Online Fixed Fraction Policies in Energy Harvesting Communication Systems

Ahmed Arafa, *Member, IEEE*, Abdulrahman Baknina, *Student Member, IEEE*, and Sennur Ulukus, *Fellow, IEEE*

***Abstract*— We consider power scheduling policies for single-user energy harvesting communication systems, where the goal is to characterize *online policies* that maximize the long term average utility, for general concave and monotonically increasing utility functions. The transmitter relies on energy harvested from nature to send its messages to the receiver, and is equipped with a finite-sized battery to store its harvested energy. Energy packets are independent and identically distributed (i.i.d.) over time slots, and are revealed causally to the transmitter. We first characterize the optimal solution for the case of Bernoulli arrivals. Then, for general i.i.d. arrivals, we first show that *fixed fraction policies*, in which a fixed fraction of the battery state is consumed in each time slot, are within a constant *multiplicative gap* from the optimal solution for all energy arrivals and battery sizes. We then derive a set of sufficient conditions on the utility function to guarantee that fixed fraction policies are within a constant *additive gap* as well from the optimal solution. We then apply these results to a specific scenario where a sensor node collects samples from a Gaussian source and sends them to a destination node over a Gaussian channel, and the goal is to minimize the long term average distortion of the source samples received at the destination. We study two problem settings for this case: the first is when sampling is cost-free, and the second is when there is a sampling cost incurred whenever samples are collected. For the problem with sampling costs, the transmission policy can be bursty; the sensor may collect samples and transmit for only a portion of the time. Finally, we present an alternative analysis approach that is more tailored to these distortion problems to show that fixed fraction policies achieve an *additive gap* that is independent of the sampling cost.**

***Index Terms*—Energy harvesting, online optimization, general utility functions, distortion minimization, finite battery, near optimal policy.**

I. INTRODUCTION

OPTIMAL energy management in communication systems that rely on energy harvested from nature is a crucial system design aspect to provide a sustainable and efficient operation over the long run. In this paper, a single-user communication channel is considered, where the transmitter relies on energy harvested from nature to send its messages to the receiver. The transmitter has a battery of finite size to save its incoming energy, and achieves a reward for every transmitted message that is in the form of some general concave increasing utility function of the transmission power, see Fig. 1. The goal is to characterize *online* power control policies that maximize the long term average utility subject to energy causality constraints.

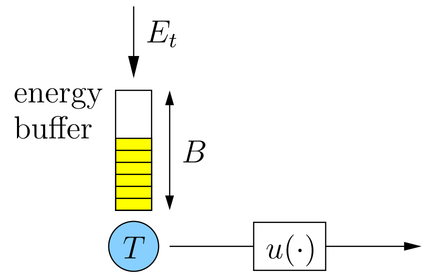

Fig. 1. Single-user energy harvesting channel with general utility function.

Power scheduling in energy harvesting communications is mainly categorized in the literature into *offline* and *online* scheduling, depending on whether the amounts/times of the harvested energy are known prior to communication. Offline scheduling has been extensively studied in the recent literature. Earlier works [1]–[4] consider the single-user setting under different battery size assumptions, with and without fading; references [5]–[11] extend this to broadcast, multiple access, and interference settings; and [12]–[15] consider two-hop and relay channels. Energy cooperation and energy sharing concepts are studied in [16] and [17]. References [18]–[22] study energy harvesting receivers, where energy harvested at the receiver is spent mainly for sampling and decoding; [23]–[27] study the impact of processing costs, i.e., the power spent for circuitry, on energy harvesting communications; and [28] studies decoding and processing costs combined in a single setting for an energy harvesting two-way channel. A source-channel coding problem with an energy harvesting transmitter is formulated in [29] to minimize the distortion of source samples sent to a destination. Impacts of processing and sampling costs are also studied, and two-dimensional water-filling interpretations are presented.

Online scheduling has been considered in the literature mainly through Markov decision processes modeling and

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A. Arafa was with the Department of Electrical and Computer Engineering, University of Maryland, College Park, MD 20742 USA. He is now with the Electrical Engineering Department, Princeton University, NJ 08544 USA (e-mail: aarafa@princeton.edu).

A. Baknina and S. Ulukus are with the Department of Electrical and Computer Engineering, University of Maryland, College Park, MD 20742 USA (e-mail: abaknina@umd.edu; ulukus@umd.edu).

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dynamic programming techniques [3], [4], [30]–[35]. Recently, however, [36] has introduced an online power control policy for a single-user energy harvesting channel that maximizes the long term average throughput under the AWGN capacity utility function $\frac{1}{2}\log(1+x)$. The proposed policy is *near optimal* in the sense that it performs within constant multiplicative and additive gaps from the optimal solution that is independent of energy arrivals and battery sizes. This constant gap approach is extended to broadcast channels in [37], multiple access channels in [38], [39], and systems with processing costs in [40].

In this paper, we generalize the approaches in [36] to work for general concave monotonically increasing utility functions for single-user channels. That is, we consider the design of online power control policies that maximize the long term average general utilities. One motivation for this setting is energy harvesting receivers. Since power consumed in decoding is modelled as a convex increasing function of the incoming rate [18], [19], [22], the rate achieved at the receiver is then a concave increasing function of the decoding power.

We first study the special case of Bernoulli energy arrivals that fully recharge the battery when harvested, and characterize the optimal online solution. Then, for the general i.i.d. arrivals, we show that the policy introduced in [36] performs within a constant multiplicative gap from the optimal solution for any general concave increasing utility function, for all energy arrivals and battery sizes. We then provide sufficient conditions on the utility function to guarantee that such policy is within a constant additive gap from the optimal solution. We note that in [36], the additive gap analysis specifically is highly dependent on properties of the $\log$ function, in particular, the fact that $\log(xy) = \log(x) + \log(y)$, which are not possessed in general by other concave utility functions. In this paper, we perform the additive gap analysis through a different technique than that in [36] by introducing auxiliary mathematical functions, derived from the utility functions considered, and based on their behavior we characterize the additive gap in terms of them. In addition, through our tools, we present sufficient conditions on utility functions such that the considered transmission policy is asymptotically optimal (as opposed to near optimal) as the battery size grows infinitely large.

We then consider a specific scenario where a sensor node collects samples from an i.i.d. Gaussian source and sends them to a destination over a Gaussian channel, and the goal is to characterize online power control policies that minimize the long term average distortion of the received samples at the destination, which is considered in [29] for the offline setting. We follow the approaches in [36]–[40] to extend the offline results in [29] to online settings. We formulate two problems: one with and the other without sampling energy consumption costs. We show that both problems can be reformulated as a maximization of a certain concave utility, and thereby the results derived for general concave utility functions are applied. In addition, we present an alternative approach, than that considered for general concave utility functions, to analyze the additive gap in a way that is tailored to the distortion minimization problems with and without sampling costs. Different from the results in [40] for single-user channels with processing costs, this approach leads to an additive gap result that is independent of the sampling cost.

We finally note that an independent result on the case with general utility functions, and concurrent with our conference versions of this paper [41], [42], has been reported in [43]. The additive gap results in there are derived for functions that satisfy a specific sub-logarithmic difference property. This allows for the usage of the properties of the $\log$ function in the same way used in [36] to analyze the additive gap. In this paper, however, the analysis approach is different. As noted above, the main technique in analyzing the additive gap is via the introduction of an auxiliary function derived from the utility function considered, and then based on its behavior the additive gap is characterized.

II. General Utility Functions

We consider a single-user channel where the transmitter relies on energy harvested from nature to send its messages to the receiver. Energy arrives (is harvested) in packets of amount E_t at the beginning of time slot t. Without loss of generality, a slot duration is normalized to one time unit. Energy packets follow an i.i.d. distribution with a given mean. Our setting is *online:* the amounts of energy are known causally in time, i.e., after being harvested. Only the mean of the energy arrivals is known a priori. Energy is saved in a battery of finite size B.

Let u be a differentiable, concave, and monotonically increasing function representing a general utility (reward) function, with $u(0) = 0$ and $u(x) > 0$ for $x > 0$, and let g_t denote the transmission power used in time slot t. By allocating power g_t in time slot t, the transmitter achieves $u\left(g_t\right)$ instantaneous reward. Denoting $\mathcal{E}^t \triangleq \{E_1, E_2, \ldots, E_t\}$, a feasible online policy $\boldsymbol{g}$ is a sequence of mappings $\{g_t : \mathcal{E}^t \rightarrow \mathbb{R}_+\}$ satisfying

$$0 \le g_t \le b_t \triangleq \min\{b_{t-1} - g_{t-1} + E_t, B\}, \quad \forall t \tag{1}$$

with $b_1 \triangleq B$ without loss of generality (using similar arguments as in [36, Appendix B]). We denote the above feasible set in (1) by $\mathcal{F}$, which represents the energy causality constraints. Given a feasible policy $\boldsymbol{g}$, we define the n-horizon average reward as

$$\mathcal{U}_n(\boldsymbol{g}) \triangleq \frac{1}{n}\mathbb{E}\left[\sum_{t=1}^{n} u\left(g_t\right)\right] \tag{2}$$

Our goal is to design online power scheduling policies that maximize the long term average reward subject to (online) energy causality constraints. That is, to characterize

$$\rho^* \triangleq \sup_{\boldsymbol{g}\in\mathcal{F}} \liminf_{n\rightarrow\infty} \ \mathcal{U}_n(\boldsymbol{g}) \tag{3}$$

We note that problem (3) can be solved by dynamic programming techniques since the underlying system evolves as a Markov decision process. However, the optimal solution using dynamic programming is usually computationally demanding with few structural insights. Therefore, in the sequel, we aim at finding relatively simple online power control policies that are provably within a constant additive and multiplicative gap from the optimal solution for all energy arrivals and battery sizes.

We assume that $E_t \leq B \ \forall t$ a.s., since any excess energy above the battery capacity cannot be saved or used. Let $\mu = \mathbb{E}[E_t]$, where $\mathbb{E}[\cdot]$ is the expectation operator, and define

$$q \triangleq \frac{\mathbb{E}[E_t]}{B} \tag{4}$$

Then, we have $0 \leq q \leq 1$ since $E_t \leq B$ a.s. We define the power control policy as follows [36]

$$\tilde{g}_t = q b_t \tag{5}$$

That is, in each time slot, the transmitter uses a fixed fraction of its available energy in the battery. Such policies were first introduced in [36], and coined *fixed fraction policies* (FFP). Clearly such policies are always feasible since $q \leq 1$. Let $\rho(\tilde{\boldsymbol{g}})$ be the long term average utility under the FFP $\{\tilde{g}_t\}$. Next, we find the *optimal* solution of problem (3) under the specific case of Bernoulli energy arrivals. After that, we discuss how the FFP performs under general i.i.d. energy arrivals.

A. Bernoulli Energy Arrivals

Let $\{\hat{E}_t\}$ be a Bernoulli energy arrival process with mean μ as follows

$$\hat{E}_t = \begin{cases} B, & \text{w.p. } p \\ 0, & \text{w.p. } 1-p \end{cases} \tag{6}$$

Note that under such specific energy arrival setting, whenever an energy packet arrives, it completely fills the battery, and resets the system. This constitutes a *renewal*. Then, by [44, Th. 3.6.1] (see also [36]), the following holds for any power control policy $\boldsymbol{g}$

$$\begin{aligned} \liminf_{n\to\infty} \ \hat{\mathcal{U}}_n(\boldsymbol{g}) &= \liminf_{n\to\infty} \ \frac{1}{n}\mathbb{E}\left[\sum_{t=1}^{n} u(g_t)\right] \\ &= \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{t=1}^{L} u(g_t)\right] \end{aligned} \tag{7}$$

where $\hat{\mathcal{U}}_n(\boldsymbol{g})$ is the n-horizon average utility under Bernoulli arrivals, and L is a random variable denoting the inter-arrival time between energy arrivals, which is geometric with parameter p, and $\mathbb{E}[L] = 1/p$.

We note that using the FFP defined in (5) in (7) gives a lower bound on the optimal long term average utility. In this case, by (6), the fraction q in (4) is now equal to p. Also, the battery state decays exponentially in between energy arrivals. To see this, observe that $b_1 = B$, and hence $g_1 = pB$. We then get that $b_2 = B - pB = (1-p)B$, and hence $g_2 = p(1-p)B$. This leads to $b_3 = (1-p)B - p(1-p)B = (1-p)^2 B$, and hence $g_3 = p(1-p)^2 B$, and so on. In general, the FFP is

$$\tilde{g}_t = p(1-p)^{t-1}B = (1-p)^{t-1}\mu \tag{8}$$

for all time slots t, where the second equality follows since $pB = \mu$.

Using (7), one can simplify the long term average utility under Bernoulli arrivals as follows (see also [36])

$$\begin{aligned} \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{t=1}^{L} u(g_t)\right] &= p\sum_{l=1}^{\infty} p(1-p)^{l-1}\sum_{t=1}^{l} u(g_t) \\ &= p^2 \sum_{t=1}^{\infty} u(g_t) \sum_{l=t}^{\infty} (1-p)^{l-1} \\ &= p^2 \sum_{t=1}^{\infty} \frac{(1-p)^{t-1}}{p} u(g_t) \end{aligned} \tag{9}$$

Whence, problem (3) in this case reduces to[1]

$$\begin{aligned} \max_{\boldsymbol{g}} \ & \sum_{t=1}^{\infty} p(1-p)^{t-1} u(g_t) \\ \text{s.t.} \ & \sum_{t=1}^{\infty} g_t \leq B, \quad g_t \geq 0, \ \forall t \end{aligned} \tag{10}$$

which is a convex optimization problem. We note that if u is linear, i.e., $u(g_t) = \kappa g_t$ for some constant $\kappa > 0$, then the solution to the above problem would directly be given by $g_1^* = B$, and $g_t^* = 0$ for $t \geq 2$, since the coefficients $p(1-p)^{t-1}$ are decreasing. This means that the optimal solution is *greedy* for linear utility functions; once the battery is recharged, it is immediately used. We therefore continue this section with the more challenging case where u is *strictly* concave. The Lagrangian for the problem in this case is,

$$\mathcal{L} = -\sum_{t=1}^{\infty} p(1-p)^{t-1} u(g_t) + \lambda\left(\sum_{t=1}^{\infty} g_t - B\right) - \sum_{t=1}^{\infty} \eta_t g_t \tag{11}$$

where λ and $\{\eta_t\}$ are Lagrange multipliers. Taking derivative with respect to g_t and equating to 0 we get[2]

$$u'(g_t) = \frac{\lambda - \eta_t}{p(1-p)^{t-1}} \tag{12}$$

Since u is strictly concave, u' is monotonically decreasing and its functional inverse $v \triangleq (u')^{-1}$ exists, and is also monotonically decreasing. By complementary slackness, we have $\eta_t = 0$ for $g_t > 0$, and the optimal power in this case is given by

$$g_t = v\left(\frac{\lambda}{p(1-p)^{t-1}}\right) \tag{13}$$

and it now remains to find the optimal λ. We note that by monotonicity of v, $\{g_t\}$ is non-increasing. We also note that if $u'(x)$ grows unboundedly as $x \to 0$, then $g_t > 0$ $\forall t$. For if $g_{t_0} = 0$ in some time slot t_0, then this would directly mean, by (13), that $u'(0) = \lambda / p(1-p)^{t_0 - 1} < \infty$; an obvious contradiction. Therefore, the optimal power allocation

[1]It can be argued [45, Th. 6.4] that there exists a stationary policy that achieves ρ^*; we find this optimal policy using the maximization problem in (10).

[2]We note that deriving the KKT conditions for the infinite number of variables considered in this problem can be handled slightly differently by, e.g., considering a finite number of variables and taking the limit as this number goes to infinity, as done in [36, Appendix C]. Such details are omitted here.

sequence is an infinite sequence in this case, and we solve the following equation for the optimal λ

$$\sum_{t=1}^{\infty} v\left(\frac{\lambda}{p(1-p)^{t-1}}\right) = B \tag{14}$$

which has a unique solution by monotonicity of v.

Now let us assume that $u'(0)$ is finite. Since u' is decreasing, it holds that

$$g_t = v\left(\frac{\lambda}{p(1-p)^{t-1}}\right) > 0 \iff \lambda < p(1-p)^{t-1}u'(0) \tag{15}$$

Thus, there exists a time slot N, after which the second inequality in (15) is violated since λ is a constant and $p(1-p)^{t-1}$ is decreasing. In this case the optimal power allocation sequence is only positive for a finite number of time slots $1 \leq t \leq N$. We note that N is the smallest integer such that

$$\lambda \geq p(1-p)^N u'(0) \tag{16}$$

One way to find the optimal N (and λ) is by first assuming N is equal to some integer $\{1, 2, 3, \ldots\}$, and solving the following equation for λ

$$\sum_{t=1}^{N} v\left(\frac{\lambda}{p(1-p)^{t-1}}\right) = B \tag{17}$$

which, again, has a unique solution by monotonicity of v. We then check if (16) is satisfied for that choice of N and λ. If it is, we stop. If not, we increase the value of N and repeat. This way, we reach a KKT point,[3] which is sufficient for optimality by convexity of the problem [46]. We note that if one can solve for λ in terms of N, then we would directly find the optimal N as the smallest integer satisfying (16). This, however, depends on the structure of v. For instance, for $u(x) = \frac{1}{2}\log(1+x)$ whose $u'(0)$ is finite, [36] was able to solve for λ in terms of N, which was termed $\tilde{N}$. We generalize their analysis for any concave increasing function u. This concludes the discussion of the optimal solution in the case of Bernoulli energy arrivals.

B. General i.i.d. Energy Arrivals

We now consider the case of a general i.i.d. energy arrival process. We first have the following two results. The proofs are in Appendices A and B, respectively.

Lemma 1: The optimal solution of problem (3) satisfies

$$\rho^* \leq u(\mu) \tag{18}$$

Theorem 1: The achieved long term average utility under the FFP in (5) satisfies

$$\frac{1}{2} \leq \frac{\rho(\tilde{\boldsymbol{g}})}{u(\mu)} \leq 1 \tag{19}$$

We note that the results in Lemma 1 and Theorem 1 indicate that the FFP in (5) achieves a long term average utility that is within a constant multiplicative gap from the optimal solution that is equal to $\frac{1}{2}$. This result is proved in [36] for $u(x) = \frac{1}{2}\log(1+x)$. Here, we are generalizing it to work for any concave increasing function u with $u(0) = 0$.

Next, we state the additive gap results. We first define the following auxiliary function that helps in assessing the gap. Its exact mathematical use will appear later on in the analysis (cf. Theorem 2). Let

$$h_\theta(x) \triangleq u(\theta x) - u(x) \tag{20}$$

for some $0 \leq \theta \leq 1$, and define the following two classes of utility functions.

Definition 1 (Utility Classes): A utility function u belongs to class (A) if $h_\theta(x)$ does not converge to 0 as $x \to \infty$, and belongs to class (B) if $\lim_{x\to\infty} h_\theta(x) = 0$.

Now let us define the following function for $0 < \theta < 1$

$$h(\theta) \triangleq \inf_x h_\theta(x) \tag{21}$$

whenever the infimum exists. Note that the infimum exists for class (B) utility functions since $h_\theta(x) < 0$ for $x > 0$ by monotonicity of u, and $h_\theta(0) = 0$. We discuss different examples of $h(\theta)$ in Section IV. The next lemma states some of its properties.

Lemma 2: $h(\theta)$ is non-positive, concave, and increasing in θ.

Proof: Since u is increasing and $0 < \theta < 1$, then $h_\theta(x) = u(\theta x) - u(x) < 0$ for all x, and hence the infimum is non-positive. Concavity follows by the concavity of u and the fact that the infimum of concave functions is also concave [46]. Finally, h is increasing since u is monotonically increasing. To see this, let $\theta_1 > \theta_2$, and let $x_i \in \arg\inf_x h_{\theta_i}(x)$, $i \in \{1, 2\}$. Then, we have $h(\theta_1) = \lim_{x\to x_1} h_{\theta_1}(x) > \lim_{x\to x_1} h_{\theta_2}(x) \geq \lim_{x\to x_2} h_{\theta_2}(x) = h(\theta_2)$, where the first inequality follows by monotonicity of u. ■

The next two theorems summarize the additive gap results for utility functions in classes (A) and (B) in Definition 1. The proofs are in Appendices C and D, respectively.

Theorem 2: If $h(\theta)$ exists, and if

$$r \triangleq (1-q)\lim_{t\to\infty} \frac{1 - \lim_{x\to\bar{x}_{t+1}} u\left((1-q)^{t+1}x\right)/u(x)}{1 - \lim_{x\to\bar{x}_t} u\left((1-q)^t x\right)/u(x)} < 1 \tag{22}$$

where $\bar{x}_t \in \arg\inf_x h_{(1-q)^t}(x)$; then the achieved long term average utility under the FFP in (5) satisfies

$$u(\mu) + \alpha \leq \rho(\tilde{\boldsymbol{g}}) \leq u(\mu) \tag{23}$$

where $\alpha \triangleq \sum_{t=0}^{\infty} q(1-q)^t h\left((1-q)^t\right)$ is finite.

Theorem 3: For class (B) utility functions, the achieved long term average utility under the FFP in (5) satisfies

$$\lim_{\mu\to\infty} \rho(\tilde{\boldsymbol{g}}) = \rho^* \tag{24}$$

We note that the results in Lemma 1 and Theorem 2 indicate that the FFP in (5) achieves a long term average utility, under some sufficient conditions, that is within a constant additive gap from the optimal solution that is equal to $\left|\sum_{t=0}^{\infty} q(1-q)^t h\left((1-q)^t\right)\right|$. One can further make this gap

[3] By KKT point, we mean a set of primal and dual variables that satisfy the KKT conditions [46].

independent of q by minimizing it over $0 \leq q \leq 1$. We discuss examples of the above results in Section IV, where we also comment on FFP performance under utility functions that do not satisfy the sufficient conditions in Theorem 2.

III. Specific Scenario: Distortion Minimization

We now focus on a specific scenario of a sensor node collecting i.i.d. Gaussian source samples, with zero-mean and variance σ_s^2, over a sequence of time slots. Samples are compressed and sent over an additive white Gaussian noise channel, with variance σ_c^2, to an intended destination. We consider a strict delay scenario where samples need to be sent during the same time slot in which they are collected. With a mean squared error distortion criterion, the average distortion of the source samples in time slot t, D_t, is given by [47]

$$D_t = \sigma_s^2 \exp\left(-2r_t\right) \tag{25}$$

where r_t denotes the sampling rate at time slot t.

The sensor uses energy harvested from nature to send its samples over the channel, with minimal distortion, and consumes energy in sampling and transmission. Depending on the physical settings, sampling energy cost can be a significant system aspect and needs to be taken into consideration [29]. We formulate two different problems for that matter: one without, and the other with sampling costs as follows.

We first consider the case of no sampling cost, where energy is consumed only in transmission. By allocating power g_t at time slot t to the Gaussian channel, the sensor achieves an instantaneous communication rate of [47]

$$r_t = \frac{1}{2}\log\left(1 + g_t/\sigma_c^2\right) \tag{26}$$

Given a feasible policy $\boldsymbol{g}$, and using (25) and (26), we define the n-horizon average distortion as follows

$$\mathcal{D}_n(\boldsymbol{g}) \triangleq \frac{1}{n}\mathbb{E}\left[\sum_{t=1}^{n} \frac{\sigma_s^2}{1 + g_t/\sigma_c^2}\right] \tag{27}$$

Our goal is to minimize the long term average distortion, subject to (online) energy causality constraints. That is, to characterize the following

$$d^* \triangleq \inf_{\boldsymbol{g}\in\mathcal{F}} \limsup_{n\to\infty} \ \mathcal{D}_n(\boldsymbol{g}) \tag{28}$$

where $\mathcal{F}$ is as defined in (1).

Now let us consider the case where sampling the source incurs an energy cost ϵ per unit time, that is a constant independent of the sampling rate. Due to the sampling cost, collecting all the source samples might not be optimal. Hence, we allow the sensor to be *on* during a $\theta_t \leq 1$ portion of time slot t, and turn off for the remainder of the time slot. The expected distortion achieved in time slot t under this setting is now given by

$$D_t^\epsilon = (1-\theta_t)\sigma_s^2 + \theta_t\sigma_s^2 \exp\left(-2r_t\right) \tag{29}$$

and the feasible set $\mathcal{F}_\epsilon$ is now given by the sequence of mappings $\{(\theta_t, g_t): \ \mathcal{E}^t \to [0,1]\times\mathbb{R}_+\}$ satisfying

$$\theta_t(\epsilon + g_t) \leq b_t \triangleq \min\{b_{t-1} - \theta_{t-1}(\epsilon + g_{t-1}) + E_t, B\}, \ \ \forall t \tag{30}$$

with $b_1 \triangleq B$; compare the feasible set in (30) with cost to the feasible set in (1) with no additional cost. We note that the problem with sampling costs is formulated slightly different in [29]. In our formulation, the expected distortion is interpreted by time sharing between not transmitting (and hence achieving σ_s^2) and transmitting with rate r_t (and hence achieving $\sigma_s^2\exp(-2r_t)$). Given a feasible policy $(\boldsymbol{\theta}, \boldsymbol{g})$, and using (26) and (29), we define the n-horizon average distortion with sampling costs as

$$\mathcal{D}_n^\epsilon(\boldsymbol{\theta}, \boldsymbol{g}) \triangleq \frac{1}{n}\mathbb{E}\left[\sum_{t=1}^{n}(1-\theta_t)\sigma_s^2 + \theta_t\frac{\sigma_s^2}{1 + g_t/\sigma_c^2}\right] \tag{31}$$

whence our goal is to characterize

$$d_\epsilon^* \triangleq \inf_{(\boldsymbol{\theta},\boldsymbol{g})\in\mathcal{F}^\epsilon} \limsup_{n\to\infty} \ \mathcal{D}_n^\epsilon(\boldsymbol{\theta}, \boldsymbol{g}) \tag{32}$$

In the next subsection (Section III-A), we discuss how problems (28) and (32) can be analyzed using the results in Section II, and then, in the following subsection (Section III-B), we propose a relatively easier approach from that considered in Section II to analyze their additive gap results. Namely, this different approach does not include the computation of the term α in Theorem 2.

A. Connection to General Utility Results

We first note that the distortion function

$$f(x) \triangleq \frac{\sigma_s^2}{1 + x/\sigma_c^2} \tag{33}$$

is convex and decreasing in x. Hence, the function $\bar{u}(x) \triangleq -\frac{\sigma_s^2}{1+x/\sigma_c^2} + \sigma_s^2$ is concave and increasing in x with $\bar{u}(0) = 0$. One can therefore apply the results of Section II to problem (28) after changing the minimization to maximization and the distortion function to the function $\bar{u}$ above.

Applying the results of Section II to the case with sampling costs, however, is not as direct. This is mainly because the optimization is over two sequences of variables $\{\theta_t\}$ and $\{g_t\}$. Towards that, we observe that the achieved distortion in a given time slot is a function of the total amount of energy allocated to that time slot, cast as an optimization problem that finds the optimal division of the energy allocated between sampling energy costs and transmission powers. Namely, for an amount of energy x allocated to a time slot, the achieved distortion with sampling costs in that time slot is given by

$$\begin{aligned} f_\epsilon(x) \triangleq \min_{\theta,\bar{g}} \ & (1-\theta)\sigma_s^2 + \theta\frac{\sigma_s^2}{1 + \frac{\bar{g}}{\theta\sigma_c^2}} \\ \text{s.t.} \ \ & \theta\epsilon + \bar{g} \leq x, \quad 0 \leq \theta \leq 1 \end{aligned} \tag{34}$$

which is basically a minimization of (29) given a total allocated energy of x, after substituting (26), and a change of variables $\bar{g} \triangleq \theta g$. We now have the following properties for f_ϵ.

Lemma 3: The function f_ϵ is convex and decreasing.

Proof: f_ϵ is decreasing since allocating more energy can strictly decrease the distortion by increasing $\bar{g}$. Now let us denote the objective function of the optimization problem by $H(\theta, \bar{g})$. This function is jointly convex in $(\theta, \bar{g})$ since the second term is the perspective function of the convex function $f(\bar{g})$, and is therefore jointly convex in $(\theta, \bar{g})$ [46]. Proceeding to show convexity of f_ϵ, let $(\theta_1, \bar{g}_1)$ and $(\theta_2, \bar{g}_2)$ be the solutions achieving $f_\epsilon(x_1)$ and $f_\epsilon(x_2)$, respectively, for some $x_1, x_2 \geq 0$. Now choose $\lambda \in [0,1]$, and let $x_\lambda \triangleq \lambda x_1 + (1-\lambda)x_2$. It is direct to see that the convex combination $(\theta_\lambda, \bar{g}_\lambda) \triangleq (\lambda\theta_1 + (1-\lambda)\theta_2, \lambda\bar{g}_1 + (1-\lambda)\bar{g}_2)$ is feasible for x_λ. Therefore,

$$\begin{aligned} f_\epsilon(x_\lambda) &\leq H(\theta_\lambda, \bar{g}_\lambda) \\ &\leq \lambda H(\theta_1, \bar{g}_1) + (1-\lambda) H(\theta_2, \bar{g}_2) \\ &= \lambda f_\epsilon(x_1) + (1-\lambda) f_\epsilon(x_2) \end{aligned} \tag{35}$$

where the second inequality follows by convexity of H. ■

In view of Lemma 3, we see that the function $\bar{u}_\epsilon(x) \triangleq -f_\epsilon(x) + \sigma_s^2$ is concave and increasing in x with $\bar{u}_\epsilon(0) = 0$. Hence, the results of Section II can be applied to problem (32) after changing the minimization to maximization and the distortion with sampling cost function to the function $\bar{u}_\epsilon$ above.

We note that while the optimization problem characterizing f_ϵ is convex, that can be solved by standard techniques [46], the function f_ϵ is not directly differentiable in its current form. Therefore, we present an explicit characterization of the optimal pair $(\theta^*, \bar{g}^*)$, and write $f_\epsilon(x)$ directly terms of them. Towards that end, we first make the substitution $\bar{g} = x - \theta\epsilon$ into the objective function. The problem now becomes

$$\min_{0 \leq \theta \leq \min\{1, x/\epsilon\}} \quad \frac{\theta}{1 - \frac{\epsilon}{\sigma_c^2} + \frac{x}{\theta\sigma_c^2}} - \theta \tag{36}$$

where the constraint $\theta \leq x/\epsilon$ ensures non-negativity of $\bar{g}$. One can show that the objective function above is convex in θ. Hence, we take the derivative, equate to 0, solve for θ, and then project the solution onto the feasible set to get the optimal solution of this problem [46]. This gives

$$\theta^* = \min\left\{\frac{x}{\epsilon + \sqrt{\epsilon\sigma_c^2}}, 1\right\}, \quad g^* = \max\left\{x - \epsilon, \sqrt{\epsilon\sigma_c^2}\right\} \tag{37}$$

where g^* is found by computing $\bar{g}^*/\theta^*$. Substituting the above into the objective function, H, gives

$$f_\epsilon(x) = \begin{cases} \sigma_s^2\left(1 - \dfrac{x}{\left(\epsilon + \sqrt{\epsilon\sigma_c^2}\right)\left(1 + \frac{\sigma_c^2}{\sqrt{\epsilon\sigma_c^2}}\right)}\right), & x < \epsilon + \sqrt{\epsilon\sigma_c^2} \\ \dfrac{\sigma_s^2}{1 + \frac{x-\epsilon}{\sigma_c^2}}, & x \geq \epsilon + \sqrt{\epsilon\sigma_c^2} \end{cases} \tag{38}$$

One can check that $f_\epsilon(x)$ is differentiable at $x = \epsilon + \sqrt{\epsilon\sigma_c^2}$, and hence differentiable on its domain. We note that if the battery size is small enough, namely $B \leq \epsilon + \sqrt{\epsilon\sigma_c^2}$, then the distortion function f_ϵ will always be linear in the allocated energy, and therefore the optimal solution under Bernoulli arrivals becomes greedy as noted in Section II-A.

We also note that $\bar{u}'_\epsilon(0)$ is finite, and therefore the results in Section II-A show that the optimal solution of (32) under Bernoulli arrivals is a finite sequence. Let us denote by N_ϵ the last time slot of transmission in this case. We have the following structural result of the optimal solution in this case, whose proof is in Appendix E.

Lemma 4: Under Bernoulli arrivals, the optimal solution of (32) can only be bursty in the final time slot of transmission, N_ϵ. That is: $\theta_t^* = 1$ for $t < N_\epsilon$; $0 < \theta_{N_\epsilon} \leq 1$; and $\theta_t^* = 0$ for $t > N_\epsilon$.

We note that similar results regarding the burstiness of the last time slot have been reported in [40] in case of single-user channels with processing costs.

It is worth noting that both $\bar{u}$ and $\bar{u}_\epsilon$ defined in this section belong to class (B) utility functions (more on this in Section IV), and hence the FFP is asymptotically optimal in the battery size by Theorem 3, and the additive gap for finite battery sizes is given by the term α in Theorem 2.

B. Alternative Additive Gap Approach

In this section, we provide a different approach than that of Theorem 2 to analyze the additive gaps of FFP in problems (28) and (32), under general i.i.d. arrivals. The approach leads to additive gaps that do not need the computation of the term α in Theorem 2. Moreover, unlike α, the gap for problem (32) is independent of the sampling cost ϵ.

For problem (28), we define the power control policy as follows [36]

$$\tilde{g}_t = qb_t \tag{39}$$

and for problem (32), we define it as

$$\tilde{\theta}_t(\epsilon + \tilde{g}_t) = qb_t \tag{40}$$

That is, for either problem, in each time slot, the sensor uses a fixed fraction of its available energy in the battery. We note that using (40) in problem (32) decouples the problem into multiple single-slot problems where the energy consumption in time slot t is qb_t. This allows for finding the optimal division of the allocated energy qb_t among $\tilde{\theta}_t$ and $\tilde{g}_t$ by (37) as

$$\tilde{\theta}_t = \min\left\{\frac{qb_t}{\epsilon + \sqrt{\epsilon\sigma_c^2}}, 1\right\}, \quad \tilde{g}_t = \max\left\{qb_t - \epsilon, \sqrt{\epsilon\sigma_c^2}\right\} \tag{41}$$

Observe that in the above assignment, for a single energy arrival, either the transmission power or the *on* time decreases over slots in a fractional manner, i.e., while one decreases the other one is fixed. This is different from the proposed FFP in [40] where both the power and the on time can decrease simultanesouly over time.

Let $d(\tilde{\boldsymbol{g}})$ and $d_\epsilon\left(\tilde{\boldsymbol{\theta}}, \tilde{\boldsymbol{g}}\right)$ denote the long term average distortion under $\{\tilde{g}_t\}$ in (39) and $\{(\tilde{\theta}_t, \tilde{g}_t)\}$ in (41), respectively. We now characterize the performance of FFP in the case of general i.i.d. arrivals in the following two theorems. The proofs are in Appendices F and G, respectively.

Theorem 4: For all i.i.d. energy arrivals with mean μ, the optimal solution of problem (28) satisfies

$$d^* \geq f(\mu) \tag{42}$$

and the FFP in (39) satisfies

$$f(\mu) \le d(\tilde{\boldsymbol{g}}) \le f(\mu) + \frac{1}{2}\sigma_s^2 \tag{43}$$

for all values of μ *and* σ_c^2*, where* f *is as defined in (33).*

Theorem 5: For all i.i.d. energy arrivals with mean μ*, the optimal solution of problem (32) satisfies*

$$d_\epsilon^* \ge f_\epsilon(\mu) \tag{44}$$

and the FFP in (41) satisfies

$$f_\epsilon(\mu) \le d_\epsilon\left(\tilde{\boldsymbol{\theta}}, \tilde{\boldsymbol{g}}\right) \le f_\epsilon(\mu) + \frac{1}{2}\sigma_s^2 \tag{45}$$

for all values of ϵ*,* μ*, and* σ_c^2*, where* f_ϵ *is as defined in (34).*

Note that the results in the two theorems above directly imply that the average long term distortion under the FFP proposed for both problems (32) and (28) lies within a constant additive gap from the optimal solution. We also see that the additive gap indicated in Theorem 5 does not depend on the sampling cost ϵ, unlike the term α in Theorem 2.

IV. Examples and Discussion

In this section we present some examples to illustrate the results of this work. We first show that the utility function $u(x) = \frac{1}{2}\log(1+x)$ considered in [36] belongs to class (A). Indeed we have $h'_\theta(x) = \frac{\theta-1}{2(1+\theta x)(1+x)}$, which is negative for all $0 < \theta < 1$, and therefore $h_\theta(x)$ is decreasing in x and does not converge to 0. We then show that the sufficient conditions of Theorem 2 are satisfied. We have the function

$$h(\theta) = \lim_{x\to\infty} \frac{1}{2}\log\frac{1+\theta x}{1+x} = \frac{1}{2}\log(\theta) \tag{46}$$

exists, and the ratio

$$\begin{aligned} r &= (1-q)\lim_{t\to\infty}\frac{1-\lim\limits_{x\to\infty}\log(1+(1-q)^{t+1}x)/\log(1+x)}{1-\lim\limits_{x\to\infty}\log(1+(1-q)^{t}x)/\log(1+x)} \\ &= 1-q \end{aligned} \tag{47}$$

is less than 1, and hence the gap α is finite. Furthermore, [36] showed that minimizing α over all q gives a constant additive gap, independent of q, that is equal to 0.72.

Next, we note that all bounded utility functions belong to class (B). These are functions u where there exists some constant $M < \infty$ such that $u(x) \le M$, $\forall x$. Examples for these include: $u(x) = 1 - e^{-\beta x}$ for some $\beta > 0$, $u(x) = x/(1+x)$, and the negative distortion function $\bar{u}(x) = -\frac{\sigma_s^2}{1+x/\sigma_c^2} + \sigma_s^2$. To see that these functions belong to class (B), observe that $\lim_{x\to\infty} u(x) = M$ by monotonicity of u, and hence $\lim_{x\to\infty} u(\theta x) - u(x) = 0$. We also note that class (B) is not only inclusive of bounded utility functions. For example, the unbounded function $u(x) = \sqrt{\log(1+x)}$ satisfies

$$\begin{aligned} &\lim_{x\to\infty}\sqrt{\log(1+\theta x)} - \sqrt{\log(1+x)} \\ &\qquad = \frac{\log(\theta)}{\lim\limits_{x\to\infty}\sqrt{\log(1+\theta x)} + \sqrt{\log(1+x)}} = 0 \end{aligned} \tag{48}$$

and therefore belongs to class (B). For such unbounded functions in class (B), the FFP is not only within a constant additive gap of the optimal solution, but it is asymptotically optimal as well, as indicated by Theorem 3.

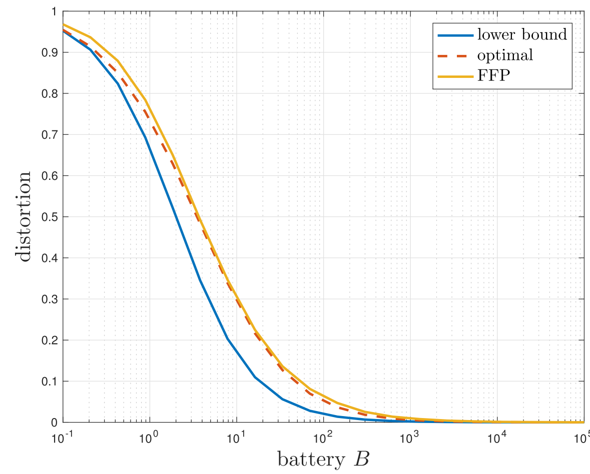

Fig. 2. Performance of the FFP with no sampling costs.

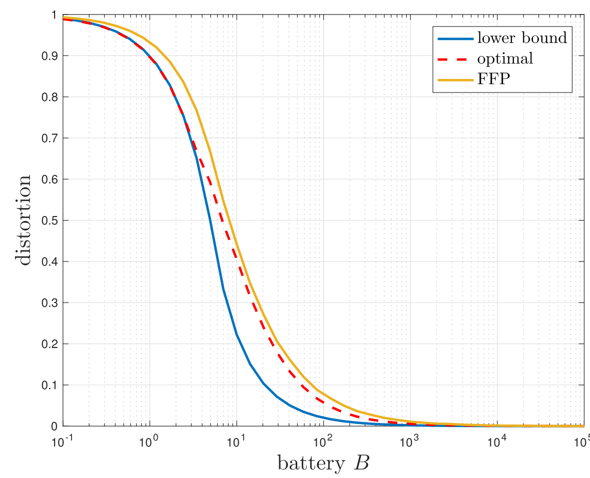

Fig. 3. Performance of the FFP with sampling costs.

Note that one can find a (strict) lower bound on $h(\theta)$ for some utility functions if it allows a more plausible bound on α, or if $h(\theta)$ itself is not direct to compute. For instance, for any bounded utility function u, the following holds: $h(\theta) \ge (\theta - 1)M$, where M is the upper bound on u. To see this, observe that by concavity of u and the fact that $u(0) = 0$ we have

$$\inf_x u(\theta x) - u(x) \ge (\theta - 1)\sup_x u(x) \tag{49}$$

This gives

$$\begin{aligned} \alpha &\ge \sum_{t=0}^{\infty} q(1-q)^t\left((1-q)^t - 1\right)M \\ &= \frac{q-1}{2-q}M \\ &\ge -\frac{1}{2}M \end{aligned} \tag{50}$$

where the second inequality follows since $\frac{q-1}{2-q}$ is minimized at $q = 0$. Another example is $u(x) = \frac{1}{2}\log\left(1+\sqrt{x}\right)$, which

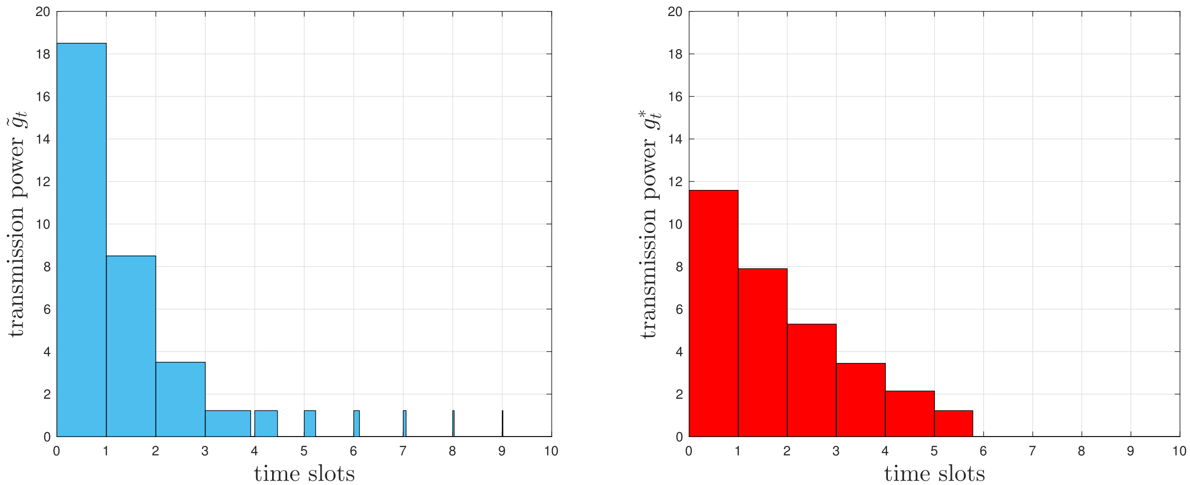

Fig. 4. FFP (left) vs. optimal policy (right) with sampling costs and one energy arrival with $B = 40$.

belongs to class (A). We observe that $h(\theta)$ in this case is lower bounded by $\frac{1}{2}\log(\theta)$. Hence, this function admits an additive gap no larger than 0.72 calculated in [36] for $u(x) = \frac{1}{2}\log(1+x)$.

Finally, we note that the conditions of Theorem 2 are only sufficient for the FFP defined in (5) to be within an additive gap from optimal. For instance, consider $u(x) = \sqrt{x}$. This function belongs to class (A) as $h_\theta(x) = \sqrt{\theta x} - \sqrt{x}$ does not converge to 0. In fact, $h_\theta(x)$ is unbounded below and $h(\theta)$ does not exist. This means that any FFP of the form $\tilde{g}_t = \theta b_t$, for any choice of $0 < \theta < 1$, is *not* within a constant additive gap from the *upper bound* $\sqrt{\mu}$. However, there exists another FFP (with a different fraction than q in (4)) that is *optimal* in the case of Bernoulli arrivals. Since $\lim_{x\to 0} u'(x) = \infty$, we use (14) to find the optimal λ, where $v(x) = 1/(4x^2)$, and substitute in (13) to get that the optimal transmission scheme is *fractional:* $g_t = \hat{p}\,(1-\hat{p})^{(t-1)}\,B,\ \forall t$, where the transmitted fraction $\hat{p} \triangleq 1-(1-p)^2$. This shows that one can pursue near optimality results under an FFP by further optimizing the fraction of power used in each time slot, and comparing the performance directly to the optimal solution instead of an upper bound. While in this work, we compared the lower bound achieved by the FFP to a universal upper bound that works for all i.i.d. energy arrivals.

Next, we present some examples regarding the distortion minimization setting. We set both σ_s^2 and σ_c^2 to unity, and consider a system with Bernoulli energy arrivals with probability $p = 0.5$. In Fig. 2, we plot the lower bound on the long term average distortion for the problem without sampling costs along with the FFP, against the battery size B. We also plot the optimal solution in this scenario. We see that the FFP performs very close to the optimal policy. We note that the empirical gap between the optimal policy and the FFP is no larger than 0.03, while the empirical gap between the lower bound and the FFP is no larger than 0.15, which is almost equal to the term α in Theorem 2, and lower than the theoretical gap of 0.5 in Theorem 4.

In Fig. 3, we plot the same curves for the problem with sampling costs. We set the sampling cost $\epsilon = 1.5$. We notice that the distortion levels are higher in general when compared to the case without sampling costs, which is mainly due to having some energy spent in sampling instead of reducing distortion. The empirical gap in this case is 0.22, which is almost equal to the term α in Theorem 3, and lower than the theoretical gap of 0.5 in Theorem 5.

In Fig. 4, we show the FFP (left hand side in blue) versus the optimal policy (right hand side in red) for $B = 40$ during only one renewal period, i.e., for one energy arrival. We plot the power and the transmit duration (burstiness) during the first 10 time slots, with the height representing power and the width representing burstiness. We see that in the FFP on the left, for time slots 1 through 3, the transmission power $\tilde{g}_t$ decreases fractionally while the value of $\tilde{\theta}_t$ is constant at unity. Starting from time slot 4 onwards, the opposite occurs; the value of $\tilde{\theta}_t$ decreases fractionally while the transmission power $\tilde{g}_t$ is constant at 1.225. As indicated by (41), either the power or the transmit duration decreases fractionally while the other is constant over time. On the other hand, in the optimal policy on the right, we see that the transmission power g_t^* is decreasing all the way to the end. In this example, the last time slot of transmission is $N_\epsilon = 6$, and the transmission is bursty only in that time slot, as indicated by Lemma 4, with $\theta_6^* = 0.78$.

V. Conclusion

We considered online power scheduling policies in single-user energy harvesting channels, where the goal is to maximize the long term average utility for a general concave increasing utility function. We showed that fixed fraction policies achieve a long term average utility that lies within a constant multiplicative gap from the optimal solution for all i.i.d. energy arrivals and battery sizes. We then derived sufficient conditions on the utility function to guarantee that fixed fraction policies are within a constant additive gap from the optimal solution as well. We then considered a specific scenario where a source is aiming at sending Gaussian samples over a Gaussian channel with minimal long term average distortion. We studied this problem with and without sampling costs, showed that they both can be reformulated in the context of concave utility maximization, and proposed a different approach to analyze the additive gaps that is tailored to the distortion minimization problems. This alternative approach is relatively easier to compute, and provides an additive gap that is independent of the sampling cost.

APPENDIX

A. Proof of Lemma 1

Following [36] and [40], we first remove the battery capacity constraint setting $B=\infty$. This way, the feasible set $\mathcal{F}$ in (1) becomes

$$\sum_{t=1}^{n} g_t \le \sum_{t=1}^{n} E_t, \quad \forall n \tag{51}$$

Then, we remove the expectation and consider the offline setting of problem (3), i.e., when energy arrivals are known a priori. Since the energy arrivals are i.i.d., the strong law of large numbers indicates that $\lim_{n\to\infty} \frac{1}{n}\sum_{t=1}^{n} E_t = \mu$ a.s., i.e., for every $\delta>0$, there exists n large enough such that $\frac{1}{n}\sum_{t=1}^{n} E_t \le \mu+\delta$ a.s., which implies by (51) that the feasible set, for such (δ,n) pair, is given by

$$\frac{1}{n}\sum_{t=1}^{n} g_t \le \mu+\delta \quad \text{a.s.} \tag{52}$$

Now fix such (δ,n) pair. The objective function is given by

$$\frac{1}{n}\sum_{t=1}^{n} u(g_t) \tag{53}$$

Since u is concave, the optimal power allocation minimizing the objective function is $g_t=\mu+\delta$, $1\le t\le n$ [46] (see also [1]). Whence, the optimal offline solution is given by $u(\mu+\delta)$. We then have $\rho^* \le u(\mu+\delta)$. Since this is true $\forall\delta>0$, we can take δ down to 0 by taking n infinitely large.

B. Proof of Theorem 1

We first derive a lower bound on the long term average utility for Bernoulli energy arrivals under the FFP as follows

$$\begin{aligned}
\liminf_{n\to\infty}\ \hat{\mathcal{U}}_n(\tilde{\boldsymbol{g}}) &\overset{(a)}{=} p\sum_{i=1}^{\infty} p(1-p)^{i-1}\sum_{t=1}^{i} u\left(\tilde{g}_t\right)\\
&= \sum_{t=1}^{\infty} p(1-p)^{t-1} u\left((1-p)^{t-1}\mu\right) \qquad (54)\\
&\overset{(b)}{\ge} \sum_{t=1}^{\infty} p(1-p)^{2(t-1)} u\left(\mu\right)\\
&= \frac{1}{2-p}u(\mu)\\
&\ge \frac{1}{2}u(\mu) \qquad (55)
\end{aligned}$$

where (a) follows by (7), (b) follows by concavity of u [46], and the last inequality follows since $0\le p\le 1$. Next, we use the above result for Bernoulli arrivals to bound the long term average utility for general i.i.d. arrivals under the FFP in the following lemma; the proof follows by concavity and monotonicity of u, along the same lines of [36, Section VII-C], and is omitted for brevity.

Lemma 5: Let $\{\hat{E}_t\}$ be a Bernoulli energy arrival process as in (6) with parameter q as in (4) and mean $qB=\mu$. Then, the long term average utility under the FFP for any general i.i.d. energy arrivals, $\rho(\tilde{\boldsymbol{g}})$, satisfies

$$\rho(\tilde{\boldsymbol{g}}) \ge \liminf_{n\to\infty}\ \hat{\mathcal{U}}_n(\tilde{\boldsymbol{g}}) \tag{56}$$

Using Lemma 1, (55), and Lemma 5, we have

$$\frac{1}{2}u(\mu) \le \rho(\tilde{\boldsymbol{g}}) \le \rho^* \le u(\mu) \tag{57}$$

C. Proof of Theorem 2

By Lemma 1 and Lemma 5, it is sufficient to study the lower bound in the case of Bernoulli arrivals. By (54) we have

$$\begin{aligned}
\liminf_{n\to\infty}\ \hat{\mathcal{U}}_n(\tilde{\boldsymbol{g}}) &= \sum_{t=1}^{\infty} p(1-p)^{t-1} u\left((1-p)^{t-1}\mu\right)\\
&\overset{(c)}{\ge} \sum_{t=1}^{\infty} p(1-p)^{t-1}\left(u\left(\mu\right)+h\left((1-p)^{t-1}\right)\right)\\
&= u(\mu)+\sum_{t=0}^{\infty} p(1-p)^{t} h\left((1-p)^{t}\right)\\
&\triangleq u(\mu)+\alpha \qquad (58)
\end{aligned}$$

where (c) follows since $h(\theta)$ exists, and is by definition no larger than $h_\theta(x)$, $\forall x,\theta$. Now to check whether α is finite, we apply the ratio test to check the convergence of the series $\sum_{t=0}^{\infty}(1-p)^t h\left((1-p)^t\right)$. That is, we compute

$$\begin{aligned}
r &\triangleq \lim_{t\to\infty}\left|\frac{(1-p)^{t+1} h\left((1-p)^{t+1}\right)}{(1-p)^{t} h\left((1-p)^{t}\right)}\right|\\
&= (1-p)\lim_{t\to\infty}\frac{\inf_x 1-u\left((1-p)^{t+1}x\right)/u(x)}{\inf_x 1-u\left((1-p)^{t}x\right)/u(x)} \qquad (59)
\end{aligned}$$

where the second equality follows by definition of h. Next, we replace $\inf_x$ by $\lim_{x\to\bar{x}_t}$ since $\bar{x}_t \in \arg\inf h_{(1-p)^t}(x)$, and take the limit inside (after the 1). Finally, if $r<1$ then α is finite; if $r>1$ then $\alpha=-\infty$; and if $r=1$ then the test is inconclusive and one has to compute $\lim_{T\to\infty}\sum_{t=0}^{T} p(1-p)^t h\left((1-p)^t\right)$ to get the value of α.

D. Proof of Theorem 3

For utility functions of class (B), we have $\lim_{x\to\infty} u(\theta x)-u(x)=0$. Thus, $\forall\epsilon>0$ there exists $\bar{\mu}$ large enough such that

$$u\left((1-p)^{t-1}\mu\right) > u\left(\mu\right)-\epsilon, \quad \forall\mu\ge\bar{\mu} \tag{60}$$

whence, for Bernoulli energy arrivals we have

$$\begin{aligned}
\liminf_{n\to\infty}\ \hat{\mathcal{U}}_n(\tilde{\boldsymbol{g}}) &= \sum_{t=1}^{\infty} p(1-p)^{t-1} u\left((1-p)^{t-1}\mu\right)\\
&\ge u\left(\mu\right)-\epsilon, \quad \forall\mu\ge\bar{\mu} \qquad (61)
\end{aligned}$$

It then follows by Lemma 1 and Lemma 5 that

$$\rho^* \geq \rho(\tilde{\boldsymbol{g}}) \geq u(\mu) - \epsilon \geq \rho^* - \epsilon, \quad \forall \mu \geq \bar{\mu} \tag{62}$$

and we can take ϵ down to 0 by taking μ infinitely large.

E. Proof of Lemma 4

Following the analysis in Section II-A, and applying the change of variables $\bar{g}_t \triangleq \theta_t g_t$, problem (32) under Bernoulli arrivals can be rewritten as

$$\begin{aligned}\min_{\boldsymbol{\theta},\boldsymbol{g}} \ & \sum_{t=1}^{\infty} p(1-p)^{t-1}\left((1-\theta_t)\sigma_s^2 + \theta_t \frac{\sigma_s^2}{1+\frac{\bar{g}_t}{\theta_t\sigma_c^2}}\right) \\ \text{s.t.} \ & \sum_{t=1}^{\infty} \bar{g}_t + \theta_t \epsilon \leq B \\ & \bar{g}_t \geq 0, \quad 0 \leq \theta_t \leq 1, \ \forall t \end{aligned} \tag{63}$$

The Lagrangian is

$$\begin{aligned}\mathcal{L} = & \sum_{t=1}^{\infty} p(1-p)^{t-1}\left((1-\theta_t)\sigma_s^2 + \theta_t \frac{\sigma_s^2}{1+\frac{\bar{g}_t}{\theta_t\sigma_c^2}}\right) \\ & + \lambda\left(\sum_{t=1}^{\infty} \bar{g}_t + \theta_t\epsilon - B\right) - \sum_{t=1}^{\infty} \eta_t \bar{g}_t \\ & - \sum_{t=1}^{\infty} \gamma_t\theta_t + \sum_{t=1}^{\infty} \omega_t(\theta_t - 1) \end{aligned} \tag{64}$$

where λ, $\{\eta_t\}$, $\{\gamma_t\}$, and $\{\omega_t\}$ are non-negative Lagrange multipliers. Taking derivative with respect to $\bar{g}_t$ and equating to 0, we get

$$\frac{\sigma_s^2 p(1-p)^{t-1}}{\sigma_c^2\left(1+\bar{g}_t/\theta_t\sigma_c^2\right)^2} = \lambda - \eta_t \tag{65}$$

which can be rewritten as follows using complementary slackness

$$\frac{\bar{g}_t}{\theta_t} = \sigma_c^2\left(\sqrt{\frac{\sigma_s^2 p(1-p)^{t-1}}{\sigma_c^2\lambda}} - 1\right)^+ \tag{66}$$

where $(x)^+ = \max\{x, 0\}$. This shows that the optimal power g_t is monotonically decreasing over time, and verifies that there exists a time slot after which there is no transmission and all powers are 0, that we denote N_ϵ. Now let us take the derivative of the Lagrangian with respect to θ_t, equate it to 0, and use (65) to get

$$\frac{\bar{g}_t}{\theta_t} = \sigma_c^2\sqrt{\frac{\lambda\epsilon - \gamma_t + \omega_t}{\sigma_c^2(\lambda - \eta_t)}} \tag{67}$$

We now argue that $\bar{g}_t = 0$ if and only if $\theta_t = 0$. Clearly $\theta_t = 0$ implies $\bar{g}_t = \theta_t g_t = 0$. To see the other direction, assume $\bar{g}_t = 0$ for some time slot t. Then, the achieved distortion in this time slot is given by σ_s^2 regardless of the value of θ_t. Therefore, setting $\theta_t = 0$ saves ϵ energy per unit time in this time slot that can be used in another time slot i to increase its transmission energy $\bar{g}_i$ and achieve lower distortion. Hence, after time slot N_ϵ, we see that $\bar{g}_t = 0$ according to (66), and hence $\theta_t^* = 0$ for $t > N_\epsilon$.

Next, let us assume that $0 < \theta_j^* < 1$ for some time slot j. By the previous argument we have $\bar{g}_j > 0$. By complementary slackness, we also have $\omega_j = \gamma_j = 0$. Hence, by (67) we have $\bar{g}_j/\theta_j = \sigma_c^2\sqrt{\epsilon/\sigma_c^2}$. Thus, whenever the transmission is bursty, the transmission power is constant. This constant can be equal to (66) at only one time slot since transmission power is decreasing. Moreover, after time slot j, the power can only decrease by increasing the value of γ_t in (67), which means by complementary slackness that $\theta_t = 0$ for $t > j$, which further implies that $\bar{g}_t = 0$ for $t > j$. Therefore, $j = N_\epsilon$.

Finally, for $t < N_\epsilon$, the power increases, going backwards in time, only by increasing the value of ω_t in (67), which means by complementary slackness that $\theta_t^* = 1$ for $t < N_\epsilon$.

F. Proof of Theorem 4

1) Lower Bounding d^*: First, we derive the lower bound in (42) by means of the offline solution along the same lines as in the proof of Lemma 1 in Appendix A. Applying the same (δ, n) argument using the strong law of large numbers, the objective function is given by

$$\frac{1}{n}\sum_{t=1}^{n} \frac{\sigma_s^2}{1+g_t/\sigma_c^2} = \frac{1}{n}\sum_{t=1}^{n} f(g_t) \tag{68}$$

It is direct to see that f is convex. Therefore, the optimal power allocation minimizing the objective function is $g_t = \mu + \delta$, $1 \leq t \leq n$ [46] (see also [1]). Whence, the optimal offline solution is given by $f(\mu + \delta)$. We then have $d^* \geq f(\mu + \delta)$. Since this is true $\forall \delta > 0$, we can take δ down to 0 by taking n infinitely large. Therefore, (42) holds.

2) Upper Bounding d^*: Bernoulli Energy Arrivals: Next, we derive an upper on d^*. Towards that, we first the study a special energy harvesting i.i.d. process: the Bernoulli process. Let $\{\hat{E}_t\}$ be a Bernoulli energy arrival process as defined in (6). Under such specific energy arrival setting, whenever an energy packet arrives, it completely fills the battery, and resets the system. This constitutes a *renewal*. Then, by [44, Th. 3.6.1], the following holds for any power control policy $\boldsymbol{g}$

$$\begin{aligned}\limsup_{n\to\infty} \ \hat{\mathcal{D}}_n(\boldsymbol{g}) &= \limsup_{n\to\infty} \ \frac{1}{n}\mathbb{E}\left[\sum_{t=1}^{n} \frac{\sigma_s^2}{1+g_t/\sigma_c^2}\right] \\ &= \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{t=1}^{L} \frac{\sigma_s^2}{1+g_t/\sigma_c^2}\right] \end{aligned} \tag{69}$$

where $\hat{\mathcal{D}}_n(\boldsymbol{g})$ is the n-horizon average distortion under Bernoulli arrivals, and L is a random variable denoting the inter-arrival time between energy arrivals, which is geometric with parameter p, and $\mathbb{E}[L] = 1/p$.

Now, substituting by the FFP (39) gives an upper bound on d^*. Note that by (6), the fraction q in (4) is now equal to p. Also note that in between energy arrivals, the battery state decays exponentially, and the FFP in (39) gives

$$\tilde{g}_t = p(1-p)^{t-1}B = (1-p)^{t-1}\mu \tag{70}$$

for all time slots t, where the second equality follows since $pB = \mu$. Therefore, using (69) and (70), we bound the distortion under the FFP as follows

$$\begin{aligned}
&\limsup_{n\to\infty}\ \hat{\mathcal{D}}_n(\tilde{\boldsymbol{g}}) \\
&\quad = \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{t=1}^{L}\frac{\sigma_s^2}{1+(1-p)^{t-1}\mu/\sigma_c^2}\right] \\
&\quad \overset{(a)}{\leq} \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{t=1}^{L}\frac{\sigma_s^2}{1+\mu/\sigma_c^2}+\left(1-(1-p)^{t-1}\right)\sigma_s^2\right] \\
&\quad = f(\mu)+\sigma_s^2\left(1-\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{t=1}^{L}(1-p)^{t-1}\right]\right) \\
&\quad = f(\mu)+\sigma_s^2\frac{p(1-p)}{1-(1-p)^2} \\
&\quad \overset{(b)}{\leq} f(\mu)+\frac{\sigma_s^2}{2}
\end{aligned} \tag{71}$$

where (a) follows since $\frac{1}{1+\lambda x} \leq \frac{1}{1+x} + (1-\lambda)$ for $0 \leq \lambda \leq 1$ and $x \geq 0$; and (b) follows since $\frac{p(1-p)}{1-(1-p)^2}$ has a maximum value of $1/2$ for $0 \leq p \leq 1$. Next, we use the above result for Bernoulli arrivals to bound the distortion for general i.i.d. arrivals under the FFP in the following lemma; the proof follows by convexity and monotonicity of f, along the same lines of [36, Section VII-C], and is omitted for brevity.

Lemma 6: Let $\{\hat{E}_t\}$ be a Bernoulli energy arrival process as in (6) with parameter q as in (4) and mean $qB = \mu$. Then, the long term average distortion under the FFP for any general i.i.d. energy arrivals, $d(\tilde{\boldsymbol{g}})$, satisfies

$$d(\tilde{\boldsymbol{g}}) \leq \limsup_{n\to\infty}\ \hat{\mathcal{D}}_n(\tilde{\boldsymbol{g}}) \tag{72}$$

Using (42), (71), and Lemma 6, we have

$$f(\mu) \leq d^* \leq d(\tilde{\boldsymbol{g}}) \leq f(\mu) + \frac{\sigma_s^2}{2} \tag{73}$$

G. Proof of Theorem 5

1) Lower Bounding d_ϵ^*: First, we derive the lower bound in (44) by means of the offline solution as done in Appendix F1. It follows by applying the same (δ, n) argument, and using convexity of the function H, introduced in Lemma 3, that the optimal power allocation minimizing the objective function is $\theta_t\epsilon + \bar{g}_t = \mu + \delta,\ 1 \leq t \leq n$ [46] (see also [1]). We denote this optimal offline solution by $f_\epsilon(\mu+\delta)$ as defined in (34). We then have $d_\epsilon^* \geq f_\epsilon(\mu+\delta)$; we take δ down to 0 by taking n infinitely large. Therefore, (44) holds.

2) Upper Bounding d_ϵ^*: Bernoulli Energy Arrivals: Next, we derive an upper bound on d_ϵ^*. Following the same steps as in Appendix F.2, we first consider Bernoulli energy arrivals as in (6). In this case we have

$$\limsup_{n\to\infty}\ \hat{\mathcal{D}}_n^\epsilon(\boldsymbol{\theta},\boldsymbol{g}) = \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{t=1}^{L}(1-\theta_t)\sigma_s^2 + \frac{\theta_t\sigma_s^2}{1+g_t/\sigma_c^2}\right] \tag{74}$$

where $\hat{\mathcal{D}}_n^\epsilon(\boldsymbol{\theta},\boldsymbol{g})$ is the n-horizon average distortion under Bernoulli arrivals. Next, we upper bound the long term average distortion in this case by substituting the FFP in (40) setting

$$\tilde{\theta}_t(\epsilon + \tilde{g}_t) = p(1-p)^{t-1}B = (1-p)^{t-1}\mu \tag{75}$$

for all time slots t. Note that the average minimal distortion in time slot t is given by $f_\epsilon\left((1-p)^{t-1}\mu\right)$.

Next, following the same steps used in showing (71), by (74) and (75), we have

$$\limsup_{n\to\infty}\ \hat{\mathcal{D}}_n^\epsilon\left(\tilde{\boldsymbol{\theta}},\tilde{\boldsymbol{g}}\right) \leq f_\epsilon(\mu) + \frac{\sigma_s^2}{2} \tag{76}$$

where step (a) in (71) follows by Lemma 3. Finally, we use the above result to bound the distortion for general i.i.d. arrivals under the FFP. We basically extend the statement of Lemma 6 to the case with sampling costs since f_ϵ is convex and monotone. We then have

$$d_\epsilon\left(\tilde{\boldsymbol{\theta}},\tilde{\boldsymbol{g}}\right) \leq \limsup_{n\to\infty}\ \hat{\mathcal{D}}_n^\epsilon\left(\tilde{\boldsymbol{\theta}},\tilde{\boldsymbol{g}}\right) \tag{77}$$

Using (44), (76), and (77), we have

$$f_\epsilon(\mu) \leq d_\epsilon^* \leq d_\epsilon\left(\tilde{\boldsymbol{\theta}},\tilde{\boldsymbol{g}}\right) \leq f_\epsilon(\mu) + \frac{\sigma_s^2}{2} \tag{78}$$

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Ahmed Arafa (S'13–M'17) received the B.Sc. degree (Hons.) in electrical engineering from Alexandria University, Egypt, in 2010, the M.Sc. degree in wireless technologies from the Wireless Intelligent Networks Center, Nile University, Egypt, in 2012, and the M.Sc. and Ph.D. degrees in electrical engineering from the University of Maryland at College Park, MD, USA, in 2016 and 2017, respectively, with his Ph.D. thesis on the design and analysis of optimal energy management policies in energy harvesting communication networks with system costs. He was a recipient of the Distinguished Dissertation Fellowship from the Department of Electrical and Computer Engineering, University of Maryland, for his Ph.D. thesis work.

In 2017, he joined the Electrical Engineering Department, Princeton University, as a Post-Doctoral Research Associate. His research interests include wireless communications, information theory, and optimization.

Abdulrahman Baknina (S'09) received the B.Sc. degree in electrical engineering and the M.Sc. degree in wireless communications from Alexandria University, Alexandria, Egypt, in 2010 and 2013, respectively. He is currently pursuing the Ph.D. degree with the Department of Electrical and Computer Engineering, University of Maryland, College Park, MD, USA. His research interests include optimization, information theory, and wireless communications.

Sennur Ulukus (S'90–M'98–SM'15–F'16) received the B.S. and M.S. degrees in electrical and electronics engineering from Bilkent University, and the Ph.D. degree in electrical and computer engineering from the Wireless Information Network Laboratory, Rutgers University. She was a Senior Technical Staff Member with AT&T Laboratories-Research. She is a Professor of electrical and computer engineering with the University of Maryland at College Park, MD, USA, where she also holds a joint appointment with the Institute for Systems Research (ISR). Her research interests include wireless communications, information theory, signal processing, and networks, with recent focus on information theoretic physical layer security, private information retrieval, energy harvesting communications, and wireless energy and information transfer.

Dr. Ulukus is a Distinguished Scholar-Teacher of the University of Maryland. She received the 2003 IEEE Marconi Prize Paper Award in Wireless Communications, the 2005 NSF CAREER Award, the 2010-2011 ISR Outstanding Systems Engineering Faculty Award, and the 2012 ECE George Corcoran Education Award. She serves on the Editorial Board of the IEEE TRANSACTIONS ON GREEN COMMUNICATIONS AND NETWORKING since 2016. She was an Editor for the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS–Series on Green Communications and Networking from 2015 to 2016, the IEEE TRANSACTIONS ON INFORMATION THEORY from 2007 to 2010, and the IEEE TRANSACTIONS ON COMMUNICATIONS from 2003 to 2007. She was a Guest Editor for the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS in 2015 and 2008, the *Journal of Communications and Networks* in 2012, and the IEEE TRANSACTIONS ON INFORMATION THEORY in 2011. She was a general TPC Co-Chair of 2017 IEEE ISIT, 2016 IEEE GLOBECOM, 2014 IEEE PIMRC, and 2011 IEEE CTW.