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Adaptive power control and MMSE interference suppression *

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Power control algorithms assume that the receiver structure is fixed and iteratively update the transmit powers of the users to provide acceptable quality of service while minimizing the total transmitter power. Multiuser detection, on the other hand, optimizes the receiver structure with the assumption that the users have fixed transmitter powers. In this study, we combine the two approaches and propose an iterative and distributed power control algorithm which iteratively updates the transmitter powers and receiver filter coefficients of the users. We show that the algorithm converges to a minimum power solution for the powers, and an MMSE multiuser detector for the filter coefficients.

1. Introduction

Code Division Multiple Access (CDMA) is a promising access scheme for future wireless systems because of its advantages such as decentralized access of the users to the channel without any need for a prior scheduling of the channel, graceful degradation of the performance of individual users as the number of users increase, and immunity to intentional jamming and multipath. However, a significant disadvantage of CDMA is the *near-far effect* which occurs as a result of the non-orthogonality of the codes with which users modulate their information bits. In near-far situations strong users can degrade the performance of the weak users significantly. In order to overcome the near-far problem, two methods are commonly used: power control and multiuser detection.

The aim of power control is to assign users with transmitter power levels so as to minimize the interference users create to each other while meeting certain quality of service objectives which are typically defined in terms of the signal to interference ratio (SIR). Earlier work [1,7,15,23] identified the power control problem as an eigenvalue problem for non-negative matrices and concentrated on determining the power vector which maximized the minimum of the SIRs or achieved a common SIR value for all users in the system. Distributed power control algorithms [4,6,13,14,24] update the transmitter power levels of the users iteratively so that the power vector converges to a minimum where all of the users satisfy their SIR based quality of service requirements. These algorithms are distributed in the sense that the users need only to know the parameters that can be measured locally such as their channel gains and interference. The power control approach assumes that a fixed receiver, usually the conventional (single user) receiver, is being used at the base stations.

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Multiuser detection [21] can be used to demodulate the signals of the users effectively in a multiple access environment. It was shown in [20] that the optimum multiuser detector has a computational complexity which increases exponentially with the number of active users. Therefore, several suboptimum detectors have been proposed to achieve a performance comparable to that of the optimum detector while keeping the complexity low. Examples of suboptimum multiuser detectors include the decorrelating detector [11], the decision feedback detector [5], the minimum mean squared error (MMSE) detector [12] and the multistage detectors [19]. Some of these multiuser detectors are also suitable for blind adaptive implementations where information about the interfering users such as their powers and signature sequences are not needed for the construction of the receiver filter of a desired user. A blind adaptive implementation of the MMSE multiuser detector is given in [9] and blind adaptive decorrelating detector implementations are presented in [18,22]. One common property of all these multiuser detectors is the assumption that the received powers of all the users are fixed.

In this work we combine the power control and multiuser detection approaches to overcome the near-far effect and propose an algorithm which controls both the transmitter powers and the receiver filters of the users. The proposed algorithm is iterative and distributed. At each iteration first the receiver filter coefficients of the users are updated to suppress the interference optimally and then the transmitter powers of the users are assigned so that each user creates the minimum possible interference to others while satisfying the quality of service requirement. The implementation of this approach will require interference measurements at each receiver. We show that the resulting power control algorithm converges to a fixed point power vector where all the users satisfy their SIR-based quality of service requirements and that the linear receiver converges to the MMSE multiuser detector. The fixed point power vector $\overline{\boldsymbol{p}}$ satisfies $\overline{\boldsymbol{p}} \leqslant \boldsymbol{p}'$ for any power vector $\boldsymbol{p}'$

for which there are filter coefficients that yield acceptable SIR for all users. In [10] a power control algorithm is proposed for a CDMA system with adaptive MMSE receivers. The algorithm in [10] and the algorithm in this paper will converge to the same minimum power solution; however, the algorithm of [10] uses measurements of the minimum mean squared error which requires the knowledge of the information bits transmitted by the users and assumes that an adaptive MMSE receiver will adjust to changes in the transmitter powers.

The organization of this paper is as follows. Sections 2 and 3 give the system model and the problem definition. The derivation of the filter coefficients for a fixed power vector is presented in section 4. The power control algorithm is proposed and its convergence is proven in section 5. In section 6, implementation issues for the proposed power control algorithm are discussed. Simulation results are presented in section 7. Finally, section 8 contains the conclusion and discussion.

2. System model

We consider the uplink of a wireless cellular system with a fixed base station assignment of N users to M base stations. We assume a synchronous CDMA scheme and BPSK modulation in order to simplify the analysis of our algorithm. For each user i, we use p_i to denote its transmitted power. The channel gain of user j to the assigned base station of user i is represented by h_{ij}.

Users have pre-assigned, unique signature sequences which they use to modulate their information bits. The signature waveform of user i, denoted by $s_i(t)$, is non-zero only in the bit interval $[0, T_\mathrm{b}]$ and is normalized to unit energy, i.e., $\int_0^{T_\mathrm{b}} s_i^2(t)\,\mathrm{d}t = 1$. The baseband received signal, $r_i(t)$, in one bit interval at the front end of the receiver filters at the assigned base of user i is given by

$$r_i(t) = \sum_{j=1}^{N} \sqrt{p_j}\sqrt{h_{ij}} b_j s_j(t) + n(t), \tag{1}$$

where b_j is the information bit transmitted by user j ($+1$ or -1 with equal probability) and $n(t)$ is an additive white Gaussian noise (AWGN) process.

We define the chip waveform to be $\psi(t)$, $t \in [0, T_\mathrm{c}]$, and 0 elsewhere, where T_c is the chip duration. Thus

$$\bigl\{\psi(t - iT_\mathrm{c}),\ i = 0, \ldots, G-1\bigr\},$$

where $G = T_\mathrm{b}/T_\mathrm{c}$ is the processing gain, is a basis for the signal space. This allows us to represent both the signature sequences and the linear receiver filters of the users with G dimensional vectors. We will use $\boldsymbol{s}_i$ and $\boldsymbol{c}_i$ to denote the pre-assigned unique signature sequence and the linear receiver filter of user i, respectively. In terms of signal vectors, the received signal at the assigned base station of user i can be written as

$$\boldsymbol{r}_i = \sum_{j=1}^{N} \sqrt{p_j}\sqrt{h_{ij}} b_j \boldsymbol{s}_j + \boldsymbol{n}, \tag{2}$$

where $\boldsymbol{n}$ is a Gaussian random vector with $E[\boldsymbol{n}\boldsymbol{n}^\mathrm{T}] = \sigma^2 \boldsymbol{I}$.

3. Problem definition

Let $\boldsymbol{c}_i$ denote the receiver filter for user i at its assigned base station. The receiver filter output of user i is

$$y_i = \sum_{j=1}^{N} \sqrt{p_j}\sqrt{h_{ij}}\bigl(\boldsymbol{c}_i^\mathrm{T}\boldsymbol{s}_j\bigr) b_j + \widetilde{n}_i, \tag{3}$$

where $\widetilde{n}_i = \boldsymbol{c}_i^\mathrm{T}\boldsymbol{n}$ is a Gaussian random variable with zero mean and variance $\sigma^2 \boldsymbol{c}_i^\mathrm{T}\boldsymbol{c}_i$. The signal to interference ratio (SIR) of user i can be written as

$$\mathrm{SIR}_i = \frac{p_i h_{ii}(\boldsymbol{c}_i^\mathrm{T}\boldsymbol{s}_i)^2}{\sum_{j\neq i} p_j h_{ij}(\boldsymbol{c}_i^\mathrm{T}\boldsymbol{s}_j)^2 + \sigma^2(\boldsymbol{c}_i^\mathrm{T}\boldsymbol{c}_i)}. \tag{4}$$

Our aim is to find optimal powers, p_i, and filter coefficients, $\boldsymbol{c}_i$ for $i = 1, \ldots, N$, such that the total transmitter power is minimized while each user i satisfies a quality of service requirement $\mathrm{SIR}_i \geqslant \gamma_i^*$, where γ_i^*, called the *target* SIR, is the minimum acceptable level of SIR for user i. Therefore, we can state the problem mathematically as

$$\begin{aligned}
&\min \textstyle\sum_{i=1}^{N} p_i\\
&\text{s.t.}\quad p_i \geqslant \frac{\gamma_i^*}{h_{ii}} \frac{\sum_{j\neq i} p_j h_{ij}(\boldsymbol{c}_i^\mathrm{T}\boldsymbol{s}_j)^2 + \sigma^2 \boldsymbol{c}_i^\mathrm{T}\boldsymbol{c}_i}{(\boldsymbol{c}_i^\mathrm{T}\boldsymbol{s}_i)^2},\\
&\qquad i = 1, \ldots, N,\\
&\qquad p_i \geqslant 0, \quad i = 1, \ldots, N,\\
&\qquad \boldsymbol{c}_i \in \boldsymbol{R}^G, \quad i = 1, \ldots, N.
\end{aligned} \tag{5}$$

The above problem statement is equivalent to the following one, where an inner optimization is inserted in the constraint set:

$$\begin{aligned}
&\min_{\boldsymbol{p}} \textstyle\sum_{i=1}^{N} p_i\\
&\text{s.t.}\quad p_i \geqslant \frac{\gamma_i^*}{h_{ii}} \min_{\boldsymbol{c}_i \in \boldsymbol{R}^G} \frac{\sum_{j\neq i} p_j h_{ij}(\boldsymbol{c}_i^\mathrm{T}\boldsymbol{s}_j)^2 + \sigma^2(\boldsymbol{c}_i^\mathrm{T}\boldsymbol{c}_i)}{(\boldsymbol{c}_i^\mathrm{T}\boldsymbol{s}_i)^2},\\
&\qquad i = 1, \ldots, N,\\
&\qquad p_i \geqslant 0, \quad i = 1, \ldots, N.
\end{aligned} \tag{6}$$

In (6) the outer optimization is defined over the power vector only, whereas the inner optimization problem assumes a fixed power vector and is defined over the filter coefficients of the individual users. Before describing the power control algorithm, we solve the inner optimization problem for the filter coefficients corresponding to a fixed power vector in the next section.

4. Derivation for the filter coefficients

We now derive the filter coefficients when the power vector is fixed and equal to $\boldsymbol{p}$. The inner optimization problem given in (6) can be written equivalently as

$$\min_{\boldsymbol{c}_i} \frac{\boldsymbol{c}_i^{\mathrm{T}}\big(\sum_{j\neq i} p_j h_{ij}\boldsymbol{s}_j\boldsymbol{s}_j^{\mathrm{T}} + \sigma^2\boldsymbol{I}\big)\boldsymbol{c}_i}{(\boldsymbol{c}_i^{\mathrm{T}}\boldsymbol{s}_i)^2}. \tag{7}$$

We define the $G\times G$ matrix $\boldsymbol{A}_i$ which is a function of the powers of all the users, except the power of user i, as

$$\boldsymbol{A}_i = \sum_{j\neq i} p_j h_{ij}\boldsymbol{s}_j\boldsymbol{s}_j^{\mathrm{T}} + \sigma^2\boldsymbol{I} \tag{8}$$

and another $G\times G$ matrix $\boldsymbol{B}_i$ as $\boldsymbol{B}_i = \boldsymbol{s}_i\boldsymbol{s}_i^{\mathrm{T}}$. This permits us to write equation (7) as

$$\min_{\boldsymbol{c}_i} \frac{\boldsymbol{c}_i^{\mathrm{T}}\boldsymbol{A}_i\boldsymbol{c}_i}{\boldsymbol{c}_i^{\mathrm{T}}\boldsymbol{B}_i\boldsymbol{c}_i}. \tag{9}$$

Since $\boldsymbol{A}_i$ is strictly positive definite due to the term $\sigma^2\boldsymbol{I}$, it can be written as $\boldsymbol{A}_i = \boldsymbol{R}_i^{\mathrm{T}}\boldsymbol{R}_i$ for some non-singular matrix $\boldsymbol{R}_i$. We define the one-to-one (since $\boldsymbol{R}_i$ is invertible) transformation $\boldsymbol{x}_i = \boldsymbol{R}_i\boldsymbol{c}_i$ and write (9) in terms of $\boldsymbol{x}_i$ as

$$\min_{\boldsymbol{x}_i} \frac{\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{x}_i}{\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{R}_i^{-\mathrm{T}}\boldsymbol{B}_i\boldsymbol{R}_i^{-1}\boldsymbol{x}_i}. \tag{10}$$

Defining a G dimensional vector $\boldsymbol{u}_i$ as $\boldsymbol{u}_i = \boldsymbol{R}_i^{-\mathrm{T}}\boldsymbol{s}_i$, (10) can equivalently be written as

$$\min_{\boldsymbol{x}_i} \frac{\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{x}_i}{\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{u}_i\boldsymbol{u}_i^{\mathrm{T}}\boldsymbol{x}_i}. \tag{11}$$

The eigenvector of matrix $\boldsymbol{u}_i\boldsymbol{u}_i^{\mathrm{T}}$ with the maximum eigenvalue attains the minimum objective function in (11) [16]. Note that the rank of $\boldsymbol{u}_i\boldsymbol{u}_i^{\mathrm{T}}$ is equal to 1. Therefore, $(G-1)$ eigenvalues of it are equal to zero and the remaining one is equal to $\boldsymbol{u}_i^{\mathrm{T}}\boldsymbol{u}_i$ with the corresponding eigenvector $\boldsymbol{u}_i$. Thus, the solution of (11) is obtained to be $\overline{\boldsymbol{x}}_i = \boldsymbol{u}_i$. Applying the inverse transformation $\overline{\boldsymbol{c}}_i = \boldsymbol{R}_i^{-1}\overline{\boldsymbol{x}}_i$ yields the solution of (9)

$$\overline{\boldsymbol{c}}_i = \boldsymbol{A}_i^{-1}\boldsymbol{s}_i \tag{12}$$

and the minimum of the objective function in (9) is equal to $(\boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{A}_i^{-1}\boldsymbol{s}_i)^{-1}$. This result is not so surprising since it is well known that for a fixed power vector $\boldsymbol{p}$, the MMSE filter coefficients maximize the SIR [12], and the optimization problem of (9) can be written using (4) as

$$\min_{\boldsymbol{c}_i} p_i h_{ii}\frac{1}{\mathrm{SIR}_i}. \tag{13}$$

Equation (13) is equivalent to

$$\max_{\boldsymbol{c}_i} \mathrm{SIR}_i \tag{14}$$

since the power vector therefore p_i is assumed fixed. The MMSE filter coefficients are given as [12]

$$\boldsymbol{c}_i^* = \frac{\sqrt{p_i}}{1 + p_i\boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{A}_i^{-1}\boldsymbol{s}_i}\boldsymbol{A}_i^{-1}\boldsymbol{s}_i. \tag{15}$$

Note that the MMSE solution $\boldsymbol{c}_i^*$ is just a scaled version of $\overline{\boldsymbol{c}}_i$ and the optimization problem of (9) is insensitive to the scaling of the vector $\boldsymbol{c}_i$. As a convention we will use the MMSE solution, $\boldsymbol{c}_i^*$, given in (15) as the solution of the inner optimization problem instead of $\overline{\boldsymbol{c}}_i$ of (12).

5. Power control algorithm

When we view (6) as a set of interference constraints on the power vector $\boldsymbol{p}$, we can define a power control algorithm in which each user i iteratively attempts to compensate for the interference. We define

$$I_i(\boldsymbol{p},\boldsymbol{c}_i) = \frac{\gamma_i^*}{h_{ii}}\frac{\sum_{j\neq i} p_j h_{ij}(\boldsymbol{c}_i^{\mathrm{T}}\boldsymbol{s}_j)^2 + \sigma^2(\boldsymbol{c}_i^{\mathrm{T}}\boldsymbol{c}_i)}{(\boldsymbol{c}_i^{\mathrm{T}}\boldsymbol{s}_i)^2}, \tag{16}$$

$$T_i(\boldsymbol{p}) = \min_{\boldsymbol{c}_i} I_i(\boldsymbol{p},\boldsymbol{c}_i) \tag{17}$$

and we propose the power control algorithm

$$\boldsymbol{p}(n+1) = \boldsymbol{T}\big(\boldsymbol{p}(n)\big), \tag{18}$$

where

$$\boldsymbol{T}(\boldsymbol{p}) = \big[T_1(\boldsymbol{p}),\ldots,T_N(\boldsymbol{p})\big]^{\mathrm{T}}. \tag{19}$$

Each power control iteration (18) includes an optimization of the filter coefficients to maximally suppress the interference. In effect, we choose the filter coefficients to minimize the required transmitter power. This is analogous to integrated power control and base station assignment algorithms [8,26] in which a user's base station assignment is iteratively chosen to minimize the transmitter power. In [25] power control algorithms of the form

$$\boldsymbol{p}(n+1) = \boldsymbol{I}\big(\boldsymbol{p}(n)\big) \tag{20}$$

are analyzed for standard interference functions $\boldsymbol{I}(\boldsymbol{p})$. The definition of standard interference functions and the theorem describing the convergence of (20) follow.

Definition 1. $\boldsymbol{I}(\boldsymbol{p})$ is a standard interference function if for all $\boldsymbol{p} \geqslant \boldsymbol{0}$ the following properties are satisfied:

- Positivity: $\boldsymbol{I}(\boldsymbol{p}) > \boldsymbol{0}$.
- Monotonicity: if $\boldsymbol{p} \geqslant \boldsymbol{p}'$ then $\boldsymbol{I}(\boldsymbol{p}) \geqslant \boldsymbol{I}(\boldsymbol{p}')$.
- Scalability: for all $\alpha > 1$, $\alpha\boldsymbol{I}(\boldsymbol{p}) > \boldsymbol{I}(\alpha\boldsymbol{p})$.

Theorem 2. If there exists $\boldsymbol{p}' \geq \boldsymbol{I}(\boldsymbol{p}')$, then for any initial power vector $\boldsymbol{p}(0)$, the sequence $\boldsymbol{p}(n) = \boldsymbol{I}(\boldsymbol{p}(n-1))$ converges to a unique fixed point $\overline{\boldsymbol{p}}$ such that $\overline{\boldsymbol{p}} \leq \boldsymbol{p}'$ for any $\boldsymbol{p}' \geq \boldsymbol{I}(\boldsymbol{p}')$.

The condition that there exists $\boldsymbol{p}' \geqslant \boldsymbol{I}(\boldsymbol{p}')$ is simply a requirement that a feasible power vector exists. The fixed point $\overline{\boldsymbol{p}}$ is a minimum power solution in that $\overline{\boldsymbol{p}} \leqslant \boldsymbol{p}'$ for any feasible power vector $\boldsymbol{p}'$. Thus, we prove the convergence of the power control algorithm (18) by proving that the transformation $\boldsymbol{T}(\boldsymbol{p})$ is standard.

Theorem 3. $\boldsymbol{T}(\boldsymbol{p})$ is a standard interference function.

Proof. From (16), for any fixed $\boldsymbol{c}_i$ we have $I_i(\boldsymbol{p}, \boldsymbol{c}_i) > 0$. Therefore, $T_i(\boldsymbol{p}) = \min_{\boldsymbol{c}_i} I_i(\boldsymbol{p}, \boldsymbol{c}_i) > 0$ and $\boldsymbol{T}(\boldsymbol{p})$ is positive. To prove monotonicity, we note for any fixed $\boldsymbol{c}_i$ that $\boldsymbol{p} \geqslant \boldsymbol{p}'$ implies $I_i(\boldsymbol{p}, \boldsymbol{c}_i) \geqslant I_i(\boldsymbol{p}', \boldsymbol{c}_i)$. If the minimum of $I_i(\boldsymbol{p}, \boldsymbol{c}_i)$ is achieved at $\boldsymbol{c}_i^*$, then

$$T_i(\boldsymbol{p}) = \min_{\boldsymbol{c}_i} I_i(\boldsymbol{p}, \boldsymbol{c}_i) \tag{21}$$

$$= I_i(\boldsymbol{p}, \boldsymbol{c}_i^*) \tag{22}$$

$$\geqslant I_i(\boldsymbol{p}', \boldsymbol{c}_i^*) \tag{23}$$

$$\geqslant \min_{\boldsymbol{c}_i} I_i(\boldsymbol{p}', \boldsymbol{c}_i) = T_i(\boldsymbol{p}'). \tag{24}$$

For scalability, we note that for any fixed $\boldsymbol{c}_i$ and $\alpha > 1$ we have $\alpha I_i(\boldsymbol{p}, \boldsymbol{c}_i) > I_i(\alpha \boldsymbol{p}, \boldsymbol{c}_i)$. Assuming again that the minimum of $I_i(\boldsymbol{p}, \boldsymbol{c}_i)$ is achieved at $\boldsymbol{c}_i^*$, we have

$$\alpha T_i(\boldsymbol{p}) = \min_{\boldsymbol{c}_i} \alpha I_i(\boldsymbol{p}, \boldsymbol{c}_i) \tag{25}$$

$$= \alpha I_i(\boldsymbol{p}, \boldsymbol{c}_i^*) \tag{26}$$

$$> I_i(\alpha \boldsymbol{p}, \boldsymbol{c}_i^*) \tag{27}$$

$$\geqslant \min_{\boldsymbol{c}_i} I_i(\alpha \boldsymbol{p}, \boldsymbol{c}_i) = T_i(\alpha \boldsymbol{p}). \tag{28}$$

□

Since $\boldsymbol{T}(\boldsymbol{p})$ is a standard interference function, the power control algorithm (18) converges to $\overline{\boldsymbol{p}} = \boldsymbol{T}(\overline{\boldsymbol{p}})$. The filter coefficients converge to $\overline{\boldsymbol{c}}_i = \arg\min_{\boldsymbol{c}_i} I_i(\overline{\boldsymbol{p}}, \boldsymbol{c}_i)$. Equivalently, the power control algorithm converges to a minimum power solution for the SIR target based power control problem with linear receiver filters; and the linear receiver filter converges to the MMSE multiuser detector.

6. Implementation of the power control algorithm

The power control algorithm (18) is implicitly a two stage algorithm. First, we adjust the filter coefficients to be the MMSE coefficients for power vector $\boldsymbol{p}$. Second, we adjust the transmitter powers to meet the SIR constraints for the chosen filter coefficients. In this section, we describe how the iteration (18) may be implemented in practice.

We will denote the matrix $\boldsymbol{A}_i$ as $\boldsymbol{A}_i(\boldsymbol{p}(n))$ below in order to emphasize its dependency on the power vector. This matrix is calculated by using (8) when $\boldsymbol{p}(n)$ is given. At iteration $n+1$, the MMSE filter $\widehat{\boldsymbol{c}}_i$ is constructed by using the current power vector $\boldsymbol{p}(n)$ and then the power vector is updated using the new filter coefficients $\widehat{\boldsymbol{c}}_i$. The resulting iterative algorithm for user i is

$$\widehat{\boldsymbol{c}}_i = \frac{\sqrt{p_i(n)}}{1 + p_i(n)\boldsymbol{s}_i^{\mathrm{T}} \boldsymbol{A}_i^{-1}(\boldsymbol{p}(n))\boldsymbol{s}_i} \boldsymbol{A}_i^{-1}\big(\boldsymbol{p}(n)\big)\boldsymbol{s}_i, \tag{29}$$

$$p_i(n+1) = \frac{\gamma_i^*}{h_{ii}} \frac{\sum_{j\neq i} p_j(n) h_{ij} (\widehat{\boldsymbol{c}}_i^{\mathrm{T}} \boldsymbol{s}_j)^2 + \sigma^2 \widehat{\boldsymbol{c}}_i^{\mathrm{T}} \widehat{\boldsymbol{c}}_i}{(\widehat{\boldsymbol{c}}_i^{\mathrm{T}} \boldsymbol{s}_i)^2}. \tag{30}$$

Equations (29) and (30) represent a deterministic iteration of the transmitter powers and filter coefficients. If the SIR targets are feasible, then starting from any initial power vector $\boldsymbol{p}$ and filter coefficients $\boldsymbol{c}_1, \ldots, \boldsymbol{c}_N$, the algorithm converges deterministically to the unique minimum power fixed point.

The theoretical properties of the iteration using equations (29) and (30) are of little practical use if the quantities needed to perform the iteration cannot be determined. Moreover, from (29) and (30), it would appear that all transmitter powers p_j and channel gains h_{ij} are needed to obtain $\boldsymbol{A}_i$ and hence $\widehat{\boldsymbol{c}}_i$. Fortunately, this is not the case. In particular, we can estimate $\boldsymbol{A}_i$ by sampling the received signal before the receiver filters and taking empirical averages. From (2), the mutual independence of the zero mean transmitted bits $\{b_n\}$ and the Gaussian noise $\boldsymbol{n}$ implies

$$E\big[\boldsymbol{r}_i \boldsymbol{r}_i^{\mathrm{T}}\big] = \boldsymbol{A}_i + p_i h_{ii} \boldsymbol{s}_i \boldsymbol{s}_i^{\mathrm{T}}. \tag{31}$$

Therefore, $\boldsymbol{r}_i \boldsymbol{r}_i^{\mathrm{T}} - p_i h_{ii} \boldsymbol{s}_i \boldsymbol{s}_i^{\mathrm{T}}$ is an unbiased estimate for $\boldsymbol{A}_i$. If at the assigned base station of user i, the uplink gain h_{ii} and transmitter power p_i are known, $\boldsymbol{A}_i$ can be estimated by a sample average of $\boldsymbol{r}_i \boldsymbol{r}_i^{\mathrm{T}}$ over multiple bit intervals. For the adjusted filter coefficients $\widehat{\boldsymbol{c}}_i$, equation (3) implies that the average squared filter output for user i under power vector $\boldsymbol{p}(n)$ is

$$E\big[y_i^2(n)\big] = \sum_{j=1}^{N} p_j(n) h_{ij} \big(\widehat{\boldsymbol{c}}_i^{\mathrm{T}} \boldsymbol{s}_j\big)^2 + \sigma^2 \widehat{\boldsymbol{c}}_i^{\mathrm{T}} \widehat{\boldsymbol{c}}_i. \tag{32}$$

Thus, from (30), the power control iteration can be written as

$$p_i(n+1) = \frac{\gamma_i^*}{h_{ii}} \frac{1}{(\widehat{\boldsymbol{c}}_i^{\mathrm{T}} \boldsymbol{s}_i)^2} \Big(E\big[y_i^2(n)\big] - p_i(n) h_{ii} \big(\widehat{\boldsymbol{c}}_i^{\mathrm{T}} \boldsymbol{s}_i\big)^2\Big). \tag{33}$$

A simple measurement based power control algorithm can use a sample average of $y_i^2(n)$ over multiple bit intervals to estimate $E[y_i^2(n)]$. We have presented these simple estimation methods not because they perform particularly well but rather to emphasize that the information needed for user i to implement the MMSE power control is available at the receiver for user i. Thus, distributed implementation is possible. We note that the simple estimation methods still require a user to estimate its own uplink gain h_{ii}. This can be done, perhaps roughly, using the downlink transmission of a base station pilot tone. Alternatively, estimating the uplink gain can be avoided by direct estimation of the SIR without separate estimates of the signal and interference components [2,3]. This is also the motivation for the MMSE power control algorithm of [10] that uses measurements of the mean squared error.

Although we have verified that the proposed power control can be implemented in a distributed manner using local measurements, we note that substitution of stochastic measurements does not preserve the deterministic convergence properties. Furthermore, the direct substitution of measured estimates in a deterministic algorithm may not be the most desirable approach. We emphasize that the need for measurements, and the consequent difficulty of analyzing the effect of measurements, is a property of all power control algorithms, whether or not those algorithms adapt the filter coefficients for interference suppression.

In particular, it may be preferable to use separate iterative algorithms for

(1) the adaptation of the filter coefficients to the MMSE coefficients $\widehat{\boldsymbol{c}}_i$;

(2) the iterative transmitter power adjustments for fixed filter coefficients.

For the first step, iterative algorithms that converge to the MMSE filter coefficients are given in [12] or the blind adaptive multiuser detector [9] can be used to converge stochastically to the MMSE solution. For fixed filter coefficients, the second step is equivalent to the conventional power control problem reviewed in section 1 of this paper. An alternate approach is the stochastic power control algorithm given in [17]. In this work, the power vector was shown to converge stochastically to the optimal power vector by using the random outputs of the fixed receiver filters. Therefore, alternating between the blind adaptive MMSE detector and the stochastic power control algorithm would yield a stochastically converging power control algorithm. However, the convergence of this two step iteration may be slow. We believe a combined stochastic optimization of filter coefficients and transmitter powers may have better convergence properties and should be investigated.

7. Simulation results

In our simulations we consider a multicell CDMA system on a rectangular grid. There are $M = 25$ base stations with (x, y) coordinates $(1000i + 500, 1000j + 500)$ for $0 \leqslant i, j \leqslant 4$. The x and y coordinates of each user are independent uniformly distributed random variables between 0 and 5,000 meters. The experiments are conducted for $N = 250, 500$ and 1000 users. Figure 1 shows the positions of users and the base stations with symbols $\times$ and $\circ$, respectively, for $N = 1000$. Each user is assigned to its nearest base station. The path loss exponent used while calculating the channel gains of the users is taken to be $\alpha = 4$. At the beginning of the iterations, power vector is initialized to zero, and the filter coefficients are initialized to the signature sequences of the users (i.e., $p_i(0) = 0$ and $\boldsymbol{c}_i(0) = \boldsymbol{s}_i$).

We chose the processing gain to be $G = 150$ and a random signature sequence of length G chips was assigned to each user. Although the convergence theorems permit individual SIR targets γ_i^* for each user i, for the simulations we chose a common SIR target $\gamma_i^* = 4$ (≈ 6 dB) for all users. The AWGN noise power equaled $\sigma^2 = 10^{-13}$, corresponding roughly to a 1 MHz bandwidth.

We compared the performance of the conventional power control algorithm which assumes a conventional detector structure composed of the filters matched to the signature sequences of the users, and the power control algorithm proposed in this paper which optimizes the filter coefficients in addition to updating the powers. Since the filter coefficients are always chosen to be the MMSE detector, we call the proposed algorithm the MMSE power control. We compared the deterministic convergence of the conventional and MMSE power control algorithms.

Figure 2 shows in log scale the total transmitter power $\sum_{i=1}^{N} p_i$, as a function of the iteration index, for the MMSE and conventional power control algorithms. We observe that the MMSE power control outperforms the conventional power control in terms of total received power, and convergence rate. Using MMSE power control, the total transmitter power is less than that needed for the conven-

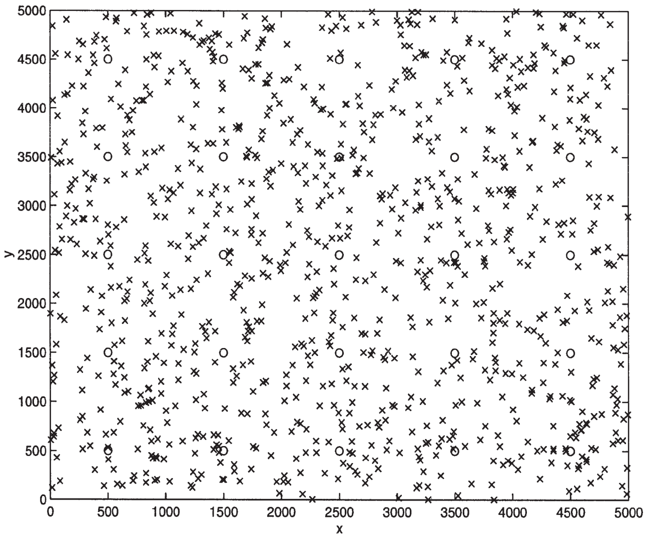

Figure 1. Simulation environment for $N = 1000$. Symbols $\circ$ and $\times$ denote the base stations and the users, respectively.

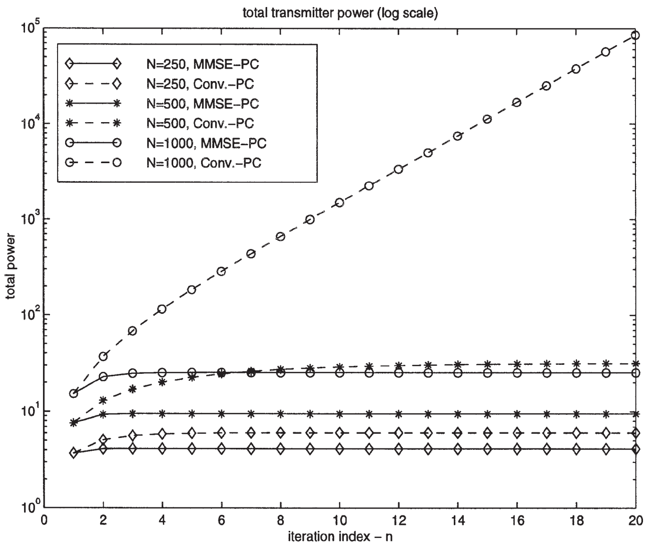

Figure 2. Total transmitter power for the conventional power control algorithm (Conv.-PC) and the MMSE power control algorithm (MMSE-PC) for $N = 250, 500$ and 1000.

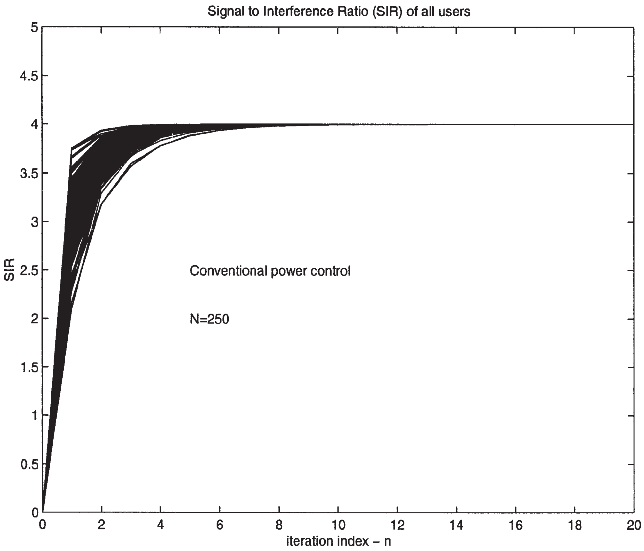

Figure 3. Signal to Interference Ratio (SIR) of all the users as a function of n for the conventional power control algorithm. SIR target value $\gamma_i^* = 4$ ($\approx$ 6 dB) for $i = 1, \ldots, N$, number of users $N = 250$.

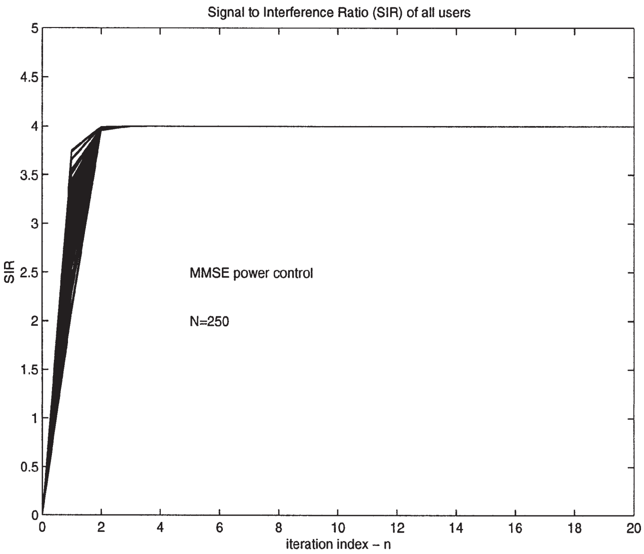

Figure 4. Signal to Interference Ratio (SIR) of all the users as a function of n for the MMSE power control algorithm. SIR target value $\gamma_i^* = 4$ ($\approx$ 6 dB) for $i = 1, \ldots, N$, number of users $N = 250$.

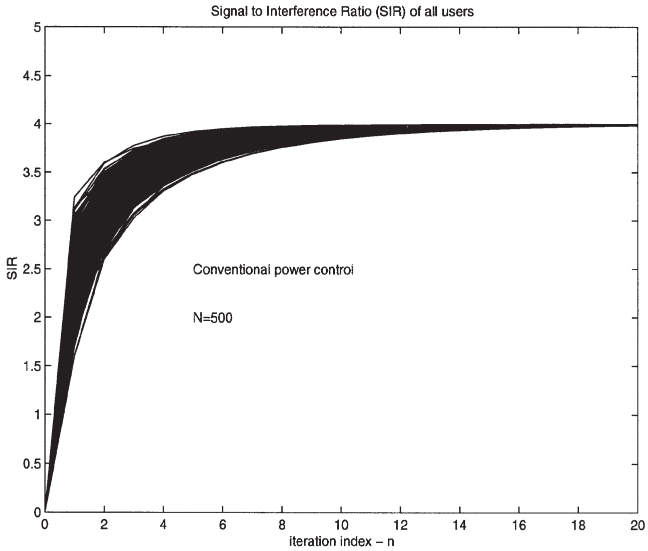

Figure 5. Signal to Interference Ratio (SIR) of all the users as a function of n for the conventional power control algorithm. SIR target value $\gamma_i^* = 4$ ($\approx$ 6 dB) for $i = 1, \ldots, N$, number of users $N = 500$.

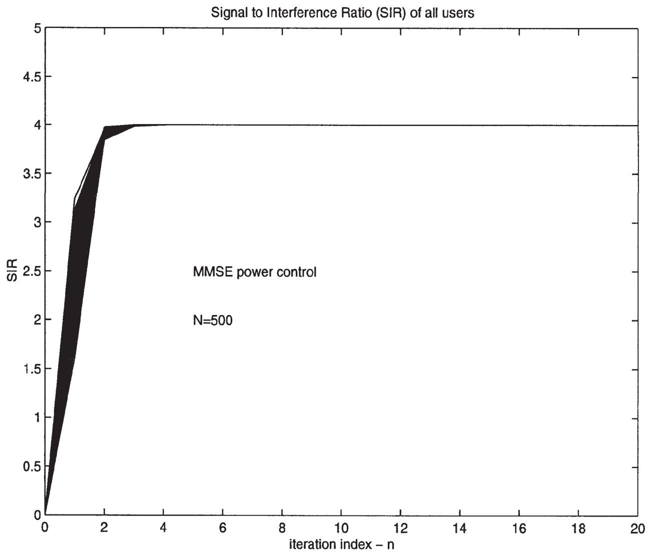

Figure 6. Signal to Interference Ratio (SIR) of all the users as a function of n for the MMSE power control algorithm. SIR target value $\gamma_i^* = 4$ ($\approx$ 6 dB) for $i = 1, \ldots, N$, number of users $N = 500$.

tional detector. The savings in total transmit power increase with increasing number of users. Also, the MMSE power control algorithm converges to the optimal power vector faster than the conventional power control algorithm.

The steadily increasing transmitter power curve for conventional power control with $N = 1000$ in figure 2 occurs because the conventional power control problem is infeasible. For this case, updating the receiver filter coefficients converted the infeasible conventional power control problem into a feasible problem.

In order to observe the convergence of the SIRs to the common target SIR, we plotted the SIRs of all of the users in figures 3 and 4 for the conventional power control algorithm and the MMSE power control algorithm, respectively, for $N = 250$ users. Figures 5, 6 and figures 7, 8 show the same graphs produced for $N = 500$ and $N = 1000$ users, respectively.

We observe from figures 3–6 that when the MMSE power control is used, the SIRs converge to the common target SIR faster than with the conventional power control algorithm. We again observe the infeasibility of the target SIR from figure 7, by noting that the SIRs of the users converge to the values which are less than the target value ($\gamma_i^* = 4$). We note that the SIRs converge to the maximum achievable common SIR target with fixed system parameters such as channel gains, cross correlations between the signature sequences; see [7,24].

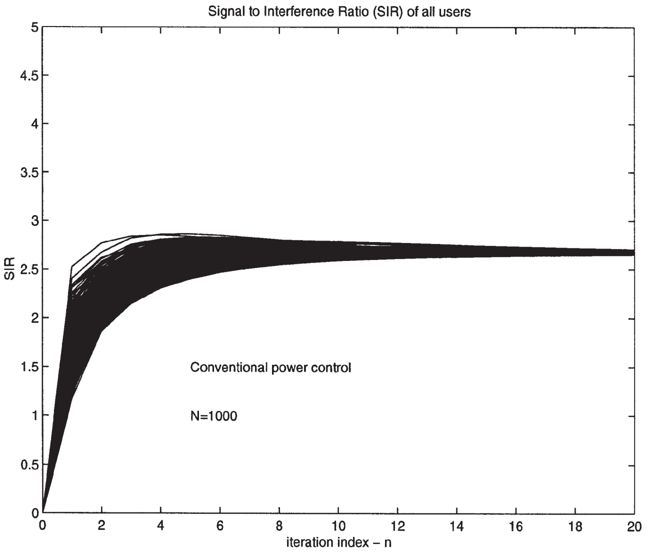

Figure 7. Signal to Interference Ratio (SIR) of all the users as a function of n for the conventional power control algorithm. SIR target value $\gamma_i^* = 4$ ($\approx$ 6 dB) for $i = 1, \ldots, N$, number of users $N = 1000$.

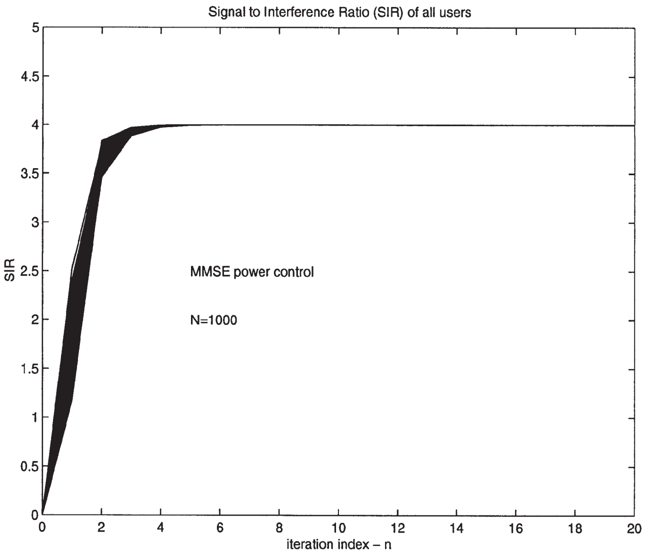

Figure 8. Signal to Interference Ratio (SIR) of all the users as a function of n for the MMSE power control algorithm. SIR target value $\gamma_i^* = 4$ ($\approx$ 6 dB) for $i = 1, \ldots, N$, number of users $N = 1000$.

8. Conclusion

We proposed an iterative and distributed power control algorithm which updates the power levels and linear receiver filters of the individual users. We showed that the proposed algorithm converges to a minimum power solution where all the users satisfy their SIR-based quality of service requirements; and that the linear receiver filter converges to an MMSE multiuser detector.

We observed that the MMSE power control is superior in terms of the total transmitter power and convergence rate when compared with the conventional power control algorithm. With MMSE power control, the same system performance is achieved with less total transmitter power, increasing the capacity of the CDMA system when compared with the conventional power control. Since MMSE power control can convert a power control problem that is infeasible with conventional power control into a feasible one, it increases the system capacity by allowing the SIR target expectations of the users to be higher, or by increasing the number of users supportable at a fixed SIR expectation level.

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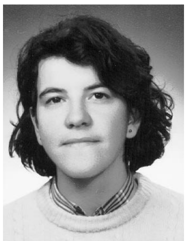

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