Interference Avoidance for Wireless Networks

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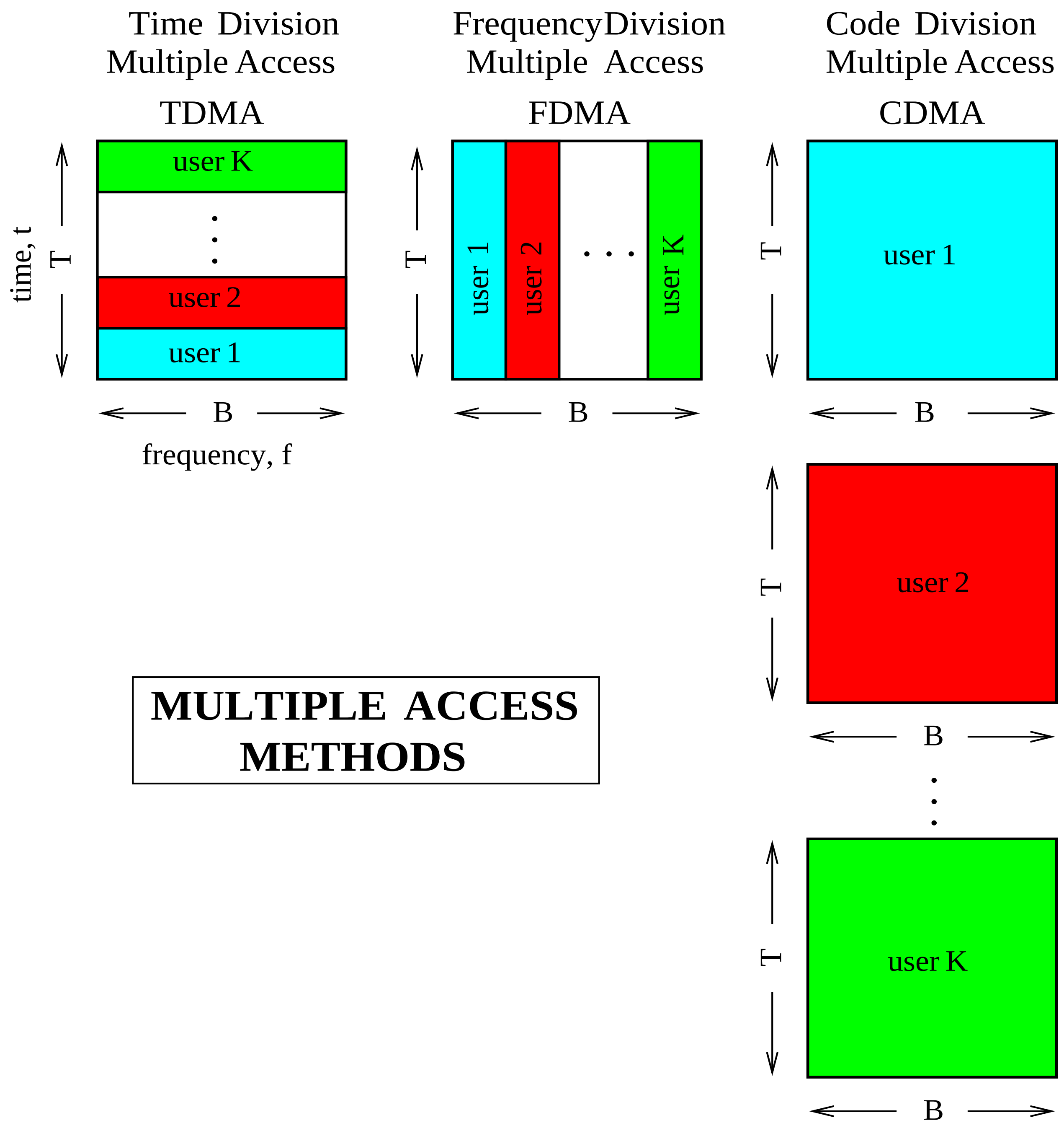
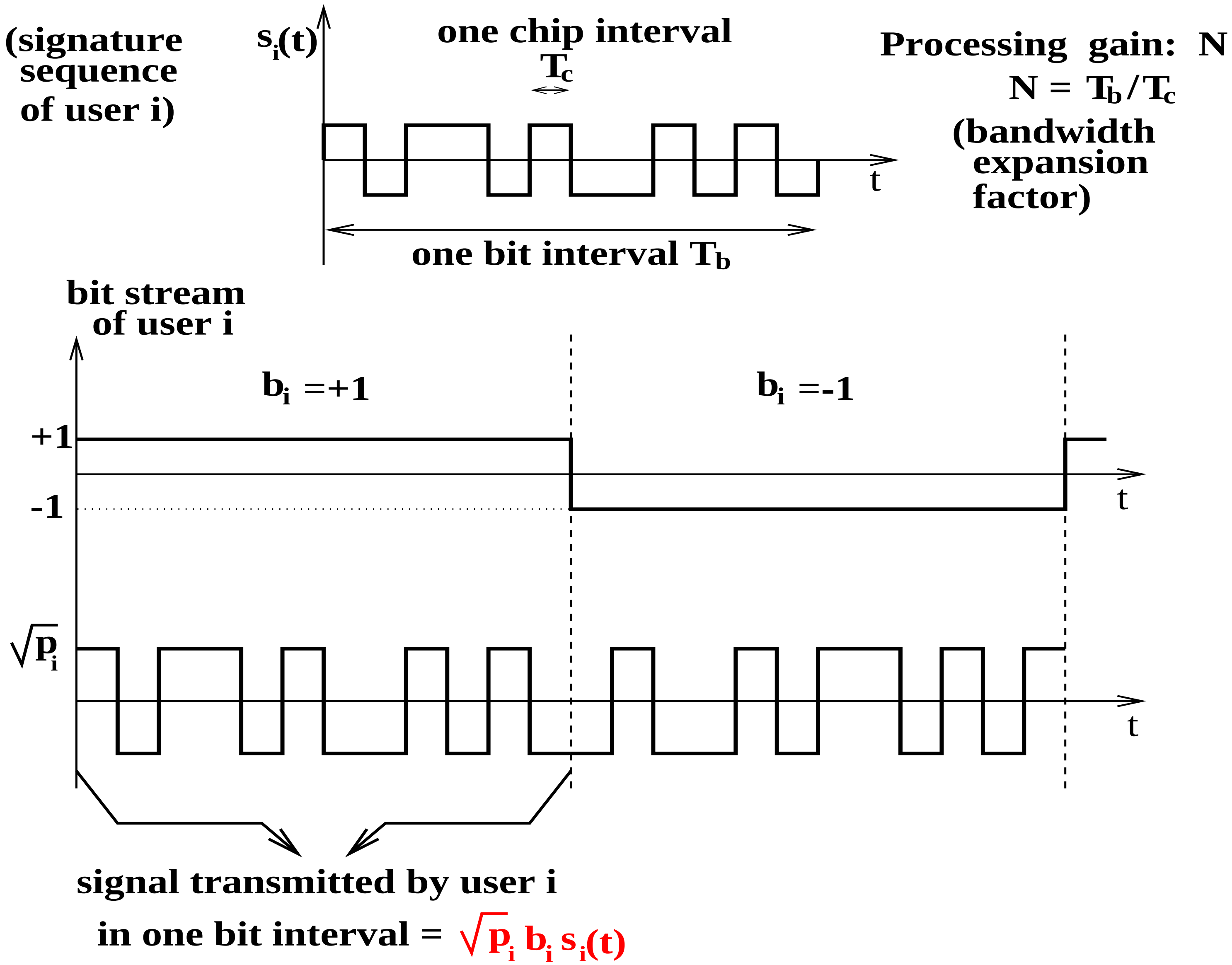
MULTIPLE ACCESS METHODS



CDMA Principles



From Continuos Signals to Signal Vectors

- Received signal in a given bit interval at the base station of user i is

$$r(t) = \sum_{j=1}^{K} \sqrt{p_j h_{ij}} b_j s_j(t) + n(t)$$

- Chip matched filtered and sampled signal

$$\mathbf{r} = \sum_{j=1}^{K} \sqrt{p_j h_{ij}} b_j \mathbf{s}_j + \mathbf{n}$$

- Detection problem:

  given the received signal $\mathbf{r}$, find out what was transmitted by each user, $\{b_1, \cdots, b_K\}$

Optimum Multiuser Detection

- Define the bit vector $\mathbf{b} = [b_1, \cdots, b_K]^\top$
- For equally likely messages, choose bits of the users according to

$$\max_{\mathbf{b}} f(\mathbf{r} \mid \mathbf{b})$$

- Jointly optimum multiuser detection (Verdú, 1984)

$$\min_{\mathbf{b} \in \{\pm 1\}^K} \mathbf{b}^\top \mathbf{H} \mathbf{b} - 2\mathbf{b}^\top \mathbf{A} \mathbf{y}$$

  where $\mathbf{H} = \mathbf{ARA}$,

  $\mathbf{R}$ is the cross correlation matrix, $R_{ij} = \mathbf{s}_i^\top \mathbf{s}_j$, and

  $\mathbf{A}$ is the (diagonal) amplitude matrix, $A_{ii} = \sqrt{p_i h_{ii}}$

- Nice convex cost function, but discrete feasible set.
- Need to check all possible 2^K bit combinations.
- The optimum multiuser detection is NP-hard (has exponential computational complexity)

Linear Multiuser Detection

- Given the received signal,

$$\mathbf{r} = \sum_{j=1}^{K} \sqrt{p_j h_{ij}} b_j \mathbf{s}_j + \mathbf{n}$$

  choose $\mathbf{c}_i$ and make bit detection according to

$$\hat{b}_i = \text{sign}(\mathbf{c}_i^\top \mathbf{r})$$

- Single-user matched filter: $\mathbf{c}_i = \mathbf{s}_i$
- Decorrelator: Choose $\mathbf{c}_i$ to $\max(\mathbf{c}_i^\top \mathbf{s}_i)^2$ while $\mathbf{c}_i^\top \mathbf{s}_j = 0$ for $j \neq i$
- MMSE receiver: Choose $\mathbf{c}_i$ to $\min E\left[(\mathbf{c}_i^\top \mathbf{r} - b_i)^2\right]$

$$\begin{aligned} \mathbf{c}_i &= \alpha_i \left(\sum_{j=1}^{K} p_j h_{ij} \mathbf{s}_j \mathbf{s}_j + \sigma^2 \mathbf{I}_N \right)^{-1} \mathbf{s}_i \\ &= \alpha_i \left(\mathbf{S}\mathbf{A}^2\mathbf{S}^\top + \sigma^2 \mathbf{I}_N \right)^{-1} \mathbf{s}_i \end{aligned}$$

Near-Far Problem

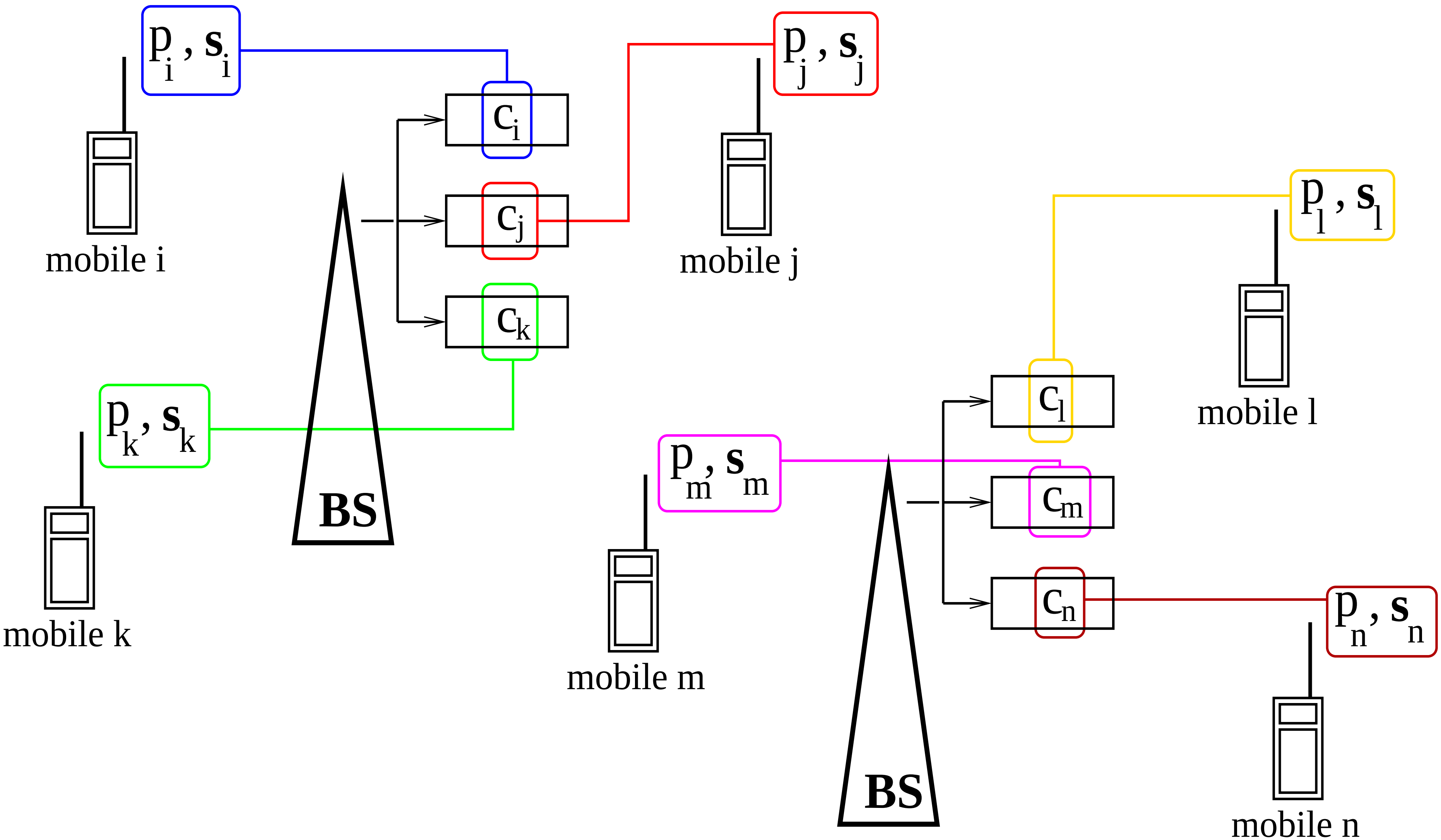
- For the ith user

$$y_i = \mathbf{c}_i^\top \mathbf{r} = \underbrace{\sqrt{p_i h_{ii}} b_i (\mathbf{c}_i^\top \mathbf{s}_i)}_{\text{desired signal}} + \underbrace{\sum_{j \neq i} \sqrt{p_j h_{ij}} b_j (\mathbf{c}_i^\top \mathbf{s}_j)}_{\text{multiaccess interference}} + \underbrace{(\mathbf{c}_i^\top \mathbf{n})}_{\text{Gaussian}}$$

- Users with high received power can degrade the quality of communication of weak users.
- Possible solutions:
  - multiuser detection: suppress the multiaccess interference and/or Gaussian noise
  - power control: balance the powers
  - signature sequence design: avoid the multiaccess interference
  - receiver beamforming: multiple antennas for diversity and interference suppression

Controllable Parameters

- Each user has a power (p_i), a signature sequence ($\mathbf{s}_i$) and a receiver filter ($\mathbf{c}_i$)



SIR Constraints

- Filter output of user *i* has signal and interference components

$$y_i = \underbrace{\sqrt{p_i h_{ii}}(\mathbf{c}_i^\top \mathbf{s}_i) b_i}_{\text{desired signal}} + \underbrace{\sum_{j \neq i} \sqrt{p_j h_{ij}}(\mathbf{c}_i^\top \mathbf{s}_j) b_j + (\mathbf{c}_i^\top \mathbf{n}_i)}_{\text{interference}}$$

- Link quality is determined by the SIR of the user

$$\text{SIR}_i = \frac{\text{Signal Power}}{\text{Interference Power}} = \frac{p_i h_{ii} (\mathbf{c}_i^\top \mathbf{s}_i)^2}{\sum_{j \neq i} p_j h_{ij} (\mathbf{c}_i^\top \mathbf{s}_j)^2 + \sigma^2 (\mathbf{c}_i^\top \mathbf{c}_i)}$$

- β_i: SIR target of user *i*.
- SIR target has a 1-to-1 relationship with the Bit Error Rate (BER)

$$\text{BER}_i \leq \varepsilon_i \qquad \Longleftrightarrow \qquad \text{SIR}_i \geq \beta_i$$

Reduction to Conventional Power Control

- In the general, SIR of user i:

$$\mathrm{SIR}_i = \frac{p_i h_{ii} (\mathbf{c}_i^\top \mathbf{s}_i)^2}{\sum_{j \neq i} p_j h_{ij} (\mathbf{c}_i^\top \mathbf{s}_j)^2 + \sigma^2 (\mathbf{c}_i^\top \mathbf{c}_i)}$$

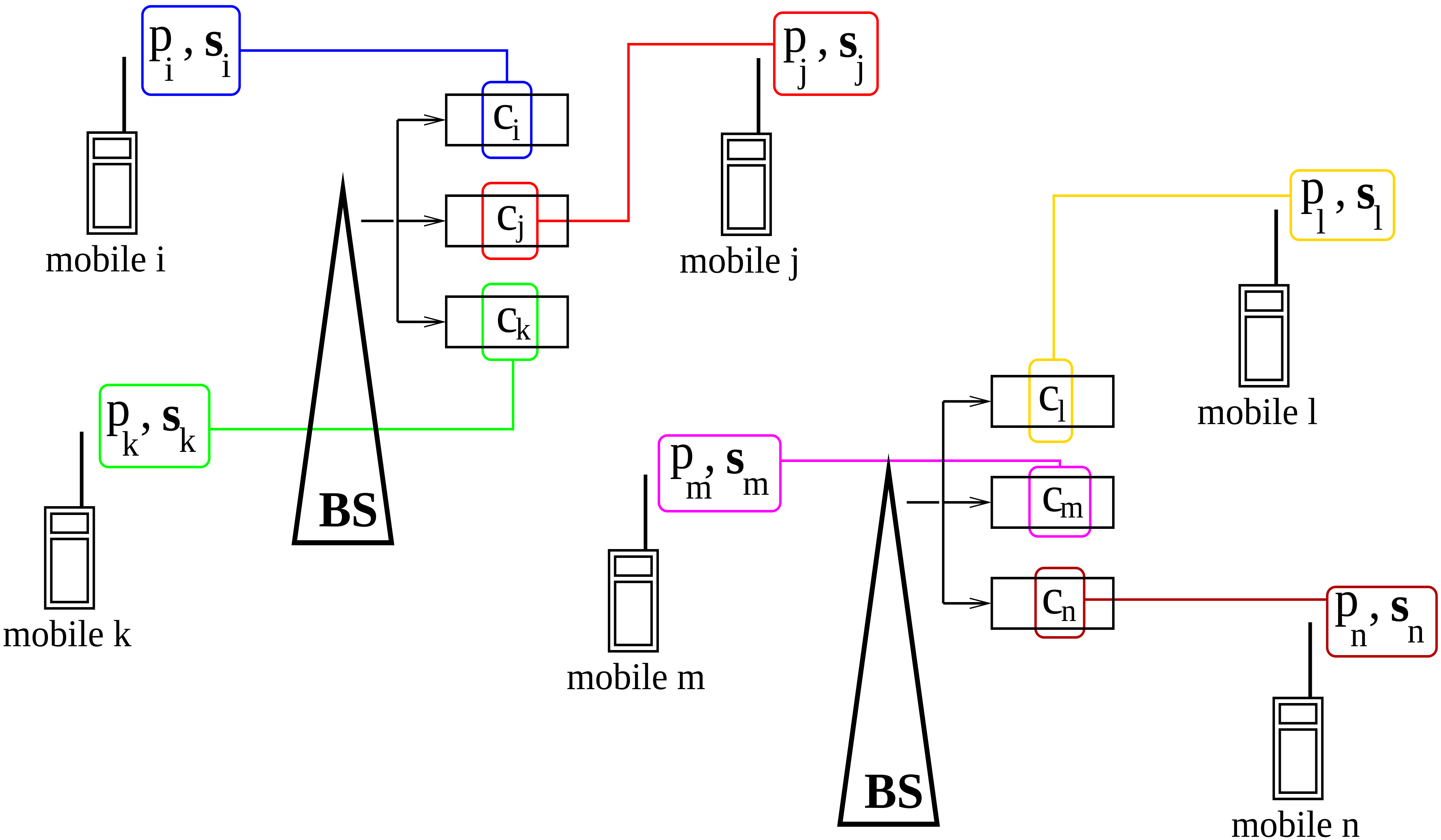
- Fix signature sequences of the users
- Then, let $\mathbf{c}_i = \mathbf{s}_i$, i.e., matched filters
- Note $\mathbf{s}_i^\top \mathbf{s}_i = 1$ and define $\rho_{ij} = \mathbf{s}_i^\top \mathbf{s}_j$

$$\mathrm{SIR}_i = \frac{p_i h_{ii}}{\sum_{j \neq i} p_j h_{ij} \rho_{ij}^2 + \sigma^2}$$

- SIR_i is a function of the power vector only

Controllable Parameters

- Each user has a power (p_i) — signature sequences and receiver filters are fixed



The Conventional Power Control Problem

- Find the *componentwise smallest* power vector such that

$$\mathrm{SIR}_i = \frac{p_i h_{ii}}{\sum_{j \neq i} p_j h_{ij} \rho_{ij}^2 + \sigma^2} \geq \beta_i \qquad\qquad i = 1, \cdots, K$$

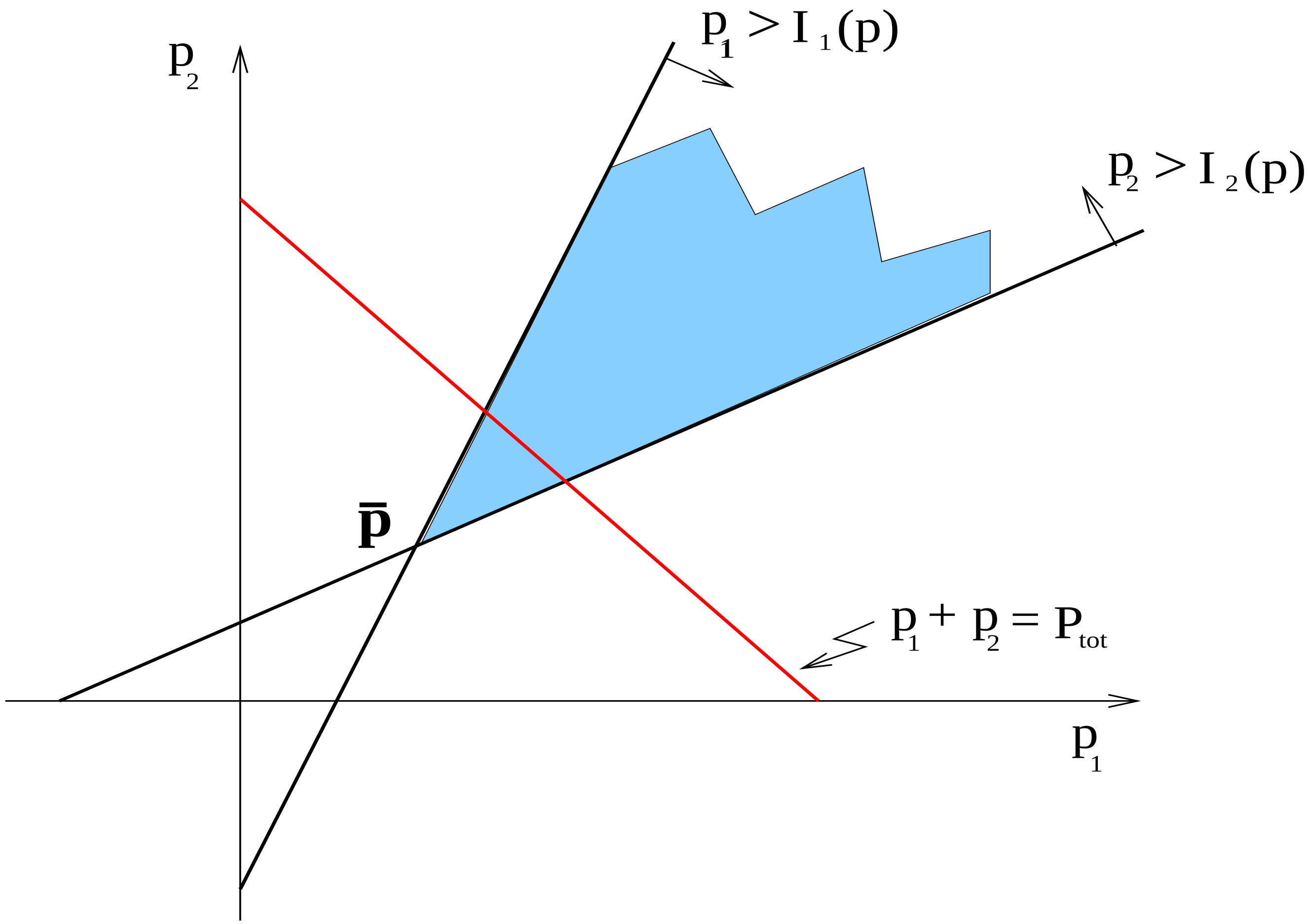
- SIR constraint for the *i*th user

$$p_i \geq \sum_{j \neq i} \frac{\beta_i h_{ij} \rho_{ij}^2}{h_{ii}} p_j + \frac{\beta_i \sigma^2}{h_{ii}} \qquad \Longleftrightarrow \qquad p_i \geq I_i(\mathbf{p})$$

- In vector notation

$$\mathbf{p} \geq \mathbf{A}\mathbf{p} + \eta \qquad \Longleftrightarrow \qquad \mathbf{p} \geq \mathbf{I}(\mathbf{p})$$

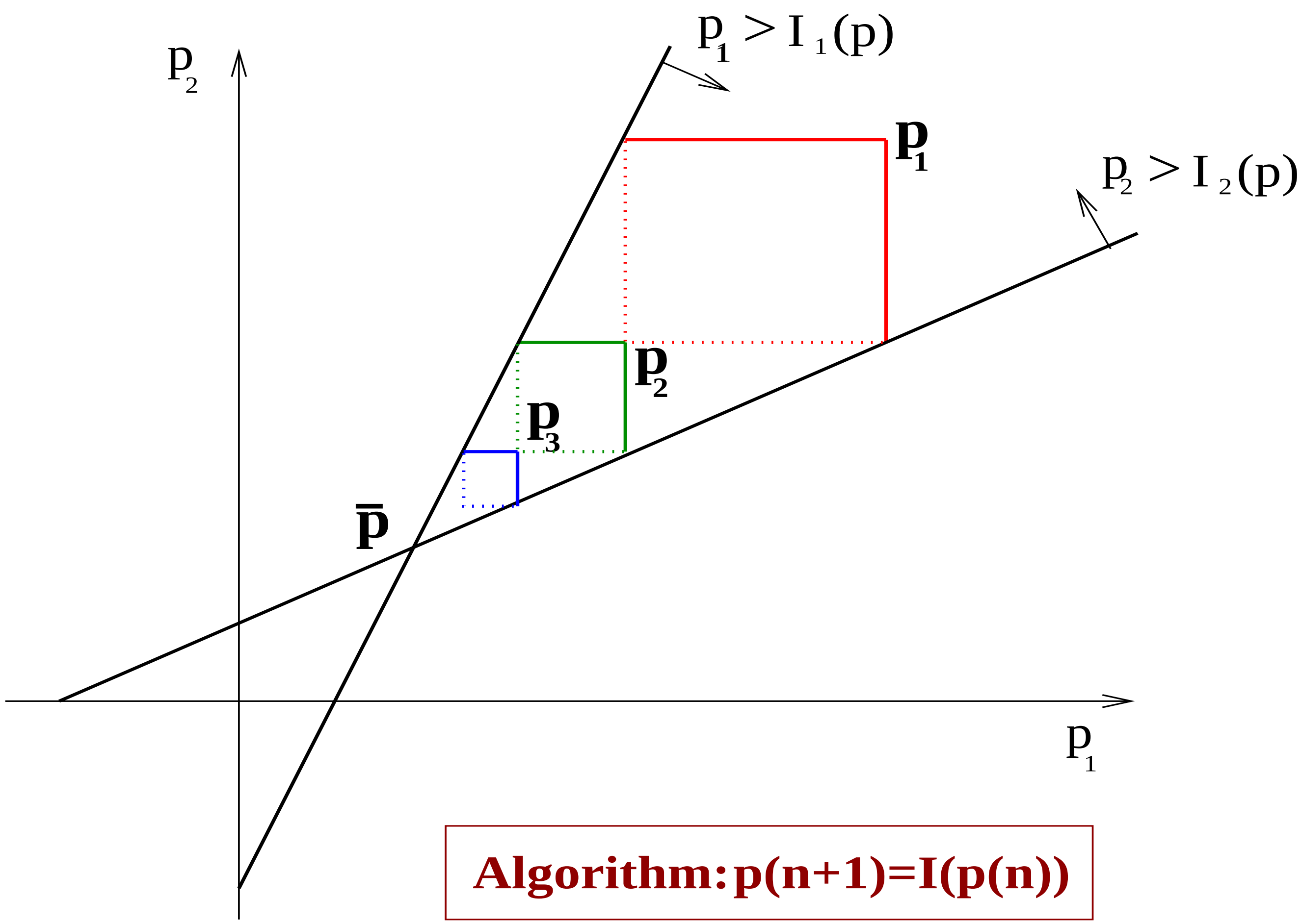
Power Control – Feasible Region



- In general optimum power vector is found at $\bar{\mathbf{p}} = \mathbf{I}(\bar{\mathbf{p}})$

$$\bar{\mathbf{p}} = \mathbf{A}\bar{\mathbf{p}} + \eta \quad \Longleftrightarrow \quad \bar{\mathbf{p}} = (\mathbf{I} - \mathbf{A})^{-1}\eta$$

Power Control – A Distributed Algorithm

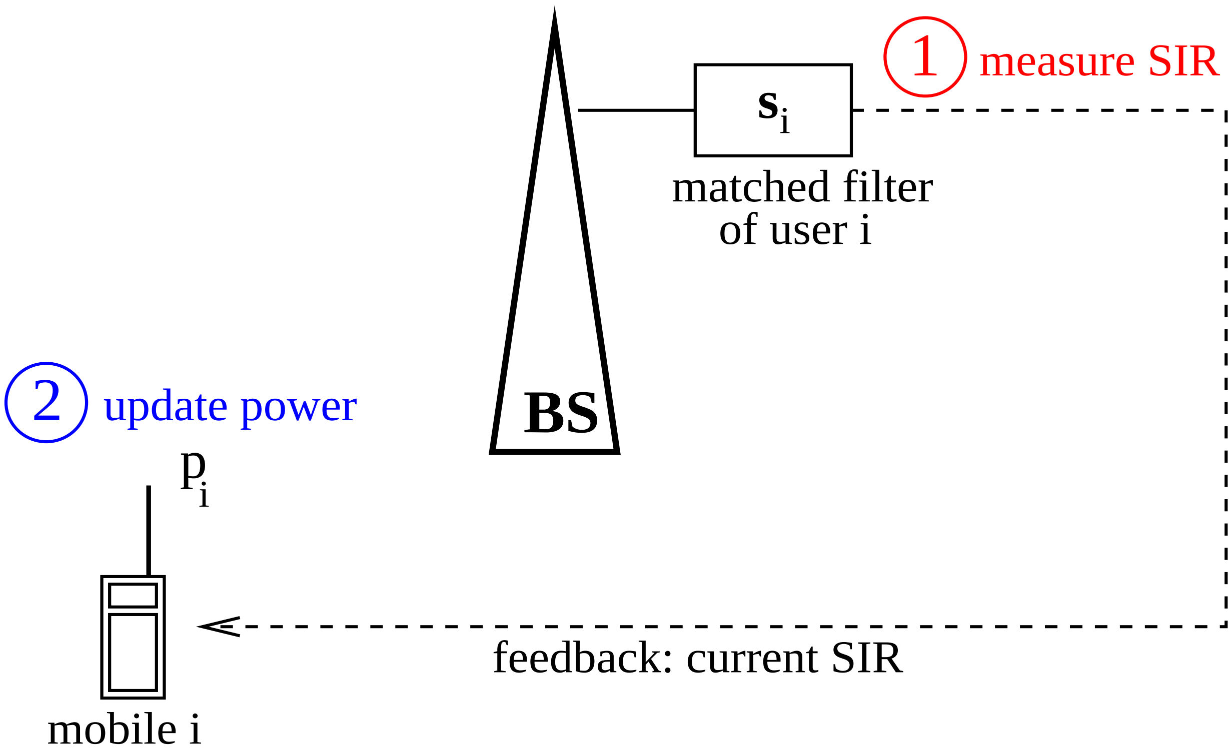


Algorithm: p(n+1)=I(p(n))

- **Theorem 1 (Yates)** *If* $\mathbf{I}(\mathbf{p})$ *is standard (positive, monotone and scalable), then* $\mathbf{p}(n+1) = \mathbf{I}(\mathbf{p}(n))$ *converges to the componentwise smallest feasible* $\mathbf{p}$.

Power Control – Implementation

$$p_i(n+1) = I_i(\mathbf{p}(n)) = \frac{\beta_i}{\mathrm{SIR}_i(n)} p_i(n)$$



Joint Power Control and Interference Suppression

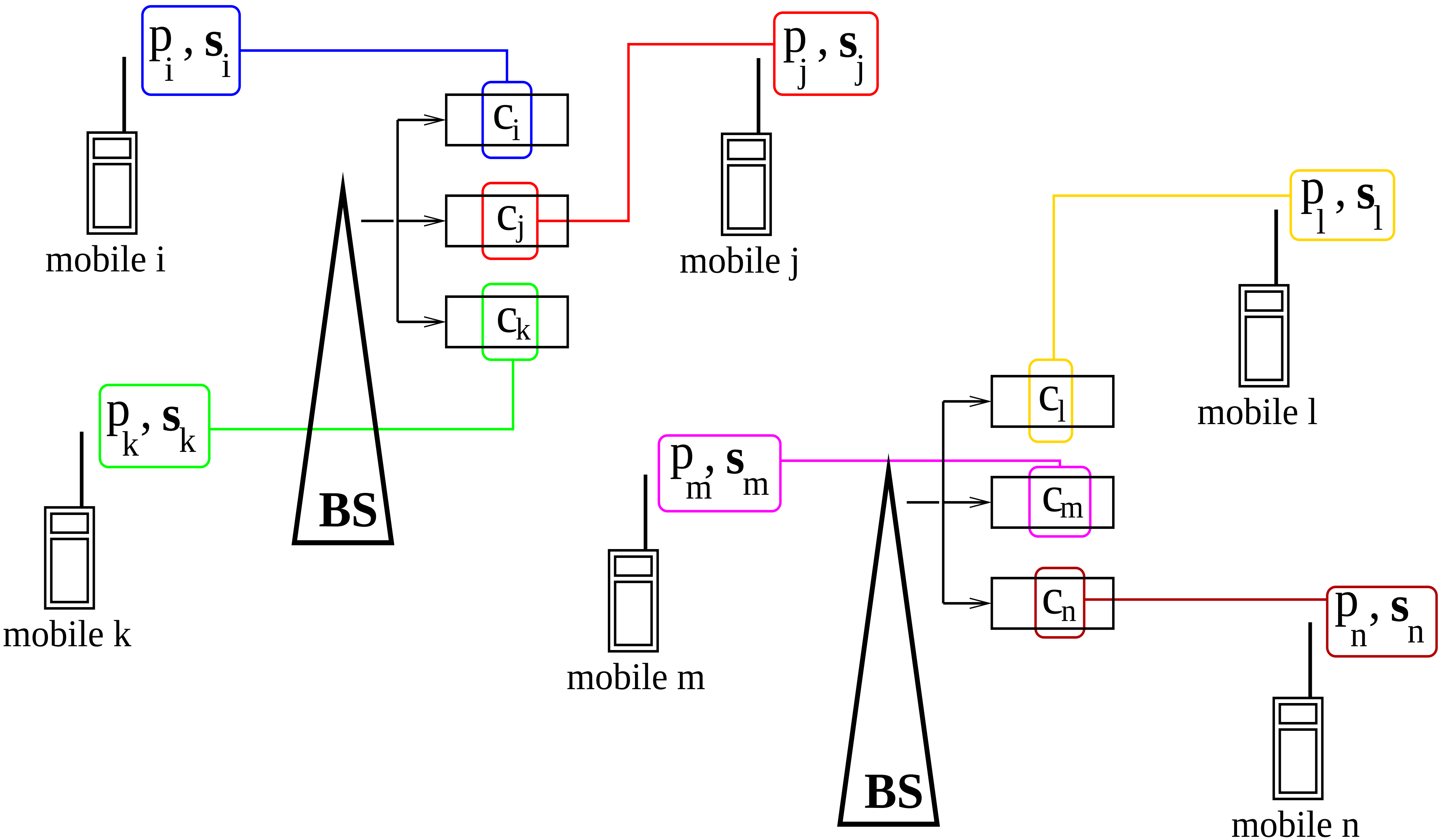
- Fix signature sequences of the users
- But, do not fix $\mathbf{c}_i$ to be the matched filter; treat $\mathbf{c}_i$ as a free-variable
- SIR of user i is a function of powers of all users and receiver filter of user i
- Find jointly optimum power vector and receiver filters such that

$$\mathrm{SIR}_i = \frac{p_i h_{ii} (\mathbf{c}_i^\top \mathbf{s}_i)^2}{\sum_{j \neq i} p_j h_{ij} (\mathbf{c}_i^\top \mathbf{s}_j)^2 + \sigma^2 (\mathbf{c}_i^\top \mathbf{c}_i)} \geq \beta_i \qquad i = 1, \cdots, K$$

with *componentwise smallest* power vector (Ulukus-Yates, 1998)

Controllable Parameters

- Each user has a power (p_i) and a receiver filter ($\mathbf{c}_i$) — signature sequences are fixed



Joint Power Control and Interference Suppression Algorithm

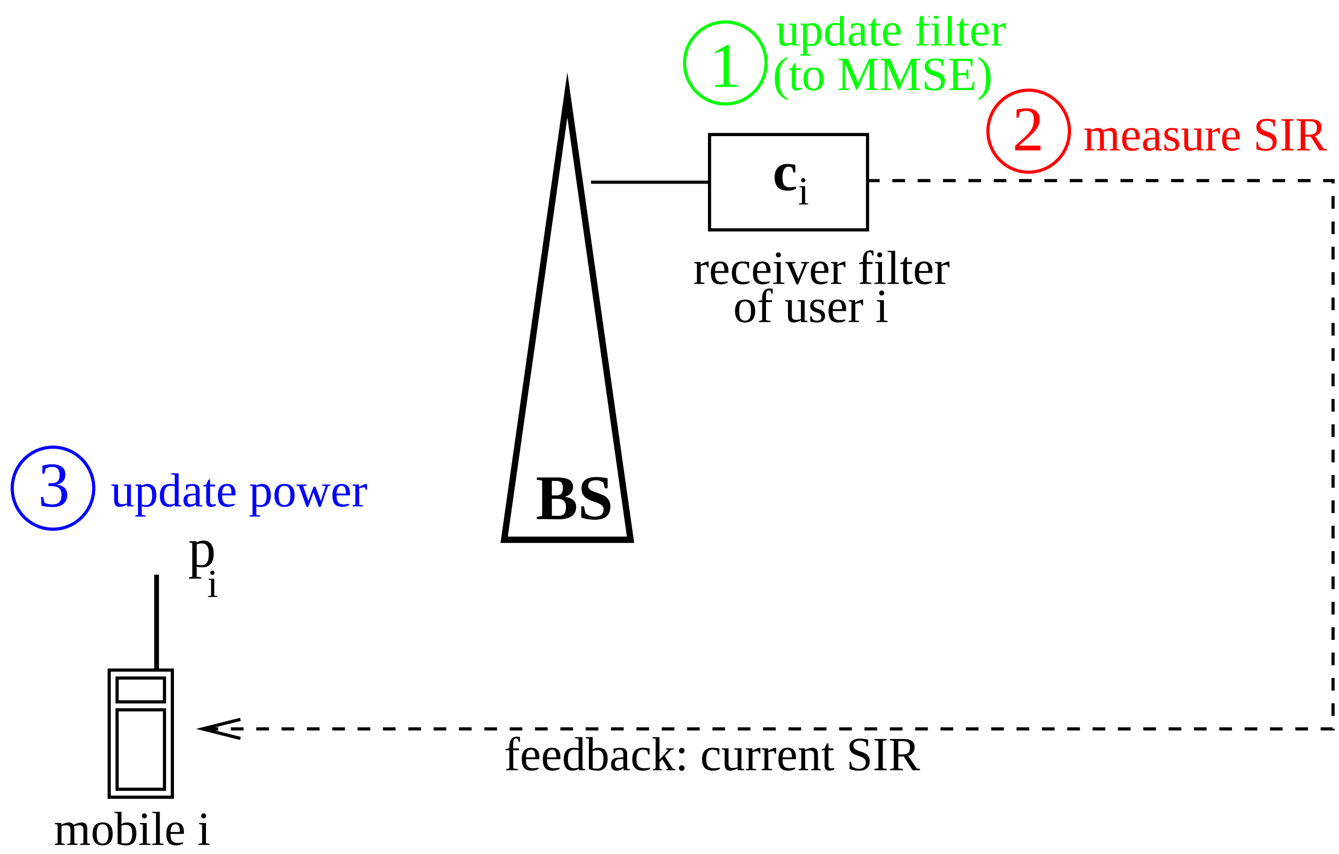
- Define interference function

$$
\begin{aligned}
I_i(\mathbf{p}) &= \min_{\mathbf{c}_i} \frac{\beta_i}{h_{ii}} \frac{\sum_{j \neq i} p_j h_{ij} (\mathbf{c}_i^\top \mathbf{s}_j)^2 + \sigma^2 (\mathbf{c}_i^\top \mathbf{c}_i)}{(\mathbf{c}_i^\top \mathbf{s}_i)^2} \\
&= \frac{\beta_i}{h_{ii}} \min_{\mathbf{c}_i} \frac{\mathbf{c}_i^\top \left(\sum_{j \neq i} p_j h_{ij} \mathbf{s}_j \mathbf{s}_j^\top + \sigma^2 \mathbf{I}_N \right) \mathbf{c}_i}{(\mathbf{c}_i^\top \mathbf{s}_i)^2}
\end{aligned}
$$

- $\mathbf{I}(\mathbf{p})$ is a standard interference function.
- Power control algorithm: $\mathbf{p}(n+1) = \mathbf{I}(\mathbf{p}(n))$
- The algorithm converges to the componentwise smallest power vector (and corresponding receiver filters) where SIR requirements of the users are satisfied.
- Inherently a two-step algorithm. At each step n,
  - For fixed transmit powers, find receiver that minimizes the interference: MMSE filter
  - Update powers to just beat the interference at the output of the updated filters

Joint Power Control and Interference Suppression – Implementation

- Update the receiver filter to be the MMSE filter
- Measure the SIR at the output of the updated filter
- Update power in the usual way: $p_i(n+1) = \frac{\beta_i}{\mathrm{SIR}_i(n)} p_i(n)$.



Joint Power Control, Interference Suppression/Avoidance

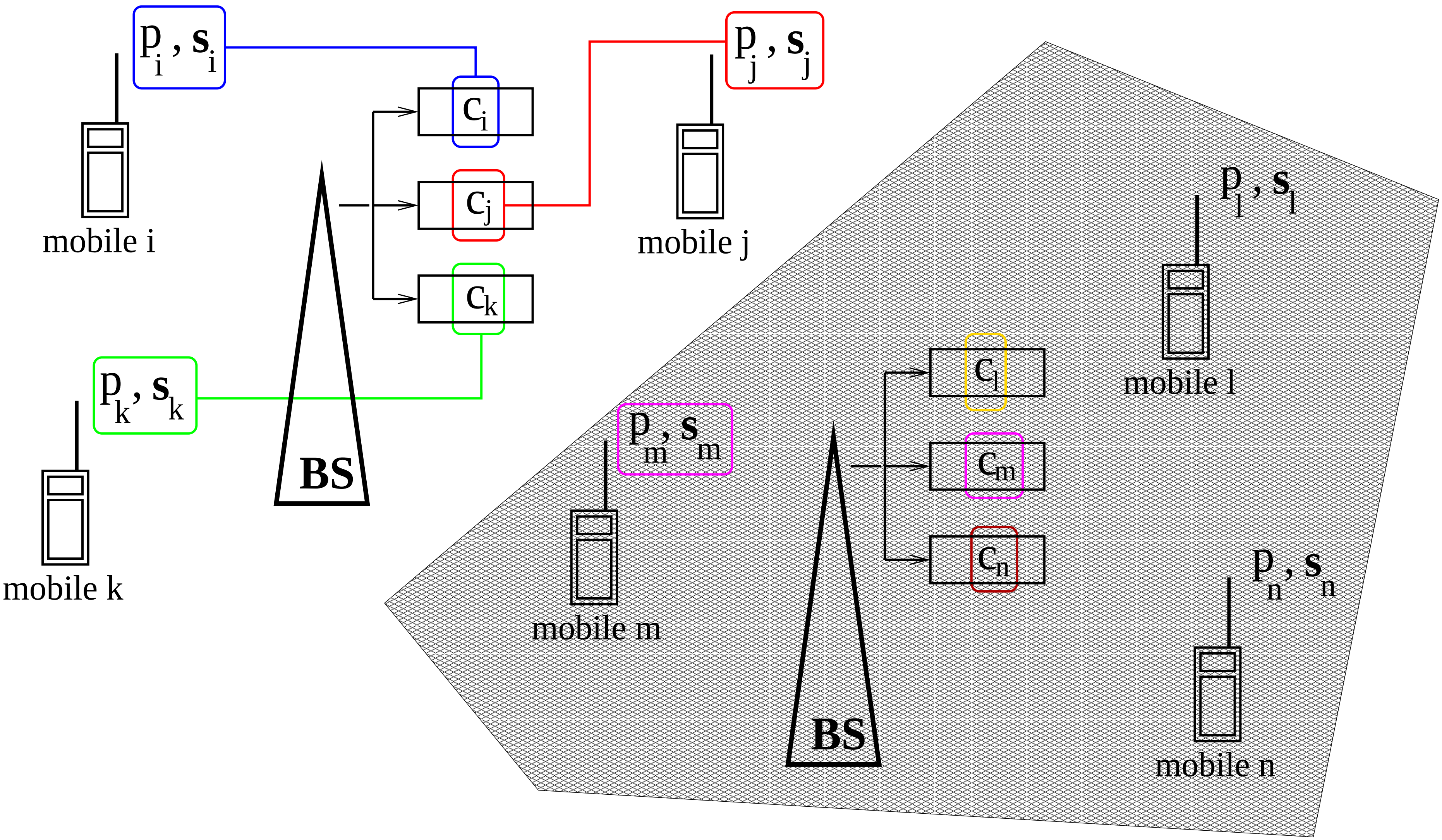
- Do not randomly choose or arbitrarily fix the signature sequences.
- Treat both signature sequences, $\mathbf{s}_i$, and receiver filters, $\mathbf{c}_i$, as free-variables.
- SIR_i is a function of all powers, all signatures and the receiver filter of user i

$$\text{SIR}_i = \frac{p_i h_{ii} (\mathbf{c}_i^\top \mathbf{s}_i)^2}{\sum_{j \neq i} p_j h_{ij} (\mathbf{c}_i^\top \mathbf{s}_j)^2 + \sigma^2 (\mathbf{c}_i^\top \mathbf{c}_i)}$$

- Choose jointly optimum powers, signature sequences and receiver filters to
  - maximize the user capacity
  - maximize the information theoretic sum capacity
- Multicell case is an open problem. Assume a single-cell system from now on.

Controllable Parameters

- Each user has a power (p_i), a signature sequence ($\mathbf{s}_i$) and a receiver filter ($\mathbf{c}_i$) — single-cell



Information Theoretic CDMA Capacity

- Let $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_K]$.
- C_{sum}: maximum total number of bits K users can transmit on the uplink (Verdú, 1986)

$$C_{\text{sum}} = \frac{1}{2} \log \left[\det \left(\mathbf{I}_N + \sigma^{-2} \mathbf{S} \mathbf{P} \mathbf{S}^\top \right) \right]$$

- For equal powers $p_i = p$ for all i,

$$C_{\text{sum}} = \frac{1}{2} \log \left[\det \left(\mathbf{I}_N + \frac{p}{\sigma^2} \mathbf{S} \mathbf{S}^\top \right) \right] = \frac{1}{2} \log \left[\det \left(\mathbf{I}_K + \frac{p}{\sigma^2} \mathbf{S}^\top \mathbf{S} \right) \right]$$

- To maximize the sum capacity (Rupf-Massey, 1994)
  - If $K \leq N$, $\mathbf{S}^\top \mathbf{S} = \mathbf{I}_K$ K orthonormal sequences
  - If $K > N$, $\mathbf{S} \mathbf{S}^\top = \frac{K}{N} \mathbf{I}_N$ K Welch Bound Equality sequences

User Capacity

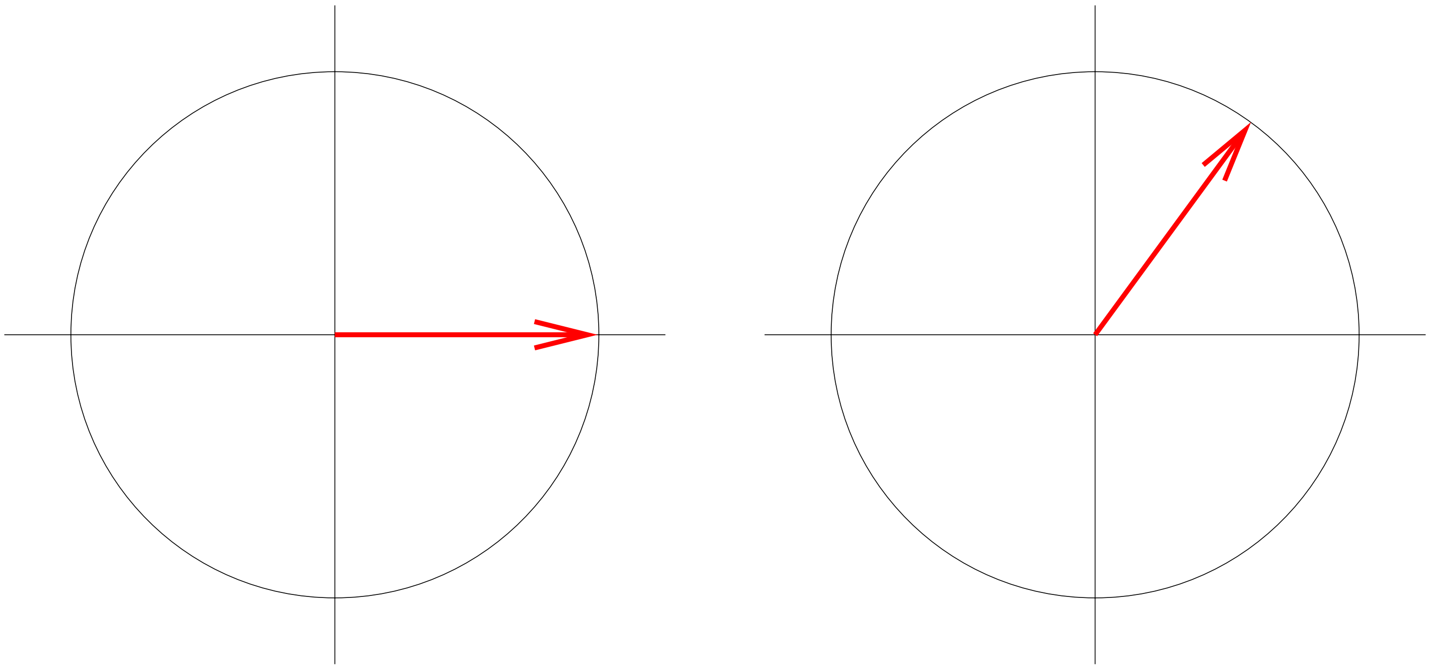
- Maximum number of admissible users given processing gain N and SIR target β
- K users are *admissible* if there are $p_i \geq 0$ and $\mathbf{s}_i, \mathbf{c}_i \in \mathbf{R}^N$ such that $\text{SIR}_i \geq \beta$, for all i
- User capacity of single-cell synchronous CDMA (Viswanath-Anantharam-Tse, 1999)

$$K < N(1 + 1/\beta)$$

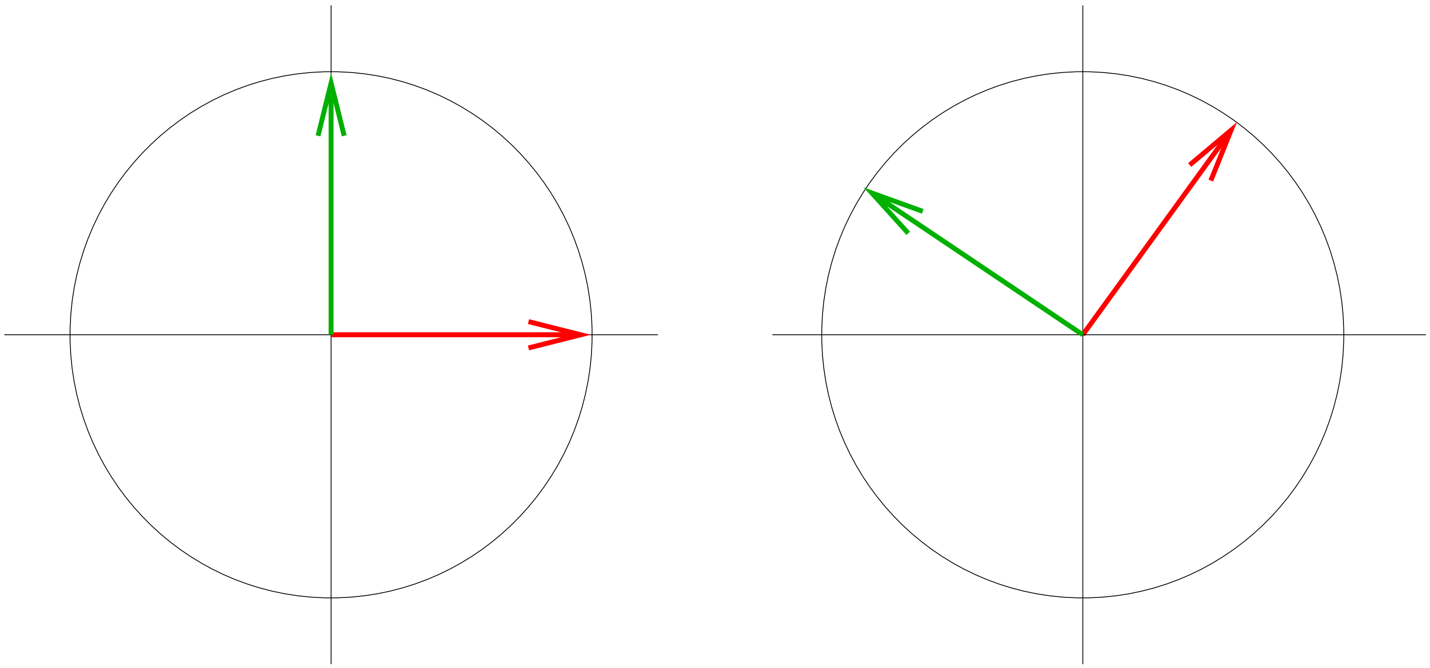
$$\beta < \frac{1}{\frac{K}{N} - 1}$$

- Optimum deterministic configuration
  - Equal received powers: $p_i = p$ for all i
  - WBE signature sequences: $\mathbf{S}\mathbf{S}^\top = (K/N)\mathbf{I}_N$
- For this configuration: MMSE receiver filters are scaled matched filters!

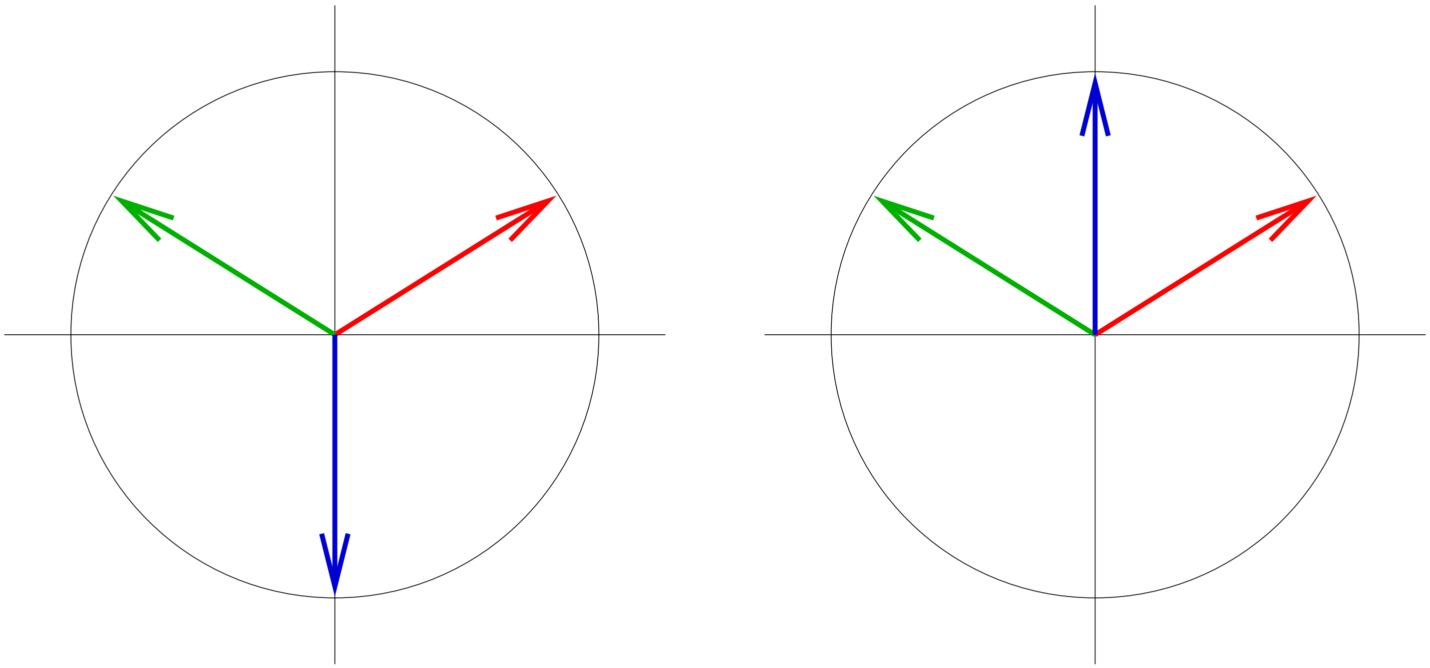
Simple example, $N = 2$, $K = 1$



Simple example, $N = 2$**,** $K = 2$



Simple example, $N = 2$, $K = 3$



Welch's Bound, Minimum TSC and Optimality

- For K unit-energy vectors $\{\mathbf{s}_1, \cdots, \mathbf{s}_K\}$, Total Squared Correlation (TSC) (Welch, 1974):

$$\text{TSC} = \sum_{i=1}^{K} \sum_{j=1}^{K} (\mathbf{s}_i^\top \mathbf{s}_j)^2 \geq \frac{K^2}{N}$$

- If $K \leq N$, the bound is loose: $\text{TSC} \geq K$. The bound is achieved iff $\mathbf{S}^\top \mathbf{S} = \mathbf{I}_K$.
- If $K > N$, the bound is achieved iff $\mathbf{S}\mathbf{S}^\top = \frac{K}{N}\mathbf{I}_N$ (Massey-Mittelholzer, 1991).
- Minimum TSC $\Longleftrightarrow$ Optimum signature sequences
- Goal: A simple algorithm to construct them.

TSC Reduction

- Separate terms that depend on $\mathbf{s}_i$ in TSC

$$\text{TSC} = (\mathbf{s}_i^\top \mathbf{s}_i)^2 + 2\mathbf{s}_i^\top \left(\sum_{j \neq i} \mathbf{s}_j \mathbf{s}_j^\top \right) \mathbf{s}_i + \sum_{k \neq i} \sum_{l \neq i} (\mathbf{s}_k^\top \mathbf{s}_l)^2$$

- Many ways to reduce TSC.

- MMSE update: (Ulukus-Yates, 1998) replace $\mathbf{s}_i$ with

$$\mathbf{c}_i = \alpha_i \mathbf{A}_i^{-1} \mathbf{s}_i$$

  where $\mathbf{A}_i = \sum_{j \neq i} \mathbf{s}_j \mathbf{s}_j^\top + a^2 \mathbf{I}_N$, and α_i is the normalizing factor.

- $\mathbf{c}_i$ is a generalized normalized MMSE filter for user i.

- Eigen update: (Rose-Ulukus-Yates, 2000) replace $\mathbf{s}_i$ with the minimum eigenvector of $\mathbf{A}_i$.

Algorithm Properties

- Replace $\mathbf{s}_i$ with normalized MMSE filter (or eigenvector) $\mathbf{c}_i$

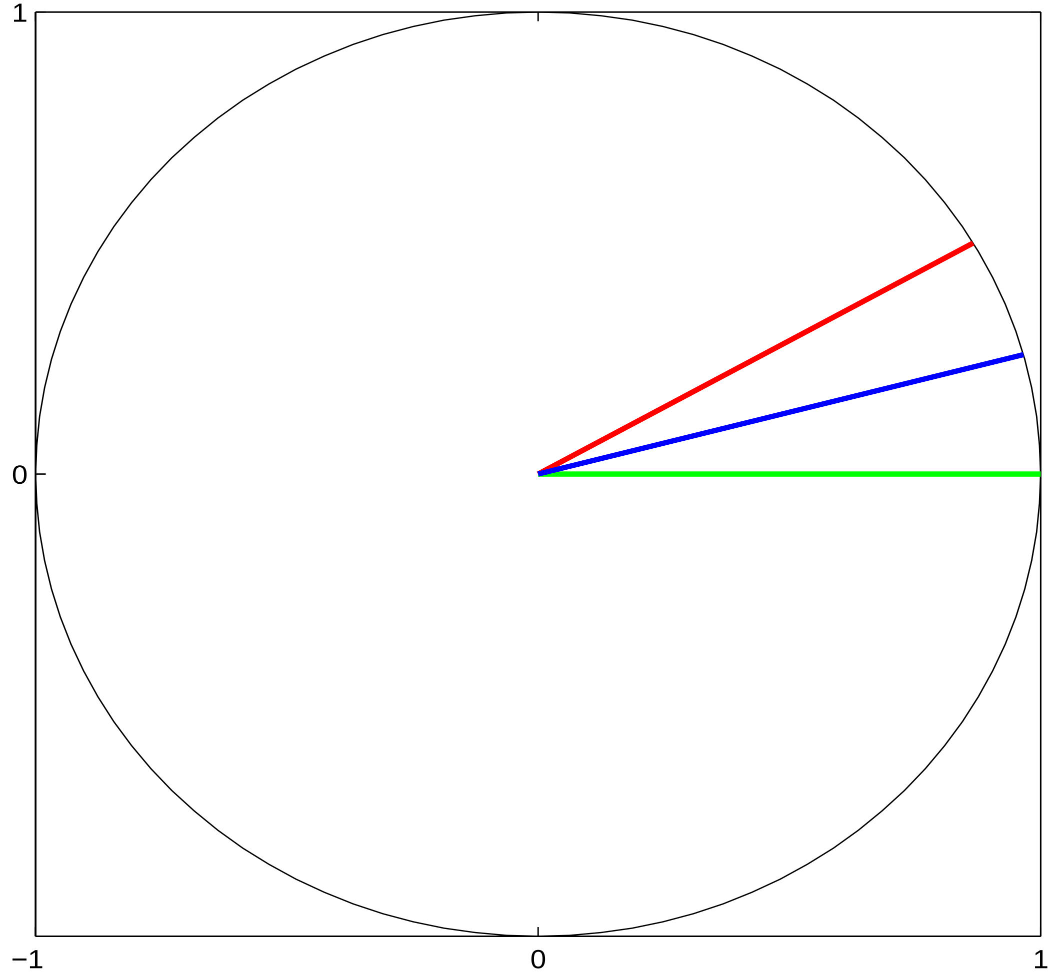
$$\mathbf{S} = [\mathbf{s}_1, \cdots, \mathbf{s}_{i-1}, \mathbf{s}_i, \mathbf{s}_{i+1}, \cdots, \mathbf{s}_K] \quad \longrightarrow \quad \mathbf{S}' = [\mathbf{s}_1, \cdots, \mathbf{s}_{i-1}, \mathbf{c}_i, \mathbf{s}_{i+1}, \cdots, \mathbf{s}_K]$$

- The proposed updates decrease the TSC, and increases the sum capacity:

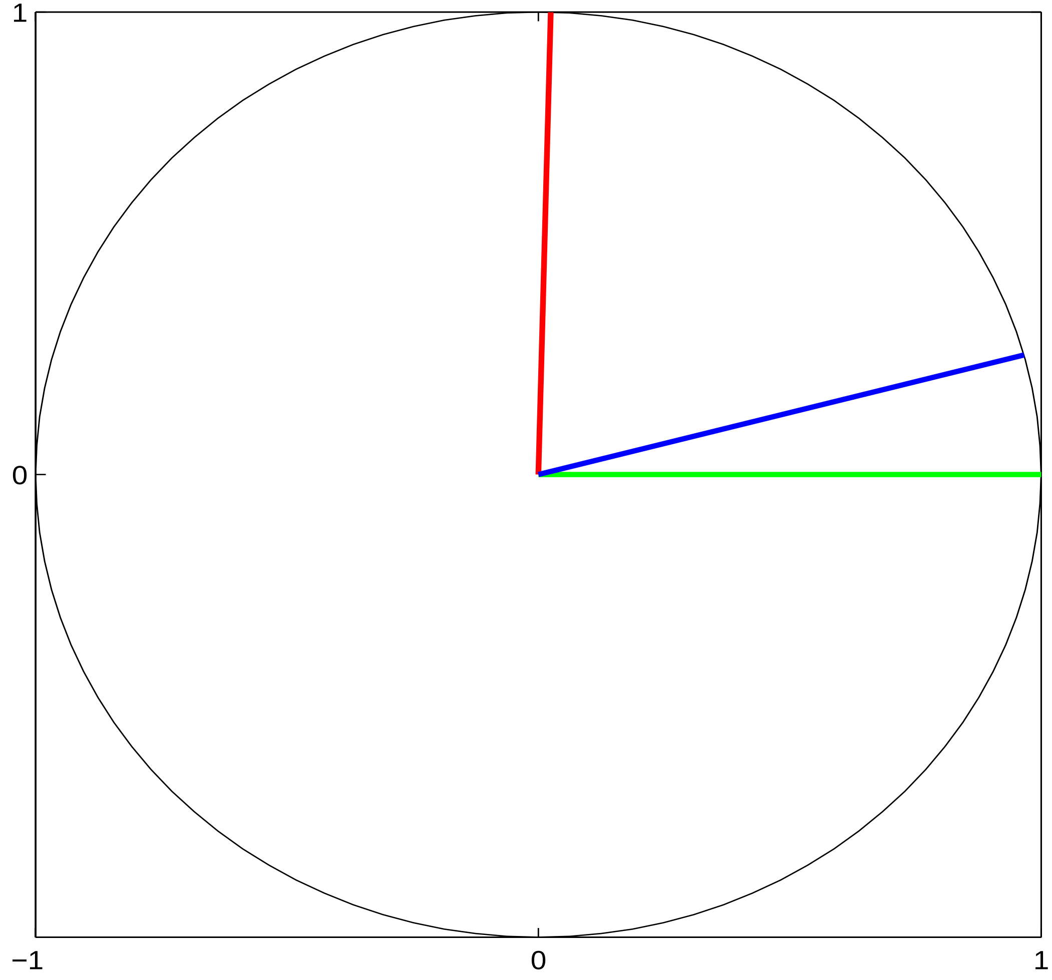
$$\begin{aligned} \mathrm{TSC}(\mathbf{S}') &\leq \mathrm{TSC}(\mathbf{S}) \\ C_{\mathrm{sum}}(\mathbf{S}') &\geq C_{\mathrm{sum}}(\mathbf{S}) \end{aligned}$$

- Let users update their signatures sequentially (one user at a time).
- The algorithm produces progressively better signature sequence sets and converges to an optimum set under mild conditions on the initial set.
- Need sequential updates condition for the convergence proof.

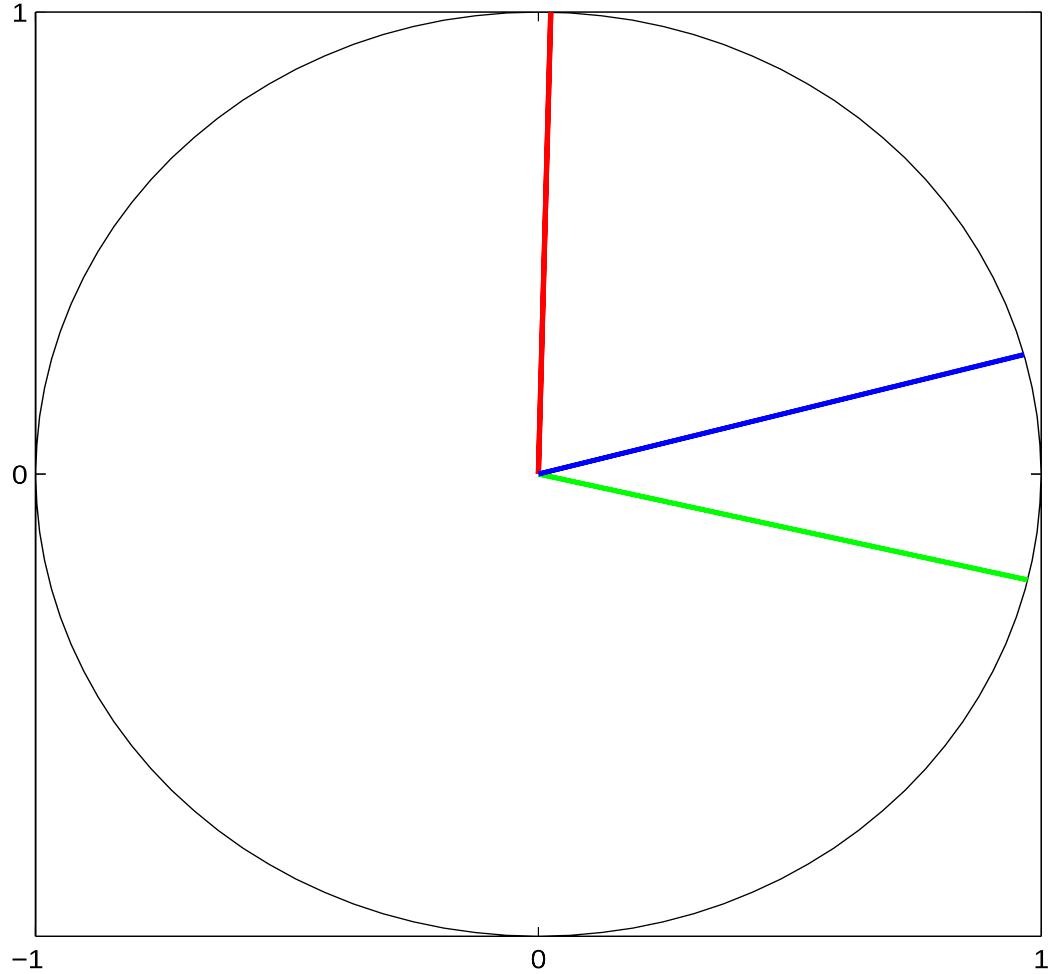
A Sample Run – start



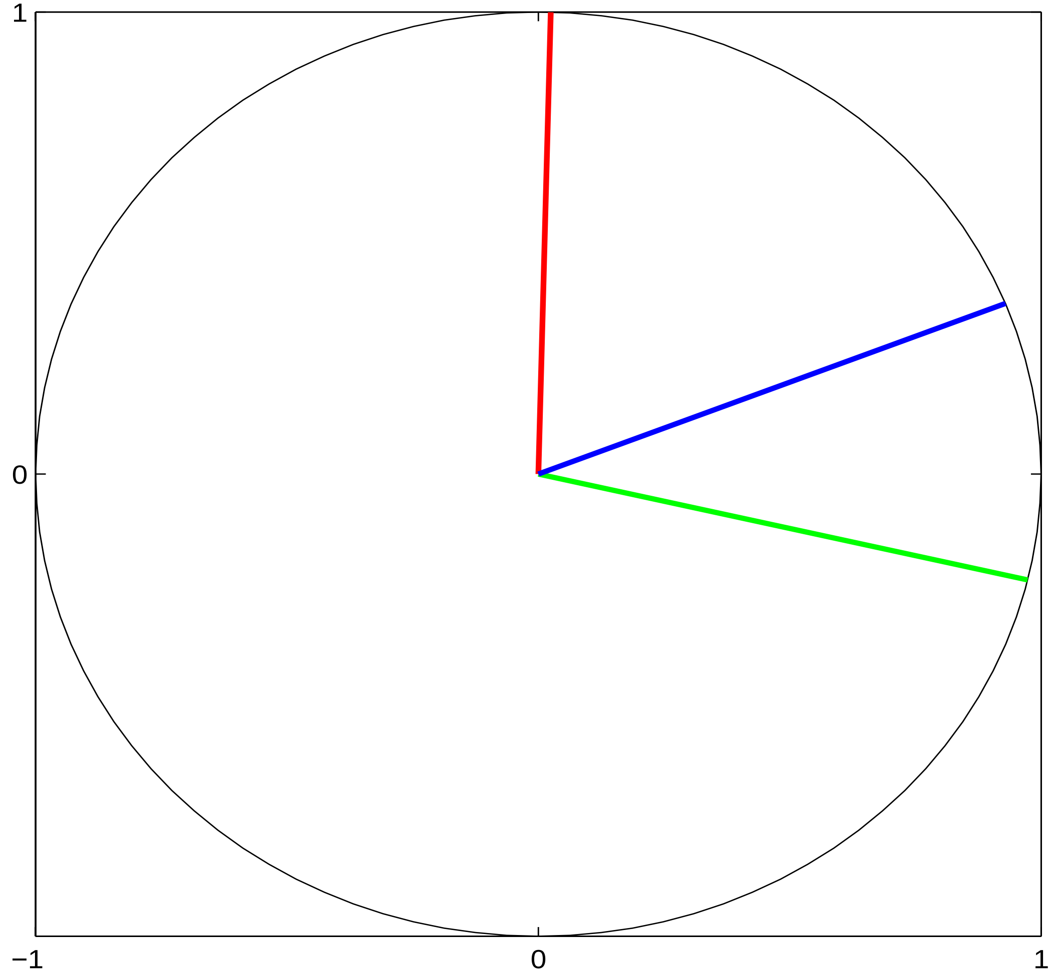
A Sample Run – 1st user updates



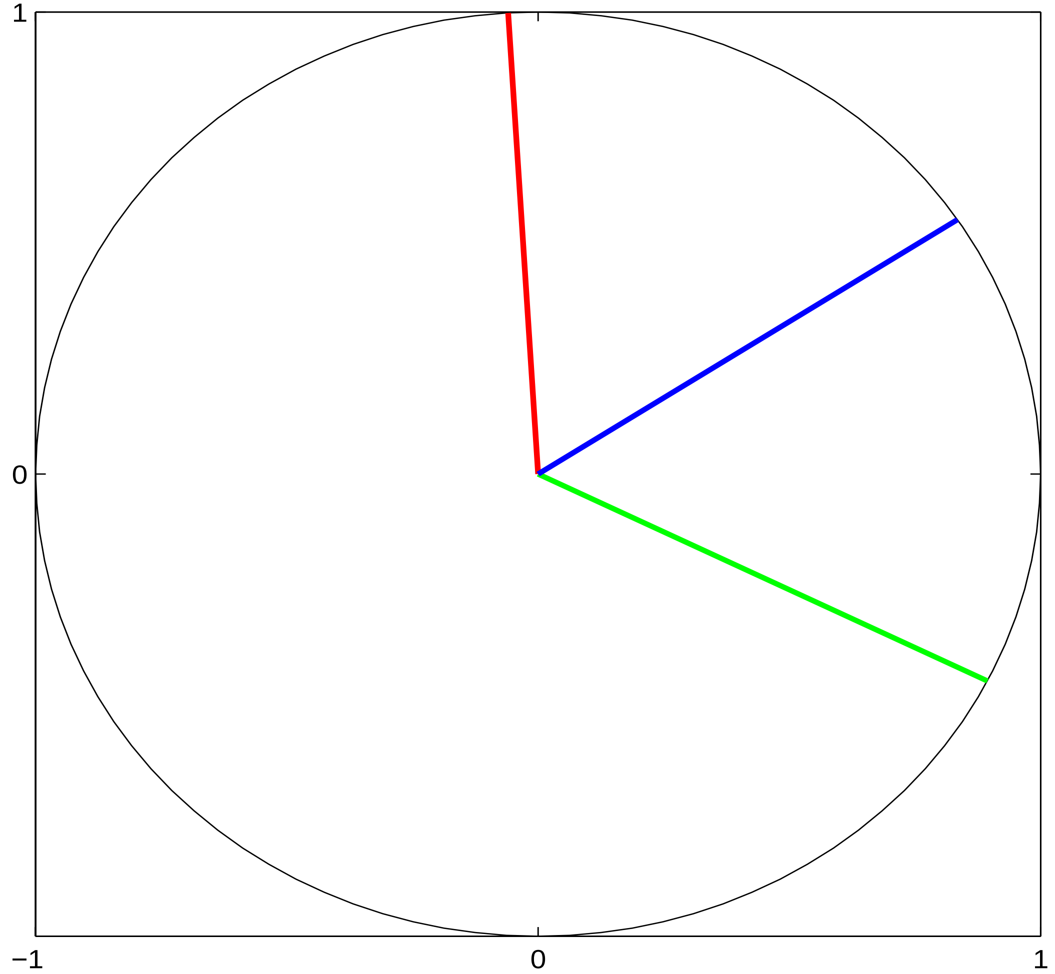
A Sample Run – 2nd user updates



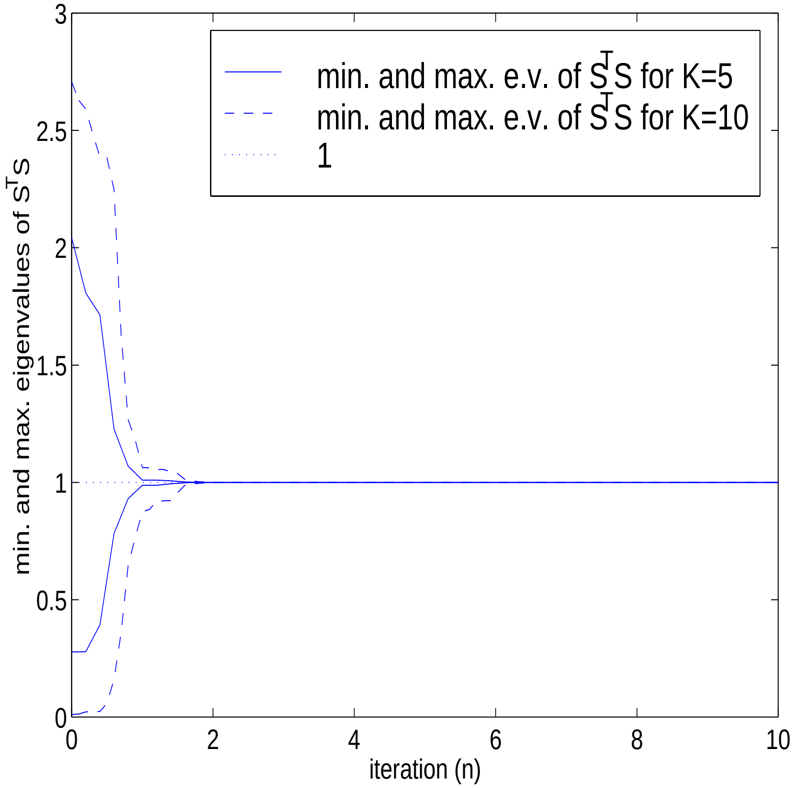
A Sample Run – 3rd user updates

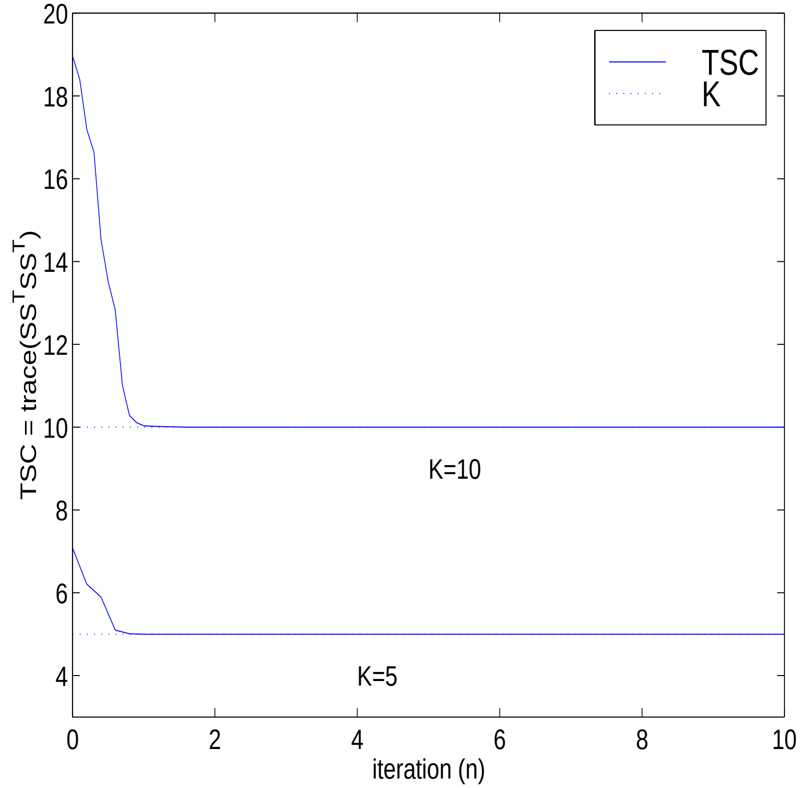


A Sample Run – after convergence



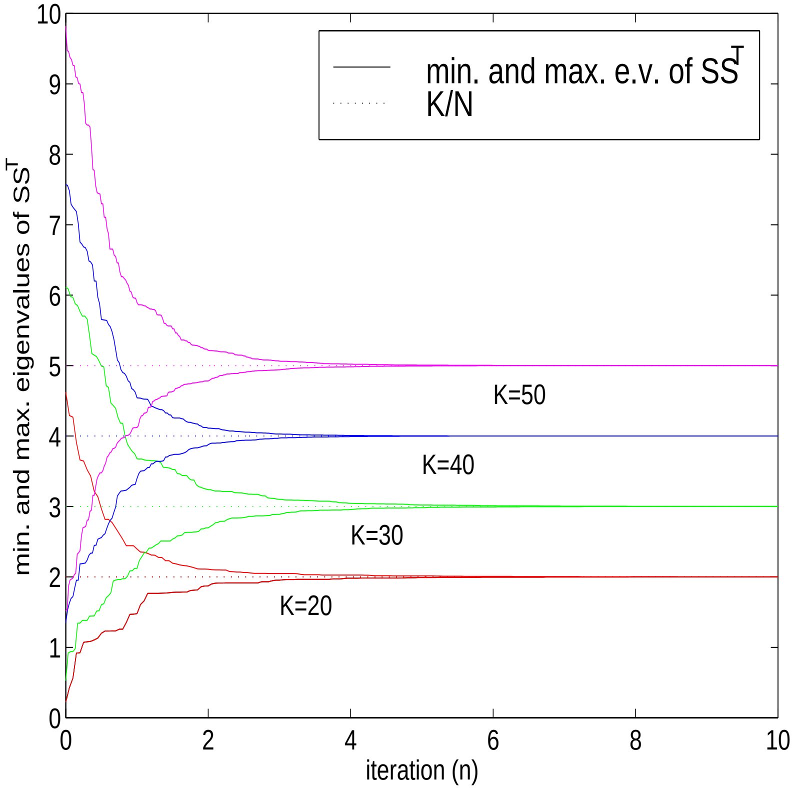
Min/Max Eigenvalues of $S^\top S$ and TSC – MMSE Update

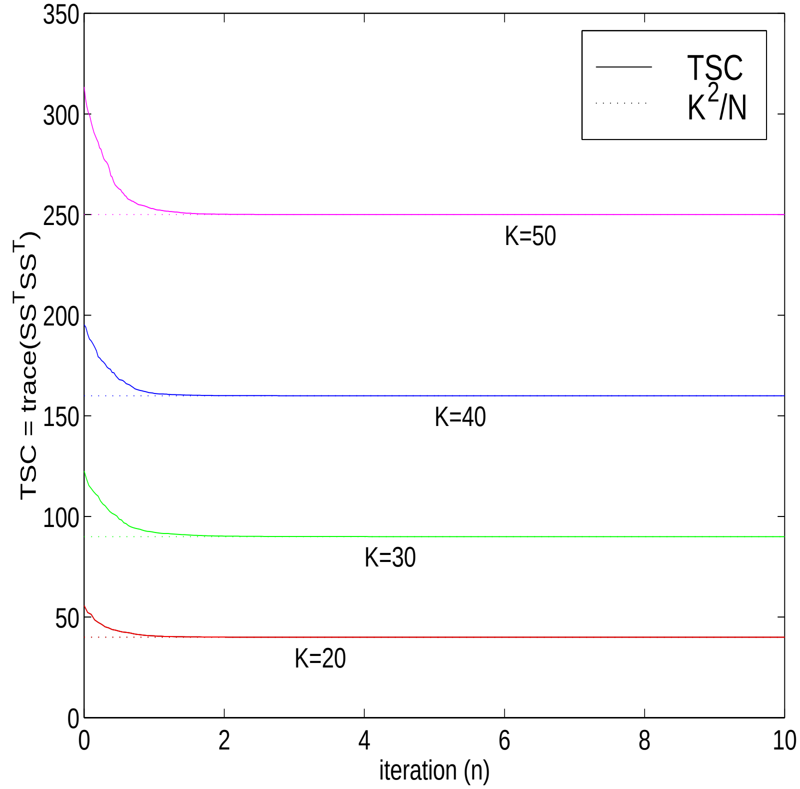




$\underline{K \leq N \text{ case:}}$ $K = 5, 10$ and $N = 10$

Min/Max Eigenvalues of $SS^\top$ and TSC – MMSE Update

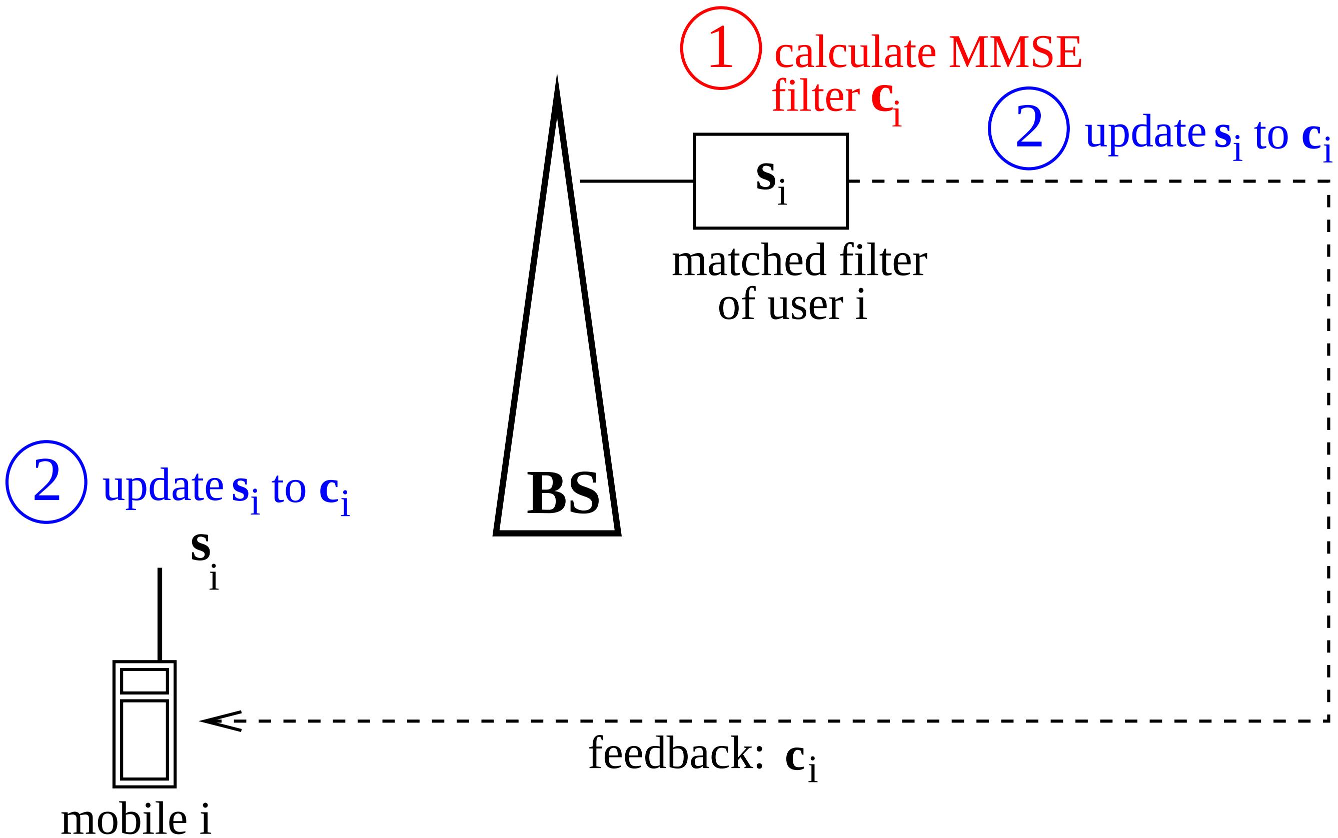




<u>$K > N$ case:</u> $K = 20, 30, 40, 50$ and $N = 10$

Interference Avoidance – Implementation

- Calculate the receiver filter to be the MMSE filter
- Update the signature sequence and the receiver simultaneously

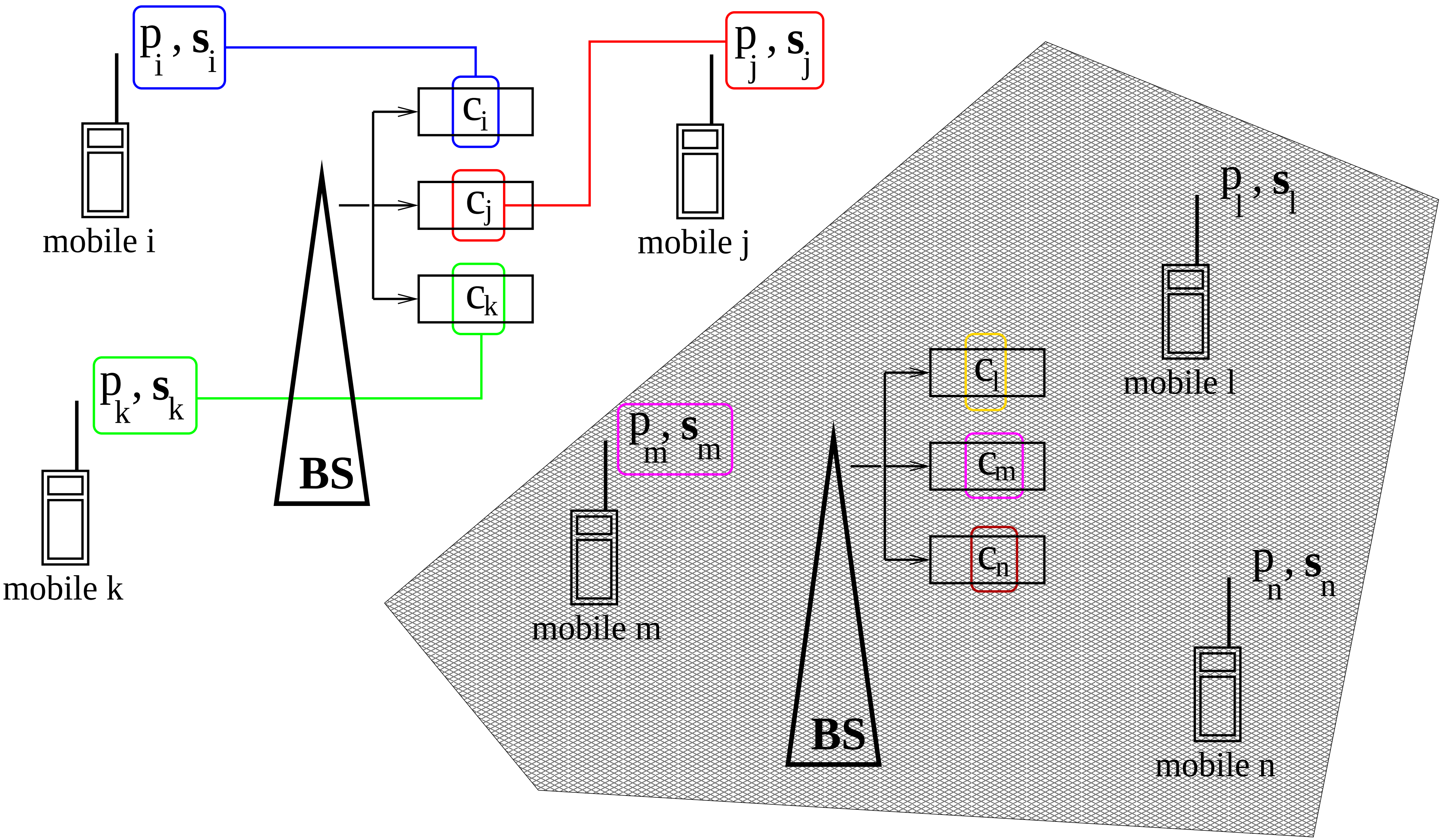


Further Improvements

- Possible shortcomings
  - Requires immediate feedback and synchronization between transmitter and receiver
  - Or, run signature adaptation algorithm off-line, when done, update transmitter once
  - For convergence proof, users need to take turns for signature updates, i.e., sequential
- Is an on-line, parallel algorithm without synchronization possible?
- Joint transmitter-receiver optimization (Ulukus-Yener, 2002)

Controllable Parameters

- Each user has a power (p_i), a signature sequence ($\mathbf{s}_i$) and a receiver filter ($\mathbf{c}_i$) — single-cell



Related Performance Measures

- Need to introduce receiver filters into the optimization
- Remember C_{sum} and TSC are functions of signature sequences only
- Look at the MSE of user i, MSE_i

$$\text{MSE}_i = \mathbf{c}_i^\top \left(\mathbf{S}\mathbf{S}^\top + \sigma^2 \mathbf{I}_N\right) \mathbf{c}_i - 2\mathbf{c}_i^\top \mathbf{s}_i + 1$$

- Define the total MSE

$$\text{MSE} = \sum_{i=1}^{K} \text{MSE}_i = \text{tr}\left[\mathbf{C}^\top \left(\mathbf{S}\mathbf{S}^\top + \sigma^2 \mathbf{I}_N\right) \mathbf{C} - 2\mathbf{C}^\top \mathbf{S} + \mathbf{I}_K\right]$$

where $\mathbf{C} = [\mathbf{c}_1, \cdots, \mathbf{c}_K]$.

- MSE is a function of signature sequences and receiver filters.

Equivalent Optimization Problems

- Using MMSE filters as receivers, we get the system-wide MMSE

$$
\begin{aligned}
\text{MMSE} = K - \text{tr}\left[\mathbf{S}\mathbf{S}^\top\left(\mathbf{S}\mathbf{S}^\top + \sigma^2\mathbf{I}_N\right)^{-1}\right] &= K - \sum_{i=1}^{N}\frac{\lambda_i}{\lambda_i + \sigma^2} \\
\text{TSC} = \text{tr}\left[(\mathbf{S}\mathbf{S}^\top)^2\right] &= \sum_{i=1}^{N}\lambda_i^2 \\
C_{\text{sum}} = \frac{1}{2}\log\left[\det\left(\mathbf{I}_N + \sigma^{-2}\mathbf{S}\mathbf{S}^\top\right)\right] &= \frac{1}{2}\sum_{i=1}^{N}\log\left(1 + \frac{\lambda_i}{\sigma^2}\right)
\end{aligned}
$$

- C_{sum} is Schur-concave, and MMSE and TSC are Schur-convex functions of $\{\lambda_i\}_{i=1}^{N}$.
- Signature set that maximizes C_{sum} minimizes MMSE and TSC, and vice versa.

MSE Minimization

- Minimize MSE over receivers and unit energy signatures

$$\min_{\{\mathbf{c}_i,\mathbf{s}_i\}} \quad \operatorname{tr}\left[\mathbf{C}^\top\left(\mathbf{S}\mathbf{S}^\top+\sigma^2\mathbf{I}_N\right)\mathbf{C}-2\mathbf{C}^\top\mathbf{S}+\mathbf{I}_K\right]$$

$$\text{s.t.} \quad \mathbf{s}_i^\top\mathbf{s}_i=1 \qquad i=1,\cdots,K$$

- An iterative algorithm using alternating minimization

- Receiver update for user i:

$$\mathbf{c}_i=\left(\mathbf{S}\mathbf{S}^\top+\sigma^2\mathbf{I}_N\right)^{-1}\mathbf{s}_i$$

- Users can update their receivers independently, i.e., in parallel

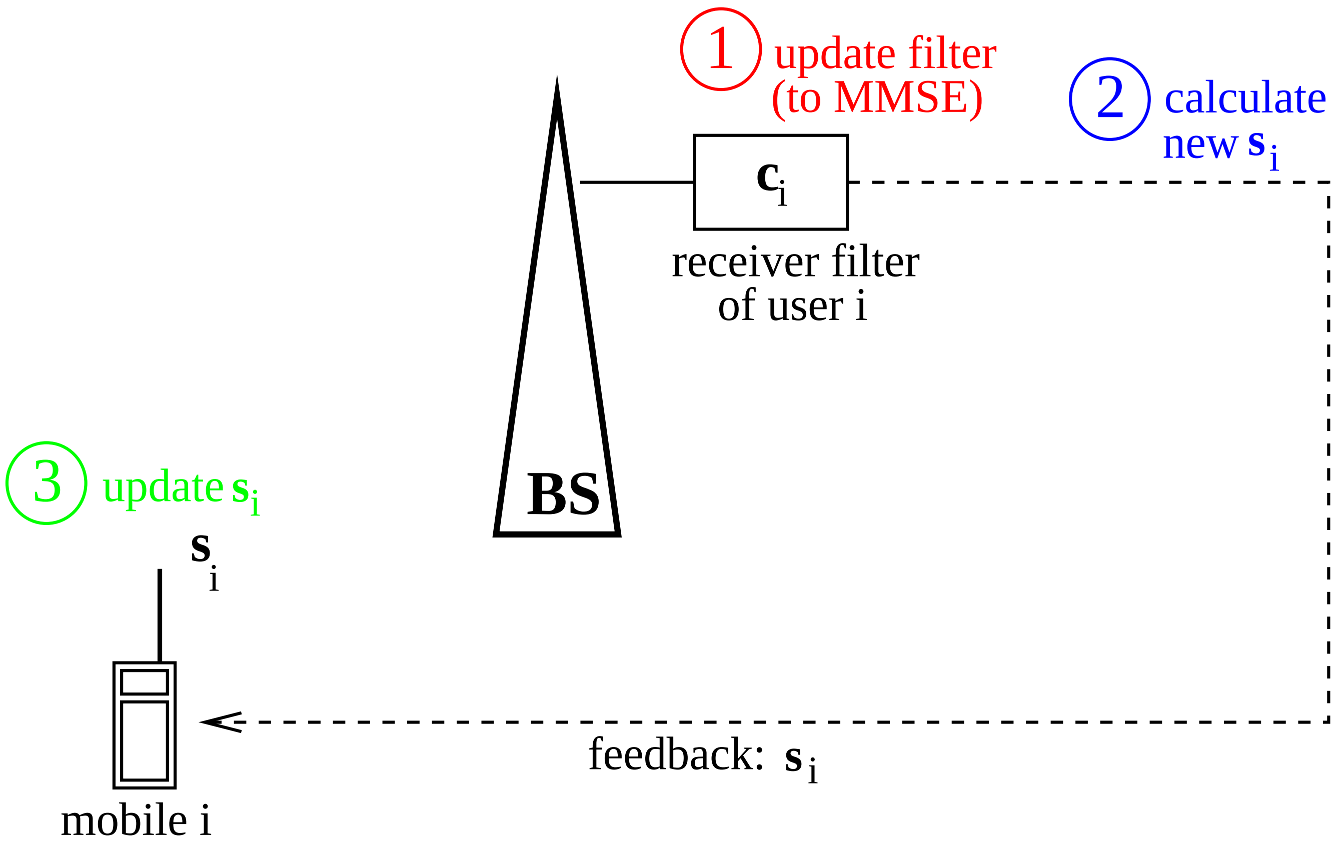
- Transmitter update for user i:

$$\mathbf{s}_i=\left(\mathbf{C}\mathbf{C}^\top+\alpha_i\mathbf{I}_N\right)^{-1}\mathbf{c}_i$$

- Users can update transmitters independently, i.e., in parallel

Joint Transmitter/Receiver Optimization – Implementation

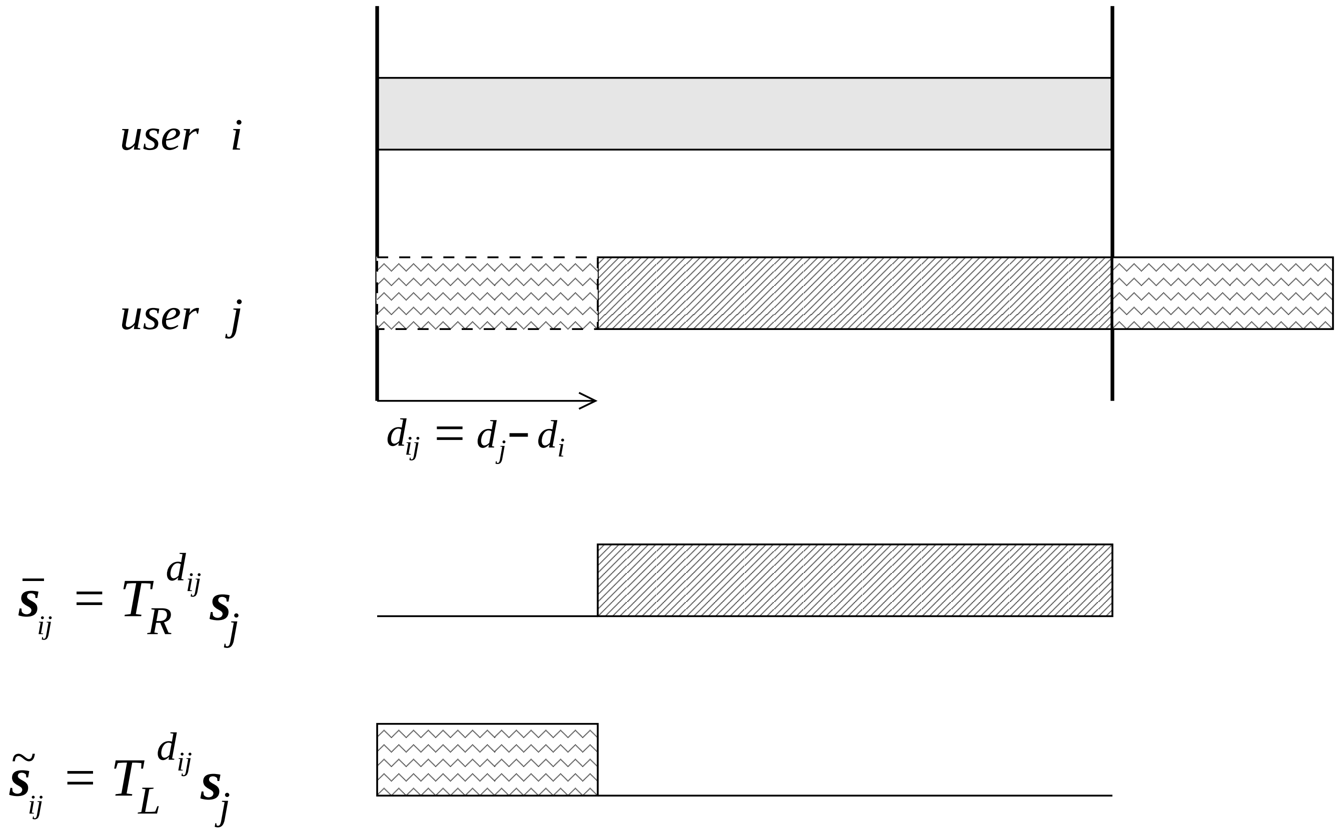
- Update the receiver filter to be the MMSE filter
- Calculate new signature sequence with the updated filter
- Update the signature sequence



User Capacity in Asynchronous System

- What is the user capacity of a single-cell asynchronous CDMA system?
- How do we choose signature sequences? Optimality criteria?
- Traditionally: choose signatures with *uniformly good* auto and cross correlations for all delay profiles [Pursley, Pursley-Sarwate, Massey-Uhran, Mow].
- In every wireless system, for some time, the delays of the users are fixed
- Design signature sequences that are *best* for the current fixed delay profile
- .... without having to know the current delay profile!

Asynchronous Interference Representation



- Received signal of the *i*th user in one symbol interval

$$\mathbf{r}_i = \sqrt{p_i} b_i \mathbf{s}_i + \sum_{j \neq i} \sqrt{p_j} \left(\bar{b}_j \bar{\mathbf{s}}_{ij} + \tilde{b}_j \tilde{\mathbf{s}}_{ij} \right) + \mathbf{n}_i$$

Asynchronous cross-correlations

- Define total squared asynchronous correlation (Ulukus-Yates, 2000)

$$\text{TSAC} = \sum_{i=1}^{K} \sum_{j=1}^{K} \left\{ (\mathbf{s}_i^\top \bar{\mathbf{s}}_{ij})^2 + (\mathbf{s}_i^\top \tilde{\mathbf{s}}_{ij})^2 \right\}$$

- Optimum signature sequences minimize the TSAC.
- Optimum powers are all equal.
- The user capacity of the asynchronous system is the same as the synchronous system.
- There is no loss in user-capacity due to asynchrony.
- With optimum signature sequences, single-user, one-shot matched filters are optimum.

TSAC Reduction

- Optimum signature sequences minimize the TSAC
- Similar to the synchronous case, separate the terms that depend on $\mathbf{s}_i$

$$\text{TSAC} = (\mathbf{s}_i^\top \mathbf{s}_i)^2 + 2\mathbf{s}_i^\top \left(\sum_{j \neq i} \bar{\mathbf{s}}_{ij} \bar{\mathbf{s}}_{ij}^\top + \tilde{\mathbf{s}}_{ij} \tilde{\mathbf{s}}_{ij}^\top \right) \mathbf{s}_i + \sum_{k \neq i} \sum_{l \neq i} R_{kl}$$

- Many TSAC reduction algorithms are possible
- Asynchronous MMSE update:

$$\mathbf{c}_i = \alpha_i \mathbf{A}_i^{-1} \mathbf{s}_i$$

  where $\mathbf{A}_i = \sum_{j \neq i} \bar{\mathbf{s}}_{ij} \bar{\mathbf{s}}_{ij}^\top + \tilde{\mathbf{s}}_{ij} \tilde{\mathbf{s}}_{ij}^\top + a^2 \mathbf{I}_N$.

- Asynchronous eigen update:

$$\mathbf{c}_i = \text{normalized eigenvector of } \mathbf{A}_i \text{ with the smallest eigenvalue}$$

Some Open Problems

- Multipath channel without ISI (some initial results available)
- Multipath channel with ISI
- Fading channels
- Channels with multiple transmit and/or multiple receive antennas
  - interactions between spatial and temporal signatures
- Multicell channel
- Signal design for security:
  - definition of secure signature sequences
- Signal design for cross-layer interactions:
  - e.g., jointly optimum signatures and route/flow assignments