^{b*} = \ell_i$, and $p_{1i}^{b*} \neq 0$, then $\delta_i = 0$.*

Proof: Note that $t_i^{sc*} > 0$ as energy is first allocated to the SC. Hence, $p_i^{sc*} > 0$ and $p_{1i}^{b*} > 0$. By the slackness condition in (4.30), $\rho_{1i} = \rho_{2i} = 0$. From (4.21)-(4.22), we have $\sum_{j=i}^{N} \lambda_j - \sum_{j=i}^{N-1} \mu_j = \frac{1}{1+p_i^{sc*}+p_{1i}^{b*}} = \sum_{j=i}^{N} \nu_j$. Using this in (4.24), we have $\xi_i = (1-\eta)\left(\frac{1}{1+p_i^{sc*}+p_{1i}^{b*}}\right) + \eta\nu_i > 0$ as $\nu_i \geq 0$ and $0 < \eta < 1$. This, from the corresponding slackness condition, implies $\gamma_i = 0$. As $t_i^{sc*} > 0$, we get $\delta_i^* = 0$. ■

Lemma 4.2.3 *If $t_i^{sc*} + t_i^{b*} = \ell_i$, $t_i^{b*} \neq 0$ and $p_{1i}^{b*} \neq 0$, then $p_i^{sc*} + p_{1i}^{b*} = p_{2i}^{b*} \geq p^*$.*

Proof: By (4.30), we have $\rho_{1i} = \rho_{2i} = \rho_{3i} = 0$. From (4.22)-(4.23), we have $\sum_{j=i}^{N} \nu_i = \frac{1}{1+p_i^{sc*}+p_{1i}^{b*}} = \frac{1}{1+p_{2i}^{b*}}$. The second equality will be satisfied only when $p_i^{sc*} + p_{1i}^{b*} = p_{2i}^{b*}$. Next, since $t_i^{sc*} + t_i^{b*} = \ell_i$, from slackness condition in (4.31), $z_i \geq 0$. Also, from (4.21), we have $\sum_{j=i}^{N} \lambda_i - \sum_{j=i}^{N-1} \mu_i = \frac{1}{1+p_i^{sc*}+p_{1i}^{b*}}$. Using these together in (4.25), and by the fact that $z_i \geq 0$, we get $\log\left(1 + p_i^{sc*} + p_{1i}^{b*}\right) \geq \left(\frac{\epsilon+p_i^{sc*}+p_{1i}^{b*}}{1+p_i^{sc*}+p_{1i}^{b*}}\right)$, which will be satisfied only when $p_i^{sc*} + p_{1i}^{b*} \geq p^*$ where p^* is the threshold power level. ■

Lemmas 4.2.1-4.2.3 provide important properties of the optimal power allocation in the presence of additive processing cost ϵ. In particular, we first determine a threshold power level p^* based only on ϵ, and determine the energy flow in time accordingly. In view of these properties, we continue our analysis for fixed $\delta_i = 0$ case in the following section. If the resulting power sequences are consistent with the optimality constraints, then we stop. Otherwise, we allow energy transfer from the SC to the battery using some additional steps.

4.2.3 Optimal Policy for Fixed $\delta_i = 0$

For fixed $\delta_i = 0$, the problem at hand reduces to throughput maximization under causality and no-energy-overflow constraints. Thus, we have the following problem:

$$\begin{aligned} \max_{\alpha_i,\beta_i,\theta_i,t_i^{sc},t_i^b \geq 0} \quad & \sum_{i=1}^{N} \frac{t_i^{sc}}{2} \log\left(1 + \frac{\alpha_i}{t_i^{sc}} + \frac{\theta_i}{t_i^{sc}}\right) + \frac{t_i^b}{2} \log\left(1 + \frac{\beta_i}{t_i^b}\right) \\ \text{s.t.} \quad & (4.14) - (4.18) \\ & \delta_i = 0, \ \ \forall i \end{aligned} \tag{4.32}$$

Parallel to Lemma 3.2.5, we next show in the following lemma that the solution of (4.32) is found by applying the directional glue-pouring algorithm in [27] only twice.

Lemma 4.2.4 *For fixed $\delta_i = 0$, let $\hat{p}_i^{sc}$ and $\hat{t}_i^{sc}$ be the outcome of directional glue-pouring given $p_i^b = 0$. Let $\hat{p}_{1i}^b$, $\hat{p}_{2i}^b$ and $\hat{t}_i^b$ be the outcome of directional glue-pouring* *given $\hat{p}_i^{sc}$ and no processing cost from the battery over the first $\hat{t}_i^{sc}$ time units. Then, $\hat{p}_i^{sc}, \hat{p}_{1i}^b, \hat{p}_{2i}^b, \hat{t}_i^{sc}$ and $\hat{t}_i^b$ are jointly optimal for (4.32).*

We provide the proof of Lemma 4.2.4 in Appendix 4.4.1. We present an illustration of the two iterations of directional glue-pouring algorithm in Fig. 4.1, where blue and red glues represent energies in the SC and the battery, respectively. In this example, $E_i^{sc} = E_{max}$ in epochs 1, 4 and 5. In the upper two figures in Fig. 4.1, we show the first directional glue-pouring where $\hat{p}_i^{sc}$ and $\hat{t}_i^{sc}$ are obtained given $p_{1i}^b = 0, p_{2i}^b = 0$. Note that if epoch length is sufficiently large, p_i^{sc*} is kept at the threshold level p^* as long as possible and is set to zero for the rest of the epoch. In the second iteration, $\hat{p}_i^{sc}$ and $\hat{t}_i^{sc}$ are fixed and we pour ηE_i^b on top of these power levels. We note that the second iteration of the directional glue-pouring algorithm is a generalized version of the one in [27] in that the processing cost drained from the battery in the initial t_i^{sc} time units of each epoch i is zero. As a result of the second iteration, we obtain $\hat{p}_{1i}^b, \hat{p}_{2i}^b$ and $\hat{t}_i^b$. These two iterations yield an optimal allocation for (4.32).

4.2.4 Determining Optimal δ_i^*

In the previous section, we have seen that for $\hat{p}_i^{sc}, \hat{p}_{1i}^b, \hat{p}_{2i}^b, \hat{t}_i^{sc}$ and $\hat{t}_i^b$, there exist Lagrange multipliers λ_i, μ_i, ν_i, ρ_{1i}, ρ_{2i} and ρ_{3i} that satisfy (4.21) - (4.26), but it is not clear if there exist ξ_i that satisfy (4.24). In this section, we propose a method to update $\hat{p}_i^{sc}$, $\hat{p}_{1i}^b$, $\hat{p}_{2i}^b$, $\hat{t}_i^{sc}$, $\hat{t}_i^b$ and corresponding Lagrange multipliers so that we obtain ξ_i and δ_i^* such that (4.21)-(4.26) are satisfied. For brevity and clarity of explanation,

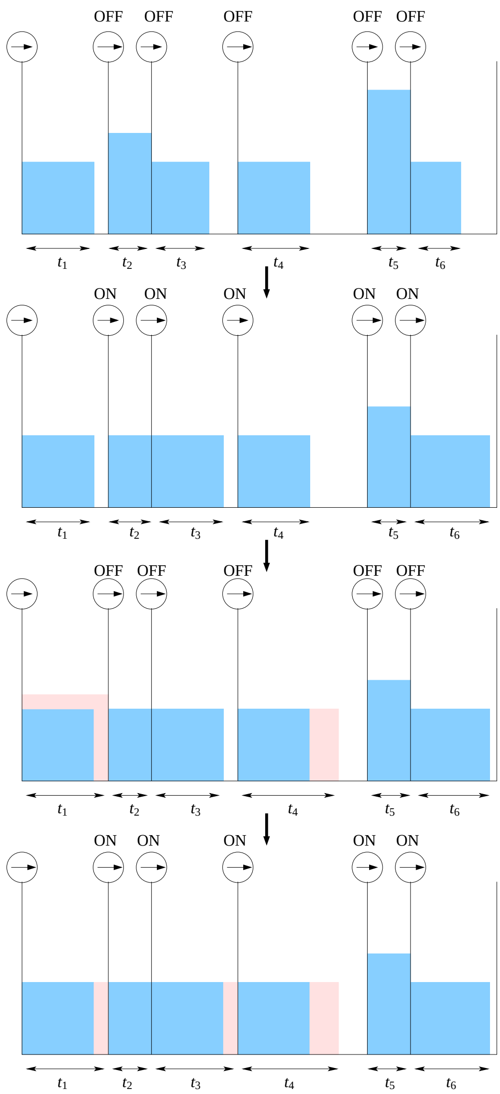

Figure 4.1: Optimal allocation with $\delta_i = 0$.

we assume without loss of generality that p_i^{sc} is higher than the threshold level p^* for $i = 1$ and equal to p^* for $i = 2, \ldots, N$.

By Lemmas 4.2.1 and 4.2.2, $\delta_i^* = 0$ if $t_i^{sc*} < \ell_i$ or $p_{1i}^{b*} > 0$. Indeed, $\delta_i > 0$, only if $\hat{p}_i^{sc} > p^*$. Thus, we first consider to update the values of z_i for those epochs where $z_i = 0$. The energy for these epochs comes from those previous epochs where $z_i \geq 0$ and $p_{1i}^{b*} = 0$.

In order to find δ_i^*, we transform the energy and water levels of isolated epochs as in Section 3.2.2. We set the bottom levels of epochs with $\hat{p}_i^{sc} > p^*$ and for the remaining epochs, we set the bottom level to $\frac{1}{\eta}$ and multiply the water level by $\frac{1}{\eta}$. In this transformed setting, if the water level is higher in an epoch where $\hat{p}_i^{sc} > p^*$ compared to the next epoch, then we transfer δ_i units of water from the SC in this epoch and $\frac{1}{\eta}\delta_i$ units of water is added to the battery in the next epoch. This way, we transfer the energy in a systematic way. In the particular case when $p_1^{sc} > p^*$ and $p_i^{sc} = p^*$ for $i = 2, \ldots, N$, energy is transferred from epoch 1 to epochs $i = 2, \ldots, N$. Note that $z_1 > 0$ and $z_i = 0$ for $i = 2, \ldots, N$ for this particular allocation. When energy is transferred, λ_i, μ_i are increased and ν_i remains unchanged until $\hat{t}_i^{sc} + t_i^b = \ell_i$ provided that sufficiently large energy is transferred. If the water level in epoch 1 is still higher than p^*, we start transferring energy to the next epoch in the transformed setting. We also note that the transferred energy can be utilized in later epochs as long as the power is kept at p^* and hence the optimal allocation is not unique. Once $t_i^{sc} + t_i^b = \ell_i$, we have to make $z_i > 0$ due to slackness condition (4.30) and raise the transmit power levels p_{1i}^b above zero and p_{2i}^b above the threshold level p^*.

We also note that if the power level of epoch 1 is lower than that in other epochs in the transformed setting, then $\delta_i = 0, i = 1, \ldots, N$. Example of such a scenario is shown in Fig. 4.2. Even though p_i^{sc} is higher than p^*, the water level in epoch 1 is lower than those of other epochs in the transformed settings. Therefore, there is no transfer from SC to the battery in this scenario.

Once $t_i^{sc} + t_i^b = \ell_i$ for all i, if the water level in epoch 1 is still higher than the levels in other epochs, we continue transferring energy. However, resulting water

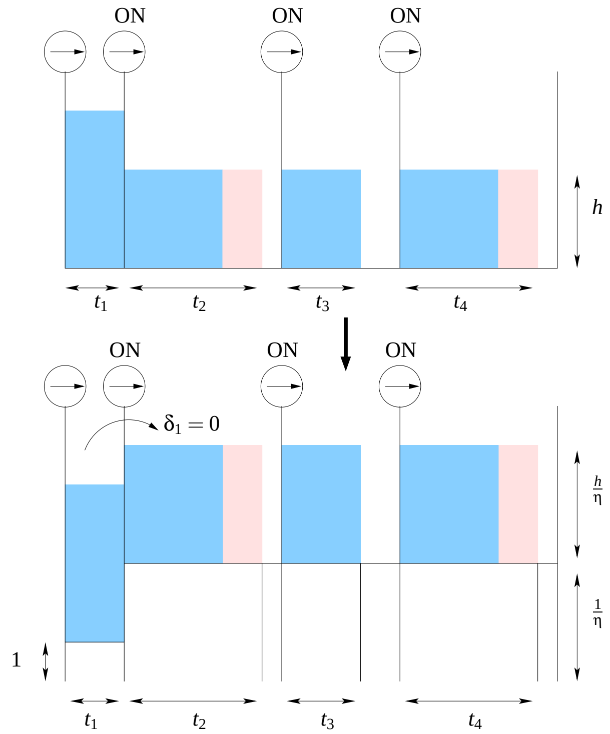

Figure 4.2: Determining δ_i in the transformed setting.

levels are now determined by classical directional water-filling [14] over the whole epoch length ℓ_i since no additional processing cost is incurred.

Finally, we note that when energy is transferred from SC to the battery in epoch i, it spreads to the future epochs $i+1, \dots, N$. The energy level in some epochs may go above the threshold level p^* as a result of this transfer. On the other hand, some energy that was already transferred may have to flow back to the battery in epochs $j < i$, resulting in a two-way flow of energy within the storage elements. To keep track of the amount of energy transferred in both directions, we measure the flow of energy across each epoch by means of meters and negate any energy that flows backward. An example of this backflow is illustrated in Fig. 4.3 where energy is transferred from epoch 1 to 2 and from epoch 3 to 5. The meters

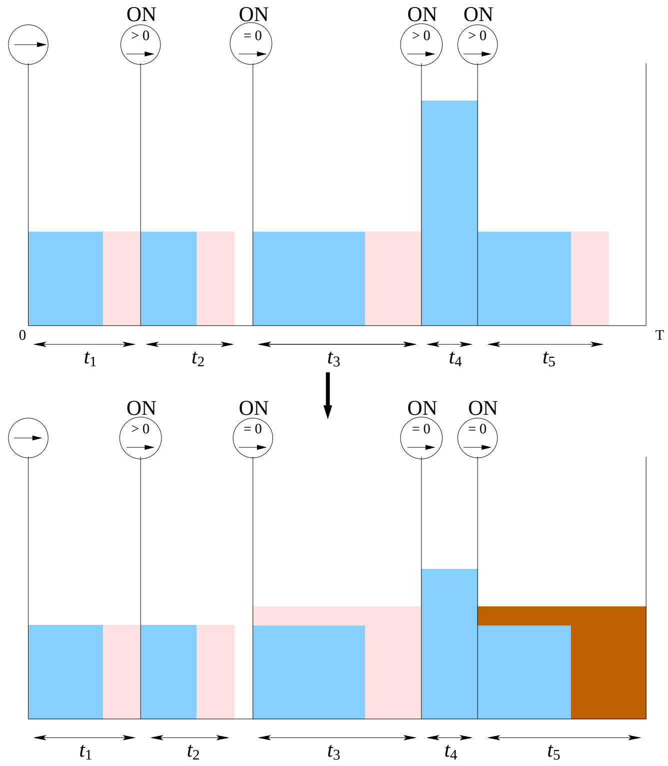

Figure 4.3: Demonstration of water backflow and energy meters.

across these epochs have positive values. When energy is transferred from the SC to the battery in epoch 4, it causes energy to flow back, and meters show zero value.

4.3 Conclusion

In this chapter, we considered the throughput maximization in an energy harvesting communication system using a hybrid energy storage system and an overhead due to processing power of the system circuitry. Utilizing the offline nature of the problem, transmission policies are described under the directional glue-pouring algorithm. It has been shown that the problem can be solved by repeated application of the directional glue-pouring algorithm, which is an extension of the directional water-filling algorithm due to the effect of a constant power consumption by system circuitry. The energy management in two different storage devices poses a different challenge and generalizes the single-stage directional glue-pouring algorithm of [27]. The solution provides insights on how the spread of energy in time is restricted due to processing power cost, maintaining a constant power level, subject to energy causality and storage capacity constraints.

4.4 Appendix

4.4.1 Proof of Lemma 4.2.4

The proof of Lemma 4.2.4 is in similar lines to that of Lemma 3.2.5. Consider the first directional glue-pouring that yields $\hat{p}_i^{sc}$, $\hat{t}_i^{sc}$, λ_i, μ_i and z_i such that

$$-\frac{t_i^{sc}}{t_i^{sc}+\alpha_i+\theta_i}+\sum_{j=i}^{N}\lambda_j-\sum_{j=i}^{N-1}\mu_j=0 \tag{4.33}$$

$$\frac{\alpha_i+\theta_i}{t_i^{sc}+\alpha_i+\theta_i}-\log\left[\frac{t_i^{sc}+\alpha_i+\theta_i}{t_i^{sc}}\right]+\epsilon\left[\sum_{j=i}^{N}\lambda_j-\sum_{j=i}^{N-1}\mu_j\right]+z_i=0 \tag{4.34}$$

Note that $\rho_{1i}=\sigma_{1i}=0$. (4.33) is equivalent to

$$-\frac{1}{1+p_i^{sc}}+\sum_{j=i}^{N}\lambda_j-\sum_{j=i}^{N-1}\mu_j=0 \tag{4.35}$$

Similarly, we can rearrange (4.34) to get

$$z_i=\log\left(1+\hat{p}_i^{sc}\right)-\left(\frac{\hat{p}_i^{sc}+\epsilon}{\hat{p}_i^{sc}+1}\right) \tag{4.36}$$

Thus, if $z_i>0$ we have $\log\left(1+\hat{p}_i^{sc}\right)>\left(\frac{\hat{p}_i^{sc}+\epsilon}{\hat{p}_i^{sc}+1}\right)$, while for $z_i=0$, we have

$$\log\left(1+\hat{p}_i^{sc}\right)=\left(\frac{\hat{p}_i^{sc}+\epsilon}{\hat{p}_i^{sc}+1}\right) \tag{4.37}$$

We note that for epochs with $z_i=0$, $\hat{p}_i^{sc}=p^*$ since (4.37) is equivalent to (4.4).

If $z_i>0$, we have $\hat{p}_i^{sc}\geq p^*$. Let i_n be the epoch indices such that $E_{i_n}^{sc}=E_{max}$. We note that the directional glue-pouring determines the energy allocation between the epochs i_n and i_{n+1} independent of the previous epochs. In the following, we consider the epochs between i_n and i_{n+1}. This causes no loss of generality as verified in the proof of Lemma 3.2.5. For epochs i between i_n and i_{n+1}, we have $\lambda_i \mu_i = 0$ and in particular, $\lambda_i > 0$ if $p_i^{sc} > p_{i-1}^{sc}$ and $\mu_i > 0$ if $p_i^{sc} < p_{i-1}^{sc}$.

In the second directional glue-pouring, $\hat{p}_i^{sc}$ and $\hat{t}_i^{sc}$ are given and the outcomes are $\hat{p}_{1i}^b$, $\hat{p}_{2i}^b$, $\hat{t}_i^b$, ν_i, ρ_{2i} and ρ_{3i} such that

$$-\frac{t_i^{sc}}{t_i^{sc} + \alpha_i + \theta_i} + \sum_{j=i}^{N} \nu_j - \rho_{2i} = 0, \quad \forall i \tag{4.38}$$

$$-\frac{t_i^b}{t_i^b + \beta_i} + \sum_{j=i}^{N} \nu_j - \rho_{3i} = 0, \quad \forall i \tag{4.39}$$

$$\frac{\beta_i}{t_i^b + \beta_i} - \log\left[\frac{t_i^b + \beta_i}{t_i^b}\right] + \epsilon \sum_{j=i}^{N} \nu_j - \sigma_{2i} + z_i = 0, \quad \forall i \tag{4.40}$$

After the second directional glue-pouring, λ_i, μ_i and z_i no longer satisfy (4.33) and (4.34) in general. We next argue that there exist $\bar{\lambda}_i$, $\bar{\mu}_i$ and $\bar{z}_i$ that satisfy (4.33)-(4.34) along with the slackness conditions (4.27)-(4.31). As in the proof of Lemma 3.2.5, existence of such $\bar{\lambda}_i$, $\bar{\mu}_i$ and $\bar{z}_i$ is sufficient to prove optimality of the claimed allocation.

Rearranging (4.38)-(4.39), we have $\frac{1}{1+\hat{p}_i^{sc}+\hat{p}_{1i}^b} = \sum_{j=i}^N \nu_j - \rho_{2i}$ and $\frac{1}{1+\hat{p}_{2i}^b} = \sum_{j=i}^N \nu_j - \rho_{3i}$ where ρ_{2i} and ρ_{3i} are determined according to the slackness condition in (4.30). If $\hat{t}_i^{sc} + \hat{t}_i^b < \ell_i$, then $\hat{p}_{1i}^b = 0$, $\hat{p}_{2i}^b = p^*$ and hence $z_i = 0$ satisfies both (4.34) and (4.40). On the other hand, if $\hat{t}_i^{sc} + \hat{t}_i^b = \ell_i$ and $\hat{p}_{1i}^b > 0$, $\hat{p}_{2i}^b > p^*$, then by (4.38)-(4.39), we have $\hat{p}_i^{sc} + \hat{p}_{1i}^b = \hat{p}_{2i}^b$. We update z_i according to (4.40) and obtain $\bar{z}_i$. As $\hat{p}_i^b$ can be selected arbitrarily when $\hat{t}_i^b = 0$, after the second directional glue-pouring, the power levels satisfy the following relations at each epoch:

$$\hat{p}_i^{sc} + \hat{p}_{1i}^b = \hat{p}_{2i}^b = \begin{cases} \max\left(\frac{1}{\sum_{j=i}^N \lambda_j - \sum_{j=i}^{N-1} \mu_j}, \frac{1}{\sum_{j=i}^N \nu_j}\right) - 1 > p^*, & \bar{z}_i > 0 \\ \frac{1}{\sum_{j=i}^N \lambda_j - \sum_{j=i}^{N-1} \mu_j} - 1 = \frac{1}{\sum_{j=i}^N \nu_j} - 1 = p^*, & \bar{z}_i = 0 \end{cases} \tag{4.41}$$

$$= \begin{cases} \frac{1}{\sum_{j=i}^N \bar{\lambda}_j - \sum_{j=i}^{N-1} \bar{\mu}_j} - 1 > p^*, & \bar{z}_i > 0 \\ \frac{1}{\sum_{j=i}^N \bar{\lambda}_j - \sum_{j=i}^{N-1} \bar{\mu}_j} - 1 = \frac{1}{\sum_{j=i}^N \nu_j} - 1 = p^*, & \bar{z}_i = 0 \end{cases} \tag{4.42}$$

where, if $\hat{p}_{i-1}^{sc} + \hat{p}_{1(i-1)}^b < \hat{p}_i^{sc} + \hat{p}_{1i}^b$:

$$\bar{\lambda}_{i-1} = \frac{1}{1 + \hat{p}_{i-1}^{sc} + \hat{p}_{1(i-1)}^b} - \frac{1}{1 + \hat{p}_i^{sc} + \hat{p}_{1i}^b}, \quad \bar{\mu}_{i-1} = 0 \tag{4.43}$$

If $\hat{p}_{i-1}^{sc} + \hat{p}_{1(i-1)}^b > \hat{p}_i^{sc} + \hat{p}_{1i}^b$:

$$\bar{\mu}_{i-1} = \frac{1}{1 + \hat{p}_i^{sc} + \hat{p}_{1i}^b} - \frac{1}{1 + \hat{p}_{i-1}^{sc} + \hat{p}_{1(i-1)}^b}, \quad \bar{\lambda}_{i-1} = 0 \tag{4.44}$$

and $\bar{\lambda}_{i_n} = \frac{1}{1+\hat{p}_i^{sc}+\hat{p}_{1i}^b}$ and $\bar{\lambda}_i = \bar{\mu}_i = 0$ otherwise.

We note that if $\hat{p}_{i-1}^{sc} + \hat{p}_{1(i-1)}^b < \hat{p}_i^{sc} + \hat{p}_{1i}^b$, then $\hat{p}_{i-1}^{sc} < \hat{p}_i^{sc}$. Similarly, if $\hat{p}_{i-1}^{sc} + \hat{p}_{1(i-1)}^b > \hat{p}_i^{sc} + \hat{p}_{1i}^b$, then $\hat{p}_{i-1}^{sc} > \hat{p}_i^{sc}$. Following the same lines as in the proof of Lemma 3.2.5, we conclude that $\bar{\lambda}$ and $\bar{\mu}$ satisfy the KKT conditions (4.21)-(4.22). Moreover, with the updated $\bar{\lambda}$ and $\bar{\mu}$, $\bar{z}_i$ is also consistent with (4.34). Hence, for fixed $\delta_i = 0$, the Lagrange multipliers $\bar{\lambda}$, $\bar{\mu}$ and $\bar{z}_i$ satisfy (4.21)-(4.26) and (4.27)-(4.31). This proves the desired result.

Chapter 5

Numerical Results and Simulations

In this chapter, we numerically study the optimal offline transmission policy in the specified hybrid energy storage model. We consider an additive white Gaussian noise channel with bandwidth $W = 1$ MHz and noise spectral density $N_0 = 10^{-19}$ W/Hz. The path loss between the transmitter and the receiver is 100 dB. This results in an instantaneous rate-power relation

$$r(t) = \log\left(1 + p_{sc}(t) + p_b(t)\right) \tag{5.1}$$

where $p_{sc}(t)$ and $p_b(t)$ are the instantaneous transmit powers drained from the SC and the battery, respectively. In particular, $r(t)$ is in Mbps and p is in mW.

5.1 Deterministic Energy Arrivals

We start our numerical study with illustrations of optimal policies under deterministic energy arrivals. The SC has a storage capacity of $E_{max} = 5$ mJ. The battery has infinite storage with efficiency $\eta = 0.75$. The specific realization of energy arrivals is $E = [4, 5, 2, 3]$ mJ at times $t = [2, 3, 8, 9]$ sec. In addition, $E_0^{sc} = 4$ mJ, $E_0^b = 0$. The deadline constraint is $T = 10$ sec. We show the energy arrivals and the resulting optimal transmission policy for this case in Fig. 5.1. Note that

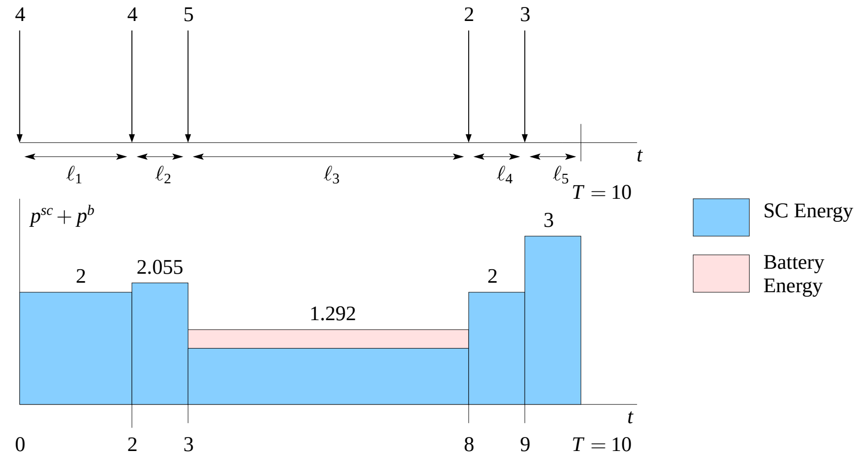

Figure 5.1: Optimal transmit powers for hybrid storage with $E = [4, 4, 5, 2, 3]$ mJ at times $t = [2, 3, 8, 9]$ sec, $\eta = 0.75$, $E_{max} = 5$ mJ and $T = 10$ sec.

in this example, the energy arrival amounts are less than E_{max} at each epoch and hence $E_i^b = 0$. However, the freedom to save energy in the battery strictly increases the throughput as it enables to spread energy in time. Specifically, in this example, the battery enables to transfer energy from epoch 2 to epoch 3 and this increases throughput. Indeed, if there was only the SC available as storage device the optimal throughput would only be 7.0385 Mbits; however, when the battery is also available, the maximum throughput is 7.1743 Mbits.

Next, we consider the effect of processing power where we fix $\epsilon = 1$ mJ/sec. The energy arrival sequence is $E = [7, 3, 5, 1, 8, 6]$ mJ at times $t = [2, 3, 5, 8, 9, 10]$ sec with initial energies $E_0^{sc} = 4$ mJ and $E_0^b = 0$. The energy arrivals and the resulting optimal transmission policy are depicted in Fig. 5.2. We note that the transmission times may be less than the corresponding epoch lengths as discussed in Section 4.2.3.

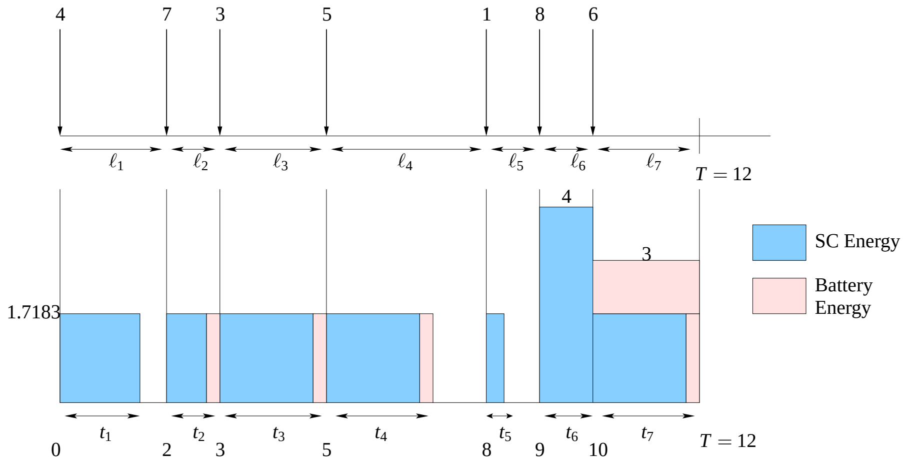

Figure 5.2: Optimal transmit powers for $E = [4, 7, 3, 5, 1, 8, 6]$ mJ at times $t = [2, 3, 5, 8, 9, 10]$ sec, $\eta = 0.75$, $E_{max} = 5$ mJ, $\epsilon = 1$ and $T = 12$ sec.

5.2 Stochastic Energy Arrivals

In this section, we consider stochastic energy arrivals. We compare the performance of the optimal offline policy with those of three heuristic event-based online policies. In particular, these policies take action only when an energy arrival event occurs.

Recall that the actions of the transmitter are determining the portions of the incoming energy saved in the SC and the battery and the transmit power. In our analysis of the optimal offline policies, we equivalently considered fixing the portions of energies allocated to the SC and the battery first and then transferring energy from the SC to the battery. In the online policies, we no longer use this formulation. We note that for optimal operation, an online policy has to first fill the space in the SC due to its perfect storage efficiency and then save the remaining energy in the battery. Due to the same reason, power must be drained from the SC first and then from the battery if the energy in the SC is run out. This way, the space available in the SC for future energy arrivals is maximized. Hence, specifying the total power level $p = p^b + p^{sc}$ at each time is sufficient to describe the online policy. Without losing optimality, we can restrict the policies to satisfy $p^{sc}(t)p^b(t) = 0$. Note that optimal policies for the offline throughput maximization problem with and without processing cost are not unique. The ones with the specified energy storage strategy and $p^{sc}(t)p^b(t) = 0$ are just a class of optimal policies which is different from the optimal policy that is found by the application of directional water-filling. Indeed, this class of optimal policies are easier to analyze in the online regime.

1) Constant Power Policy: This policy transmits with a constant power equal to the average recharge rate, $\mathbb{E}[E_i]$. The transmission continues until the hybrid storage unit runs out of energy. This policy uses the mean value of the energy arrival process.

2) Energy Adaptive Transmission Policy: This policy transmits with power equal to the instantaneously available energy at each energy arrival instant, i.e. $p_i = E_{current}$. Note that available energy is the sum of energies in the SC and the battery: $E_{current} = E^{sc} + \eta E^b$. Similar to the constant power policy, the transmitter remains active as long as the power level p_i can be maintained and otherwise it is silent.

3)Time-Energy Adaptive Transmission Policy: A variant of the energy adaptive transmission policy is obtained by adapting the transmission power to the total energy level and the time remaining till the deadline T. The power level is determined by $p_i = \frac{E_{current}}{T - s_i}$, where s_i is the time of the most recent energy arrival.

We also consider comparing the performances of the policies with upper bounds.

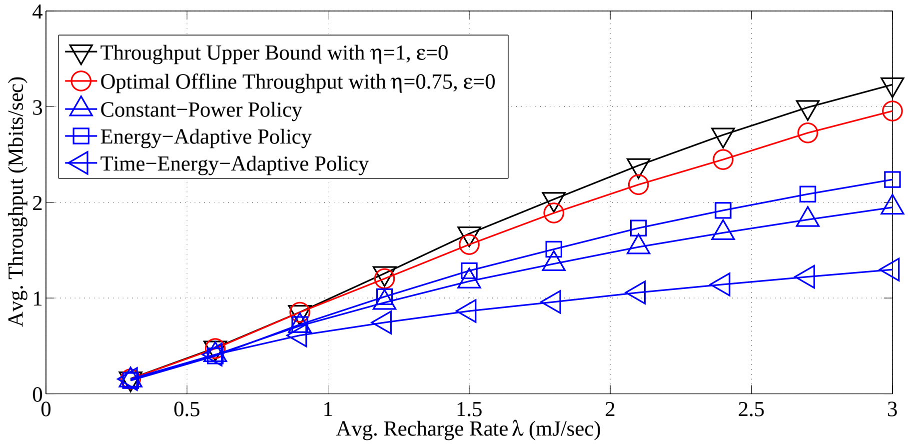

Figure 5.3: Performances of the proposed policies for varying energy arrival rates λ with $\eta = 0.6$, $E_{max} = 2$ mJ and $T = 10$ sec.

In the no processing energy case, we consider the offline optimal throughput when the battery efficiency is $\eta = 1$ as an upper bound. Note that this is essentially the offline optimal throughput with an infinite storage, whose solution is known due to [12]. In the nonzero processing energy case, we consider the offline throughput with zero processing energy as an upper bound.

We select the energy arrivals as a compound Poisson process with uniform density f_e over the interval $[0, 2P_{avg}]$ where P_{avg} is the average power. We perform simulations for 500 randomly generated realizations of the energy arrivals. The rate λ_e of the Poisson marking process is taken to be 1 mark per second so that the average recharge rate $\mathbb{E}[E_i]$ is equal to P_{avg} throughout the simulations.

We start by examining the performance of the hybrid storage system with zero processing cost. We simulate different scenarios by varying the energy arrival rate, battery efficiency and transmission deadline constraint. As a baseline, we choose the storage capacity of SC as $E_{max} = 1$ mJ, the battery efficiency as $\eta = 0.6$ and the deadline constraint as $T = 10$ sec. We vary these values as necessary. In Fig. 5.3, we show the average throughput with respect to the average recharge rate P_{avg}. We observe monotone increases in the performances of the policies as energy recharge rate increases. Note that the optimal offline throughput with the inefficient battery is close to the upper bound, i.e., the optimal offline throughput with perfectly efficient battery. Hence, the loss incurred due to inefficiency of the battery is relatively small in the offline regime. On the other side, we observe that the constant power policy performs worse compared to the energy adaptive policy in the online regime. Note that energy adaptive policy is viewed as an inferior policy in [14] as it cannot properly spread the energy for future use; however, in the hybrid energy storage setting this policy performs well in the online regime. The constant power policy also suffers from low SC storage capacity and hence cannot spread the energy properly compared to higher storage capacities studied in [14]. Similar comparisons are made in Figs. 5.4 and 5.5 with respect to varying battery efficiency and deadline. Note that time-energy adaptive performs well in small deadlines; however, as the deadline is increased the loss incurred due to saving energy in the battery significantly deteriorates its performance. We also remark that other online policies retain an almost constant throughput with regard to varying deadline.

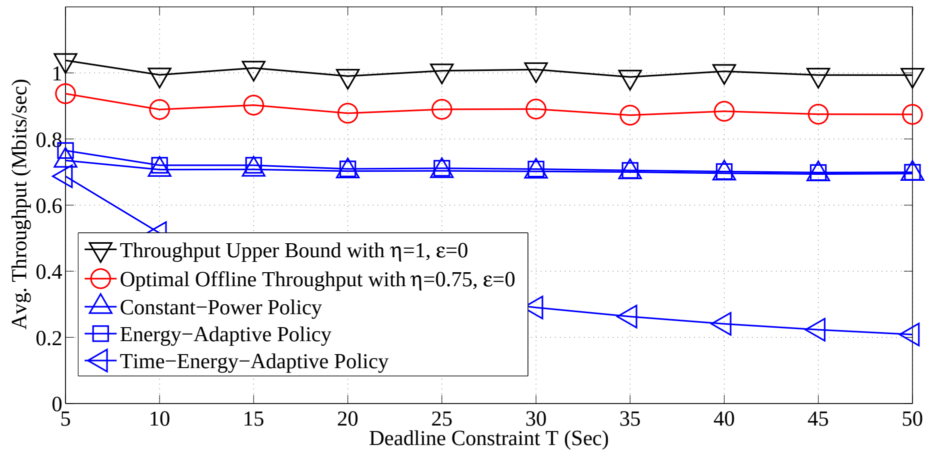

Figure 5.4: Performances of the proposed policies for varying transmission deadline constraint T with $\eta = 0.6$, $E_{max} = 1$ mJ and $\lambda = 1$ mJ/sec.

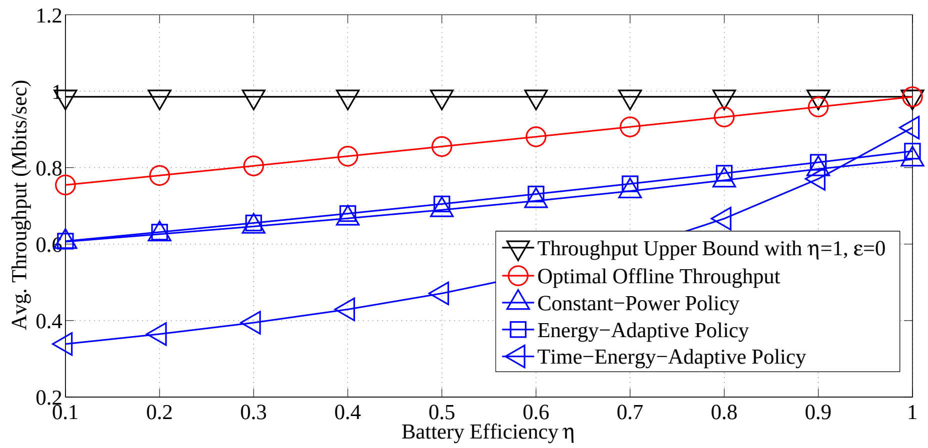

Figure 5.5: Performances of the proposed policies for varying battery efficiency η with $E_{max} = 1$ mJ, $\lambda = 1$ mJ/sec and $T = 10$ sec.

Next, we consider the average throughput performances of the transmission policies with hybrid energy storage and processing cost $\epsilon = 1$ mJ/sec. We obtain performance comparisons of the policies with respect to varying energy recharge rate and transmission deadline constraints and present resulting plots in Figs. 5.6

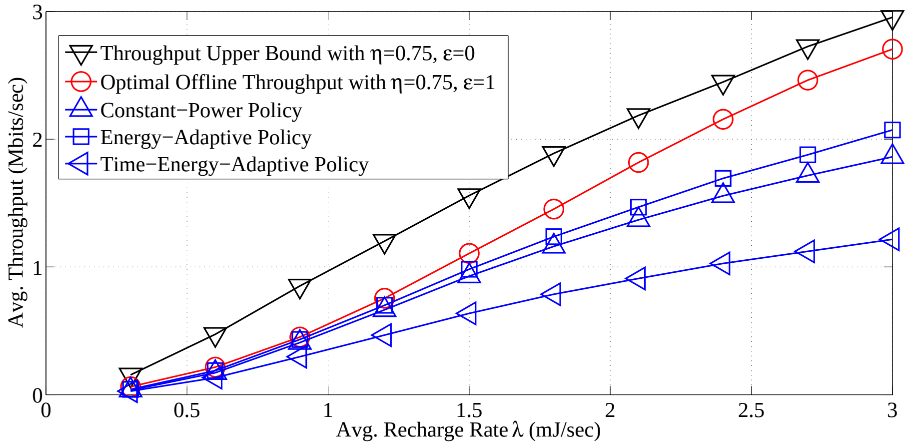

Figure 5.6: Performances of the proposed policies for varying energy arrival rates λ with non-ideal processing power using $\eta = 0.6$, $E_{max} = 2$ mJ, $\epsilon = 1$ mJ/sec and $T = 10$ sec.

and 5.7. Moreover, we compare the performances of the policies with another upper bound, which is the optimal offline throughput with zero processing energy. Note that plots of this upper bound indeed match those of the optimal offline throughput in the zero processing energy in Figs. 5.3 and 5.4. We observe that processing cost significantly diminishes the throughput particularly in the high energy arrival regime. The significant performance loss of time-energy adaptive policy as deadline increases in the zero processing cost case is observed as a milder performance loss in the presence of processing cost. Moreover, remaining online policies still retain an almost constant throughput as the deadline is increased in the presence of processing energy.

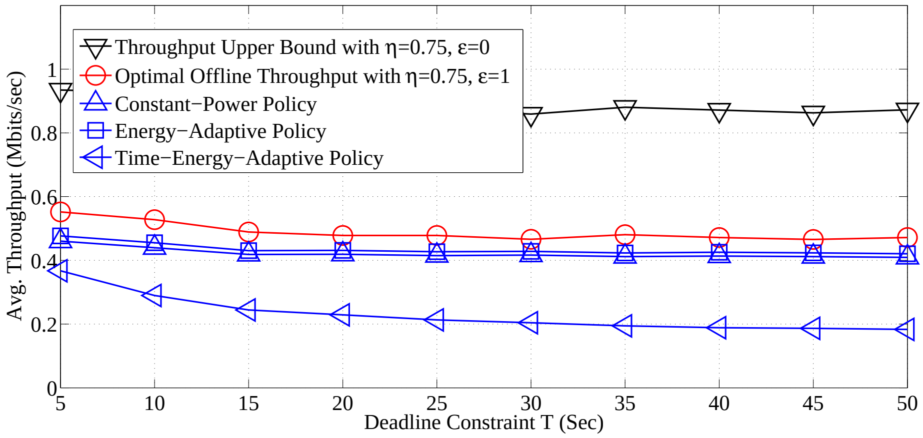

Figure 5.7: Performances of proposed policies for varying deadline constraint T with non-ideal processing power using $\eta = 0.6$, $E_{max} = 1$ mJ, $\epsilon = 1$ mJ/sec and $\lambda = 1$ mJ/sec.

Chapter 6

Conclusions

In this thesis, we consider optimal transmission schemes for energy harvesting transmitters in wireless communication. The transmitter is equipped with a hybrid energy storage unit, using a combination of a super-capacitor (SC) and a battery.

We first solve the optimization problem of maximizing the system throughput in an AWGN channel under causality and storage capacity constraints, as imposed by the energy harvesting profile. We analyze internal energy dynamics and their adjustment in the hybrid energy storage. Since the SC suffers from a storage capacity limitation while battery has a low power density, the combination of both these elements offers better performance from a practical point of view. Although the energy variables in both storage elements are highly coupled in this situation, we show that the optimal solution is found by repeated application of the directional water-filling algorithm. We generalize the single stage energy storage scenario and observe how energy is spread in time subject to the aforementioned energy constraints.

Next, we extend the hybrid energy storage setup to include the processing power consumed by the system circuitry. The processing power is assumed to be constant and independent of the transmit power level. We observe that in presence of a constant processing cost, the optimal transmission scheme is bursty in nature and the directional water-filling algorithm gets transformed into a directional glue-pouring algorithm. The bursty nature of communication is to transmit for a certain fraction of the available time and for the rest of the time, the transmitter remains inactive. Finally, we present numerical illustrations of the optimal policies and performance comparisons with heuristic online transmission policies.

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