Power Control for User Cooperation

Şennur Ulukuş

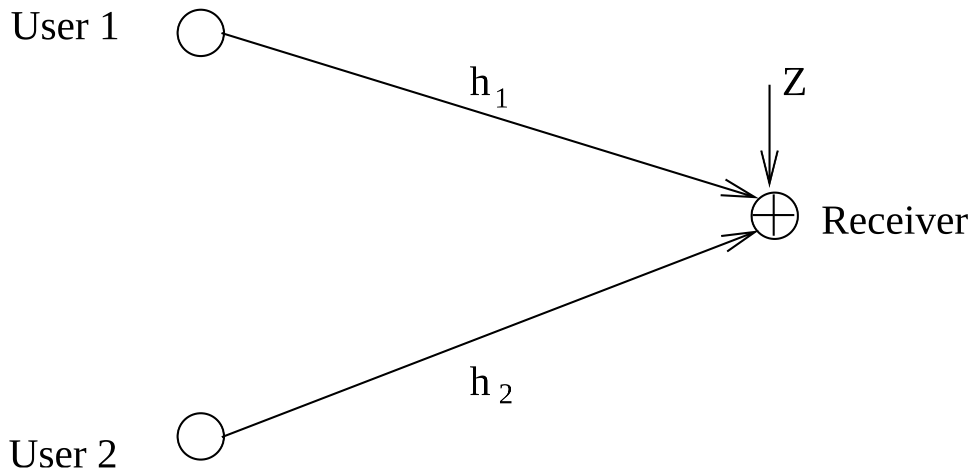
Department of ECE

University of Maryland

ulukus@umd.edu

Joint work with Onur Kaya.

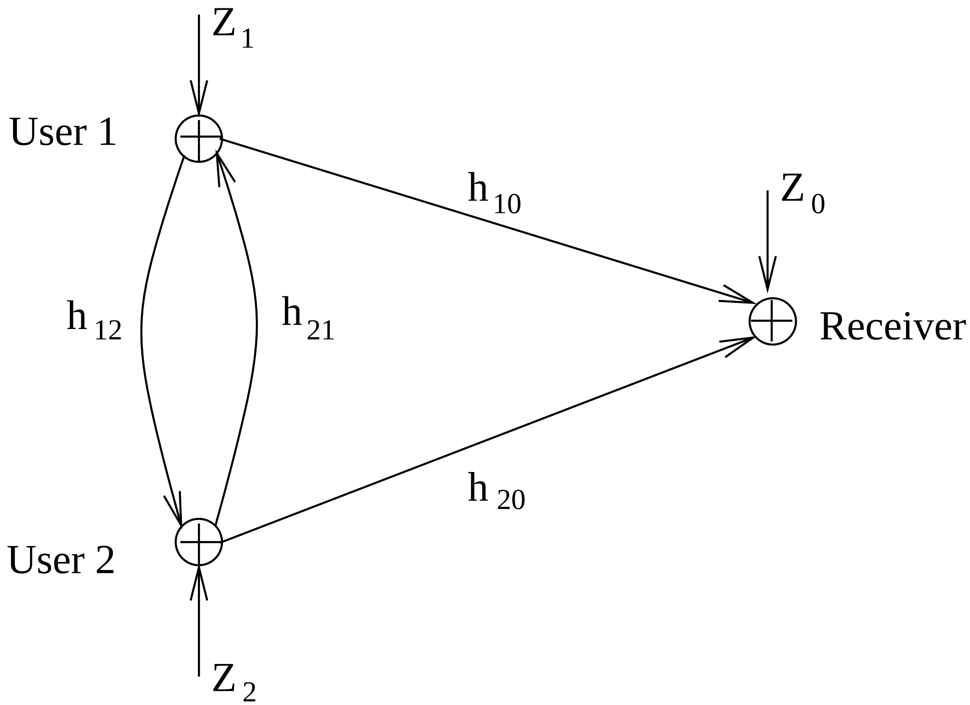
Traditional Multiple Access



$$Y = \sqrt{h_1}X_1 + \sqrt{h_2}X_2 + Z$$

- Users talk only to the receiver, do not listen or talk to each other.
- This models the single cell of a cellular system:
  - Fixed roles for nodes (base station or mobile)
  - Strict and inflexible transmission scheme (uplink and downlink)
  - Single hop of wireless communication
- Information theoretically: multiple access channel; the capacity region is known.

User Cooperation



$$Y_0 = \sqrt{h_{10}} X_1 + \sqrt{h_{20}} X_2 + Z_0$$

$$Y_1 = \sqrt{h_{21}} X_2 + Z_1$$

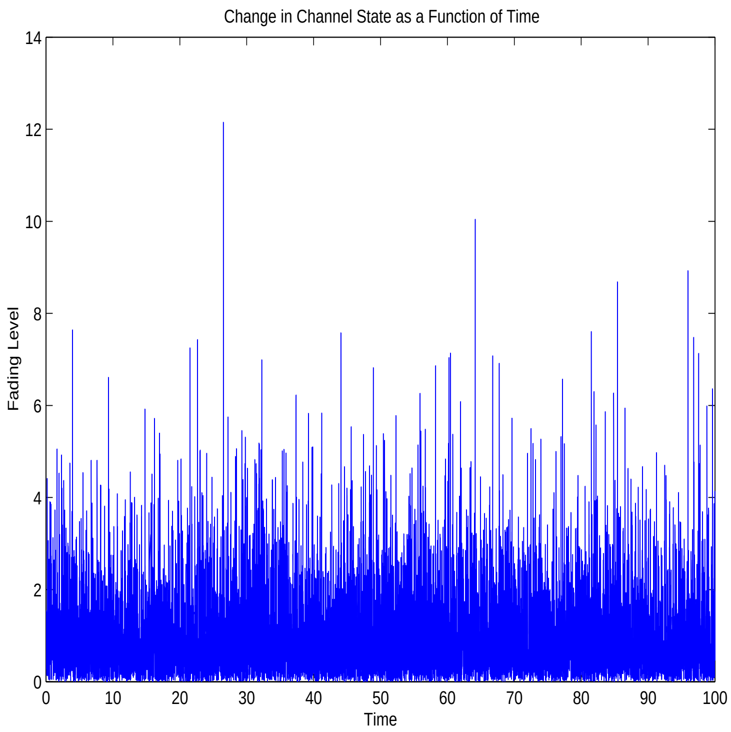
$$Y_2 = \sqrt{h_{12}} X_1 + Z_2$$

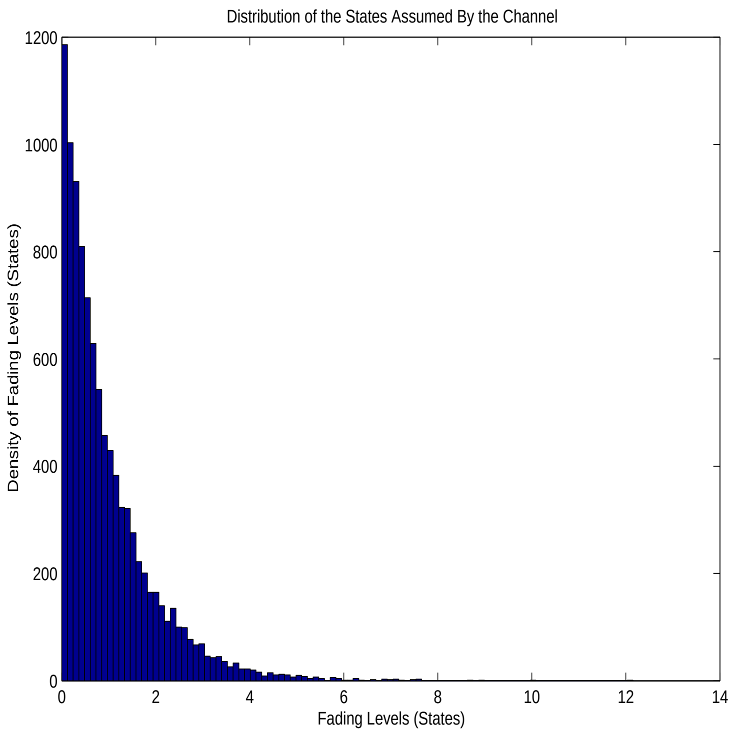
- Interference is information.
- Some versions of all transmitted signals are received by all nodes.
- Exploit overheard information to jointly design encoding, transmission, routing policies.
- Building block towards the analysis of ad-hoc and sensor networks.
- Information theoretically: multiple access channel with generalized feedback (capacity open).

Fading

- Fading: random fluctuations in channel gains.

$$Y = \sqrt{h}X + Z$$



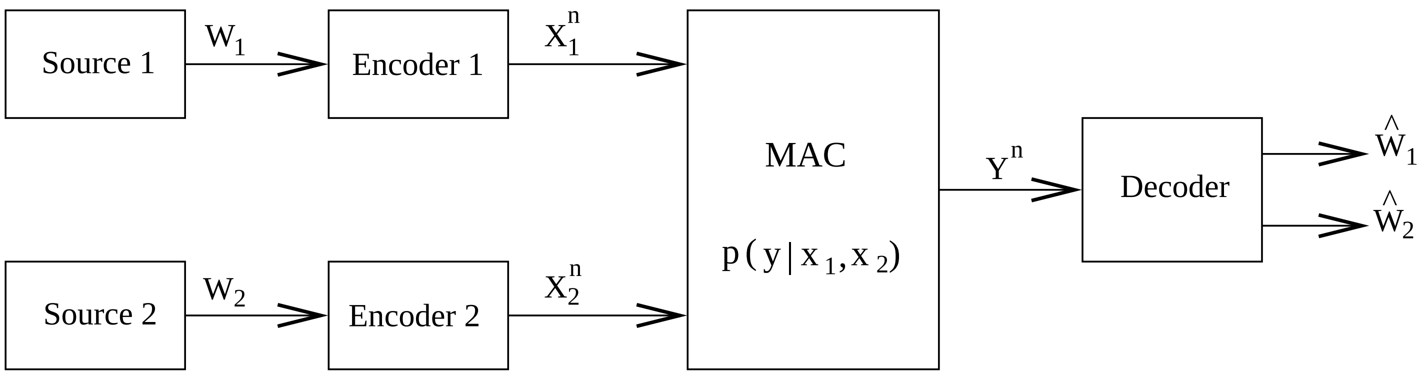


Power Control

- Fading: random fluctuations in channel gains.
- If perfect channel state information (CSI) is available at transmitters
  - Dynamic resource allocation to improve quality-of-service or capacity
- Quality-of-service based
  - Provide all users with desired SIR levels
  - Compensate for channel fading; more power if bad channel, less if good channel
- Capacity based
  - Maximize information theoretic ergodic capacity subject to average power constraints
  - Exploit variations; more power if good channel, less if bad, no power if very bad
- Underlying motivation/justification:
  - Voice communication: delay intolerant; guarantee quality-of-service at all times.
  - **Data communication: delay tolerant; we may wait for the channel to be good**.

Multiple Access Channel

- Two user multiple-access channel:



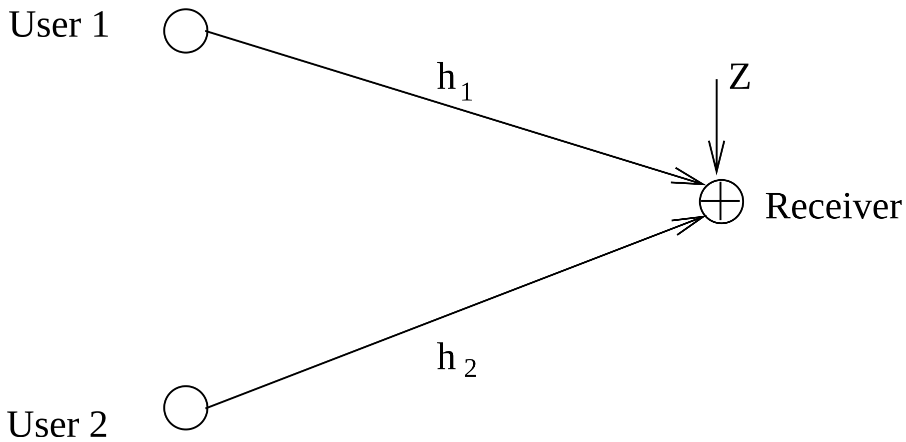
- Capacity region: closure of convex hull of all (R_1, R_2) satisfying

$$R_1 < I(X_1;Y|X_2)$$
$$R_2 < I(X_2;Y|X_1)$$
$$R_1 + R_2 < I(X_1,X_2;Y)$$

for some product distribution $p(x_1,x_2) = p(x_1)p(x_2)$.

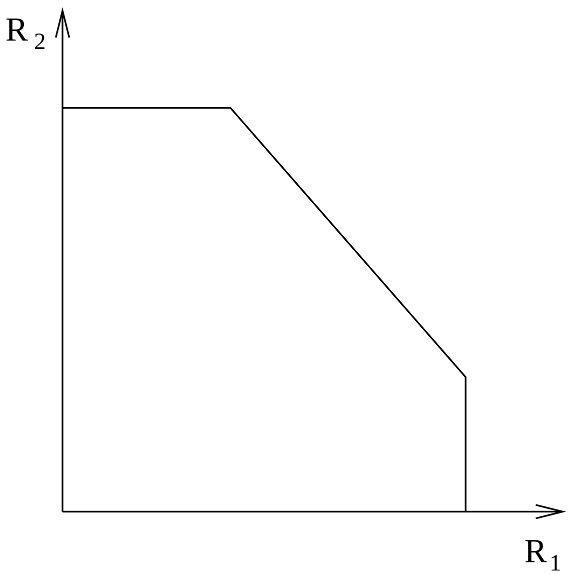
- Closure of convex hull (union) of many pentagons.

Gaussian Multiple Access Channel



$$Y = \sqrt{h_1} X_1 + \sqrt{h_2} X_2 + Z$$

- There is one outer-most pentagon that includes all others; $X_1 \sim N(0, P_1)$ and $X_2 \sim N(0, P_2)$.



$$R_1 < \frac{1}{2} \log\left(1 + \frac{P_1 h_1}{\sigma^2}\right)$$

$$R_2 < \frac{1}{2} \log\left(1 + \frac{P_2 h_2}{\sigma^2}\right)$$

$$R_1 + R_2 < \frac{1}{2} \log\left(1 + \frac{P_1 h_1 + P_2 h_2}{\sigma^2}\right)$$

Fading Single User Channel (Goldsmith-Varaiya 1994)

- Channel capacity for single user

$$C = \frac{1}{2}\log\left(1+\frac{p}{\sigma^2}\right)$$

- In the presence of fading, for a fixed channel state h

$$C(h) = \frac{1}{2}\log\left(1+\frac{p(h)h}{\sigma^2}\right)$$

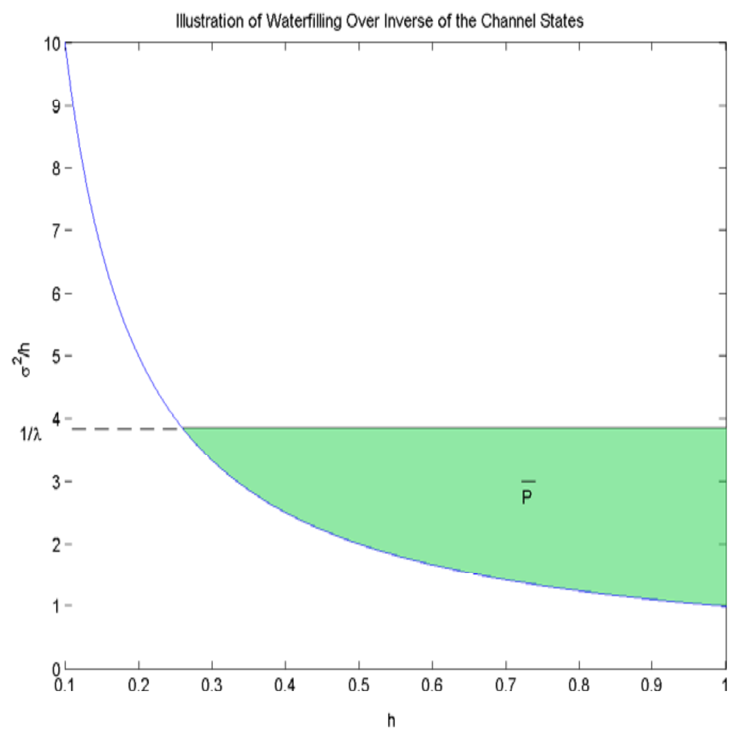
- Maximize the ergodic capacity, given an average power constraint

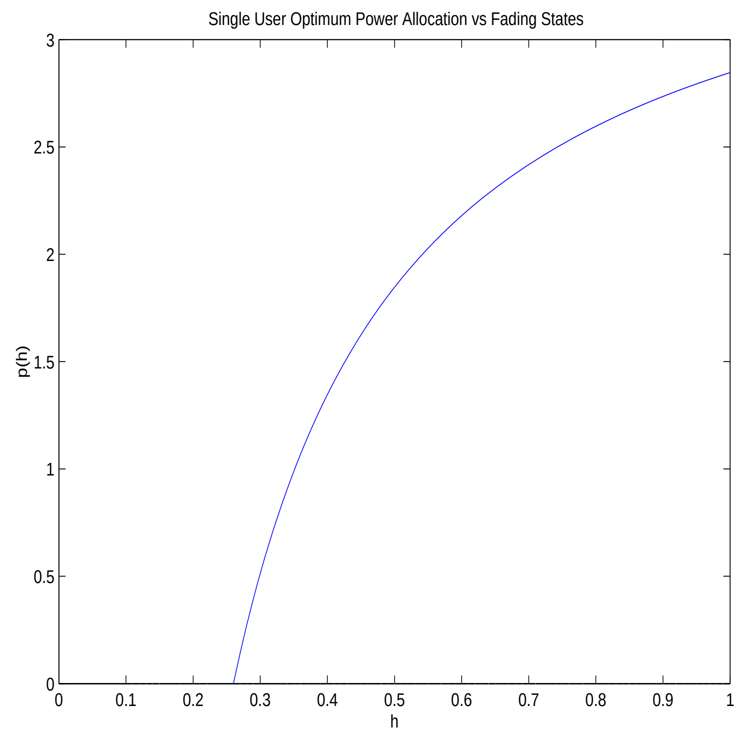
$$\begin{aligned}\max_{p(h)} \quad & E_h\left[\log\left(1+\frac{p(h)h}{\sigma^2}\right)\right] \\ \text{s.t.} \quad & E_h[p(h)] \leq P \\ & p(h) \geq 0, \quad \forall h\end{aligned}$$

Fading Single User Channel Solution: Waterfilling

- Optimal power allocation: waterfilling of power over time

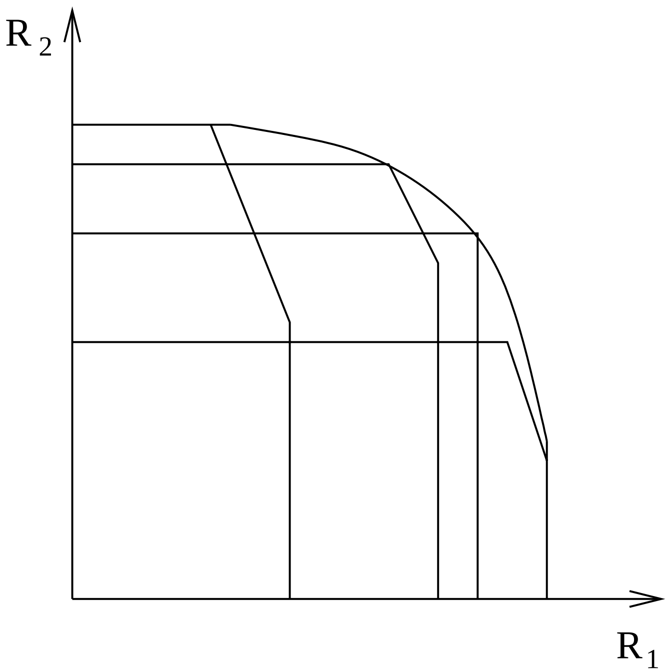
$$p(h) = \left(\frac{1}{\lambda} - \frac{\sigma^2}{h}\right)^+$$





- More power to better channel states; no power to very poor channel states.

Fading Gaussian Multiple Access Channel (Tse-Hanly 1998)



Union of pentagons

$$R_1 < E\left[\frac{1}{2}\log\left(1+\frac{p_1(\mathbf{h})h_1}{\sigma^2}\right)\right] \quad (\triangleq C_1)$$

$$R_2 < E\left[\frac{1}{2}\log\left(1+\frac{p_2(\mathbf{h})h_2}{\sigma^2}\right)\right] \quad (\triangleq C_2)$$

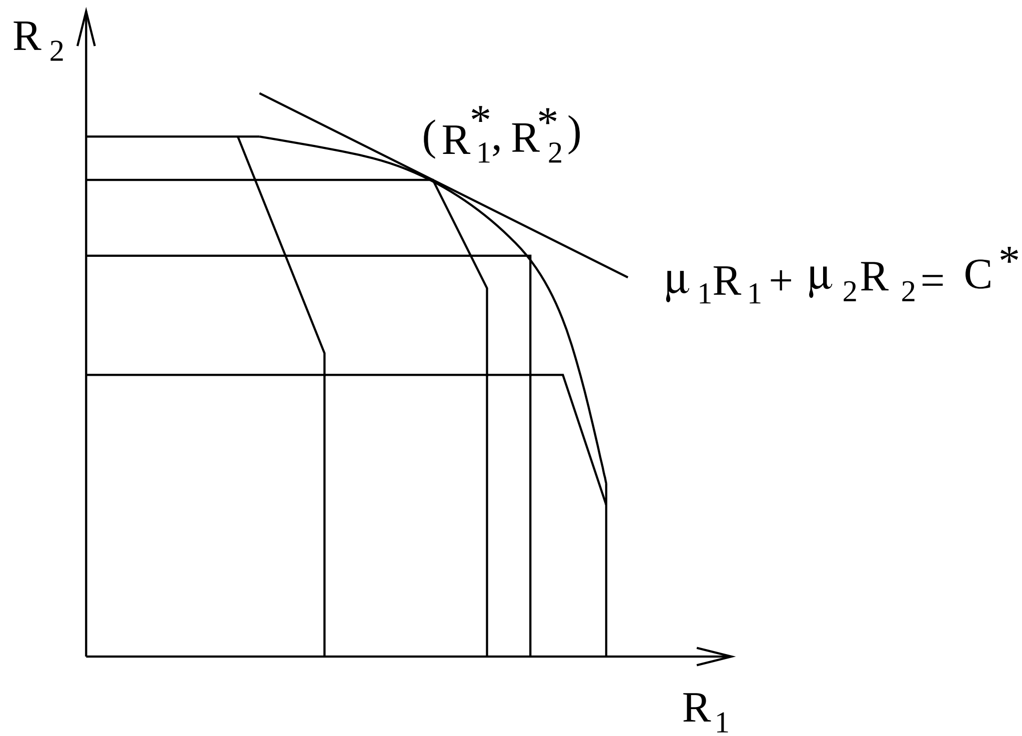
$$R_1 + R_2 < E\left[\frac{1}{2}\log\left(1+\frac{p_1(\mathbf{h})h_1 + p_2(\mathbf{h})h_2}{\sigma^2}\right)\right] \quad (\triangleq C_s)$$

over all feasible power distributions

$$E\left[p_1(\mathbf{h})\right] \le P_1$$

$$E\left[p_2(\mathbf{h})\right] \le P_2$$

Determining the Boundary of the Capacity Region



- The power control policy that corresponds to the rate pair (R_1^*, R_2^*) can be found by maximizing $\mu_1 R_1 + \mu_2 R_2$ subject to the average power constraints, for some μ_1, μ_2.
- Any (R_1^*, R_2^*) on the curved portion of the boundary is a corner of one of the pentagons.
- If $\mu_2 > \mu_1$ then the upper corner; if $\mu_1 > \mu_2$ then the lower corner.

Achieving Arbitrary Rate Tuples on the Boundary

- For given priorities μ_i, maximize $C_\mu \triangleq \mu_1 R_1 + \mu_2 R_2$ subject to $E_{\mathbf{h}}[p_i(\mathbf{h})] \leq P_i$, $\mathbf{R} \in C$.
- Wlog, assume $\mu_2 > \mu_1$. For a given power control policy, the optimum $\mathbf{R}$ is the upper corner.
- The coordinates of the upper corner are:

$$R_2 = C_2, \qquad R_1 = C_s - C_2$$

and

$$\begin{aligned} C_\mu &= \mu_1(C_s - C_2) + \mu_2 C_2 \\ &= (\mu_2 - \mu_1)C_2 + \mu_1 C_s \end{aligned}$$

- Therefore, the optimum power allocation policy is the solution to:

$$\begin{aligned} \max_{\mathbf{p}(\mathbf{h})} \quad & E_{\mathbf{h}}\left[(\mu_2 - \mu_1)\log\left(1 + \frac{p_2(\mathbf{h})h_2}{\sigma^2}\right) + \mu_1 \log\left(1 + \frac{p_1(\mathbf{h})h_1 + p_2(\mathbf{h})h_2}{\sigma^2}\right)\right] \\ \text{s.t.} \quad & E_{\mathbf{h}}[p_i(\mathbf{h})] \leq P_i, \qquad i = 1,2 \\ & p_i(\mathbf{h}) \geq 0, \quad \forall\, \mathbf{h}, \quad i = 1,2 \end{aligned}$$

- The objective function is concave, and the constraint set is convex in powers.

Optimum Power Allocation - Generalized Iterative Waterfilling

- $\mathbf{p}^*(\mathbf{h})$ achieves the global maximum of C_μ iff it satisfies the extended KKT conditions,

$$\sum_{i=1}^{k} \frac{(\mu_i - \mu_{i-1})\, h_k}{\sum_{j=1}^{i} p_j(\mathbf{h}) h_j + p_k(\mathbf{h}) h_k} \leq \lambda_k, \quad \forall\, \mathbf{h}, k = 1, \cdots, K$$

  with equality at $\mathbf{h}$, if $p_k(\mathbf{h}) > 0$.

- Solution based on utilities (Tse-Hanly 1998).
- Solve the KKTs iteratively: generalized iterative waterfilling (Kaya-Ulukus)
  - One-user-at-a-time algorithm, converges to the optimum.
  - Gradually increase power allocated to channel states, in a "specific" order.
  - For user k, power levels can be obtained by solving kth order polynomial equations.

Special Case: Sum Capacity

- All equal μ_i, i.e., $\mu_i = \mu = 1$.

- Maximize ergodic sum capacity, given average power constraints

$$\max_{\mathbf{p}(\mathbf{h})} \quad E_{\mathbf{h}}\left[\log\left(1 + \frac{p_1(\mathbf{h})h_1 + p_2(\mathbf{h})h_2}{\sigma^2}\right)\right]$$
$$\text{s.t.} \quad E_{\mathbf{h}}[p_i(\mathbf{h})] \le P_i, \qquad p_i(\mathbf{h}) \ge 0, \qquad i = 1,2$$

- KKT conditions

$$\frac{h_k}{p_1(\mathbf{h})h_1 + p_2(\mathbf{h})h_2} \le \lambda_k, \quad \forall\, \mathbf{h}, k = 1,2$$

  with equality at $\mathbf{h}$, if $p_k(\mathbf{h}) > 0$.

- For both users to transmit simultaneously at channel state $\mathbf{h} = (h_1, h_2)$, we need

$$\frac{h_1}{h_2} = \frac{\lambda_1}{\lambda_2}$$

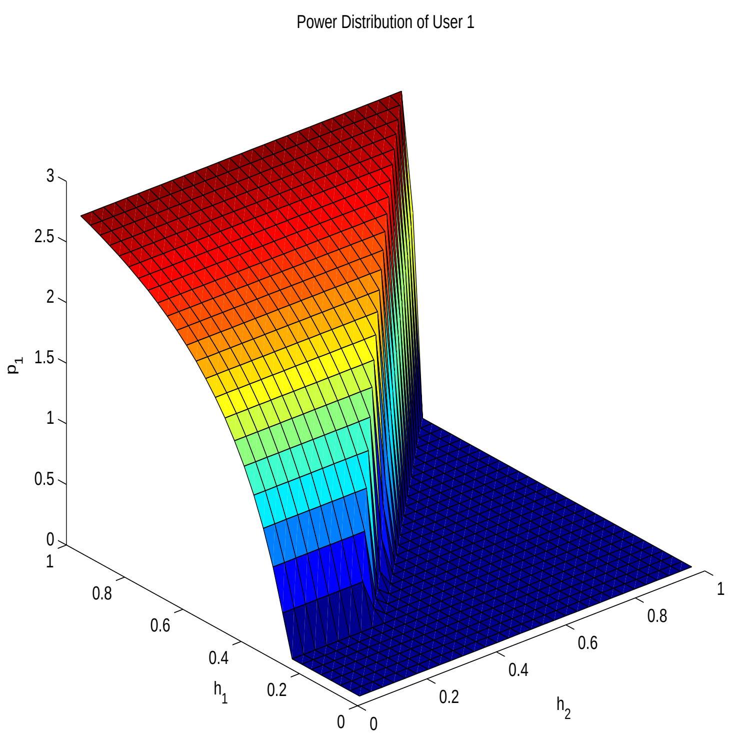
- For continuous channel gains, this is a zero-probability event.

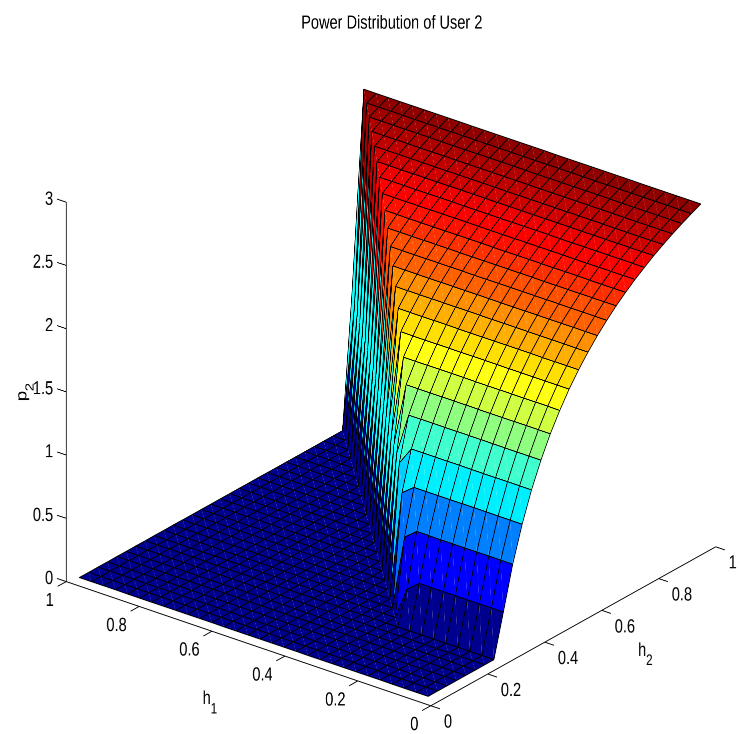
- Only the strongest (after some scaling) user transmits at any given time.

Sum Capacity – Closed-form Solution (Knopp-Humblet 1995)

- Closed-form optimal power allocation: single-user waterfilling over favorable channel states

$$p_k(\mathbf{h}) = \begin{cases} \left(\frac{1}{\lambda_k} - \frac{\sigma^2}{h_k}\right)^+, & \text{if} \quad h_k/\lambda_k > h_j/\lambda_j, \quad \forall j \neq k \\ 0, & \text{otherwise} \end{cases}$$



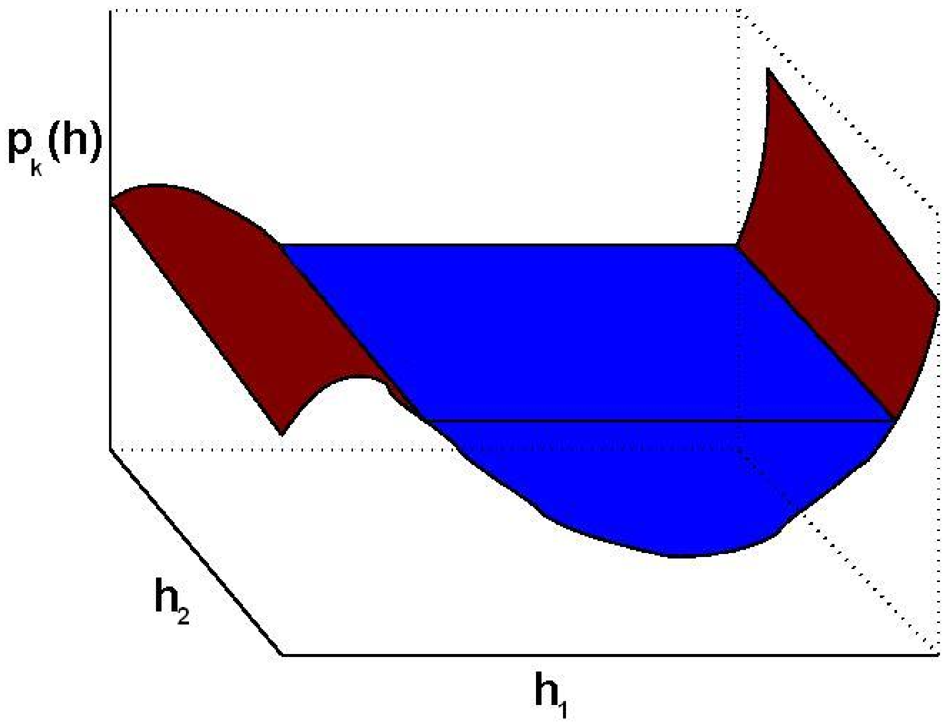


Sum Capacity – Algorithmic Solution

- One-user-at-a-time iterative waterfilling (Kaya-Ulukus)

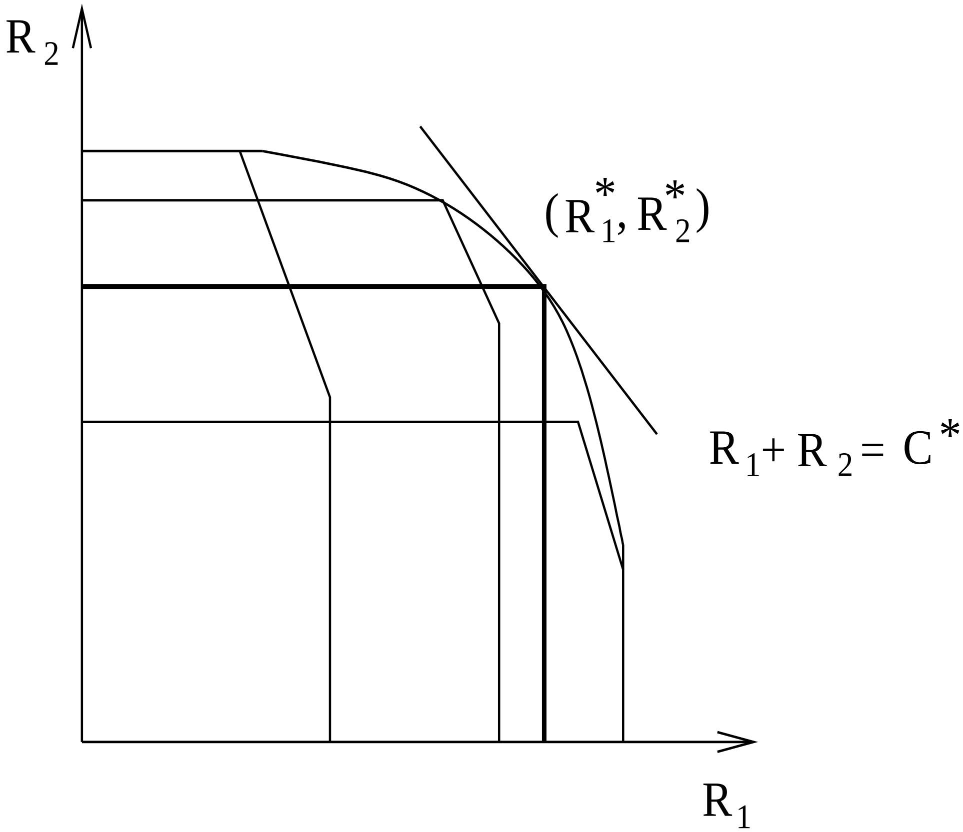
$$p_1(\mathbf{h}) = \left(\frac{1}{\lambda_1} - \frac{p_2(\mathbf{h})h_2 + \sigma^2}{h_1}\right)^+$$

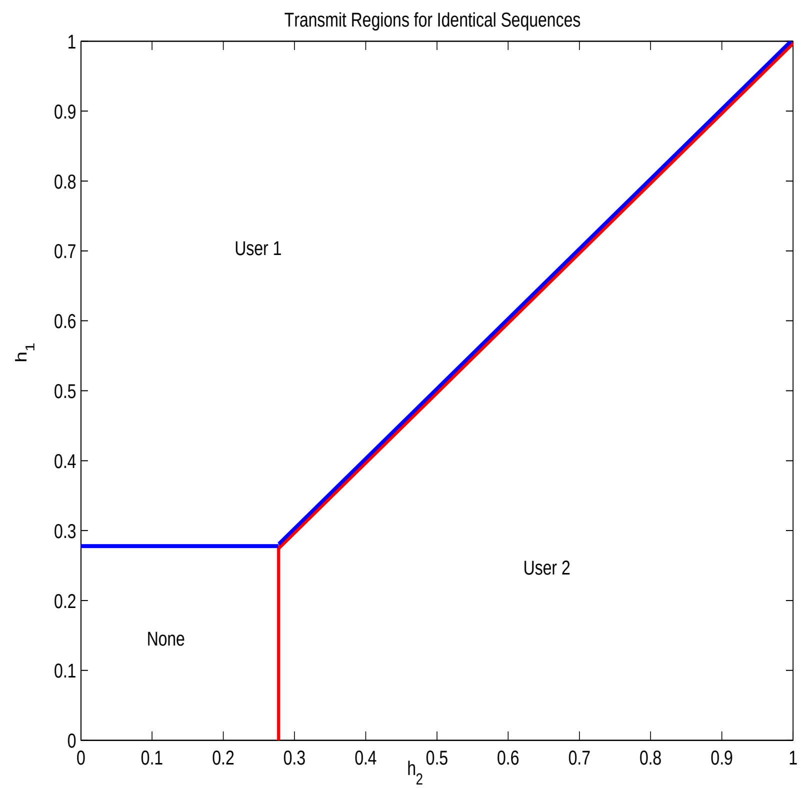
$$p_2(\mathbf{h}) = \left(\frac{1}{\lambda_2} - \frac{p_1(\mathbf{h})h_1 + \sigma^2}{h_2}\right)^+$$



Issue of Simultaneous Transmissions Revisited

- Sum capacity achieving pentagon: a rectangle (orthogonal transmissions)

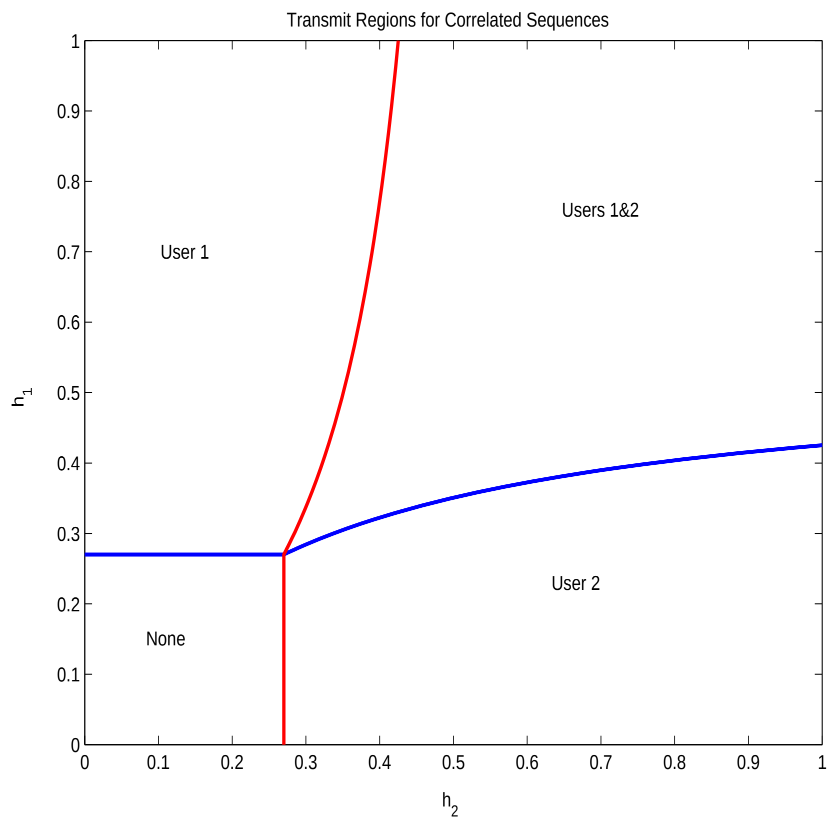




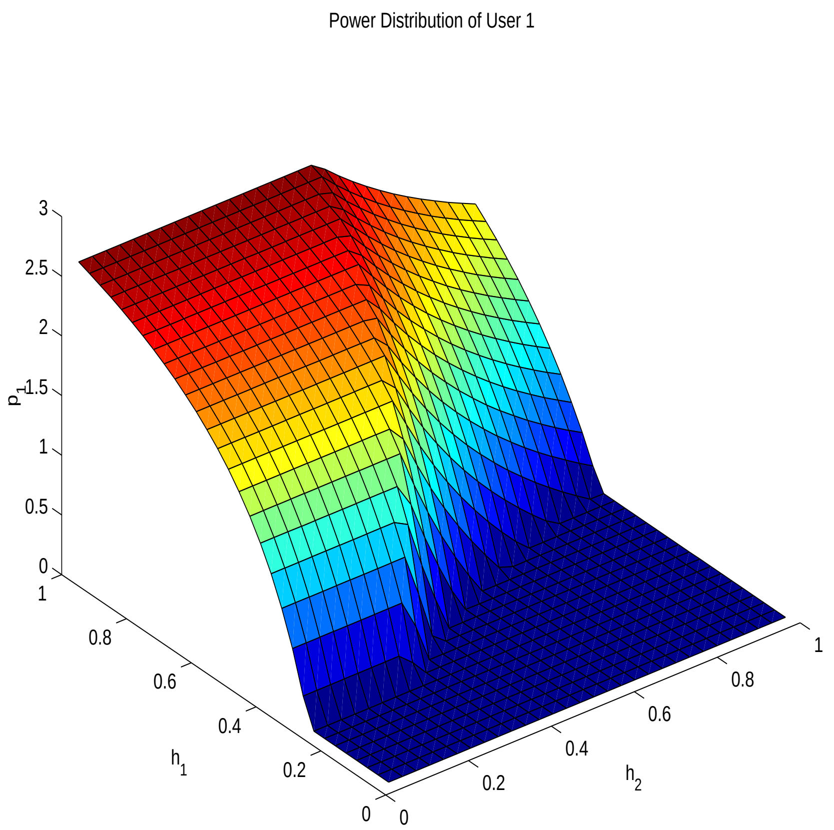
- "Only the strongest user transmits" result is specific to **sum capacity** and **scalar channel**.

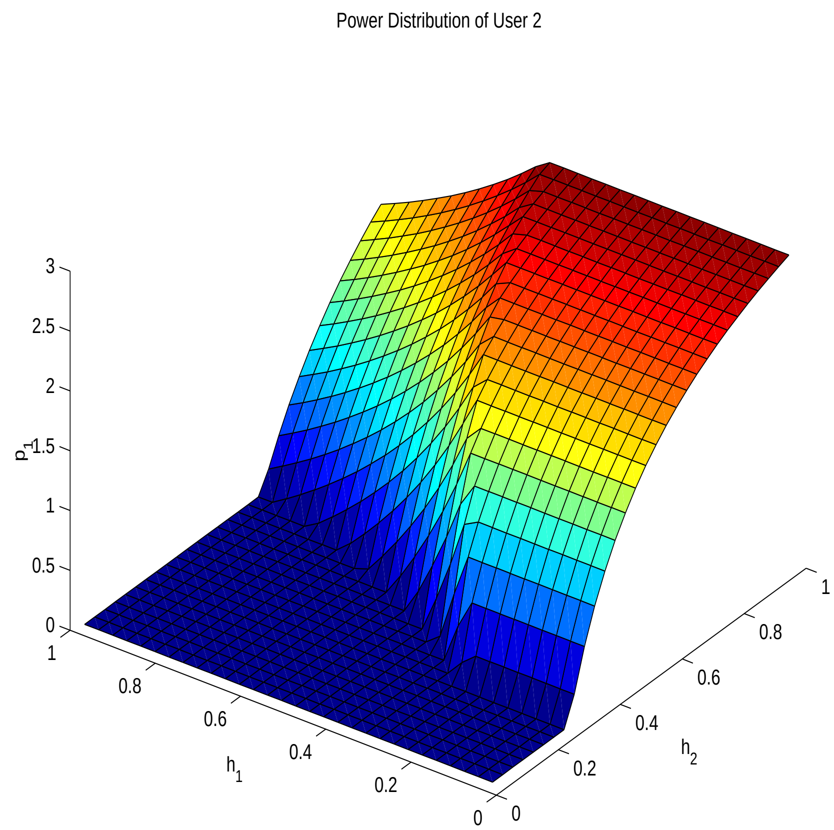
Simultaneous Transmit Condition in CDMA (A Vector Channel)

- There exists a non-zero probability region of fading states **h** where all K users in the system transmit simultaneously, if and only if $\{\mathbf{s}_i\mathbf{s}_i^\top\}_{i=1}^K$ are linearly independent.
- When $K \leq N$, for a set of K linearly independent signature sequences, there always exists a non-zero probability region of channel states where all K users transmit simultaneously.



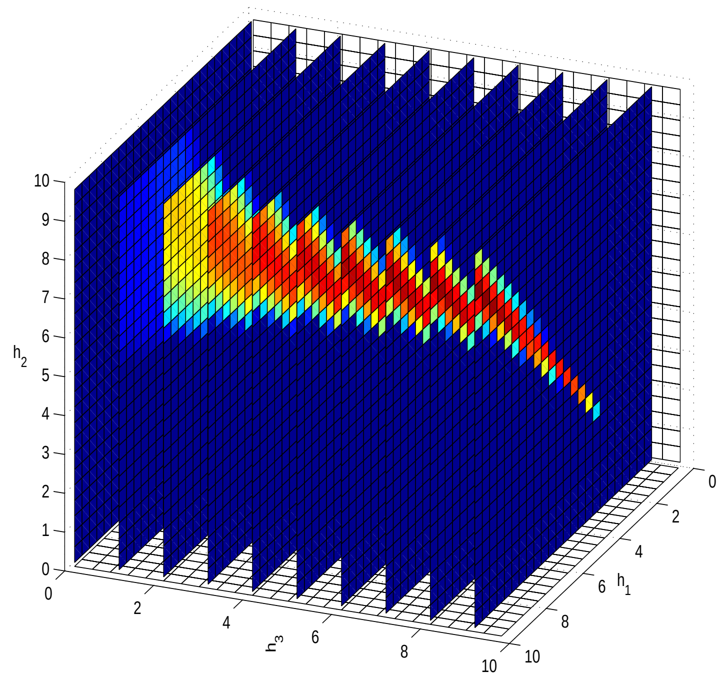
Transmit Powers: Arbitrary Signatures (CDMA)





Maximum Number of Simultaneous Transmissions (CDMA)

- For a set of K signature sequences and processing gain N, the number of users that can transmit simultaneously cannot be larger than $N(N+1)/2$.

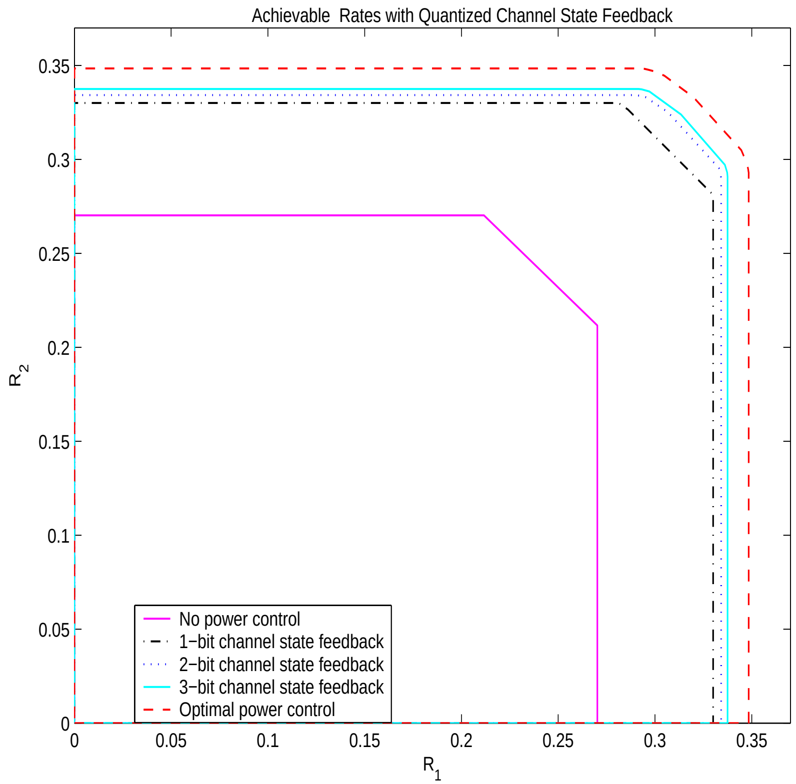


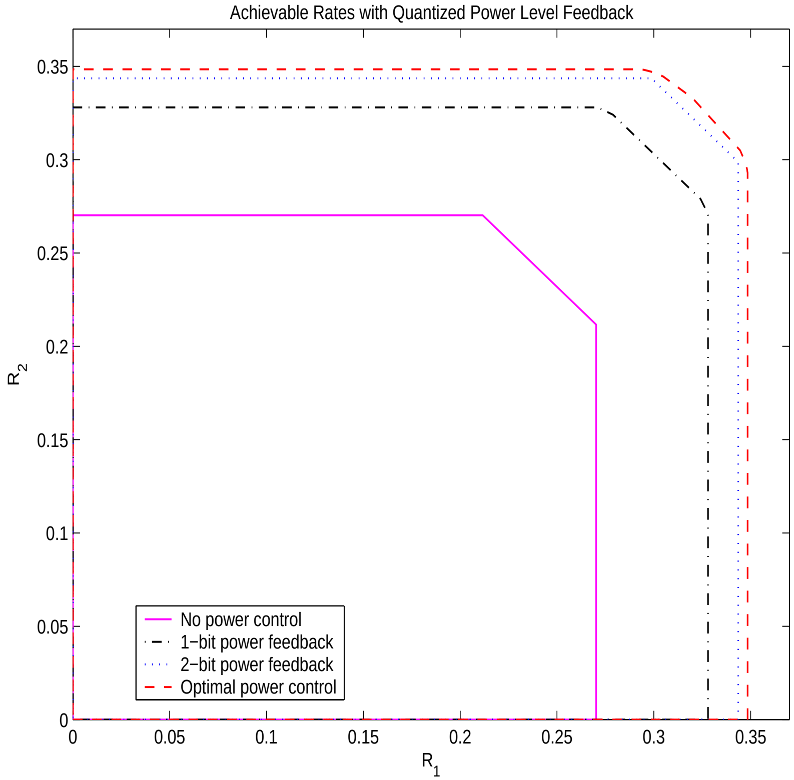
- Signature sequences $\{\mathbf{s}_i\}_{i=1}^{K}$ are linearly dependent, but $\{\mathbf{s}_i\mathbf{s}_i^\top\}_{i=1}^{K}$ are linearly independent.

Power Control with Limited Feedback

- In practice, CSI feedback from the receiver to the transmitters is very limited.
- Can formulate an optimal vector quantization problem to find the quantization bins for $\mathbf{h}$ and quantizer output levels $p(\mathbf{h})$ for given feedback rate constraints, difficult to solve.
- Two simple approaches to reduce the feedback information
  - Quantize the channel states to the nearest (MMSE sense) discrete levels, feed them back, design power control w.r.t. the quantized discrete random variable.
  - Obtain optimal power levels $p(\mathbf{h})$, find their distribution, quantize w.r.t. this induced distribution to the nearest (MMSE sense) discrete power levels, and feed back.

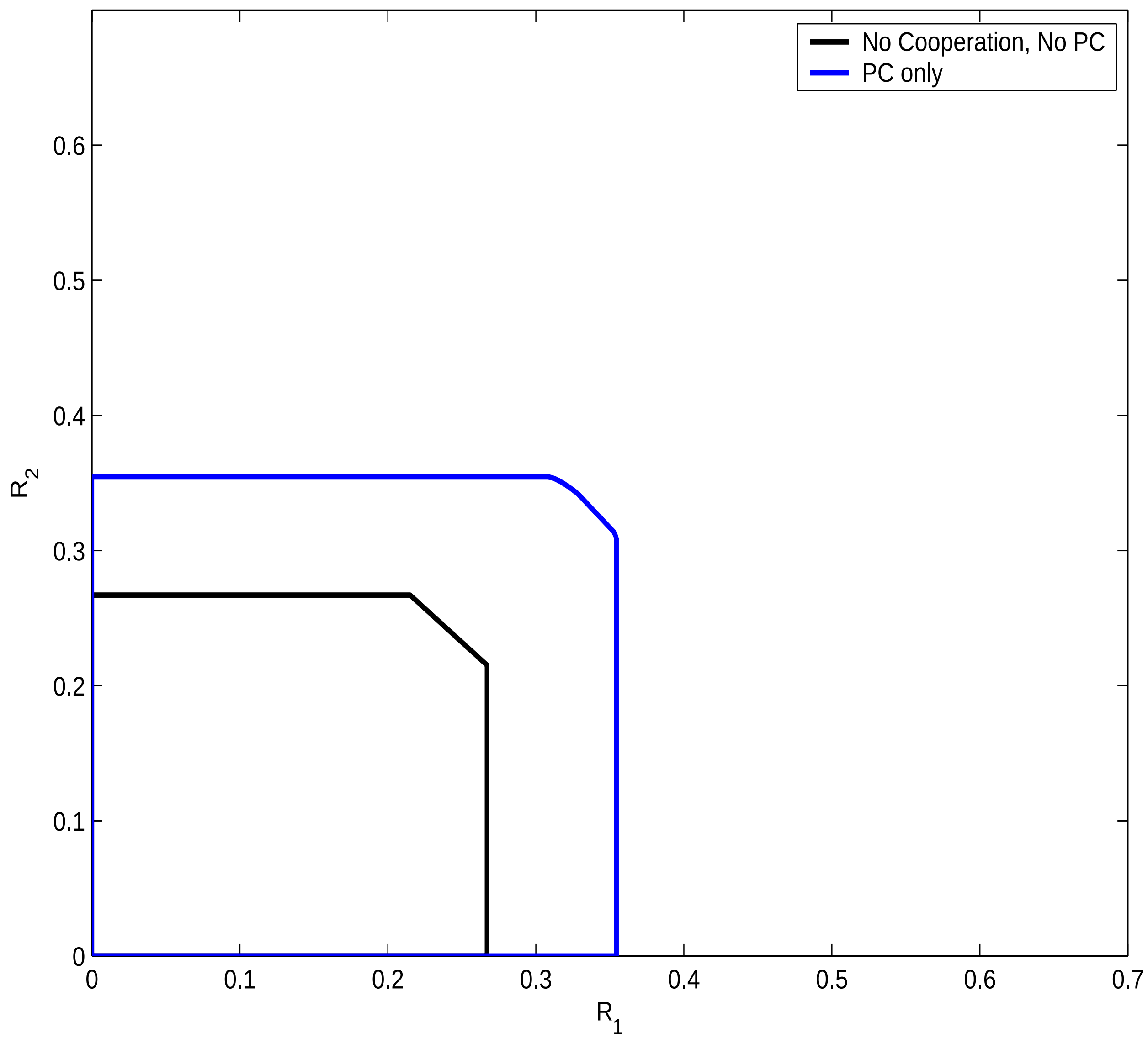
Achievable Rate Regions with Limited Feedback



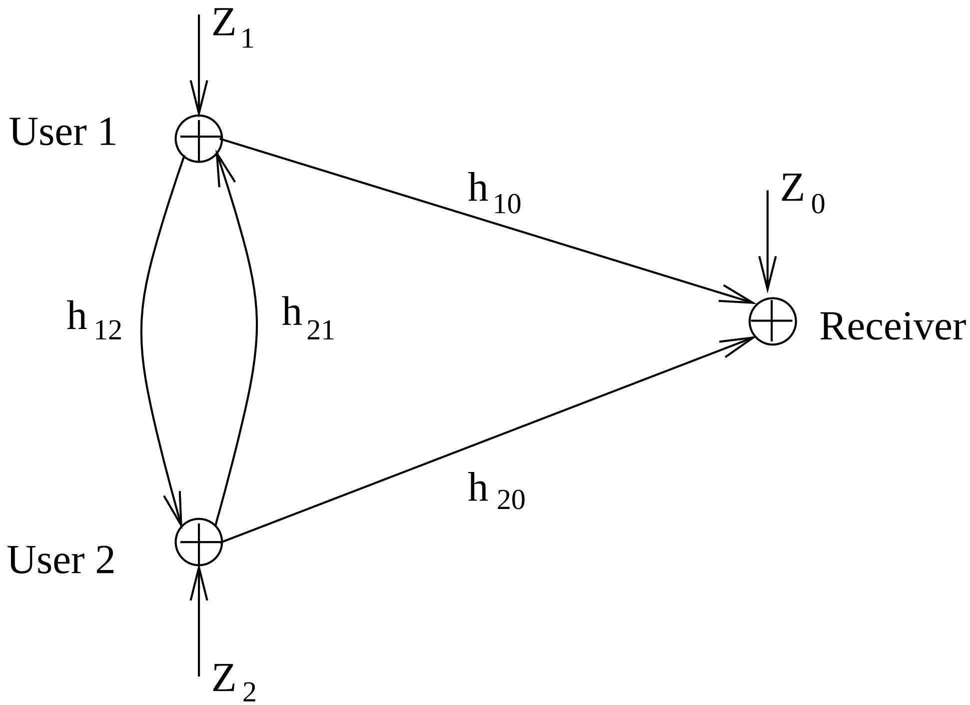


- We obtain rates very close to the capacity with perfect CSI, with very little feedback per user.

Conclusion: Rate Region of Fading MAC with Power Control



User Cooperation

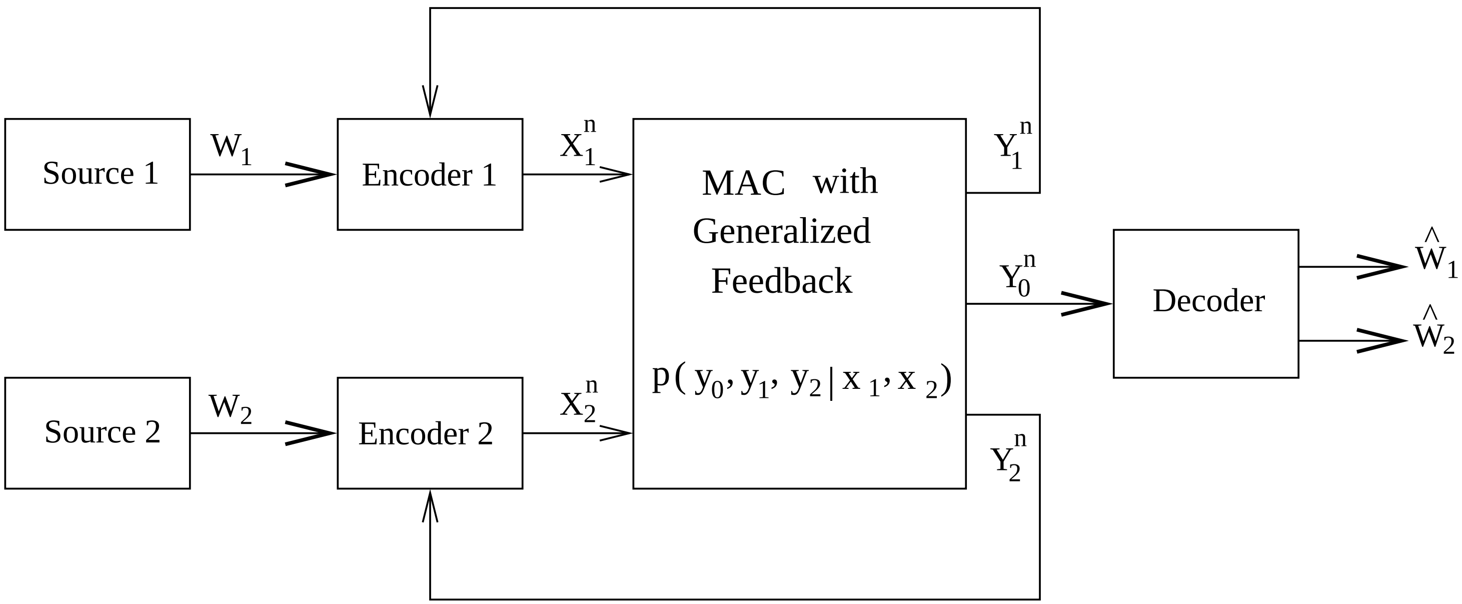


$$Y_0 = \sqrt{h_{10}} X_1 + \sqrt{h_{20}} X_2 + Z_0$$
$$Y_1 = \sqrt{h_{21}} X_2 + Z_1$$
$$Y_2 = \sqrt{h_{12}} X_1 + Z_2$$

- Interference is information.
- Some versions of all transmitted signals are received by all nodes.
- Exploit overheard information to jointly design encoding, transmission, routing policies.
- Building block towards the analysis of ad-hoc and sensor networks.
- Information theoretically: multiple access channel with generalized feedback (capacity open).

MAC with Generalized Feedback

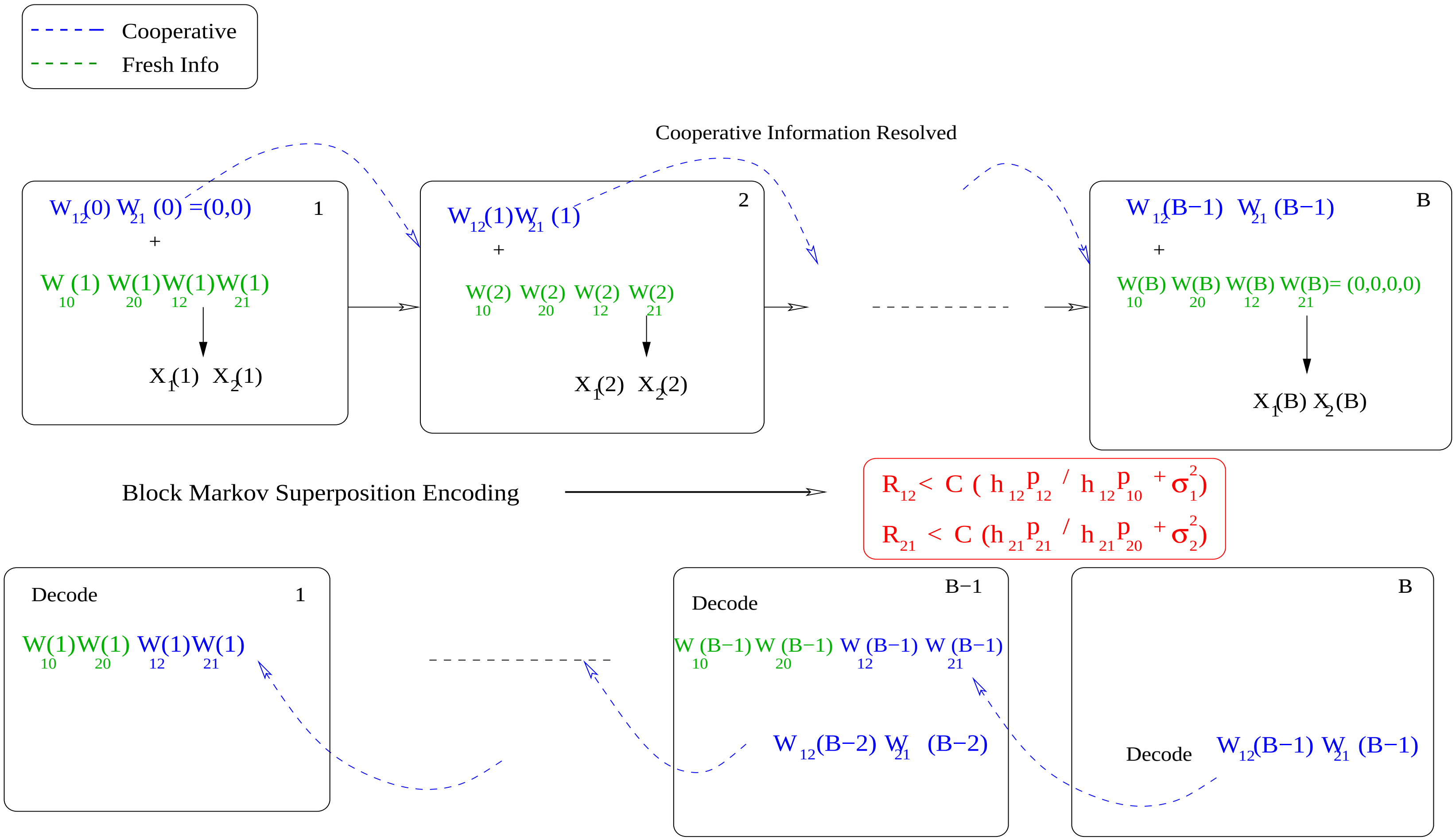
- Gaussian MAC with cooperating encoders (Sendonaris, Erkip, Aazhang 2003)
- Special case of MAC with generalized feedback (Willems, van der Meulen, Schalkwijk 1983)



MAC with Generalized Feedback – Achievable Rate Region

- An achievable rate region is obtained by employing
  - Block Markov superposition encoding
    - Inject high rate fresh information to be resolved with the help of upcoming blocks.
    - Send resolution information for previous blocks.
  - Backward decoding
    - After receiving all blocks, decode the resolution information in the last block.
    - Using previously decoded resolution information, sequentially decode earlier blocks.

Block Markov Superposition Encoding and Backward Decoding



$$R_{12} < C\left(h_{12}P_{12} / (h_{12}P_{10} + \sigma_1^2)\right)$$
$$R_{21} < C\left(h_{21}P_{21} / (h_{21}P_{20} + \sigma_2^2)\right)$$

$$R_{10} < C\left(h_{10}P_{10} / \sigma_0^2\right) \qquad R_{20} < C\left(h_{20}P_{20} / \sigma_0^2\right)$$
$$R_{10} + R_{20} < C\left((h_{10}P_{10} + h_{20}P_{20}) / \sigma_0^2\right)$$
$$R_{10} + R_{20} + R_{12} + R_{21} < C\left((h_{10}P_1 + h_{20}P_2 + 2\sqrt{h_{10}h_{20}P_{u1}P_{u2}}) / \sigma_0^2 \right)$$

Gaussian MAC with User Cooperation – No Resource Allocation

- Block Markov superposition coding (Sendonaris, Erkip, Aazhang 2003)
  - Build common information (X_{12}, X_{21})
  - Cooperatively send (U)
  - Inject new information (X_{10}, X_{20})

$$X_1 = \sqrt{p_{10}} X_{10} + \sqrt{p_{12}} X_{12} + \sqrt{p_{u1}} U$$
$$X_2 = \sqrt{p_{20}} X_{20} + \sqrt{p_{21}} X_{21} + \sqrt{p_{u2}} U$$

- Amplitude of the each channel's gain is assumed to be known at the corresponding receiver.
- Phases of all channel gains are assumed to be known at the receiver and the transmitters
  - Coherent combining
- **Amplitudes of the channel gains are not known at the transmitters**.

Achievable Region with Block Markov Encoding/Backward Decoding

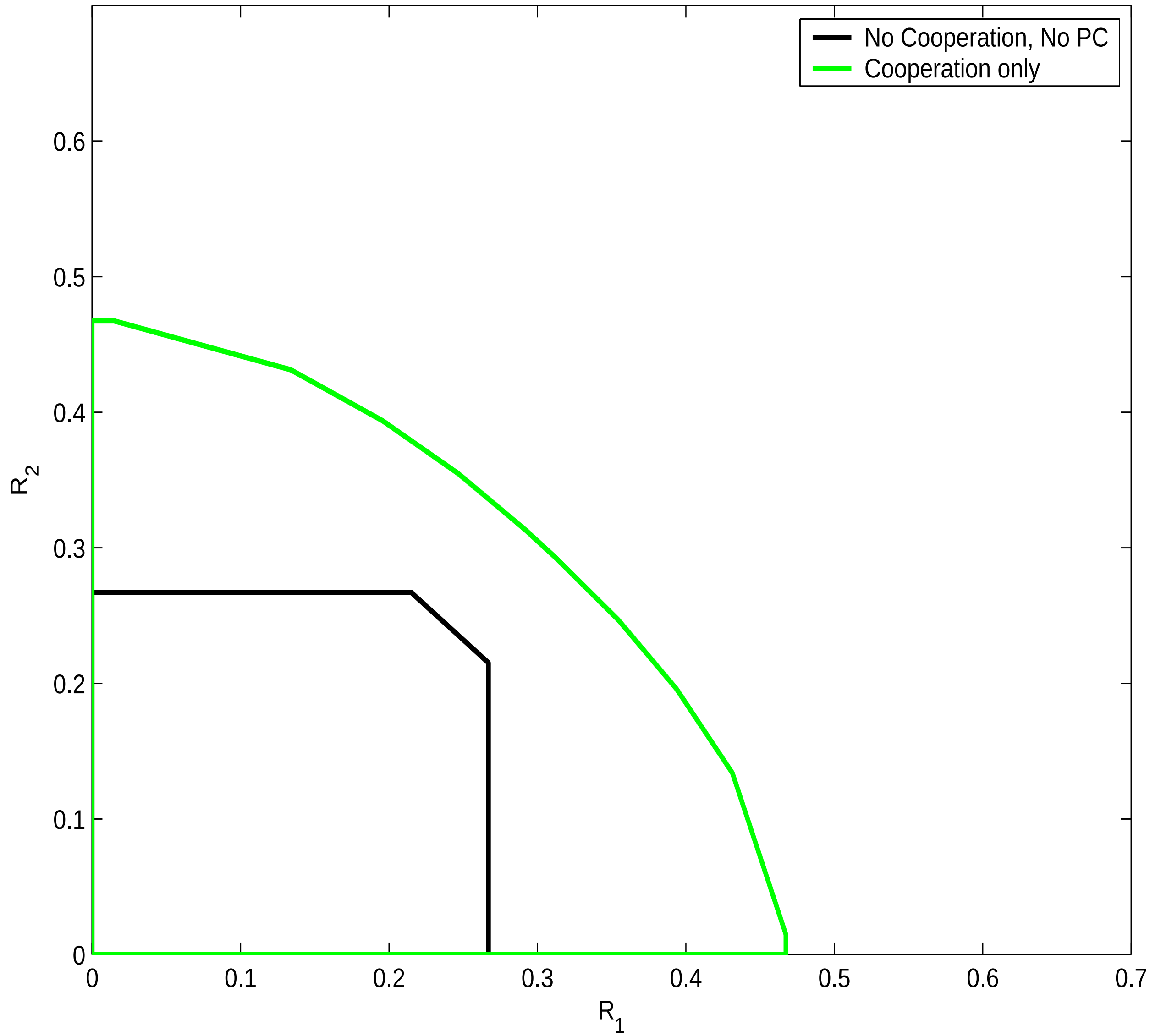
- Union of regions (pentagons or triangles)

$$
\begin{aligned}
R_1 &< E\left[\log\left(1+\frac{h_{12}p_{12}}{h_{12}p_{10}+\sigma_2^2}\right)+\log\left(1+\frac{h_{10}p_{10}}{\sigma_0^2}\right)\right]\\
R_2 &< E\left[\log\left(1+\frac{h_{21}p_{21}}{h_{21}p_{20}+\sigma_1^2}\right)+\log\left(1+\frac{h_{20}p_{20}}{\sigma_0^2}\right)\right]\\
R_1+R_2 &< \min\left\{E\left[\log\left(1+\frac{h_{10}p_1+h_{20}p_2+2\sqrt{h_{10}h_{20}p_{U_1}p_{U_2}}}{\sigma_0^2}\right)\right],\right.\\
&\left. E\left[\log\left(1+\frac{h_{12}p_{12}}{h_{12}p_{10}+\sigma_2^2}\right)+\log\left(1+\frac{h_{21}p_{21}}{h_{21}p_{20}+\sigma_1^2}\right)+\log\left(1+\frac{h_{10}p_{10}+h_{20}p_{20}}{\sigma_0^2}\right)\right]\right\}
\end{aligned}
$$

over all valid (**channel-independent**) power distributions

$$
\begin{aligned}
p_{10}+p_{12}+p_{U_1} &\le P_1\\
p_{20}+p_{21}+p_{U_2} &\le P_2
\end{aligned}
$$

Achievable Rate Region for MAC with User Cooperation



Gaussian MAC with User Cooperation – Resource Allocation

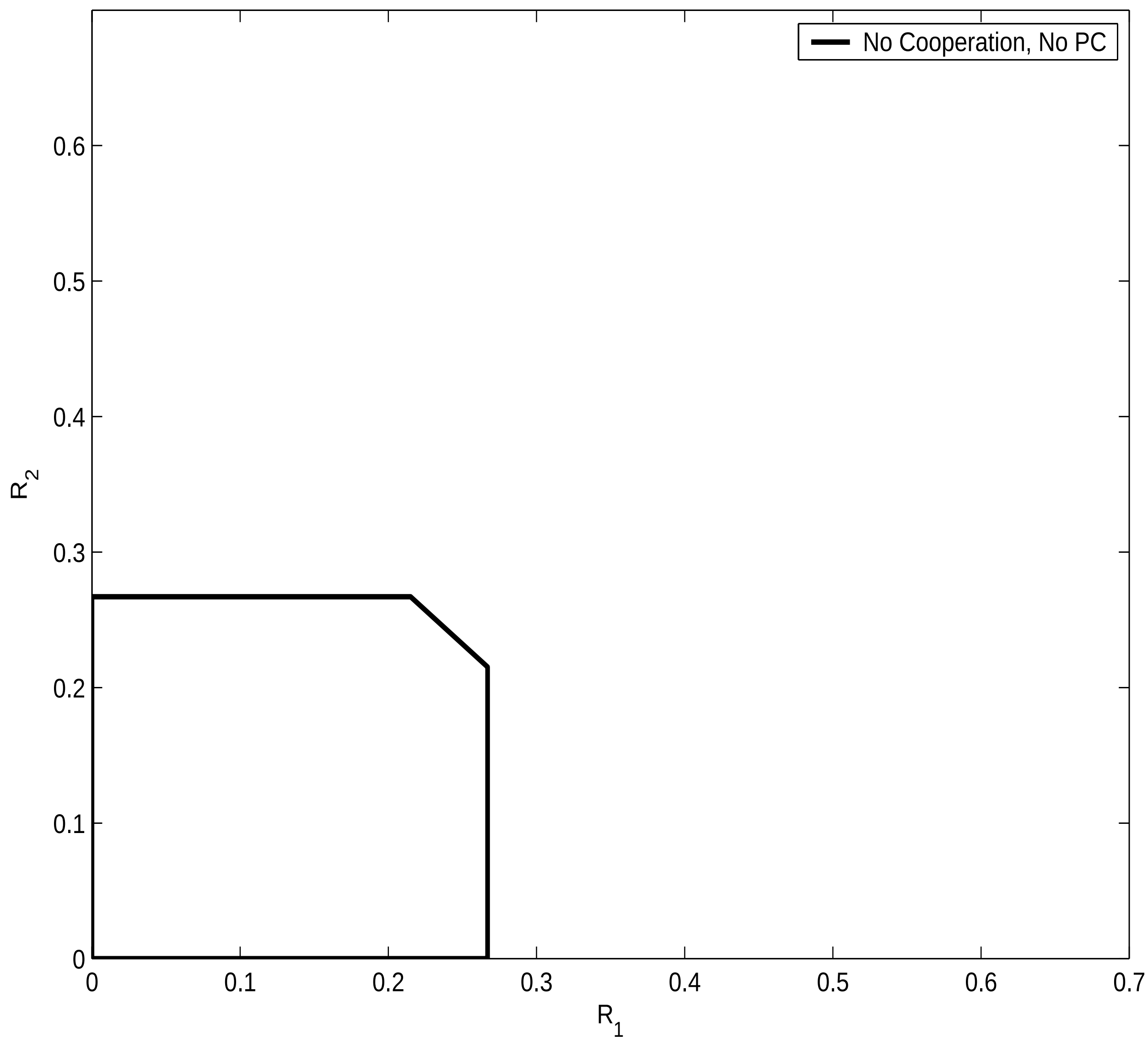
- Choose transmit powers as functions of the channel state

$$X_1 = \sqrt{p_{10}(\mathbf{h})}X_{10} + \sqrt{p_{12}(\mathbf{h})}X_{12} + \sqrt{p_{u1}(\mathbf{h})}U$$
$$X_2 = \sqrt{p_{20}(\mathbf{h})}X_{20} + \sqrt{p_{21}(\mathbf{h})}X_{21} + \sqrt{p_{u2}(\mathbf{h})}U$$

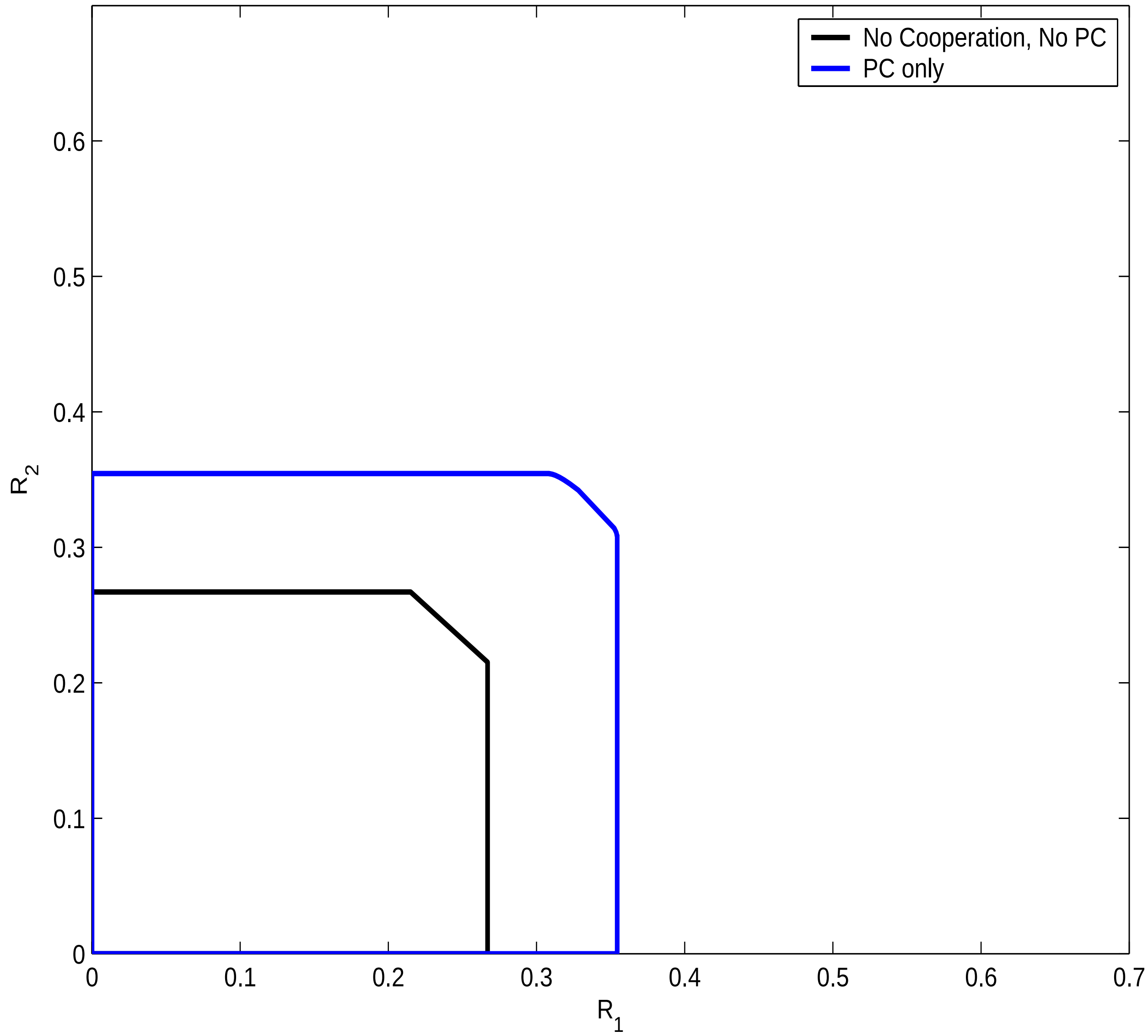
- Complete channel state information at the transmitters and the receiver.
  - Only additional CSI: channel amplitudes at the transmitters.
- Transmitted codewords are modulated by channel adaptive power levels
  - Opportunistic cooperation and transmission – use available average power efficiently.
- A special case is power control only

$$X_1 = \sqrt{p_{10}(\mathbf{h})}X_{10}$$
$$X_2 = \sqrt{p_{20}(\mathbf{h})}X_{20}$$

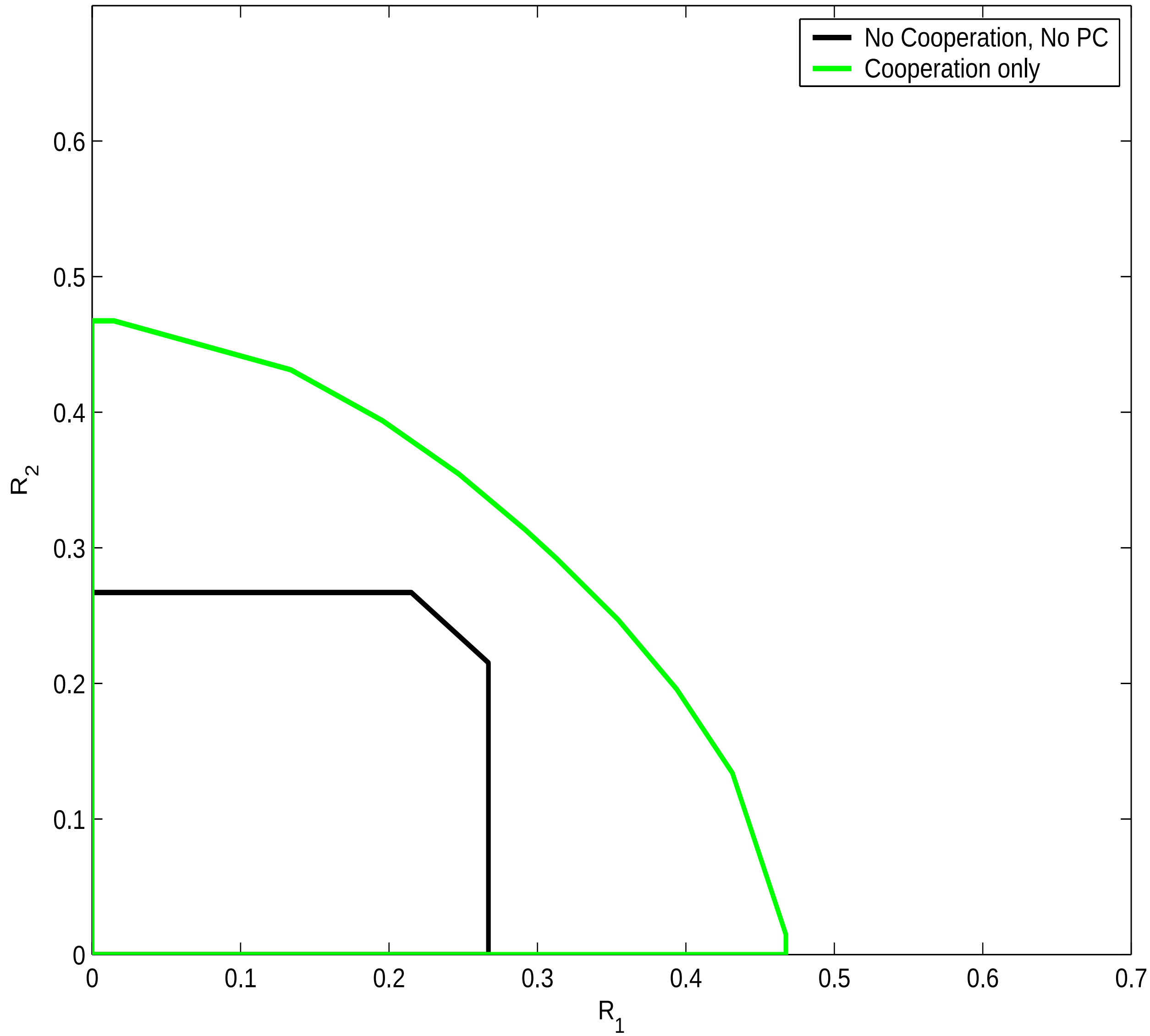
No Cooperation, No Power Control



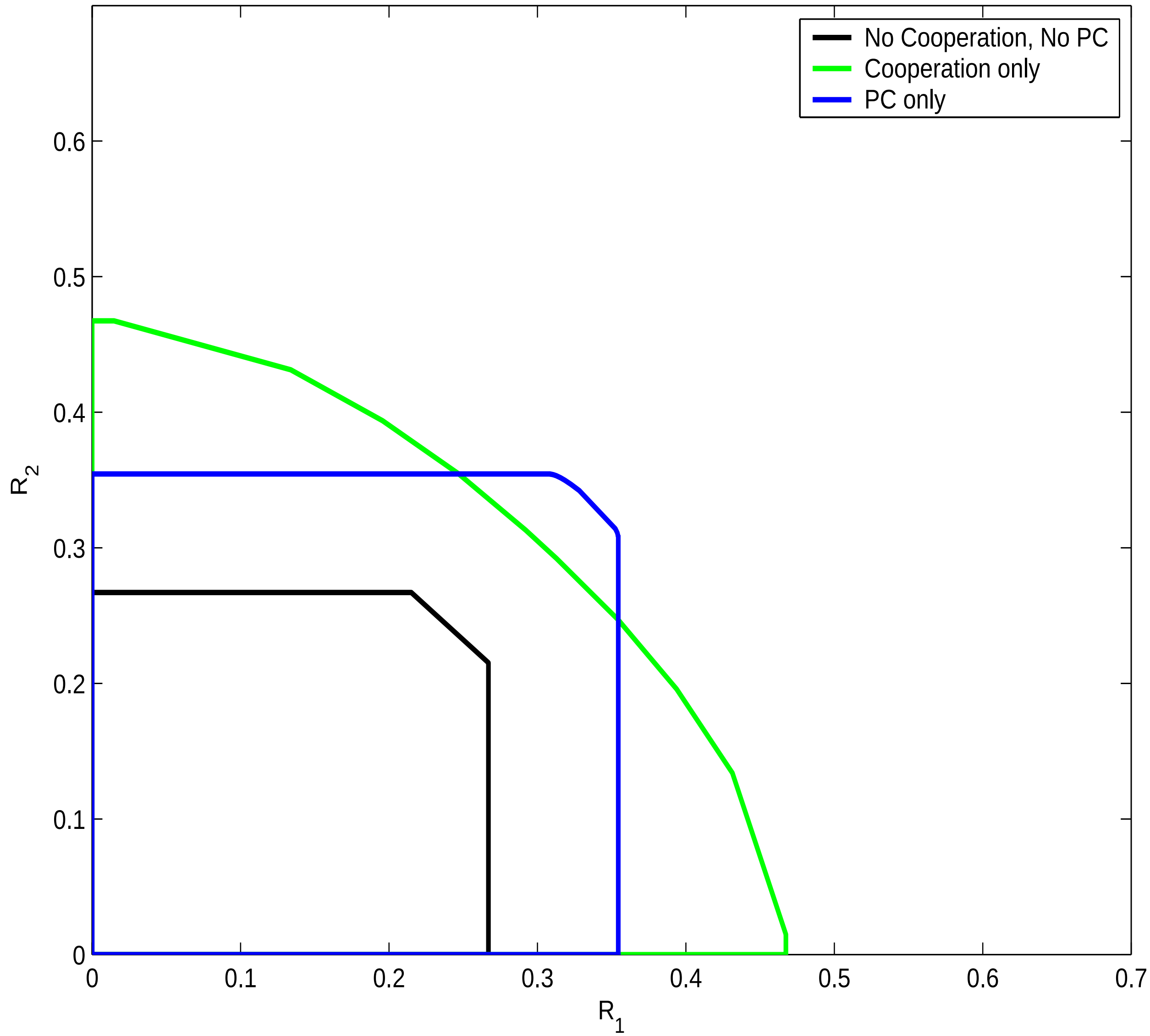
Power Control Only



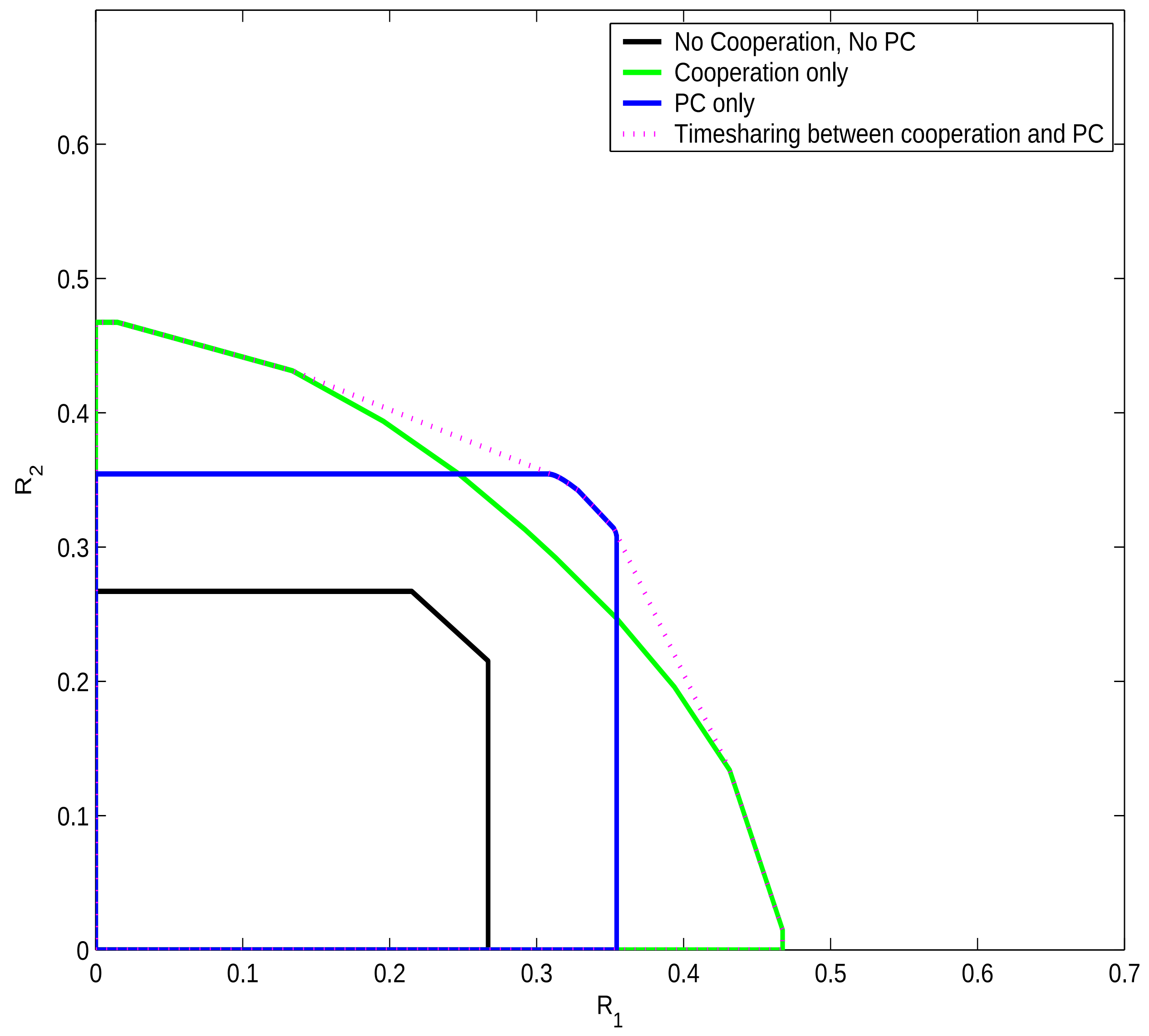
User Cooperation Only



Power Control Only and User Cooperation Only



Power Control Only, User Cooperation Only and Timesharing



Achievable Region of Rates with Power Control

- Union of regions (pentagons and triangles)

$$R_1 < E\left[\log\left(1+\frac{h_{12}p_{12}(\mathbf{h})}{h_{12}p_{10}(\mathbf{h})+\sigma_2^2}\right)+\log\left(1+\frac{h_{10}p_{10}(\mathbf{h})}{\sigma_0^2}\right)\right]$$

$$R_2 < E\left[\log\left(1+\frac{h_{21}p_{21}(\mathbf{h})}{h_{21}p_{20}(\mathbf{h})+\sigma_1^2}\right)+\log\left(1+\frac{h_{20}p_{20}(\mathbf{h})}{\sigma_0^2}\right)\right]$$

$$R_1+R_2 < \min\left\{E\left[\log\left(1+\frac{h_{10}p_1(\mathbf{h})+h_{20}p_2(\mathbf{h})+2\sqrt{h_{10}h_{20}p_{U_1}(\mathbf{h})p_{U_2}(\mathbf{h})}}{\sigma_0^2}\right)\right],\right.$$
$$\left. E\left[\log\left(1+\frac{h_{12}p_{12}(\mathbf{h})}{h_{12}p_{10}(\mathbf{h})+\sigma_2^2}\right)+\log\left(1+\frac{h_{21}p_{21}(\mathbf{h})}{h_{21}p_{20}(\mathbf{h})+\sigma_1^2}\right)+\log\left(1+\frac{h_{10}p_{10}(\mathbf{h})+h_{20}p_{20}(\mathbf{h})}{\sigma_0^2}\right)\right]\right\}$$

over all **(channel-dependent)** feasible power control policies

$$E[p_{10}(\mathbf{h})+p_{12}(\mathbf{h})+p_{U_1}(\mathbf{h})] \le P_1$$
$$E[p_{20}(\mathbf{h})+p_{21}(\mathbf{h})+p_{U_2}(\mathbf{h})] \le P_2$$

- Bounds are not concave in power vector $\mathbf{p}(\mathbf{h}) = [p_{10}(\mathbf{h})\ p_{12}(\mathbf{h})\ p_{U_1}(\mathbf{h})\ p_{20}(\mathbf{h})\ p_{21}(\mathbf{h})\ p_{U_2}(\mathbf{h})]$

Properties of the Sum-Rate-Optimal Power Allocation

- Let the effective channel gains normalized by the noise powers be defined as $s_{ij} = h_{ij}/\sigma_j^2$. Then, for the power control policy $\mathbf{p}^*(\mathbf{h})$ that maximizes the sum rate, we need
  - $p_{10}^*(\mathbf{h}) = p_{20}^*(\mathbf{h}) = 0$, if $s_{12} > s_{10}$ and $s_{21} > s_{20}$
  - $p_{10}^*(\mathbf{h}) = p_{21}^*(\mathbf{h}) = 0$, if $s_{12} > s_{10}$ and $s_{21} \leq s_{20}$
  - $p_{12}^*(\mathbf{h}) = p_{20}^*(\mathbf{h}) = 0$, if $s_{12} \leq s_{10}$ and $s_{21} > s_{20}$
  - $$\left.\begin{array}{c} p_{12}^*(\mathbf{h}) = p_{21}^*(\mathbf{h}) = 0 \\ \text{OR} \\ p_{10}^*(\mathbf{h}) = p_{21}^*(\mathbf{h}) = 0 \\ \text{OR} \\ p_{12}^*(\mathbf{h}) = p_{20}^*(\mathbf{h}) = 0 \end{array}\right\} \text{if } s_{12} \leq s_{10} \text{ and } s_{21} \leq s_{20}$$

Implications of the Optimal Power Allocation

- Block Markov superposition coding is simpler than originally thought.
  - Each transmitter either sends a cooperation signal or fresh information, but not both!
- The choice at each channel state “only” depends on the channel state.
  - Channel statistics, power constraints play no role in deciding which signals to transmit.
  - Except for the tiny little last case ... which usually has very insignificant probability.
- The achievable rate expressions are greatly simplified, and are now concave.
- This simplified coding policy not only maximizes the sum rate, but also the individual rate constraints on R_1 and R_2, and is optimal in terms of the entire rate region.
- Concave optimization problem over a convex constraint set, but non-differentiable.

Simplified Rate Region – Example

- Assume $s_{12} > s_{10}$, $s_{21} > s_{20}$ to illustrate the simplified rate region.

$$
\begin{aligned}
R_1 &< E\left[\log\left(1 + s_{12} p_{12}(\mathbf{h})\right)\right] \\
R_2 &< E\left[\log\left(1 + s_{21} p_{21}(\mathbf{h})\right)\right] \\
R_1 + R_2 &< \min\left\{ E\left[\log\left(1 + s_{10} p_1(\mathbf{h}) + s_{20} p_2(\mathbf{h}) + 2\sqrt{s_{10} s_{20} p_{U_1}(\mathbf{h}) p_{U_2}(\mathbf{h})}\right)\right], \right. \\
&\qquad \left. E\left[\log\left(1 + s_{12} p_{12}(\mathbf{h})\right) + \log\left(1 + s_{21} p_{21}(\mathbf{h})\right)\right] \right\}
\end{aligned}
$$

- Inequalities define either a pentagon like in the traditional MAC, or a triangle.
- All bounds are concave in powers, and so is any weighted sum $\mu_1 R_1 + \mu_2 R_2$ at the corners.
- Sum rate is not differentiable where the arguments of the min are equal.

Rate Maximization Using Subgradient Method

- Points on the rate region boundary can be obtained by maximizing $C_\mu = \mu_1 R_1 + \mu_2 R_2$.
- The optimization problem for arbitrary priorities μ_1 and μ_2 is given by

$$\begin{aligned} \max_{\mathbf{p}(\mathbf{h})} \quad & \mu_1 R_1 + \mu_2 R_2 \\ \text{s.t.} \quad & E_{3,4}\left[p_{10}(\mathbf{h})\right] + E_{1,2}\left[p_{12}(\mathbf{h})\right] + E\left[p_{U_1}(\mathbf{h})\right] \leq P_1 \\ & E_{2,4}\left[p_{20}(\mathbf{h})\right] + E_{1,3}\left[p_{21}(\mathbf{h})\right] + E\left[p_{U_2}(\mathbf{h})\right] \leq P_2 \end{aligned}$$

- $\{R_1, R_2\}$ is the corner of the pentagon obtained for a given power allocation policy.
- Gradient of the objective function does not exist everywhere, find subgradient $\mathbf{g}$ instead

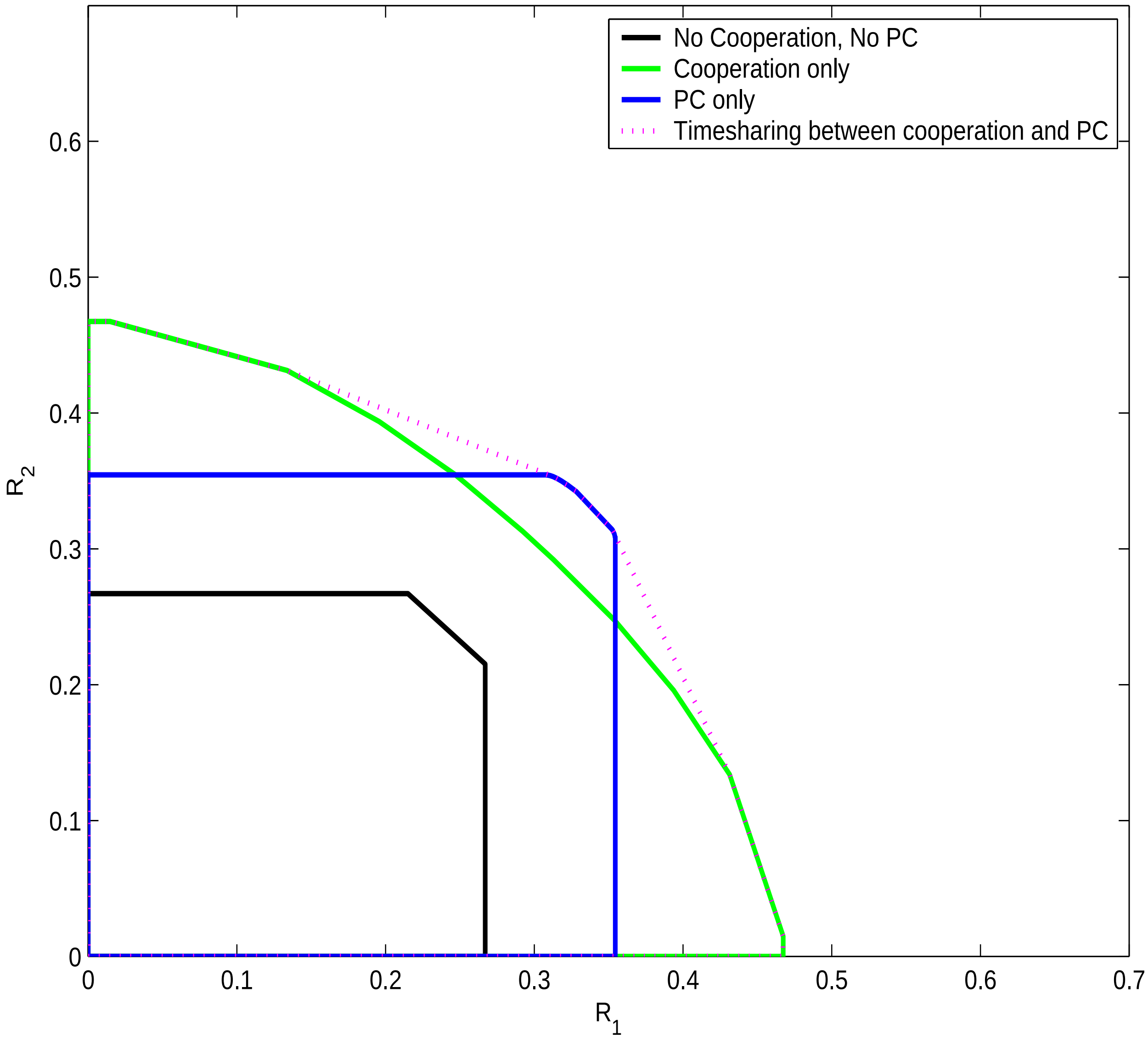
$$C_\mu(\mathbf{p}') \leq C_\mu(\mathbf{p}) + (\mathbf{p}' - \mathbf{p})^\top \mathbf{g}$$

- Use projected subgradient method to maximize C_μ

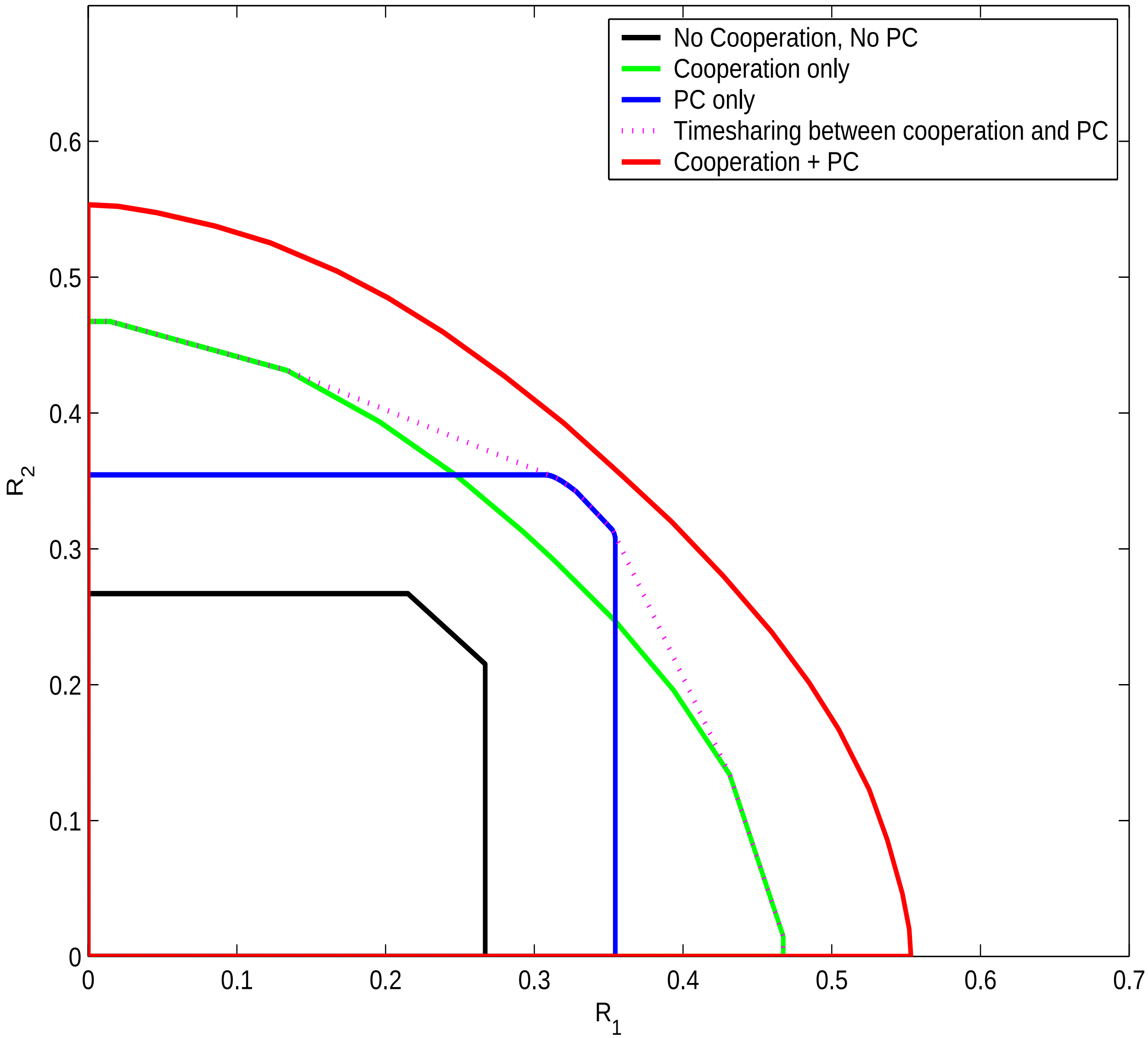
$$\mathbf{p}(k+1) = \left[\mathbf{p}(k) + \alpha_k \mathbf{g}_k\right]^+$$

- Provably converges for a diminishing stepsize α_k sequence (Shor).

Power Control Only, User Cooperation Only and Timesharing



Achievable Rate Region for Joint Power Control and User Cooperation



Summary

- Power control only and user cooperation only are techniques that improve capacity.
- User cooperation exploits spatial diversity and power control exploits time diversity.
- Joint power control and user cooperation exploits both: major improvements in capacity.
- Encoding and decoding is significantly simplified w.r.t. user cooperation only.
  - Transmitters send either cooperation or fresh information signals, but not both.
- The solution: joint power allocation, coding, medium accessing, and routing.
  - An instance of cross-layer design.
  - Trade-offs between complexity and capacity.
  - Value of the CSI.