Optimal Packet Scheduling in an Energy Harvesting Communication System

Jing Yang, *Member, IEEE*, and Sennur Ulukus, *Member, IEEE*

***Abstract*—We consider the optimal packet scheduling problem in a single-user energy harvesting wireless communication system. In this system, both the data packets and the harvested energy are modeled to arrive at the source node randomly. Our goal is to adaptively change the transmission rate according to the traffic load and available energy, such that the time by which all packets are delivered is minimized. Under a deterministic system setting, we assume that the energy harvesting times and harvested energy amounts are known before the transmission starts. For the data traffic arrivals, we consider two different scenarios. In the first scenario, we assume that all bits have arrived and are ready at the transmitter before the transmission starts. In the second scenario, we consider the case where packets arrive during the transmissions, with known arrival times and sizes. We develop optimal off-line scheduling policies which minimize the time by which all packets are delivered to the destination, under causality constraints on both data and energy arrivals.**

***Index Terms*—Energy harvesting, rechargeable wireless networks, transmission completion time minimization.**

I. Introduction

We consider wireless communication networks where nodes are able to harvest energy from nature. The nodes may harvest energy through solar cells, vibration absorption devices, water mills, thermoelectric generators, microbial fuel cells, etc. In this work, we do not focus on how energy is harvested, instead, we focus on developing transmission methods that take into account the *arrivals* of the *data packets* as well as the *arrivals* of the *harvested energy* during the course of transmission. As shown in Fig. 1, the transmitter node has two queues. The data queue stores the data arrivals, while the energy queue stores the energy harvested from the environment. In general, the data arrivals and the harvested energy can be represented as two independent random processes. Then, the optimal scheduling policy becomes that of adaptively changing the transmission rate and power according to the instantaneous data and energy queue lengths.

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J. Yang was with the Department of Electrical and Computer Engineering, University of Maryland, College Park, MD 20742 USA. She is now with the Department of Electrical and Computer Engingeering, University of Wisconsin-Madison, WI 53706 USA (e-mail: yangjing@ece.wisc.edu).

S. Ulukus is with the Department of Electrical and Computer Engineering, University of Maryland, College Park, MD 20742 USA (e-mail: ulukus@umd.edu).

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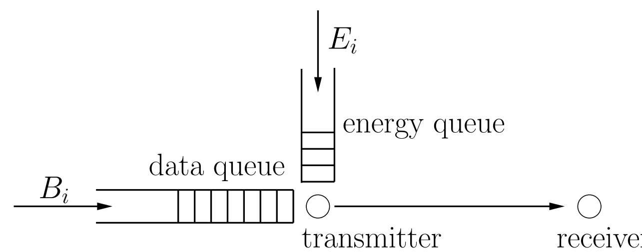


Fig. 1. An energy harvesting communication system model.

In this work, we consider the *off-line* problem where we assume the availability of the off-line knowledge of energy and data arrivals at the transmitter. Our goal is to determine the structural properties of an optimal off-line schedular. In this paper, we determine the properties of the optimal off-line solution, and develop an optimal off-line algorithm. We incorporate channel fading and random energy arrivals into our formulation, and develop the corresponding dynamic programming based and simpler heuristic *on-line* algorithms in [2]–[4].

In this paper, we consider a single node shown in Fig. 2. We assume that packets arrive at times marked with $\times$ and energy arrives (is harvested) at points in time marked with $\circ$. In Fig. 2, B_i denotes the number of bits in the ith arriving data packet, and E_i denotes the amount of energy in the ith energy arrival (energy harvesting). Our goal then is to develop methods of transmission to minimize the time, T, by which all of the data packets are delivered to the destination. The most challenging aspect of our optimization problem is the *causality* constraints introduced by the packet and energy arrival times, i.e., a packet may not be delivered before it has arrived and energy may not be used before it is harvested.

The trade-off relationship between delay and energy has been well investigated in traditional battery powered (unrechargeable) systems. References [5]–[10] investigate energy minimization problems with various deadline constraints. Reference [5] considers the problem of minimizing the energy in delivering all packets to the destination by a deadline. It develops a *lazy scheduling algorithm*, where the transmission times of all packets are equalized as much as possible, subject to the deadline and causality constraints, i.e., all packets must be delivered by the deadline and no packet may be transmitted before it has arrived. This algorithm also elongates the transmission time of each packet as much as possible, hence the name, *lazy* scheduling. Under a similar system setting, [6] proposes an interesting novel calculus approach to solve the energy minimization problem with individual deadlines

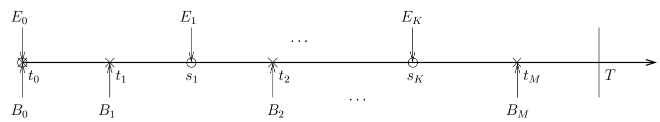


Fig. 2. System model with random packet and energy arrivals. Data packets arrive at points denoted by $\times$ and energies arrive (are harvested) at points denoted by $\circ$.

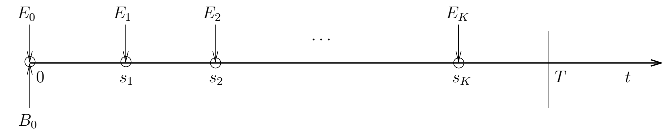


Fig. 3. System model with all bits available at the beginning. Energies arrive at points denoted by $\circ$.

for each packet. Reference [7] develops dynamic programming formulations and determines optimality conditions for a situation where channel gain varies stochastically over time. Reference [8] considers energy-efficient packet transmission with individual packet delay constraints over a fading channel, and develops a recursive algorithm to find an optimal off-line schedule. This optimal off-line scheduler equalizes the energy-rate derivative function as much as possible subject to the deadline and causality constraints. References [9] and [10] extend the single-user problem to multi-user scenarios. Under a setting similar to [5], we investigate the average delay minimization problem with a given amount of energy, and develop iterative algorithms and analytical solutions under various data arrival assumptions in [11]. References [12]–[18] investigate delay optimal resource allocation problems under various different settings. References [12]–[14] consider average power constrained delay minimization problem for a single-user system, while [15]–[18] minimize the average delay through rate allocation in a multiple access channel.

In this paper, we consider a single-user communication channel with an energy harvesting transmitter. We assume that an initial amount of energy is available at $t = 0$. As time progresses, certain amounts of energies will be harvested. In this paper, we assume that the energy harvesting procedure can be precisely predicted, i.e., that, at the beginning, we know exactly when and how much energy will be harvested. For the data arrivals, we consider two different scenarios. In the first scenario, we assume that packets have already arrived and are ready to be transmitted at the transmitter before the transmission starts. In the second scenario, we assume that packets arrive during the transmissions. As in the case of energy arrivals, we assume that we know exactly when and in what amounts data will arrive. Subject to the energy and data arrival constraints, our goal is to minimize the time by which all packets are delivered to the destination through controlling the transmission rate and power.

This is similar to the energy minimization problem in [5], where the objective is to minimize the energy consumption with a given *deadline* constraint. In this paper, minimizing the transmission completion time is akin to minimizing the deadline in [5]. However, the problems are different, because, we do not know the exact amount of energy to be used in the transmissions, even though we know the times and amounts of harvested energy. This is because, intuitively, using more energy reduces the transmission time, however, using more energy entails waiting for energy arrivals, which increases the total transmission time. Therefore, minimizing the transmission completion time in the system requires a sophisticated utilization of the harvested energy. To that end, we develop an algorithm, which first obtains a good lower bound for the final total transmission duration at the beginning, and performs rate and power allocation based on this lower bound. The procedure works progressively until all of the transmission rates and powers are determined. We prove that the transmission policy obtained through this algorithm is globally optimum.

II. Scenario I: Packets Ready Before Transmission Starts

We assume that there are a total of B_0 bits available at the transmitter at time $t = 0$. We also assume that there is E_0 amount of energy available at time $t = 0$, and at times s_1, s_2, …, s_K, we have energies harvested with amounts E_1, E_2, …, E_K, respectively. This system model is shown in Fig. 3. Our objective is to minimize the transmission completion time, T.

We assume that the transmitter can adaptively change its transmission power and rate according to the available energy and the remaining number of bits. We assume that the transmission rate and transmit power are related through a continuous function, $g(p)$, i.e., $r = g(p)$. We assume that $g(p)$ satisfies the following properties: i) $g(0) = 0$ and $g(p) \to \infty$ as $p \to \infty$, ii) $g(p)$ increases monotonically in p, iii) $g(p)$ is strictly concave in p, iv) $g(p)$ is continuously differentiable, and v) $g(p)/p$ decreases monotonically in p. Properties i)-iii) guarantee that $g^{-1}(r)$ exists and is strictly convex. Property v) can be derived from properties ii) and iii). It implies that for a fixed amount of energy, the number of bits that can be transmitted increases as the transmission duration increases. It can be verified that these properties are satisfied in many systems with realistic encoding/decoding schemes, such as optimal random coding in single-user additive white Gaussian noise channel, where $g(p) = \frac{1}{2}\log(1 + p)$.

Assuming the transmitter changes its transmission power N times before it finishes the transmission, let us denote the sequence of transmission powers as p_1, p_2, …, p_N, and the corresponding transmission durations of each rate as l_1, l_2, …, l_N, respectively; see Fig. 4. Then, the energy consumed up to time t, denoted as $E(t)$, and the total number of bits departed up to time t, denoted as $B(t)$, can be related through the function g as follows:

$$E(t) = \sum_{i=1}^{\bar{i}} p_i l_i + p_{\bar{i}+1}\left(t - \sum_{i=1}^{\bar{i}} l_i\right) \tag{1}$$

$$B(t) = \sum_{i=1}^{\bar{i}} g(p_i) l_i + g(p_{\bar{i}+1})\left(t - \sum_{i=1}^{\bar{i}} l_i\right) \tag{2}$$

where $\bar{i} = \max\{i : \sum_{j=1}^{i} l_j \leq t\}$.

Then, the transmission completion time minimization prob-

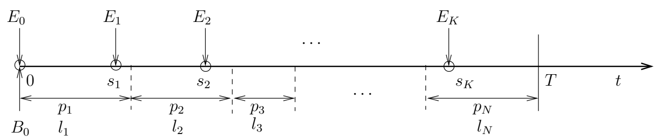


Fig. 4. The sequence of transmission powers and durations.

lem can be formulated as:

$$\begin{aligned} \min_{\mathbf{p},\mathbf{l}} \quad & T \\ \text{s.t.} \quad & E(t) \leq \sum_{i:s_i<t} E_i, \qquad 0 \leq t \leq T \\ & B(T) = B_0 \end{aligned} \tag{3}$$

Throughout the paper, we assume that a finite solution to this optimization problem exists, i.e., that the given harvested energy sequence is sufficient to deliver the given number of bits within a finite time. First, we determine the properties of the optimum solution in the following three lemmas. The proofs of these lemmas are given in Appendices A, B and C.

Lemma 1 *Under the optimal policy, the transmit powers increase monotonically, i.e.,* $p_1 \leq p_2 \leq \cdots \leq p_N$.

Lemma 2 *Under the optimal policy, the transmission power/rate remains constant between energy harvests, i.e., the power/rate only potentially changes when new energy arrives.*

Lemma 3 *Under the optimal policy, whenever the transmission rate changes, the energy consumed up to that instant equals the energy harvested up to that instant.*

Based on Lemmas 1, 2 and 3, we can characterize the optimal policy in the following way. For given energy arrivals, we plot the total amount of harvested energy as a function of t, which is a staircase curve as shown in Fig. 5. The total energy consumed up to time t can also be represented as a continuous curve, as shown in Fig. 5. In order to satisfy the feasibility constraints on the energy, energy consumption curve must lie below the energy harvesting curve at all times. Based on Lemma 2, we know that the optimal energy consumption curve must be linear between any two consecutive energy harvesting instants, and the slope of the segment corresponds to the transmit power level during that segment. Lemma 3 implies that whenever the slope changes, the energy consumption curve must touch the energy harvesting curve at that energy harvesting instant. Therefore, the first linear segment of the energy consumption curve must be one of the lines connecting the origin and any corner point on the energy harvesting curve before $t = T$ (including the point at $t = T$). Because of the monotonicity property of the power given in Lemma 1, among those lines, we should always pick the one with the minimal slope, as shown in Fig. 5. Otherwise, either the feasibility constraints on the energy will not be satisfied, or the monotonicity property given in Lemma 1 will be violated. For example, if we choose the line ending at the corner point at s_3, this will violate the feasibility constraint, as

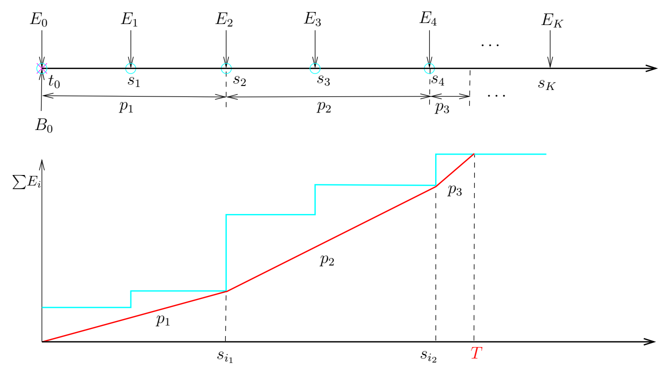


Fig. 5. An interpretation of transmission policies satisfying Lemmas 1, 2 and 3.

the energy consumption curve will surpass the energy arrival curve. On the other hand, if we choose the line ending at the corner point at s_1, then the monotonicity property in Lemma 1 will be violated, because in that case, the slope of the following segment would be smaller. These properties must hold similarly for p_2, p_3, …, p_N. We also observe that, for given T, the optimal transmission policy is the tightest string below the energy harvesting curve connecting the origin and the total harvested energy by time T. This is similar to the structure in [6].

We state the structure of the optimal policy formally in the following theorem. In order to simplify the expressions, we let $i_0 = 0$, and let $s_{m+1} = T$ if the transmission completion time T lies between s_m and s_{m+1}.

Theorem 1 *For a given* B_0*, consider a transmission policy with power vector* $\mathbf{p} = [p_1, p_2, \ldots, p_N]$ *and corresponding duration vector* $\mathbf{l} = [l_1, l_2, \ldots, l_N]$*. This policy is optimal if and only if it has the following structure:*

$$\sum_{n=1}^{N} g(p_n) l_n = B_0 \tag{4}$$

and for $n = 1, 2, \ldots, N$,

$$i_n = \arg \min_{\substack{i: s_i \leq T \\ s_i > s_{i_{n-1}}}} \left\{ \frac{\sum_{j=i_{n-1}}^{i-1} E_j}{s_i - s_{i_{n-1}}} \right\} \tag{5}$$

$$p_n = \frac{\sum_{j=i_{n-1}}^{i_n-1} E_j}{s_{i_n} - s_{i_{n-1}}} \tag{6}$$

$$l_n = s_{i_n} - s_{i_{n-1}} \tag{7}$$

where $T = \sum_{n=1}^{N} l_n$*, and* i_n *is the index of the energy arrival epoch when the power* p_n *switches to* p_{n+1}*, i.e., at* $t = s_{i_n}$*,* p_n *switches to* p_{n+1}.

The proof of this theorem is given in Appendix D.

Therefore, we conclude that if the overall transmission duration T is known, then the optimal transmission policy is known via Theorem 1. In particular, optimal transmission policy is the one that yields the tightest piecewise linear energy consumption curve that lies under the energy harvesting curve at all times and touches the energy harvesting curve at $t = T$. On the other hand, the overall transmission time T is what we

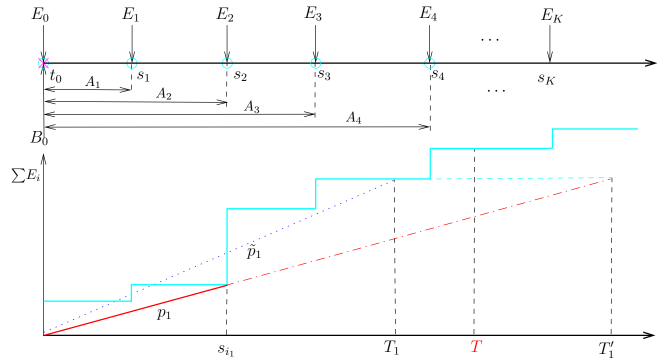


Fig. 6. An illustration of the algorithm.

want to minimize, and we do not know its optimal value up front. Consequently, we do not know up front which energy harvests will be utilized. For example, if the number of bits is small, and E_0 is large, then, we can empty the data queue before the arrival of E_1, thus, the rest of the energy arrivals are not necessary. Therefore, as a first step, we first obtain a good lower bound on the optimal transmission duration.

We first illustrate our algorithm through an example in Fig. 6. We first compute the minimal energy required to finish the transmission before s_1. We denote it as A_1, and it equals

$$A_1 = g^{-1}\left(\frac{B_0}{s_1}\right) s_1 \tag{8}$$

Then, we compare it with E_0. If $A_1 < E_0$, it implies that we can complete the transmission before the arrival of the first energy harvest, thus E_1 is not necessary for the transmission. We allocate E_0 evenly to B_0 bits, and the duration A_1 is the minimum transmission duration. On the other hand, if $A_1 > E_0$, which is the case in the example, the final transmission completion time should be longer than s_1. Thus, we move on and compute A_2, A_3, A_4, and find that $A_2 > \sum_{i=0}^{1} E_i$, $A_3 > \sum_{i=0}^{2} E_i$ and $A_4 < \sum_{i=0}^{3} E_i$. This means that the total transmission completion time will be larger than s_3 and energies $E_0, \ldots, E_3$ will surely be utilized. Then, we allocate $\sum_{i=0}^{3} E_i$ evenly to B_0 bits and obtain a constant transmission power $\tilde{p}_1$, which is the dotted line in the figure. The corresponding transmission duration is T_1. Based on our allocation, we know that the final optimal transmission duration T must be greater than T_1. This is because, this allocation assumes that all $E_0, \ldots, E_3$ are available at the beginning, i.e., at time $t = 0$, which, in fact, are not. Therefore, the actual transmission time will only be larger. Thus, T_1 is a lower bound for T.

Next, we need to check the feasibility of $\tilde{p}_1$. Observing the figure, we find that $\tilde{p}_1$ is not feasible since it is above the staircase energy harvesting curve for some duration. Therefore, we connect all the corner points on the staircase curve before $t = T_1$ with the origin, and find the line with the minimum slope among those lines. This corresponds to the red solid line in the figure. Then, we update $\tilde{p}_1$ with the slope p_1, and the duration for p_1 is $l_1 = s_{i_1}$. We repeat this procedure at $t = s_{i_1}$ and obtain p_2, and continue the procedure until all of the bits are finished.

We state our algorithm for the general scenario in Algorithm 1. We search for the optimal p_n in a sequential way. Specifically, in round n, we first compute the amounts of energy required to finish the transmission of B bits before $s_{i_{n-1}+1}, s_{i_{n-2}+2}, \ldots, s_K$, respectively, at a constant rate. We denote these as A_i. Then, we compare A_i with $\sum_{j=i_{n-1}}^{i-1} E_j$, and find the smallest i such that $A_i \leq \sum_{j=i_{n-1}}^{i-1} E_j$. We denote this i as $\tilde{i}_n$. If no such $\tilde{i}_n$ exists, we let $\tilde{i}_n = K + 1$.

Algorithm 1 The algorithm to minimize the transmission completion time

Initialization: $i_0 = 0, B = B_0, n = 0$
while $B > 0$ **do**
  $n = n + 1$;
  for $i = i_{n-1} + 1, i_{n-1} + 2, \ldots, K$ **do**
    $A_i = g^{-1}\left(\frac{B}{s_i - s_{i_{n-1}}}\right)(s_i - s_{i_{n-1}})$;
    if $A_i \leq \sum_{j=i_{n-1}}^{i-1} E_j$ **then**
      $\tilde{i}_n = i$;
      break;
    else
      $\tilde{i}_n = i + 1$;
    end if
  end for
  Solve $g\left(\frac{\sum_{j=i_{n-1}}^{\tilde{i}_n - 1} E_j}{T_n - s_{i_{n-1}}}\right)(T_n - s_{i_{n-1}}) = B$;
  $\tilde{p}_n = \frac{\sum_{j=i_{n-1}}^{\tilde{i}_n - 1} E_j}{T_n - s_{i_{n-1}}}$;
  $i_n = \arg\min_{i_{n-1} < i < \tilde{i}_n} \left\{\frac{\sum_{j=i_{n-1}}^{i-1} E_j}{s_i - s_{i_{n-1}}}\right\}$;
  $p_n = \frac{\sum_{j=i_{n-1}}^{i_n - 1} E_j}{s_{i_n} - s_{i_{n-1}}}$;
  if $\tilde{p}_n \leq p_n$ **then**
    $p_n = \tilde{p}_n$, $s_{i_n} = T_n$;
    break;
  else
    $B = B - g(p_n)(s_{i_n} - s_{i_{n-1}})$;
  end if
end while

Now, we assume that we can use $\sum_{j=i_{n-1}}^{\tilde{i}_1 - 1} E_j$ to transmit all B bits at a constant rate. We allocate the energy evenly to these bits, and obtain the overall transmission time T and the corresponding constant transmit power $\tilde{p}_n$. Next, we compare $\tilde{p}_n$ with $\frac{\sum_{j=i_{n-1}}^{i-1} E_j}{s_i - s_{i_{n-1}}}$ for every $i_{n-1} < i < \tilde{i}_n$. If $\tilde{p}_n$ is smaller than every term, then, maintaining $\tilde{p}_n$ is feasible, therefore, we have $p_n = \tilde{p}_n$, the transmission duration equals T_n, and the iteration terminates. Otherwise, maintaining $\tilde{p}_n$ is infeasible under the given energy arrival realization. Thus, we update i_n and p_n accordingly, i.e., over the duration $[s_{i_{n-1}}, s_{i_n})$, we choose to transmit with power p_n to make sure that the energy consumption is feasible. Then, at time $t = s_{i_n}$, the total number of bits departed is $g(p_n)(s_{i_n} - s_{i_{n-1}})$, and the remaining number of bits is $B - g(p_n)(s_{i_n} - s_{i_{n-1}})$. Subsequently, with initial number of bits $B - g(p_n)(s_{i_n} - s_{i_{n-1}})$, we start from s_{i_n}, and repeat the procedure above. Through this procedure, we obtain $p_2, p_3, \ldots, p_N$, and the corresponding $i_2, i_3, \ldots, i_N$, until we finish transmitting all of the bits.

Based on our allocation algorithm, we know that p_1 is optimum up to time T_1, since it corresponds to the minimal slope line passing through the origin and any corner point before $t = T_1$. However, the algorithm also implies that the final transmission duration T will be larger than T_1. The question then is, whether p_1 is still the minimum slope line up to time T. If we can prove that p_1 is lower than the slopes of the lines passing through the origin and any corner point in $[T_1, T]$, then, using Theorem 1, we will claim that p_1 is the optimal transmission policy, not only between $[0, T_1]$, but also between $[0, T]$.

The fact that this will be the case can be illustrated through the example in Fig. 6. We note that, clearly, T_1 is a lower bound on the eventual T. If we keep transmitting at power p_1, if no additional energy arrives, the energy harvested up until $s_{\tilde{i}_1}$, i.e., $\sum_{i=0}^{\tilde{i}_1-1} E_i$, will be depleted by time T_1'. We will next prove that T_1' is an upper bound on T. Because of property v) of the function $g(p)$, we can prove that under this policy, the number of bits departed up to time T_1' is greater than B_0. Therefore, since potentially additional energy will arrive, T_1' provides an upper bound. Thus, we know that the optimal T lies between T_1 and T_1'. We next note that if we connect the origin with any corner point of the staircase curve between T_1 and T_1', the slope of the resulting line will be larger than p_1, thus, p_1 will be the smallest slope not only up to time T_1, which is a lower bound, but also up to time T_1', which is an upper bound. This proves that while we do not know the optimal T, if we run the algorithm with respect to the lower bound on T, i.e., T_1, it will still yield an optimal policy, in that the resulting policy will satisfy Theorem 1.

We prove the optimality of the algorithm formally in the following theorem.

Theorem 2 *The allocation procedure in Algorithm 1 gives the optimal transmission policy.*

The proof of this theorem is given in Appendix E.

III. Scenario II: Packets Arrive During Transmissions

In this section, we consider the situation where packets arrive during transmissions. We assume that there is an E_0 amount of energy available at time $t = 0$, and at times s_1, s_2, …, s_K, energy is harvested in amounts E_1, E_2, …, E_K, respectively, as in the previous section. We also assume that at $t = 0$, we have B_0 bits available, and at times t_1, t_2, …, t_M, bits arrive in amounts B_1, B_2, …, B_M, respectively. This system model is shown in Fig. 2. Our objective is again to minimize the transmission completion time, T, which again is the time by which the last bit is delivered to the destination.

Let us denote the sequence of transmission powers by p_1, p_2, …, p_N, and the corresponding transmission durations by l_1, l_2, …, l_N. Then, the optimization problem becomes:

$$\begin{aligned} \min_{\mathbf{p},\mathbf{l}} \quad & T \\ \text{s.t.} \quad & E(t) \le \sum_{i:s_i<t} E_i, \quad 0 \le t \le T \\ & B(t) \le \sum_{i:t_i<t} B_i, \quad 0 \le t \le T \end{aligned}$$

$$B(T) = \sum_{i=0}^{M} B_i \tag{9}$$

where $E(t)$, $B(t)$ are defined in (1) and (2). We again determine the properties of the optimal transmission policy in the following three lemmas. The proofs of these lemmas are given in Appendices F, G and H.

Lemma 4 *Under the optimal policy, the transmission rates increase in time, i.e., $r_1 \le r_2 \le \cdots \le r_N$.*

Lemma 5 *Under the optimal policy, the transmission power/rate remains constant between two event epoches, i.e., the rate only potentially changes when new energy is harvested or a new packet arrives.*

Lemma 6 *Under the optimal policy, if the transmission rate changes at an energy harvesting epoch, then the energy consumed up to that epoch equals the energy harvested up to that epoch; if the transmission rate changes at a packet arrival epoch, then, the number of packets departed up to that epoch equals the number of packets arrived up to that epoch; if the transmission rate changes at an event epoch that has both energy and data arrivals at the same time, then, one of the causality constraints must be met with equality.*

Based on Lemmas 4, 5 and 6, we can identify the structure of the unique optimal transmission policy as stated in the following theorem. In order to simplify the notation, we define u_i to be the time epoch when the ith arrival (energy or data) happens, i.e.,

$$\begin{aligned} u_1 &= \min\{s_1, t_1\} \\ u_i &= \min\{s_i, t_j : s_i > u_{i-1}, t_j > u_{i-1}\}, \quad i = 2, 3, \ldots \end{aligned}$$

We also let $u_{m+1} = T$ if the transmission completion time T lies between u_m and u_{m+1}. In order to simplify the notation, we define $Er(t)$, $Br(t)$ as the total received energy and traffic up to time t^-. Specifically, we have

$$Er(u_i) = \sum_{j:u_0 \le s_j < u_{i-1}} E_j \tag{10}$$

$$Br(u_i) = \sum_{j:u_0 \le t_j < u_{i-1}} B_j, \qquad i = 1, 2, 3, \ldots \tag{11}$$

Theorem 3 *For a given energy harvesting and packet arrival profile, the transmission policy with a transmission rate vector $\mathbf{r} = [r_1, r_2, \ldots, r_N]$ and the corresponding duration vector $\mathbf{l} = [l_1, l_2, \ldots, l_N]$ is optimal, if and only if it has the following structure:*

$$\sum_{i=1}^{N} r_i l_i = \sum_{i=0}^{M} B_i \tag{12}$$

and for $n = 1, 2, \ldots, N$,

$$i_n = \arg \min_{i:u_{i_{n-1}} < u_i \le T} \left\{ g\left(\frac{Er(u_i) - E(u_{i_{n-1}})}{u_i - u_{i_{n-1}}}\right), \frac{Br(u_i) - B(u_{i_{n-1}})}{u_i - u_{i_{n-1}}} \right\} \tag{13}$$

$$r_n = \min_{i: u_{i_{n-1}} < u_i \leq T} \left\{ g\left(\frac{Er(u_i) - E(u_{i_{n-1}})}{u_i - u_{i_{n-1}}}\right), \frac{Br(u_i) - B(u_{i_{n-1}})}{u_i - u_{i_{n-1}}} \right\} \tag{14}$$

$$l_n = u_{i_n} - u_{i_{n-1}} \tag{15}$$

where $i_0 = 0$, $T = \sum_{i=1}^{N} l_i$, $Er(t)$ and $Br(t)$ are defined in (10), and $E(t)$, $B(t)$ are defined in (1),(2).

The proof of this theorem is given in Appendix I.

For a given optimal transmission duration, T, the optimal policy which has the structure in Theorem 3 is unique. However, since we do not know the exact transmission duration up front, we obtain a lower bound on T first, as in the previous section. In this case also, we develop a similar procedure to find the optimal transmission policy. The basic idea is to keep the transmit power/rate as constant as possible throughout the entire transmission duration. Because of the additional causality constraints due to data arrivals, we need to consider both the average data arrival rate as well as the average power the system can support for feasibility.

If $s_K \leq t_M$, i.e., bits have arrived after the last energy harvest, then, all of the harvested energy will be used. The procedure to obtain the optimal scheduling policy is stated in Algorithm 2. First, we calculate the transmission duration which will result if we can use these energies to maintain a constant rate. This will be the solution of

$$g\left(\frac{\sum_{j=0}^{K} E_j}{T}\right) T = \sum_{j=0}^{M} B_j \tag{16}$$

Then, we check whether this constant power/rate is feasible. We check the availability of the energy, as well as the available number of bits. We compare $g^{-1}(r_1)$ with $\tilde{p}_1$. If the former is greater than the latter, then the constant transmit power $\tilde{p}_1$ is feasible. Thus, we achieve the minimum possible transmission completion time T. Otherwise, constant-power transmission is not feasible. We choose the transmit power to be $g^{-1}(r_1)$, and the duration to be the one associated with this transmit power. We repeat this procedure until all of the bits are transmitted.

If $s_K > t_M$, then, as in the first scenario where packets have arrived and are ready before the transmission starts, some of the harvested energy may not be utilized to transmit the bits. In this case also, we need to get a lower bound for the final transmission completion time. The procedure is stated in Algorithm 3. Let $u_{\bar{M}}$ be the energy harvesting epoch right after t_M. Then, starting from $u_{\bar{M}}$, we compute the energy required to transmit $\sum_{j=0}^{M} B_j$ bits at a constant rate by u_i, $u_{\bar{M}} \leq u_i \leq u_{K+M}$, and compare them with the total energy harvested up to that epoch, i.e., $Er(u_i)$. We identify the smallest i such that the required energy is smaller than the total harvested energy, and denote it by $\tilde{i}_1$. If no such $\tilde{i}_1$ exists, we let $\tilde{i}_1 = M+K+1$.

Now, we assume that we can use $Er(u_{\tilde{i}_1})$ to transmit $\sum_{j=0}^{M} B_j$ bits at a constant rate. We allocate the energy evenly to these bits, and the overall transmission time T_1 is the solution of

$$g\left(\frac{Er(u_{\tilde{i}_1})}{T_1}\right) T_1 = \sum_{j=0}^{M} B_j \tag{17}$$

Algorithm 2 The algorithm to minimize the transmission completion time when $s_K \leq t_M$

Initialization: $i_0 = 0, B = \sum_{j=0}^{M} B_j, n = 0$;
while $B > 0$ **do**
  $n = n + 1$;
  Solve $g\left(\frac{\sum_{j=0}^{K} E_j - E(u_{i_{n-1}})}{T_n - u_{i_{n-1}}}\right)(T_n - u_{i_{n-1}}) = B$;
  $\tilde{p}_n = \frac{\sum_{j=0}^{K} E_j - E(u_{i_{n-1}})}{T_n - u_{i_{n-1}}}$;
  Update i_n and r_n according to (13) and (14), where T is replaced by t_M;
  if $\tilde{p}_n \leq g^{-1}(r_1)$ **then**
    $p_n = \tilde{p}_n$, $t_n = g(p_n)$, $u_{i_n} = T_n$;
  else
    $p_n = g^{-1}(r_n)$;
  end if
  $B = B - g(p_n)(u_{i_n} - u_{i_{n-1}})$;
end while

Algorithm 3 The algorithm to minimize the transmission completion time when $s_K > t_M$

Initialization: $i_0 = 0, B = \sum_{j=0}^{M} B_j, n = 0$. Let $u_{\bar{M}}$ be the energy harvesting epoch right after t_M.
while $B > 0$ **do**
  $n = n + 1$;
  for $i = \bar{M}, \bar{M}+1, \ldots, K+M$ **do**
    $A_i = g^{-1}\left(\frac{B}{u_i - u_{i_{n-1}}}\right)(u_i - u_{i_{n-1}})$;
    if $A_i \leq Er(u_i) - E(u_{i_{n-1}})$ **then**
      $\tilde{i}_n = i$;
      break;
    else
      $\tilde{i}_n = i + 1$;
    end if
  end for
  Solve $g\left(\frac{Er(u_{\tilde{i}_n}) - E(u_{i_{n-1}})}{T_n - u_{i_{n-1}}}\right)(T_n - u_{i_{n-1}}) = B$;
  $\tilde{p}_n = \frac{Er(u_{\tilde{i}_n}) - E(u_{i_{n-1}})}{T_n - u_{i_{n-1}}}$;
  Update i_n and r_n according to (13) and (14), where T is replaced by $u_{\tilde{i}_n}$;
  if $\tilde{p}_n < g^{-1}(r_n)$ **then**
    $p_n = \tilde{p}_n$, $r_n = g(p_n)$, $u_{i_n} = T_n$;
  else
    $p_n = g^{-1}(r_n)$;
  end if
  $B = B - g(p_n)(u_{i_n} - u_{i_{n-1}})$;
end while

and the corresponding constant transmit power is $\tilde{p}_1$. Next, we compare $\tilde{p}_1$ with $\frac{Er(u_i)}{u_i}$ and $g^{-1}\left(\frac{Br(u_i)}{u_i}\right)$ for every $i < \tilde{i}_1$. If $\tilde{p}_1$ is smaller than all of these terms, then, maintaining $\tilde{p}_1$ is feasible from both energy and data arrival points of view. The optimal policy is to keep a constant transmission rate at $g(\tilde{p}_1)$ with duration T_1, which yields the smallest possible transmission completion time. Otherwise, maintaining $\tilde{p}_1$ is not feasible under the given energy and data arrival realizations. This infeasibility is due to the causality constraints on either the energy or the data arrival, or both. Next, we identify the tightest constraint, and update the transmit power to be the power associated with that constraint. We repeat this procedure until all of the bits are delivered.

Theorem 4 *The transmission policy obtained through Algorithms 2 and 3 is optimal.*

The proof of this theorem is given in Appendix J.

IV. Simulation Results

We consider a band-limited additive white Gaussian noise channel, with bandwidth $W = 1$MHz and the noise power spectral density $N_0 = 10^{-19}$W/Hz. We assume that the distance between the transmitter and the receiver is 1km, and the path loss h is about 110dB. Then, we have $g(p) = W\log_2\left(1+\frac{ph}{N_0W}\right) = \log_2\left(1+\frac{p}{10^{-2}}\right)$Mbps. It is easy to verify that this function has the properties assumed at the beginning of Section II. For the energy harvesting process, we assume that at times $\mathbf{t} = [0, 2, 5, 6, 8, 9, 11]$s, we have energy harvested with amounts $\mathbf{E} = [10, 5, 10, 5, 10, 10, 10]$mJ, as shown in Fig. 7. We assume that at $t = 0$, we have 5.44Mbits to transmit. We choose the numbers in such a way that the solution is expressable in simple numbers, and can be potted conveniently. Then, using our algorithm, we obtain the optimal transmission policy, which is shown in Fig. 7. We note that the powers change only potentially at instances when energy arrives (Lemma 2); when the power changes, energy consumed up to that point equals energy harvested (Lemma 3); and power sequence is monotonically increasing (Lemma 1). We also note that, for this case, the active transmission is completed by time $T = 9.5$s, and the last energy harvest at time $t = 11$s is not used.

Next, we consider the scenario where data packets arrive during the transmissions. We consider a smaller time scale, where each unit consists of 10ms. We assume that at times $\mathbf{t} = [0, 5, 6, 8, 9]$, energies arrive with amounts $\mathbf{E} = [5, 5, 5, 5, 5] \times 10^{-2}$mJ, while at times $\mathbf{t} = [0, 4, 10]$, packets arrive with equal size 10kbits, as shown in Fig. 8. We observe that the transmitter changes its transmission power during the transmissions. The first change happens at $t = 5$ when energy arrives, and the energy constraint at that instant is satisfied with equality, while the second change happens at $t = 10$ when new bits arrive, and the traffic constraint at that time is satisfied with equality.

V. Conclusions

In this paper, we investigated the transmission completion time minimization problem in an energy harvesting communication system. We considered two different scenarios, where in the first scenario, we assume that packets have already arrived and are ready to be transmitted at the transmitter before the transmission starts, and in the second scenario, we assume that packets may arrive during the transmissions. We first analyzed the structural properties of the optimal transmission policy, and then developed an algorithm to obtain a globally optimal off-line scheduling policy, in each scenario.

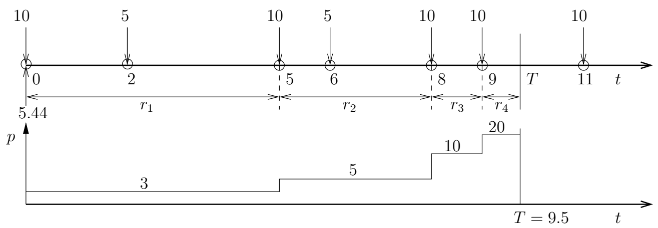


Fig. 7. Optimal transmit powers $\mathbf{p} = [3, 5, 10, 20]$mW, with durations $\mathbf{l} = [5, 3, 1, 0.5]$s.

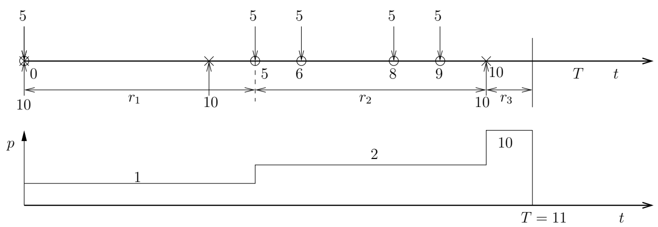


Fig. 8. Optimal transmit powers $\mathbf{p} = [1, 2, 10]$mW, with durations $\mathbf{l} = [5, 5, 1] \times 10^{-2}$s.

Appendix

A. Proof of Lemma 1

Assume that the powers do not increase monotonically, i.e., that we can find two powers such that $p_i > p_{i+1}$. The total energy consumed over this duration is $p_il_i + p_{i+1}l_{i+1}$. Let

$$p_i' = p_{i+1}' = \frac{p_il_i + p_{i+1}l_{i+1}}{l_i + l_{i+1}} \tag{18}$$

$$r_i' = r_{i+1}' = g\left(\frac{p_il_i + p_{i+1}l_{i+1}}{l_i + l_{i+1}}\right) \tag{19}$$

Then, we have $p_i' \leq p_i$, $p_{i+1}' \geq p_{i+1}$. Since $p_i'l_i \leq p_il_i$, the energy constraint is still satisfied, and thus, the new energy allocation is feasible. We use r_i', r_{i+1}' to replace r_i, r_{i+1} in the transmission policy, and keep the rest of the rates the same. Then, the total number of bits transmitted over the duration $l_i + l_{i+1}$ becomes

$$\begin{aligned} & r_i'l_i + r_{i+1}'l_{i+1} \\ & = g\left(\frac{p_il_i + p_{i+1}l_{i+1}}{l_i + l_{i+1}}\right)(l_i + l_{i+1}) \\ & \geq g(p_i)\frac{l_i}{l_i + l_{i+1}}(l_i + l_{i+1}) + g(p_{i+1})\frac{l_{i+1}}{l_i + l_{i+1}}(l_i + l_{i+1}) \\ & = r_il_i + r_{i+1}l_{i+1} \end{aligned} \tag{20}$$

where the inequality follows from the fact that $g(p)$ is concave in p. Therefore, the new policy departs more bits by time $\sum_{j=1}^{i+1} l_j$. Keeping the remaining transmission rates the same, the new policy will finish the entire transmission over a shorter duration. Thus, the original policy could not be optimal. Therefore, the optimal policy must have monotonically increasing powers (and rates).

B. Proof of Lemma 2

Assume that the transmitter changes its transmission rate between two energy harvesting instances s_i, s_{i+1}. Denote the

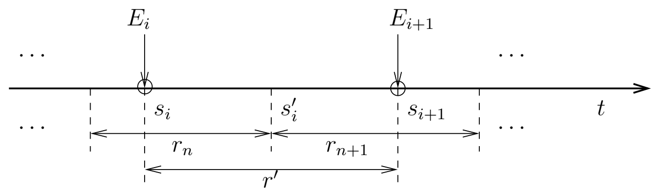


Fig. 9. The rate must remain constant between energy harvests.

rates as r_n, r_{n+1}, and the instant when the rate changes as s_i', as shown in Fig. 9. Now, consider the duration $[s_i, s_{i+1})$. The total energy consumed during the duration is $p_n(s_i' - s_i) + p_{n+1}(s_{i+1} - s_i')$. Let

$$p' = \frac{p_n(s_i' - s_i) + p_{n+1}(s_{i+1} - s_i')}{s_{i+1} - s_i} \tag{21}$$

$$r' = g\left(\frac{p_n(s_i' - s_i) + p_{n+1}(s_{i+1} - s_i')}{s_{i+1} - s_i}\right) \tag{22}$$

Now let us use r' as the new transmission rate over $[s_i, s_{i+1})$, and keep the rest of the rates the same. It is easy to check that the energy constraints are satisfied under this new policy, thus this new policy is feasible. On the other hand, the total number of bits departed over this duration under this new policy is

$$\begin{aligned} & r'(s_{i+1} - s_i) \\ & = g\left(\frac{p_n(s_i' - s_i) + p_{n+1}(s_{i+1} - s_i')}{s_{i+1} - s_i}\right)(s_{i+1} - s_i) \\ & \geq \left(g(p_n)\frac{s_i' - s_i}{s_{i+1} - s_i} + g(p_{n+1})\frac{s_{i+1} - s_i'}{s_{i+1} - s_i}\right)(s_{i+1} - s_i) \\ & = r_n(s_i' - s_i) + r_{n+1}(s_{i+1} - s_i') \end{aligned} \tag{23}$$

where the inequality follows from the fact that $g(p)$ is concave in p. Therefore, the total number of bits departed under the new policy is larger than that under the original policy. If we keep all of the remaining rates the same, the transmission will be completed at an earlier time. This conflicts with the optimality of the original policy.

C. Proof of Lemma 3

From Lemma 2, we know that the transmission rate can change only at certain energy harvesting instances. Assume that the transmission rate changes at s_i, however, the energy consumed by s_i, which is denoted by $E(s_i)$, is less than $\sum_{j=0}^{i-1} E_j$. We denote the energy gap as $\Delta \triangleq \sum_{j=0}^{i-1} E_j - E(s_i)$. Let us denote the rates before and after s_i by r_n, r_{n+1}. Now, we can always have two small amounts of perturbations δ_n, $\delta_{n+1} > 0$ on the corresponding transmit powers, such that

$$p_n' = p_n + \delta_n \tag{24}$$

$$p_{n+1}' = p_{n+1} - \delta_{n+1} \tag{25}$$

$$\delta_n l_n = \delta_{n+1} l_{n+1} \tag{26}$$

We also make sure that δ_n and δ_{n+1} are small enough such that $\delta_n l_n < \Delta$, and $p_n' \leq p_{n+1}'$. If we keep the transmission rates over the rest of the duration the same, under the new transmission policy, the energy allocation will still be feasible. The total number of bits departed over the duration

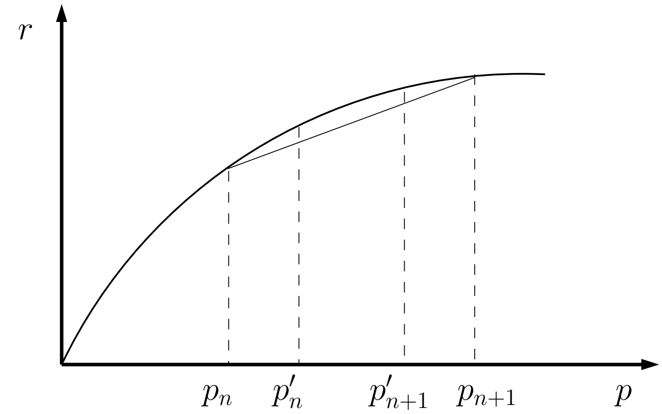


Fig. 10. $g(p)$ is concave in p.

$\left(\sum_{i=1}^{n-1} l_i, \sum_{i=1}^{n+1} l_i\right)$ is

$$g(p_n')l_n + g(p_{n+1}')l_{n+1} \geq g(p_n)l_n + g(p_{n+1})l_{n+1} \tag{27}$$

where the inequality follows from the concavity of $g(p)$ in p, and the fact that $p_n l_n + p_{n+1} l_{n+1} = p_n' l_n + p_{n+1}' l_{n+1}$, $p_n \leq p_n' \leq p_{n+1}' \leq p_{n+1}$, as shown in Fig. 10. This conflicts with the optimality of the original allocation.

D. Proof of Theorem 1

We will prove the necessariness and the sufficiency of the stated structure separately. First, we prove that the optimal policy must have the structure given above. We prove this through contradiction. Assume that the optimal policy, which satisfies Lemmas 1, 2 and 3, does not have the structure given above. Specifically, assume that the optimal policy over the duration $[0, s_{i_{n-1}})$ is the same as the policy described in Theorem 1, however, the transmit power right after $s_{i_{n-1}}$, which is p_n, is not the smallest average power possible starting from $s_{i_{n-1}}$, i.e., we can find another $s_{i'} \leq s_{i_N}$, such that

$$p_n > \frac{\sum_{j=i_{n-1}}^{i'-1} E_j}{s_{i'} - s_{i_{n-1}}} \triangleq p' \tag{28}$$

Based on Lemma 3, the energy consumed up to $s_{i_{n-1}}$ is equal to $\sum_{j=0}^{i_{n-1}-1} E_j$, i.e., there is no energy remaining at $t = s_{i_{n-1}}^-$.

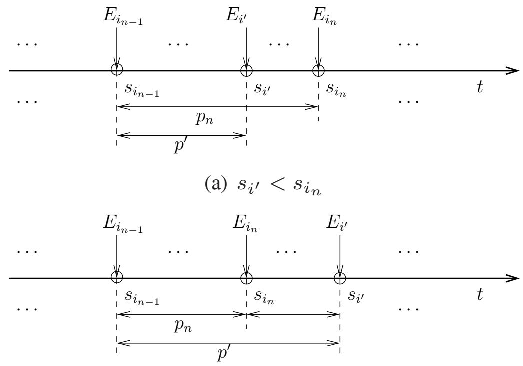
We consider two possible cases here. The first case is that $s_{i'} < s_{i_n}$, as shown in Fig. 11(a). Under the optimal policy, the energy required to maintain a transmit power p_n over the duration $[s_{i_{n-1}}, s_{i'})$ is $p_n(s_{i'} - s_{i_{n-1}})$. Based on (28), this is greater than the total amount of energy harvested during $[s_{i_{n-1}}, s_{i'})$, which is $\sum_{j=i_{n-1}}^{i'-1} E_j$. Therefore, this energy allocation under this policy is infeasible.

On the other hand, if $s_{i'} > s_{i_n}$, as shown in Fig. 11(b), then the total amount of energy harvested over $[s_{i_n}, s_{i'})$ is $\sum_{j=i_n}^{i'-1} E_j$. From (28), we know

$$p_n = \frac{\sum_{j=i_{n-1}}^{i_n-1} E_j}{s_{i_n} - s_{i_{n-1}}} > \frac{\sum_{j=i_{n-1}}^{i'-1} E_j}{s_{i'} - s_{i_{n-1}}} \tag{29}$$

Since

$$\frac{\sum_{j=i_{n-1}}^{i'-1} E_j}{s_{i'} - s_{i_{n-1}}} = \frac{\sum_{j=i_{n-1}}^{i_n-1} E_j}{s_{i_n} - s_{i_{n-1}}} \frac{s_{i_n} - s_{i_{n-1}}}{s_{i'} - s_{i_{n-1}}}$$



(a) $s_{i'} < s_{i_n}$

(b) $s_{i'} > s_{i_n}$

Fig. 11. Two different cases in the proof of Theorem 1.

$$+\frac{\sum_{j=i_n}^{i'-1} E_j}{s_{i'} - s_{i_n}} \frac{s_{i'} - s_{i_n}}{s_{i_n} - s_{i_{n-1}}} \tag{30}$$

Combining (29) and (30), we have

$$p_n > \frac{\sum_{j=i_{n-1}}^{i'-1} E_j}{s_{i'} - s_{i_{n-1}}} > \frac{\sum_{j=i_n}^{i'-1} E_j}{s_{i'} - s_{i_n}} \tag{31}$$

Thus, under any feasible policy, there must exist a duration $l \subseteq [s_{i_n}, s_{i'})$, such that the transmit power over this duration is less than p_n. This contradicts with Lemma 1. Therefore, this policy cannot be optimal.

Next, we prove that if a policy with power vector $\mathbf{p}$ and duration vector $\mathbf{l}$ has the structure given above, then, it must be optimal. We prove this through contradiction. We assume that there exists another policy with power vector $\mathbf{p}'$ and duration vector $\mathbf{l}'$, and the transmission completion time T' under this policy is smaller.

We assume both of the policies are the same over the duration $[0, s_{i_{n-1}})$, however, the transmit policies right after $s_{i_{n-1}}$, which are p_n and p'_n, with durations l_n and l'_n, respectively, are different. Based on the assumption, we must have $p_n < p'_n$.

If $l_n < l'_n$, from Lemma 3, we know that the total energy available over $[s_{i_{n-1}}, s_{i_n})$ is equal to $p_n l_n$. Since $p_n < p'_n$, p'_n is infeasible over $[s_{i_{n-1}}, s_{i_n})$. Thus, policy $\mathbf{p}'$ cannot be optimal. Then, we consider the case when $l_n > l'_n$. Since the power-rate function g is concave, with the sum of energy available over $[s_{i_{n-1}}, s_{i_n})$ fixed, the total number of bits departed over $[s_{i_{n-1}}, s_{i_n})$ under $\mathbf{p}$ is greater than any other policy. Therefore, $T' \geq s_{i_n}$. Then, the total energy spent over $[s_{i_{n-1}}, s_{i_n})$ under $\mathbf{p}'$ is greater than $p_n l_n$, since $p'_n > p_n$, and $p'_{n+1} > p'_n$ based on Lemma 1. Thus, policy $\mathbf{p}'$ is infeasible and cannot be optimal.

In summary, a policy is optimal if and only if it has the structure given above, completing the proof.

E. Proof of Theorem 2

Let T be the final transmission duration given by the allocation procedure. Then, we have $B(T) = B_0$. In order to prove that the allocation is optimal, we need to show that the final transmission policy has the structure given in Theorem 1. We first prove that p_1 satisfies (6). Then, we can similarly prove that p_2, p_3, $\ldots$ satisfy (6).

We know that if $T = T_1$, then it is the minimum possible transmission completion time. We know that this transmit policy will satisfy the structural properties in Theorem 1. Otherwise, the final optimal transmission time T is greater than T_1, and more harvested energy may need to be utilized to transmit the remaining bits. From the allocation procedure, we know that

$$p_1 \leq \frac{\sum_{j=0}^{i-1} E_j}{s_i}, \quad \forall i < \tilde{i}_1 \tag{32}$$

In order to prove that p_1 satisfies (6), we need to show that

$$p_1 \leq \frac{\sum_{j=0}^{i-1} E_j}{s_i}, \quad \forall i : s_{\tilde{i}_1} \leq s_i \leq T \tag{33}$$

If we keep transmitting with power p_1, then at $T'_1 = \frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{p_1}$, the total number of bits departed will be

$$g(p_1)T'_1 \geq g\left(\frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{T_1}\right) T_1 = B_0 \tag{34}$$

where the inequality follows from the assumption that $g(p)/p$ decreases in p. Then, (32) guarantees that this is a feasible policy. Thus, under the optimal policy, the transmission duration T will be upper bounded by T'_1, i.e.,

$$T \leq \frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{p_1} \tag{35}$$

which implies

$$p_1 \leq \frac{\sum_{j=0}^{\tilde{i}_1-1} E_j}{T} \tag{36}$$

If $T \leq s_{\tilde{i}_1}$, as shown in Fig. 12(a), no future harvested energy is utilized for the transmissions. Then, (36) guarantees that (33) is satisfied.

If $T > s_{\tilde{i}_1}$, as shown in Fig. 12(b), additional energy harvested after $s_{\tilde{i}_1}$ should be utilized to transmit the data. We next prove that (33) still holds through contradiction. Assume that there exists i' with $s_{\tilde{i}_1} \leq s_{i'} \leq T$, such that (33) is not satisfied, i.e.,

$$p_1 > \frac{\sum_{j=0}^{i'-1} E_j}{s_{i'}} \triangleq p' \tag{37}$$

Then,

$$\frac{\sum_{j=0}^{i'-1} E_j}{p_1} < s_{i'} \tag{38}$$

Combining this with (35), we have $T < s_{i'}$, which contradicts with the assumption that $s_{i'} \leq T$. Thus, (33) holds, p_1 satisfies the requirement of the optimal structure in (32).

We can then prove using similar arguments that p_2, p_3, $\ldots$ also satisfy the properties of the optimal solution. Based on Lemma 1, this procedure gives us the unique optimal policy.

F. Proof of Lemma 4

First, note that since the relationship between power and rate, $r = g(p)$, is monotone, stating that the rates increase monotonically is equivalent to stating that the powers increase

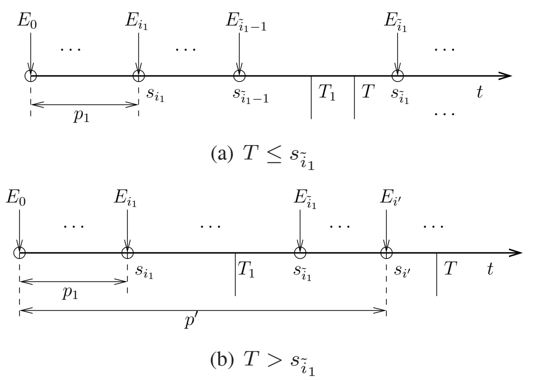


Fig. 12. Two different cases in the proof of Theorem 2.

monotonically. We follow steps similar to those in the proof of Lemma 1 to prove this lemma. Assume that the rates do not increase monotonically, i.e., that we can find two rates such that $r_i > r_{i+1}$, with duration l_i, l_{i+1}, respectively. If $i+1 \neq N$, then, let

$$r_i' = r_{i+1}' = \frac{r_i l_i + r_{i+1} l_{i+1}}{l_i + l_{i+1}} \tag{39}$$

$$p_i' = p_{i+1}' = g^{-1}\left(\frac{r_i l_i + r_{i+1} l_{i+1}}{l_i + l_{i+1}}\right) \tag{40}$$

Since $r_i > r_i' = r_{i+1}' > r_{i+1}$, $p_i > p_i' = p_{i+1}' > p_{i+1}$, it is easy to verify that the new policy is feasible up to the end of l_{i+1}, from both the data and energy arrival points of view. On the other hand, based on the convexity of g^{-1}, the energy spent over the duration $l_i + l_{i+1}$ is smaller than $p_i l_i + p_{i+1} l_{i+1}$. If we allocate the saved energy over to the last transmission duration, without conflicting any energy or data constraints, the transmission will be completed in a shorter duration. If $i+1 = N$, then, we let

$$p_i' = p_{i+1}' = \frac{p_i l_i + p_{i+1} l_{i+1}}{l_i + l_{i+1}} \tag{41}$$

$$r_i' = r_{i+1}' = g\left(\frac{p_i l_i + p_{i+1} l_{i+1}}{l_i + l_{i+1}}\right) \tag{42}$$

Then, from (20), under the new policy, the last bit will depart before the end of l_{i+1}. The energy and data arrival constraints are satisfied over the whole transmission duration. Consequently, the original policy could not be optimal. Therefore, the optimal policy must have monotonically increasing rates (and powers).

G. *Proof of Lemma 5*

This lemma can be proved through a procedure similar to that in Lemma 2. The properties i)-iii) of $g(p)$ implies that $g^{-1}(r)$ exists, and it is convex in r. If power/rate is not constant between two event epoches, then, by equalizing the rate over the duration while keeping the total departures fixed, the total amount of energy spent is reduced because of the convexity of $g^{-1}(r)$. Allocating the saved energy to the last transmission duration, we can shorten the whole transmission duration. Thus, if power/rate is not constant between two event epoches, the policy cannot be optimal.

H. *Proof of Lemma 6*

This lemma can be proved through contradiction using techniques similar to those used in the proof of Lemma 3. When the transmission rate changes at a packet arrival epoch, if the total number of departures is not equal to the total number of bits arrive before that epoch, then, without conflicting the data and energy causality constraints, we can always increase the rate before that epoch a little and decrease the rate after that epoch a little while keeping the total departures fixed. This policy would save some energy which can be used to shorten the transmission durations afterwards. Thus, the data constraint at that epoch must be satisfied as an equality. The remaining situations can be proved similarly.

I. *Proof of Theorem 3*

First, we prove that for the optimal transmission policy, r_1 must satisfy (14). We prove this through contradiction. If r_1 does not satisfy (14), then, we can always find another $u_{i'}$, such that

$$r_1 > \min\left\{g\left(\frac{Er(u_{i'})}{u_{i'}}\right), \frac{Br(u_{i'})}{u_{i'}}\right\} \tag{43}$$

First, we assume that $g\left(\frac{Er(u_{i'})}{u_{i'}}\right) < \frac{Br(u_{i'})}{u_{i'}}$. Then, if $u_{i'} < u_{i_1}$, clearly r_1 is not feasible over the duration $[0, u_{i'})$, because of the energy constraint. If $u_{i'} > u_{i_1}$, then, the transmitter cannot maintain a transmission rate that is always greater than r_1 over $[u_i, u_{i'})$, from the energy point of view. This contradicts with Lemma 4. Similarly, if $g\left(\frac{Er(u_{i'})}{u_{i'}}\right) > \frac{Br(u_{i'})}{u_{i'}}$, the "bottleneck" is the data constraint. We can prove that r_1 is not feasible. Thus, r_1 must be the smallest feasible rate starting from $t=0$, as in (14). We can also prove that $r_2, r_3, \ldots$ must have the same structure, in the same way. Next, we can prove that any policy has the structure described above is optimal. We can prove this through contradiction. Assume that there exists another policy with a shorter transmission completion time. Based on Lemmas 4 and 6, we can prove that this policy could not be feasible.

J. *Proof of Theorem 4*

We focus on the scenario when $s_K \geq t_M$. When $s_K < t_M$, the optimality of Algorithm 2 can be proved as a special case of Algorithm 3 where $\tilde{i}_1 = K + M + 1$.

First we prove that r_1 obtained through Algorithm 3 satisfies (14). If $T = T_1$, i.e., the constant rate is achievable throughout the transmission, then it is the shortest transmission duration we can get, thus, it is optimal. If $T \neq T_1$, from the procedure, we have

$$r_1 \leq \min_{1 \leq i < \tilde{i}_1}\left\{g\left(\frac{Er(u_i)}{u_i}\right), \frac{Br(u_i)}{u_i}\right\} \tag{44}$$

We need to prove that

$$r_1 \leq \min\left\{g\left(\frac{Er(u_i)}{u_i}\right), \frac{Br(u_i)}{u_i}\right\} \quad \text{for } u_{\tilde{i}_1} \leq u_i \leq T. \tag{45}$$

We note that $u_{\tilde{i}_1} \geq u_{\bar{M}} > t_M$, thus, for $u_i \geq u_{\tilde{i}_1}$, we have $Br(u_i) = \sum_{j=0}^{M} B_j$.

Considering the policy with a constant power $p_1 = g^{-1}(r_1)$, then, at $T_1' = \frac{Er(u_{\tilde{i}_1})}{p_1}$, the total number of bits departed will be

$$g(p_1)T_1' \geq g\left(\frac{Er(u_{\tilde{i}_1})}{T_1}\right)T_1 = \sum_{j=0}^{M} B_j \qquad (46)$$

where the inequality follows from the assumption that $g(p)/p$ decreases in p.

On the other hand, since $r_1 \leq \frac{Br(T_M)}{T_M}$, keeping rate r_1 after T_M until $T_1'' = \frac{\sum_{j=0}^{M} B_j}{r_1}$ is also feasible from data arrival perspective. Therefore, maintaining a transmission rate r_1 until the last bit departs the system is feasible from both the energy and data arrival points of view. Thus, under the optimal policy, the transmission duration T will be upper bounded by $\min\{T_1', T_1''\} = T_1''$.

Since

$$r_1 = \frac{\sum_{j=0}^{M} B_j}{T_1''} \leq \frac{Br(t)}{t}, \quad \text{for } T_M < t \leq T_1'' \qquad (47)$$

r_1 is always optimal from data arrival's perspective. To complete the proof, we only need to check the optimality of r_1 from energy point of view. The proof directly follows a similar procedure in the proof of Theorem 2.

Thus, (45) holds, r_1 satisfies the requirement of the optimal structure in (14). We can then prove using a similar argument that $r_2, r_3, \ldots$ also satisfy the structure of the optimal solution. Based on Theorem 3, this procedure gives us the unique optimal transmission policy.

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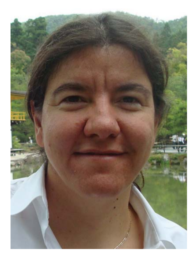
Jing Yang received the B.S. degree in electronic engineering and information science from the University of Science and Technology of China, Hefei, China in 2004, and the M.S. and Ph.D. degrees in electrical and computer engineering from the University of Maryland, College Park in 2010. Since October 2010, she has been a research associate in the department of electrical and computer engineering at the University of Wisconsin-Madison. Her research interests are in wireless communication theory and networking, multi-user information theory, queueing theory, optimization in wireless networks, and statistical signal processing.



Sennur Ulukus is a Professor of Electrical and Computer Engineering at the University of Maryland at College Park, where she also holds a joint appointment with the Institute for Systems Research (ISR). Prior to joining UMD, she was a Senior Technical Staff Member at AT&T Labs-Research. She received her Ph.D. degree in electrical and computer engineering from the Wireless Information Network Laboratory (WINLAB), Rutgers University, and B.S. and M.S. degrees in electrical and electronics engineering from Bilkent University. Her research interests are in wireless communication theory and networking, network information theory for wireless communications, signal processing for wireless communications, physical-layer information-theoretic security for wireless networks, and energy-harvesting wireless communications.

Dr. Ulukus received the 2003 IEEE Marconi Prize Paper Award in Wireless Communications, the 2005 NSF CAREER Award, and the 2010-2011 ISR Outstanding Systems Engineering Faculty Award. She served as an Associate Editor for the IEEE TRANSACTIONS ON INFORMATION THEORY between 2007-2010, as an Associate Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS between 2003-2007, as a Guest Editor for the *Journal of Communications and Networks* for the special issue on energy harvesting in wireless networks, as a Guest Editor for the IEEE TRANSACTIONS ON INFORMATION THEORY for the special issue on interference networks, and as a Guest Editor for the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS for the special issue on multiuser detection for advanced communication systems and networks. She served as the TPC co-chair of the Communication Theory Symposium at the 2007 IEEE Global Telecommunications Conference, the Medium Access Control (MAC) Track at the 2008 IEEE Wireless Communications and Networking Conference, the Wireless Communications Symposium at the 2010 IEEE International Conference on Communications, the 2011 Communication Theory Workshop, the Physical-Layer Security Workshop at the 2011 IEEE International Conference on Communications, and the Physical-Layer Security Workshop at the 2011 IEEE Global Telecommunications Conference. She was the Secretary of the IEEE Communication Theory Technical Committee (CTTC) in 2007-2009.