Mobile Energy Harvesting Nodes: Offline and Online Optimal Policies

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***Abstract*—We consider a mobile energy harvesting transmitter where movement is motivated by trying to find better energy harvesting locations. Movement comes with an energy cost expenditure, and hence there exists a *throughput-movement tradeoff*. On one hand, the transmitter may opt not to move and use all its available energy for transmission; on the other hand, it can choose to move to a potentially better location, spending some of its available energy during the movement process, and yet harvest larger amounts of energy at the new location and achieve higher throughput. In this paper, we characterize this tradeoff by designing throughput optimal power allocation policies subject to energy causality constraints and moving costs. In our setup, the transmitter moves along a straight line, where two energy sources are located at the opposite ends of the line. We first study the offline version of this problem where the goal is to maximize the throughput by a given deadline. We find a closed form solution for the case of single energy arrival at each source, and provide an iterative solution for the case of multiple energy arrivals. Then, we study the online version of this problem with independent and identically distributed (i.i.d.) energy arrivals at each source, and the goal is to maximize the long term average throughput. We propose an optimal *move-then-transmit* scheme where the transmitter first moves towards the source with higher mean energy arrival, stays at that source, and then starts transmission.**

***Index Terms*—Mobile energy harvesting nodes, moving costs, throughput-movement tradeoff, online best effort, online near optimal.**

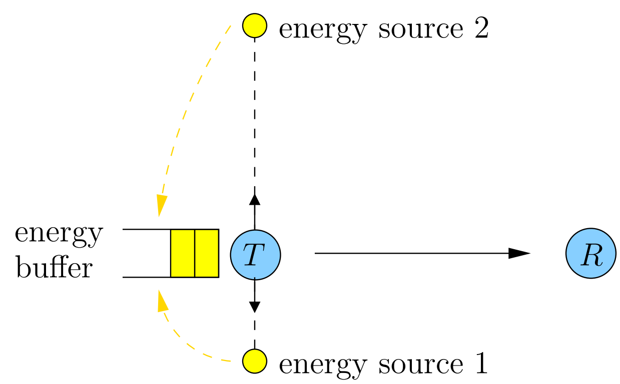

Fig. 1. Mobile energy harvesting node moving along a straight line between two energy sources. The position of the node determines how much energy it harvests from each source.

I. Introduction

WE CONSIDER an energy harvesting single-user channel where the transmitter uses its harvested energy in data transmission and to move to different locations in search for better energy harvesting spots, see Fig. 1. We design both offline and online power and movement scheduling policies that maximize the throughput subject to energy causality constraints.

Offline energy management policies in energy harvesting communication networks have been considered extensively in the recent literature. Earlier works [1]–[4] consider the single-user setting with various battery size assumptions, with and without fading. References [5]–[11] extend this to broadcast, multiple access, and interference settings; [12]–[15] consider two-hop and relay channels; and [16], [17] study two-way channels. Energy sharing and energy cooperation concepts are studied in [18]–[20]. Most of the above works focus on transmitter-side energy harvesting.

References [21]–[27] study energy harvesting receivers, where energy harvested at the receiver is spent mainly for sampling and decoding. Other works [28]–[33] study the impact of processing costs, i.e., the power spent for circuitry, on energy harvesting communications. Depending on energy availability and system parameters, the above references show that considering decoding and processing costs can change the characteristics of optimal power policies.

Near optimal online energy management policies have been recently studied in [34]–[40] for single-user channels, multiuser channels, channels with processing costs, general utility functions, and distortion minimization in sensor nodes. The proposed policies are near optimal in the sense that they perform within constant multiplicative and additive gaps from the optimal solution that are independent of energy arrivals and battery sizes (for examples of other online approaches see [41]–[43]).

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In this paper, we study another aspect of power consumption in energy harvesting sensor nodes, namely, the power consumed in the process of harvesting energy. That is, there is a cost to taking actions to harvest energy. In this paper, we model this cost via the energy consumed in physical movement. We consider an energy harvesting transmitter with the ability to move along a straight line. Two energy sources are located at the opposite ends of the line, and the amount of energy harvested at the transmitter from each source depends on its distance from the two sources, see Fig. 1. Movement is thus motivated by finding better energy harvesting locations. However, the transmitter incurs a moving cost per unit distance travelled. Therefore, a tradeoff arises between staying in the same position and using all available energy in transmission, and spending some of the available energy to move to another location where it harvests higher energy. In this work, we characterize this tradeoff optimally, by designing throughput optimal power and movement policies. We note that related system models are considered in [44] and [45] where some devices (energy-rich sources) move through a sensor network and refill the batteries of the sensors with RF radiation.

In this paper, we study both offline and online settings. In the offline setting, our goal is to maximize the throughput by a given deadline. We first study the case where each energy source has a single energy arrival, and then generalize it to the case of multiple energy arrivals. Although our problem formulation is non-convex, we are able to solve it optimally for the single energy arrival scenario. For the multiple energy arrivals scenario, we design an iterative algorithm with guaranteed convergence to a local optimal solution of our optimization problem. For each iteration, we first show that given the optimal movement energy expenditure in a given time slot, the movement policy is *greedy;* the transmitter moves to the better location (energy-wise) in that time slot only without considering future time slots. We then find optimal movement energy consumption using a water-filling algorithm.

In the online setting, we model the energy arrival processes at the two source as two independent and i.i.d. processes. Only the means of the two processes are known before communication. Our goal is to maximize the long term average throughput. To that end, we propose an optimal *move-then-transmit* scheme where the transmitter first uses all its harvested energy to move towards the source with higher energy harvesting mean. After that, it stays at that source's position and starts communicating with the receiver. We show that the energy used in movement does not affect the throughput in the long term average sense. If the transmitter has an infinite battery, we use the best effort transmission strategy to optimally manage the harvested energy in transmission [46]. In this policy, the transmitter sends with the average harvesting rate whenever feasible and stays silent otherwise. On the other hand, if the transmitter has a finite battery, we use the fixed fraction policy [34], where the transmitter uses a fixed fraction of the amount of energy available in its battery for transmission in every time slot, to achieve a long term average rate that lies within constant additive and multiplicative gaps from the optimal solution for all energy arrival patterns and battery sizes.

II. System Model and Problem Formulation

We consider a single-user AWGN channel with an energy harvesting transmitter with moving abilities. The transmitter has the ability to relocate itself to different positions in search for better energy harvesting spots. Movement is along a straight line, and energy is harvested from two energy sources located at the two opposite ends of the line, see Fig. 1. The transmitter's position determines how much energy is harvested from each source: the closer the transmitter is to one source, the larger the amounts of energy it harvests from that source compared to the other. In our setting, the transmitter-receiver distance is much larger than the distance between the two energy sources so as to ensure that the transmitter-receiver channel characteristics are not affected by the transmitter's movement.

We consider a time-slotted model, where the transmitter is allowed to move during a fixed portion of time at the beginning of each slot, and then starts communicating. Without loss of generality, we assume that the remaining portion of the time slot where the transmitter communicates is normalized to one time unit, so that we may use energy and power interchangeably. Throughout most of this paper, we will consider the case where the transmitter is equipped with an infinite-sized battery to save its harvested energy. However, in some cases we will extend our analysis to the finite battery case as well. Energy arrives in packets of amounts E_{1i} and E_{2i} in slot i at the first and the second energy source, respectively. At the beginning of slot i, the transmitter relocates itself to some position x_i, and harvests energy from both sources simultaneously according to the following relationship [47]–[49]

$$E(i, x_i) = \frac{E_{1i}}{(x_i + \ell)^{\alpha}} + \frac{E_{2i}}{(L - x_i + \ell)^{\alpha}} \tag{1}$$

where α is the path loss factor, L is the distance between the two energy sources, and $\ell \geq 1$ is a parameter added to adjust the Friis' free space equation for short distance communication, that is, to keep the harvested energy bounded when the transmitter lies at either ends of the line. Note that $E(i, x_i)$ is the actual amount of harvested energy that enters into the battery of the transmitter.

The transmitter incurs moving costs whenever it relocates itself to a different position. We model the total moving cost up to slot k as follows

$$c_m(k) \triangleq \epsilon_m \sum_{i=1}^{k} |x_i - x_{i-1}| \tag{2}$$

where x_0 is the initial position of the transmitter, $\sum_{i=1}^{k} |x_i - x_{i-1}|$ represents the total distance moved by the transmitter up to slot k, and ϵ_m is the cost of movement in energy per unit distance. Since movement is not cost-free, a tradeoff arises between spending energy to move into better spots (in the sense of energy availability), and staying at the same location and spending all the available energy in communicating. We design power and movement policies that capture the optimal tradeoff of this setting.

A. Offline Problem

We first consider an offline scenario, where energy amounts are known to the transmitter prior to the start of communication. Our goal in this setting is to maximize the total number of bits delivered to the receiver by a given deadline N, subject to energy causality constraints and moving costs. The physical layer is Gaussian with unit noise power, and the transmitter uses power p_i for transmission in time slot i. We formulate the problem as follows

$$\begin{aligned} \max_{\boldsymbol{p},\boldsymbol{x}} \quad & \sum_{i=1}^{N} \frac{1}{2}\log(1+p_i) \\ \text{s.t.} \quad & c_m(1) \leq E_0 \\ & c_m(k+1) + \sum_{i=1}^{k} p_i \leq E_0 + \sum_{i=1}^{k} E(i, x_i), \quad 1 \leq k \leq N \\ & 0 \leq x_i \leq L, \quad p_i \geq 0, \quad \forall i \end{aligned} \tag{3}$$

where $c_m(N+1) \triangleq c_m(N)$, and E_0 is the initial energy available at the transmitter. This initial energy enables the transmitter to relocate itself during the first slot (if needed). Note that if the transmitter needs to move in slot $k+1$, then it needs to save some energy by the end of slot k for that purpose. In other words, it should not consume all its energy in transmission by the end of slot k. That is why the energy incurred for moving up to slot $k+1$ is bounded by the residual energy remaining after slot k : $E_0 + \sum_{i=1}^{k} E(i, x_i) - p_i$. We solve problem (3) in Section III.

B. Online Problem

We then consider an online scenario, where energy amounts are only revealed to the transmitter causally over time; the amount of energy harvested at time slot t is only known *after* moving to position x_t. We assume that energy harvesting processes at the two sources $\{E_{1i}\}$ and $\{E_{2i}\}$ follow two independent i.i.d. distributions with means μ_1 and μ_2, respectively. Only the means of the two processes are known to the transmitter prior to the start of communication. Let b_t represent the amount of energy in the battery at time slot t, and let $\mathcal{E}^t \triangleq \{E_1, E_2, \ldots, E_t\}$. A feasible online power control and movement policy $\{\boldsymbol{p}, \boldsymbol{x}\}$ is a sequence of mappings $\{x_t : \mathcal{E}^{t-1} \to [0, L]\}$ and $\{p_t : \mathcal{E}^t \to \mathbb{R}_+\}$ satisfying

$$\epsilon_m |x_1 - x_0| \leq E_0 \tag{4}$$

$$\epsilon_m |x_t - x_{t-1}| + p_t \leq b_t, \quad t \geq 2 \tag{5}$$

where b_t denotes the amount of energy in the battery in time slot t, which evolves as follows

$$b_t = b_{t-1} - \epsilon_m |x_{t-1} - x_{t-2}| - p_{t-1} + E(t, x_t) \tag{6}$$

with $b_1 \triangleq E_0$. Let us denote the above feasible set by $\mathcal{F}$. Our goal is to maximize the long term average throughput

$$r^* \triangleq \max_{\{\boldsymbol{p},\boldsymbol{x}\} \in \mathcal{F}} \lim_{T \to \infty} \frac{1}{T} \mathbb{E}\left[\sum_{t=1}^{T} \frac{1}{2}\log(1+p_t)\right] \tag{7}$$

We solve problem (7) in Section IV, where we also discuss the case where the transmitter is equipped with a finite battery of size B.

III. Offline Setting: Problem (3)

In this section, we characterize the optimal solution of problem (3). We first note the following necessary optimality conditions.

Lemma 1: In the optimal solution of (3), powers are non-decreasing over time.

Proof: We show this by contradiction. Assume that at the optimal policy $\{\boldsymbol{p}^*, \boldsymbol{x}^*\}$, there exists a time slot k such that $p_k^* > p_{k+1}^*$. Keeping the movement policy $\boldsymbol{x}^*$ the same, we define another power policy $\tilde{\boldsymbol{p}}$ where only the kth and $(k+1)$st powers change to $\tilde{p}_k = \tilde{p}_{k+1} = \frac{p_k^* + p_{k+1}^*}{2}$. It is direct to see that $\{\tilde{\boldsymbol{p}}, \boldsymbol{x}^*\}$ is a feasible policy. By concavity of the log, this new policy strictly increases the objective function, and hence the original policy $\{\boldsymbol{p}^*, \boldsymbol{x}^*\}$ cannot be optimal. ■

Lemma 2: In the optimal solution of (3), the transmitter consumes all its harvested energy by the end of communication.

Proof: We show this by contradiction. If the statement of the lemma were not true, then we can increase the value of p_N until all energy is consumed. This strictly increases the objective function. ■

A. Single Energy Arrival

In this section we study the case where each energy source has only one energy packet arrival. That is, we have only one pair of variables (p, x) to optimize. By Lemma 2, we have

$$p(x) = E_0 + \frac{E_1}{(x+\ell)^\alpha} + \frac{E_2}{(L-x+\ell)^\alpha} - \epsilon_m |x - x_0| \tag{8}$$

and therefore, by monotonicity of the log, problem (3) becomes

$$\begin{aligned} \max_{x} \quad & \frac{E_1}{(x+\ell)^\alpha} + \frac{E_2}{(L-x+\ell)^\alpha} - \epsilon_m |x - x_0| \\ \text{s.t.} \quad & \epsilon_m |x - x_0| \leq E_0 \\ & 0 \leq x \leq L \end{aligned} \tag{9}$$

Therefore, the problem now reduces to finding the optimal position x^*.

Note that there are two possible movement strategies the transmitter can make: move forward to some $x \geq x_0$, or move backward to some $x < x_0$. The transmitter chooses the movement strategy that gives the maximum objective function (and hence power/rate). To that end, we next solve the case of moving forward. The problem in this case becomes

$$\begin{aligned} \max_{x} \quad & \frac{E_1}{(x+\ell)^\alpha} + \frac{E_2}{(L-x+\ell)^\alpha} - \epsilon_m x \\ \text{s.t.} \quad & x_0 \leq x \leq \min\left\{\frac{E_0}{\epsilon_m} + x_0, L\right\} \triangleq x_{\max} \end{aligned} \tag{10}$$

Now observe that the objective function is a convex function in x that is maximized over an interval. It then follows that the optimal solution x^* has to be at the boundary of the feasible set [50], i.e.,

$$x^* \in \{x_0, x_{\max}\} \tag{11}$$

Hence, we pick x^* that gives the higher value after substituting in (8), i.e., after comparing $p(x_0)$ and $p(x_{\max})$.

Similarly, the problem in the case of moving backward is given by

$$\begin{aligned} \max_{x} \quad & \frac{E_1}{(x+\ell)^\alpha} + \frac{E_2}{(L-x+\ell)^\alpha} + \epsilon_m x \\ \text{s.t.} \quad & x_{\min} \triangleq \max\left\{x_0 - \frac{E_0}{\epsilon_m}, 0\right\} \le x \le x_0 \end{aligned} \tag{12}$$

which again, by convexity of the objective function, yields a solution at the boundary. That is

$$x^* \in \{x_{\min}, x_0\} \tag{13}$$

Hence, we pick x^* that gives the higher value after substituting in (8), i.e., after comparing $p(x_0)$ and $p(x_{\min})$.

Based on the previous analysis, the optimal position in the single energy arrival scenario can only have three possible values: $x^* \in \{x_{\min}, x_0, x_{\max}\}$. This means that if the transmitter decides to move, it moves to the furthest possible distance (forward or backward) allowed by its available initial energy E_0 and the physical length of the straight line L. Therefore, the optimal power is given by

$$p^* = \max\{p(x_{\min}), p(x_0), p(x_{\max})\} \tag{14}$$

and x^* is the corresponding maximizing argument.

B. Multiple Energy Arrivals

In this section we study the multiple energy arrivals setting. We note that problem (3) is not a convex optimization problem due to the convexity of the energy harvesting function $E(i, x_i)$ in (1). We therefore follow a majorization maximization argument to find a local optimal solution for this problem via successive convex optimization. Namely, we approximate $E(i, x_i)$ around some feasible point to get a convex problem, whose solution is then used to (better) approximate $E(i, x_i)$ in the next iteration. Approximate functions should be chosen carefully such that iterations converge to a local optimal solution of the original problem [51], [52]. In particular, in the $(j+1)$st iteration, we solve the following problem

$$\begin{aligned} \max_{\boldsymbol{p},\boldsymbol{x}} \quad & \sum_{i=1}^{N} \frac{1}{2}\log(1+p_i) \\ \text{s.t.} \quad & c_m(1) \le E_0 \\ & c_m(k+1) + \sum_{i=1}^{k} p_i \le E_0 + \sum_{i=1}^{k} f^{(j)}(i, x_i), \quad \forall k \\ & 0 \le x_i \le L, \quad p_i \ge 0, \quad \forall i \end{aligned} \tag{15}$$

where $f^{(j)}(i, x_i)$ is the first order Taylor series approximation of $E(i, x_i)$ around $x_i^{(j)}$, the solution of the approximate problem in the jth iteration. That is, we have

$$f^{(j)}(i, x_i) \triangleq b_i^{(j)} + m_i^{(j)} x_i \tag{16}$$

where

$$\begin{aligned} b_i^{(j)} \triangleq & \frac{E_{1i}}{\left(x_i^{(j)}+\ell\right)^\alpha} + \frac{E_{2i}}{\left(L - x_i^{(j)}+\ell\right)^\alpha} \\ & + \left(\frac{\alpha E_{1i}}{\left(x_i^{(j)}+\ell\right)^{\alpha+1}} - \frac{\alpha E_{2i}}{\left(L-x_i^{(j)}+\ell\right)^{\alpha+1}}\right) x_i^{(j)} \end{aligned} \tag{17}$$

and

$$m_i^{(j)} \triangleq -\frac{\alpha E_{1i}}{\left(x_i^{(j)}+\ell\right)^{\alpha+1}} + \frac{\alpha E_{2i}}{\left(L-x_i^{(j)}+\ell\right)^{\alpha+1}} \tag{18}$$

By convexity of $E(i, x_i)$, it is direct to see that $f^{(j)}(i, x_i)$ satisfies the conditions stated in [51] that guarantee convergence of the iterative solution of problem (15) to a local optimal point of problem (3). Namely, it holds that

$$f^{(j)}(i, x_i) \le E(i, x_i), \ \forall x_i \tag{19}$$

$$f^{(j)}\left(i, x_i^{(j)}\right) = E\left(i, x_i^{(j)}\right) \tag{20}$$

$$\frac{df^{(j)}\left(i, x_i^{(j)}\right)}{dx_i} = \frac{dE\left(i, x_i^{(j)}\right)}{dx_i} \tag{21}$$

We focus on problem (15) in the remainder of this section. In particular, we introduce some auxiliary variables $\{\delta_i\}$ to denote the amount of energy used for movement in the ith slot. That is, we have

$$\epsilon_m |x_i - x_{i-1}| = \delta_i, \quad \forall i \tag{22}$$

This allows us to rewrite the optimization problem as follows

$$\begin{aligned} \max_{\boldsymbol{p},\boldsymbol{x},\boldsymbol{\delta}} \quad & \sum_{i=1}^{N} \frac{1}{2}\log(1+p_i) \\ \text{s.t.} \quad & \sum_{i=1}^{k} p_i \le E_0 + \sum_{i=1}^{k} b_i^{(j)} + m_i^{(j)} x_i - \sum_{i=1}^{k+1} \delta_i, \quad \forall k \\ & \delta_1 \le E_0 \\ & \epsilon_m |x_i - x_{i-1}| \le \delta_i, \quad \forall i \\ & 0 \le x_i \le L, \ p_i \ge 0, \ \delta_i \ge 0, \quad \forall i \end{aligned} \tag{23}$$

where the relaxation of the equality in (22) to an inequality in the above problem does not change the solution. To see this, note that if there exists some slot k such that $\delta_k^* > \epsilon_m |x_k^* - x_{k-1}^*|$, then one can simply decrease the value of δ_k^* until equality holds while keeping the values of x_k^* and x_{k-1}^* the same. This strictly increases the feasible set and thereby potentially increases the objective function. Also note that we set $\delta_{N+1} \triangleq 0$. We now have the following lemma.

Lemma 3: In the optimal solution of problem (23), if $\delta_i^* > 0$ then the transmitter should move forward (resp. backward) during slot i if $m_i^{(j)}$ is positive (resp. negative). Conversely, if $m_i^{(j)} = 0$, then there exists an optimal policy with $\delta_i^* = 0$.

Proof: We show this by contradiction. Assume that we have $\delta_i^* > 0$ and $m_i^{(j)} > 0$ but the transmitter moves backward during time slot i, i.e., $x_i^* < x_{i-1}^*$. Now consider the following alternative policy. Let $\delta_i = 0$, i.e., $x_i = x_{i-1}^*$, and let $\delta_{i+1} = \delta_i^* + \delta_{i+1}^*$. Since the cost to move is linear with distance, this new policy reaches the position x_{i+1}^* from x_{i-1}^* with the same cost. At the same time, since $m_i^{(j)} > 0$, this new policy harvests higher energy at slot i, and thereby achieves higher rates. Thus, the transmitter should move forward. The case where $m_i^{(j)} < 0$ implies that the transmitter should move backward can be shown using similar arguments. This proves the first part of the lemma.

To show the second part, note that since $m_i^{(j)} = 0$, moving during slot i does not make the transmitter gain any energy. Hence, by linearity of the moving cost, given any optimal policy with $\delta_i^* > 0$, setting $\delta_i = 0$ and $\delta_{i+1} = \delta_i^* + \delta_{i+1}^*$ in that case makes the transmitter harvest the same amount of energy, and reach x_{i+1}^* with the same moving cost. ■

Lemma 3 indicates that given the optimal amount of movement energy, the optimal movement policy is *greedy*. That is, if the transmitter moves during some time slot i, it moves towards the higher energy location in slot i without considering upcoming slots' energies. Next, we find the optimal greedy policy by decomposing problem (23) into inner and outer problems as follows.

1) Inner Problem: We first fix a feasible choice for $\{\delta_i\}$ and solve an *inner problem* for the pair $\{p_i, x_i\}$. We denote the solution of the inner problem by $R(\boldsymbol{\delta})$. By Lemma 3, once $\boldsymbol{\delta}$ is fixed, the position $\boldsymbol{x}$ is determined according to the sign of $\boldsymbol{m}^{(j)}$. Whence, the power $\boldsymbol{p}$ is found via directional water-filling [3]. Note that the choice of δ_i should be such that it is equal to 0 if $m_i^{(j)} = 0$, according to Lemma 3. In addition, we note that if we have some $\delta_i > 0$ while the greedy movement is not feasible, i.e., moving forward/backward with δ_i energy gets the transmitter outside the straight line boundaries, then surely this δ_i choice is not optimal and needs to change. How to optimally find $\{\delta_i^*\}$ is handled next.

2) Outer Problem: After we solve the inner problem, we find the optimal $\{\delta_i^*\}$ by solving an *outer problem* by maximizing $R(\boldsymbol{\delta})$ over the feasible choices of δ_i. We have the following lemma regarding this problem.

Lemma 4: $R(\boldsymbol{\delta})$ is a concave function in $\boldsymbol{\delta}$.

Proof: Let us pick two feasible points $\boldsymbol{\delta}^{(1)}$ and $\boldsymbol{\delta}^{(2)}$ and denote the solutions of the inner problem for these two choices by $\{\boldsymbol{p}^{(1)}, \boldsymbol{x}^{(1)}\}$ and $\{\boldsymbol{p}^{(2)}, \boldsymbol{x}^{(2)}\}$, respectively. Now let $\boldsymbol{\delta}^{\theta} \triangleq \theta\boldsymbol{\delta}^{(1)} + (1-\theta)\boldsymbol{\delta}^{(2)}$ for some $0 \le \theta \le 1$. Next, observe that by linearity of the feasible set, the pair $\boldsymbol{p}^{(\theta)} \triangleq \theta\boldsymbol{p}^{(1)} + (1-\theta)\boldsymbol{p}^{(2)}$ and $\boldsymbol{x}^{(\theta)} \triangleq \theta\boldsymbol{x}^{(1)} + (1-\theta)\boldsymbol{x}^{(2)}$ is feasible in the inner problem for the choice of $\boldsymbol{\delta}^{(\theta)}$. Therefore, we have

$$\begin{aligned} R\left(\boldsymbol{\delta}^{(\theta)}\right) &\ge \sum_{i=1}^{N} \frac{1}{2}\log\left(1 + p_i^{(\theta)}\right) \\ &\ge \sum_{i=1}^{N} \frac{\theta}{2}\log\left(1 + p_i^{(1)}\right) + \frac{1-\theta}{2}\log\left(1 + p_i^{(2)}\right) \\ &= \theta R\left(\boldsymbol{\delta}^{(1)}\right) + (1-\theta) R\left(\boldsymbol{\delta}^{(2)}\right) \end{aligned} \tag{24}$$

where the second inequality follows by concavity of the log. This concludes the proof. ■

We now solve the following outer problem

$$\begin{aligned} \max_{\boldsymbol{\delta}} \quad & R(\boldsymbol{\delta}) \\ \text{s.t.} \quad & \delta_1 \le E_0 \\ & \sum_{i=1}^{k+1} \delta_i \le E_0 + \sum_{i=1}^{k} b_i^{(j)} + \left[m_i^{(j)} L\right]^+, \quad \forall k \\ & \delta_i \ge 0, \quad \forall i \end{aligned} \tag{25}$$

with $\delta_{N+1} \triangleq 0$, and $[y]^+ \triangleq \max(y, 0)$. Note that the term $[m_i^{(j)} L]^+$ ensures that all the feasible range of $\{\boldsymbol{\delta}\}$ is covered in the outer problem, and that the inner problem is energy-feasible. By Lemma 4, the outer problem is a convex optimization problem [50]. However, not all the available energy should be used in movement, or else we achieve zero throughput. Hence, we follow an iterative water-filling algorithm to solve the outer problem similar to the one proposed in [20] and [26] that we summarize next. We add an extra $(N+1)$st slot where unused energy can be discarded. Initially, each slot is filled up by its own energy arrival and the extra $(N+1)$st slot is left empty. We allow energy/water to move to the right only if this increases the objective function. Meters are put in between slots to measure the amount of water moving forward. This allows us to pull water back to their sources if this increases the objective function. Eventually, all the water in the $(N+1)$st slot will be discarded but can be pulled back also during the iterations if necessary. Since the objective function increases with each water flow, problem feasibility is maintained during iterations, and by convexity of the problem, iterations converge to the optimal solution. This solution is a KKT point for the original problem in (3) by the proper choice of the approximate function f in (16), and is at least a local optimal solution, that could also be global depending on the initial conditions of the iterations. We summarize the multiple energy arrivals solution approach in Algorithm 1.

Algorithm 1 Offline Problem Solution

```
1: repeat
2:     Approximate E(i, x_i) around the (j − 1)st iteration's
       location solution x_i^(j−1) using (16)-(18), ∀i.
3:     Fix a feasible movement energy allocation δ.
4:     repeat
5:         Solve inner problem for R(δ) as in Section III-B1.
6:         Solve outer problem for δ* as in Section III-B2.
7:     until Convergence of movement energy water levels.
8: until ‖(x^(j), p^(j)) − (x^(j−1), p^(j−1))‖ is small enough.
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IV. Online Setting: Problem (7)

In this section we discuss the solution of problem (7). Note that the transmitter needs to decide on both the movement and the transmission energy for each time slot during the course of communication given only causal knowledge of the harvested energy. In particular, since the energy at time slot t is revealed *after* the transmitter relocates itself to position x_t, this means that the transmitter decides on where to relocate *blindly*, i.e., before knowing what amount of energy it will harvest. We now derive an upper bound on the optimal long term average throughput under such conditions in the following lemma.

Lemma 5: The optimal solution, r^*, of problem (7) satisfies

$$r^* \le \frac{1}{2}\log(1 + \max\{\bar{\mu}_1, \bar{\mu}_2\}) \tag{26}$$

where $\bar{\mu}_1 \triangleq \frac{\mu_1}{\ell^\alpha} + \frac{\mu_2}{(L+\ell)^\alpha}$ and $\bar{\mu}_2 \triangleq \frac{\mu_1}{(L+\ell)^\alpha} + \frac{\mu_2}{\ell^\alpha}$.

Proof: First, let us take $\epsilon_m = 0$. This enlarges the feasible set $\mathcal{F}$ since now the transmitter can move without energy cost. Since $E(i, x_i)$ is convex in x_i, the movement policy in this case should be extremal; the transmitter should only be positioned at either ends of the line to harvest maximal energy. Let us assume that the transmitter chooses to be at the first source's position, i.e., at $x = 0$, for θ fraction of the time. This allows us to construct a set of time slot indices $J_1(n) \subseteq \{1, \dots, n\}$ with $x_i = 0$ for $i \in J_1(n)$, and $\lim_{n\to\infty}|J_1(n)|/n = \theta$. Similarly, we can define $J_2(n)$ to be the time slot indices where the transmitter located at the second source's position, $x_i = L$, with $\lim_{n\to\infty}|J_2(n)|/n = 1-\theta$. Using Jensen's inequality [50] we have

$$r^* \le \lim_{T\to\infty} \frac{1}{2}\log\left(1 + \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} p_t\right]\right) \tag{27}$$

$$\le \lim_{T\to\infty} \frac{1}{2}\log\left(1 + \mathbb{E}\left[\frac{1}{T}\sum_{t\in J_1(T)} E(t,0) + \frac{1}{T}\sum_{t\in J_2(T)} E(t,L)\right]\right) \tag{28}$$

$$= \lim_{T\to\infty} \frac{1}{2}\log\left(1 + \frac{|J_1(T)|}{T}\bar{\mu}_1 + \frac{|J_2(T)|}{T}\bar{\mu}_2\right) \tag{29}$$

$$= \frac{1}{2}\log(1 + \theta\bar{\mu}_1 + (1-\theta)\bar{\mu}_2) \tag{30}$$

$$\le \frac{1}{2}\log(1 + \max\{\bar{\mu}_1, \bar{\mu}_2\}) \tag{31}$$

where (28) follows by definitions of the feasible set $\mathcal{F}$. ■

Next, we propose an online feasible energy management policy and show that it achieves the upper bound in the previous lemma, and thereby proving its optimality. Let $j \triangleq \arg\max_{i\in\{1,2\}} \bar{\mu}_i$, i.e., j denotes the energy source with higher average arrival rate μ_j (note that $\mu_1 > \mu_2$ implies $\bar{\mu}_1 > \bar{\mu}_2$ and vice-versa). Then, starting from its original position x_0, the transmitter uses all its harvested energy to move towards source j, and does not use any energy in transmission. Let us denote by n_0 the time slot at which the transmitter arrives at source j. Then, starting from time slot $n_0 + 1$ onwards, the transmitter uses all its energy in transmission, and does not use any energy in movement, i.e., it stays at source j till the end. We coin the above scheme as the *move-then-transmit* scheme. We now have the following result regarding how n_0 behaves asymptotically.

Lemma 6: For all values of $\epsilon_m > 0$, $0 < L < \infty$, and $E_0 \ge 0$, it holds that $\lim_{k\to\infty} \frac{n_0}{k} = 0$ a.s.

Proof: Let us assume without loss of generality that $\mu_2 > \mu_1$. If we assume $x_0 = 0$, then by definition, one can upper bound n_0 as follows

$$n_0 \le \min\left\{k : \sum_{i=1}^{k} E(i, x_i) \ge L\epsilon_m - E_0\right\} \tag{32}$$

$$\le \min\left\{k : \sum_{i=1}^{k} E_{1i} + E_{2i} \ge (L+\ell)^\alpha (L\epsilon_m - E_0)\right\} \tag{33}$$

where (33) follows by considering the worst case (smallest) amount of energy harvested from both sources simultaneously, i.e., assuming the transmitter is at distance L away from both sources. Now since $\{E_{1i} + E_{2i}\}$ is an i.i.d. process with mean $\mu_1 + \mu_2$, by the strong law of large numbers, we have that for fixed $\gamma, \nu > 0$, there exists a number k_0 such that $\forall k \ge k_0$ the following holds

$$\sum_{i=1}^{k} E_{1i} + E_{2i} \ge k(\mu_1 + \mu_2 - \nu) \tag{34}$$

with probability larger than $1 - \gamma$. Whence, a further upper bound on n_0, that holds with probability larger than $1 - \gamma$ is given by

$$n_0 \le \min\left\{k : k(\mu_1 + \mu_2 - \nu) \ge (L+\ell)^\alpha (L\epsilon_m - E_0), k \ge k_0\right\} \tag{35}$$

$$= \max\left\{\left\lceil \frac{(L+\ell)^\alpha (L\epsilon_m - E_0)}{\mu_1 + \mu_2 - \nu}\right\rceil, k_0\right\} \tag{36}$$

Therefore, it holds that

$$\lim_{k\to\infty} \frac{n_0}{k} \le \lim_{k\to\infty} \frac{1}{k} \max\left\{\left\lceil \frac{(L+\ell)^\alpha (L\epsilon_m - E_0)}{\mu_1 + \mu_2 - \nu}\right\rceil, k_0\right\} = 0 \tag{37}$$

with probability larger than $1 - \gamma$. Since $\gamma > 0$ was arbitrary, the above is true as $\gamma \to 0$ as well. This concludes the proof. ■

Note that while staying at source j, the transmitter is harvesting i.i.d. amount of energy with an average of $\bar{\mu}_j$. Hence, the transmitter can use, e.g., the *best effort transmission* scheme introduced and analyzed in [46] to optimally manage the amounts of its harvested energy for transmission. This best effort transmission scheme achieves the capacity of an AWGN channel with an average power constraint equal to the average energy harvesting rate by basically allowing the transmitter to send with energy equal to the average harvesting rate as long as it is feasible, and staying silent otherwise [46].

Next, we state the two main results of this section. Throughout, we assume that the amounts of energy generated at the two source are bounded, i.e., there exist some $M_1 > 0$ and $M_2 > 0$ such that $E_{1i} \le M_1$ a.s. $\forall i$ and $E_{2i} \le M_2$ a.s. $\forall i$. It is worth noting that this boundedness assumption is satisfied naturally if the transmitter is equipped with a finite battery B, since any excess energy received above the battery capacity overflows and cannot be used. We now have the following result for the infinite battery case.

Theorem 1: The move-then-transmit scheme along with best effort transmission strategy is optimal, for all values of $\epsilon_m > 0$, $0 < L < \infty$, and $E_0 \ge 0$.

Proof: Without loss of generality let us assume that $\mu_2 > \mu_1$, and hence the transmitter initially moves towards the second source and reaches there after some n_0 time slots. We then have the following energy causality constraints for transmission

$$\frac{1}{k}\sum_{i=n_0+1}^{k} p_i \le \frac{1}{k}\sum_{i=n_0+1}^{k} E(i, L), \quad \forall k \ge n_0 \tag{38}$$

Now let us examine the amounts of energy not used in transmission, i.e., during the first n_0 time slots, if the transmitter was initially located at $x_0 = L$. By Lemma 6 and the boundedness assumption, this amount behaves asymptotically as follows

$$\lim_{k\to\infty}\frac{1}{k}\sum_{i=1}^{n_0}E(i,L)\le\lim_{k\to\infty}\frac{n_0}{k}\left(\frac{M_1}{(L+r)^\alpha}+\frac{M_2}{r^\alpha}\right)=0\quad a.s. \tag{39}$$

Therefore, for fixed $\gamma,\nu>0$ we have

$$\frac{1}{k}\sum_{i=n_0+1}^{k}p_i\le\frac{1}{k}\sum_{i=1}^{k}E(i,L)-\frac{\nu}{k},\quad\forall k\ge n_0 \tag{40}$$

with probability larger than $1-\gamma$, and k large enough. Thus, as k grows infinitely large, one can take γ and ν down to 0, which means that the energy used in movement does not have a long term average effect on the energy causality constraint set. Therefore, transmitting by the average harvesting rate at the second source $\mathbb{E}[E(i,L)]=\bar{\mu}_2$ using the best effort strategy one achieves the following rate for T large enough [46]

$$\frac{T-n_0}{T}\frac{1}{2}\log(1+\bar{\mu}_2)-\kappa_T \tag{41}$$

where $\kappa_T\to 0$ as $T\to\infty$. Hence, taking the limit as $T\to\infty$, and using Lemma 6, one achieves a long term average throughput of $\frac{1}{2}\log(1+\bar{\mu}_2)$, which is equal to the upper bound stated in Lemma 5. Therefore, the proposed scheme is optimal. ■

Next, we discuss the case where the transmitter is equipped with a finite battery of size B. Under a finite battery capacity B, [34] introduced a near-optimal online policy for single-user energy harvesting channels coined the *fixed fraction policy* (FFP). Under this policy, in each time slot, the transmitter uses a fixed fraction of the amount of energy available in its battery for transmission. Such fraction is given by the average harvesting rate divided by the battery capacity. It is shown in [34] that such policy achieves a long term average throughput that lies within constant multiplicative and additive gaps from the optimal solution, for all i.i.d. energy arrival patterns and battery sizes. In our setting let us define the following fraction

$$q\triangleq\frac{\max\{\bar{\mu}_1,\bar{\mu}_2\}}{B} \tag{42}$$

and define the transmission power at time slot t to be given by

$$p_t=\begin{cases}0, & t\le n_0\\ qb_t, & t>n_0\end{cases} \tag{43}$$

Without loss of generality, we assume that $\frac{M_1}{\ell^\alpha}+\frac{M_2}{(L+\ell)^\alpha}$ and $\frac{M_1}{(L+\ell)^\alpha}+\frac{M_2}{\ell^\alpha}$ are both no larger than the battery capacity B, as any excess amount will overflow. Therefore, we have $q\le 1$, and the FFP policy above is always feasible. We now state the following result for the finite battery case.

Theorem 2: The move-then-transmit scheme along with the FFP in (43) achieve a long term average throughput that lies within an additive gap of 0.72 and a multiplicative gap of 0.5 from the optimal solution, for all values of $\epsilon_m>0$, $0<L<\infty$, and $E_0\ge 0$; and for all i.i.d. energy patterns and battery sizes.

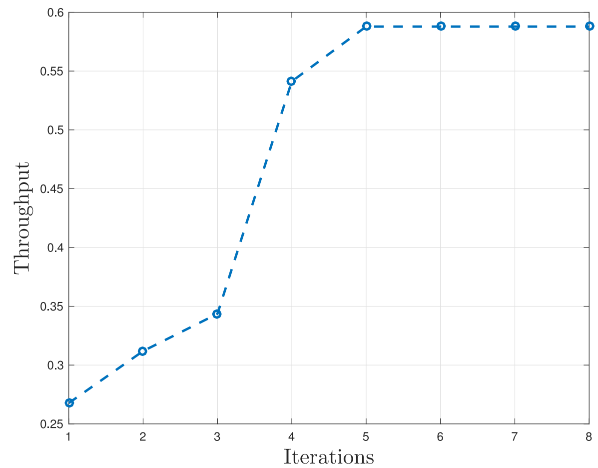

Fig. 2. Convergence of throughput over time.

Proof: The proof follows the same lines as in the proof of Theorem 1; basically, the fact that the effect of the movement strategy on the energy causality constraint set vanishes in the long term does not depend on the battery capacity, and hence it still holds. Once this is established, one can treat the problem as a single-user online problem with an energy harvesting average rate of $\bar{\mu}_j$, and use the same techniques as in [34, Th. 2] to show that the achieved rate lies within the constant gaps mentioned in the theorem from the upper bound stated in Lemma 5, and hence, the same constant gap results hold with respect to the optimal solution. ■

V. Numerical Results

A. Deterministic Arrivals

In this section, we present some numerical examples to further illustrate our results in the offline setting. We consider a system of four time slots. The transmitter has an initial amount of energy of $E_0=0.1$ energy units. The length of the straight line between the energy sources is $L=10$ distance units, and the transmitter is initially positioned at $x_0=2.5$. Energies arrive at the two energy sources with amounts $\boldsymbol{E}_1=[0,1,7,5]$ and $\boldsymbol{E}_2=[8,5,1,1]$, at the first and the second energy source, respectively. The path loss factor $\alpha=2.5$, $\ell=1.5$, and the movement energy cost per distance $\epsilon_m=0.2$.

We solve problem (15) by initially approximating the energy-position function at each time slot around x_0. We then do the problem decomposition to solve for $\{\delta_i^*\}$ and $\{p_i^*,x_i^*\}$ as discussed in Sections III-B1 and III-B2. Finally, we substitute by $\{x_i^*\}$ in problem (15) and re-iterate until convergence. For this example, it takes 5 iterations to converge to a local optimal solution of problem (3). In Fig. 2, we show the convergence of the throughput with iterations.

In Fig. 3, we plot the results of this example. We show the transmitter's position at different slots in between the two energy sources. Arrows at the sources represent the amounts of energy arriving (emitted) by each source at a given time slot. From the figure, we see that the transmitter stays at its initial position in the first time slot, i.e., $x_1^*=2.5$. This is mainly because the initial position of the transmitter is inclined

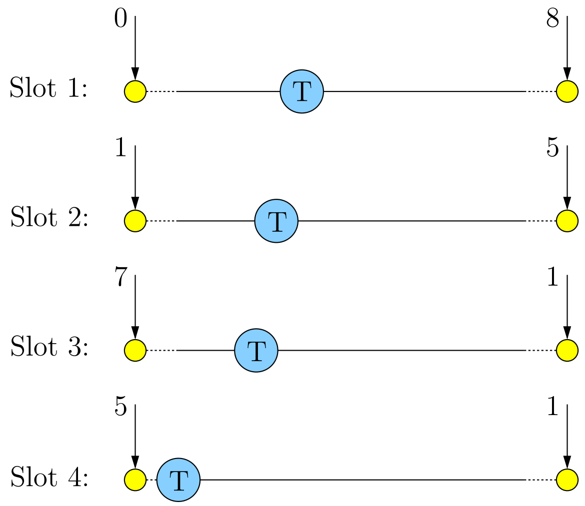

Fig. 3. Optimal transmitter location in a four-slot system.

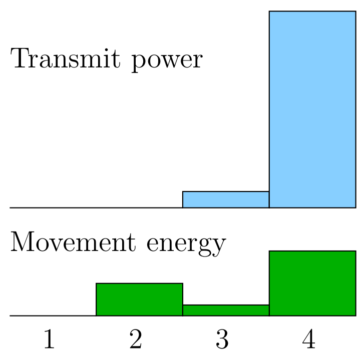

Fig. 4. Transmit power and movement energy consumptions.

towards the first source, and the fact that the energy amount at the second source is higher than that of the first source in the first time slot. One more reason for this movement behavior is that the first source receives higher amounts of energy in later slots. Therefore, we see that the transmitter moves towards the first source during slots 2 and 3 until it reaches the end of the line in slot 4. The optimal position is given by $\boldsymbol{x}^* = [2.5, 1.84, 1.5, 0]$, with powers $\boldsymbol{p}^* = [0, 0, 0.15, 1.82]$, and movement energy consumption of $[0, 0.03, 0.01, 0.06]$.

We plot the optimal transmit power and movement energy consumptions over the four time slots in Fig. 4. The height in blue and green represents the transmit power and the movement energy costs, respectively. We see that the transmitter neither moves nor transmits during the first time slot and saves all its harvested energy for later slots' movements and transmission. It starts spending some energy in movement during the second time slot while still not transmitting, and then finally during the third and fourth time slots it both moves and transmits to the receiver, achieving a throughput of 0.59.

Next, we show the effect of the movement energy cost per unit distance, ϵ_m, on the throughput. We shift the initial position to $x_0 = 3.5$ and use the same parameter values from the previous example except that we decrease ϵ_m to 0.01 and L to 7. The solution in this case is $x^* = [7, 7, 0, 0]$ with a throughput equal to 2.37. Due to the small movement energy cost, the transmitter in this case rides the energy peaks from the two sources, i.e., it harvests $E_i = \frac{1}{\ell^\alpha} \max\{E_{1i}, E_{2i}\}, \forall i$. The optimal location is shown by the hatched green transmitter in Fig. 5. We then increase ϵ_m to 3 and re-solve. In this case, we get $x^* = [3.5, 3.5, 3.5, 3.5]$ with a throughput equal to 0.28. Due to the large movement energy cost, the transmitter does not move during the course of communication and uses all of its available energy only for transmission. The optimal location in this case is shown by the solid brown transmitter in Fig. 5.

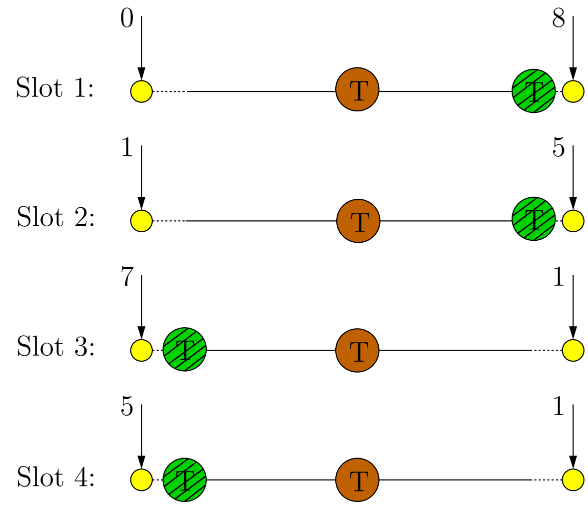

Fig. 5. Effect of moving cost on optimal location.

B. Stochastic Arrivals

In this section, we present some numerical results for the online setting. We consider a system where the energy arrivals at the first source follows a uniform distribution and that at the second source follows an exponential distribution. The system parameters are the same as in the first offline example except that we set $x_0 = 3.5$ distance units and $\epsilon_m = 10$ energy units per unit distance. In Fig. 6 we plot the long term average rate achieved by the proposed move-then-transmit and best effort policy against μ_1. We set $\mu_2 = 2\mu_1$ in this example. We also plot the theoretical upper bound obtained in Lemma 5. We see that the proposed policy achieves the theoretical upper bound and that the two curves are almost identical as stated in Theorem 1.

Finally, we consider a transmitter with finite battery capacity B. Energy arrivals follow Bernoulli distribution with parameters 0.5 and 0.3 at the first and the second source, respectively. In Fig. 7, we plot the long term average throughput achieved by the proposed move-then-transmit and FFP against μ_1. We set $\mu_2 = 2 \times \frac{0.3}{0.5}\mu_2 = 1.2\mu_2$ in this example. We also scale the battery with μ_1 and set it to $B = \max\{\bar{B}_1, \bar{B}_2\}$ where $\bar{B}_1 \triangleq \frac{0.5\mu_1}{\ell^\alpha} + \frac{0.3\mu_2}{(L+\ell)^\alpha}$ and $\bar{B}_2 \triangleq \frac{0.5\mu_1}{(L+\ell)^\alpha} + \frac{0.3\mu_2}{\ell^\alpha}$. We see that the rate achieved lies within a constant additive gap from the upper bound as stated in Theorem 2. Empirically, in this example the gap is no larger than 0.15, which is less than the 0.72 bound stated in the theorem.

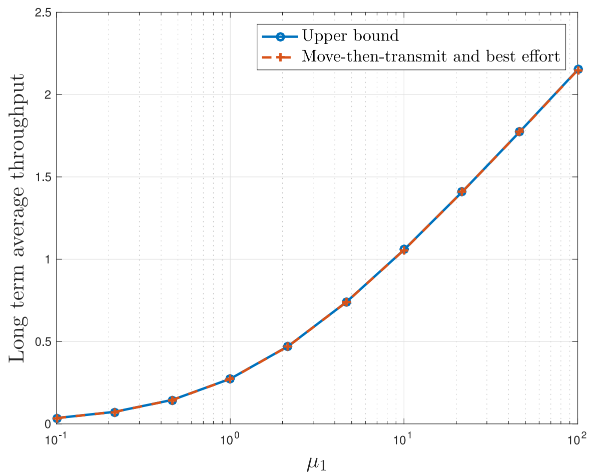

Fig. 6. Long term average rate achieved by the proposed move-then-transmit and best effort policy, and the theoretical upper bound, versus the average harvesting rate of the first source. In this example we set $\mu_2 = 2\mu_1$.

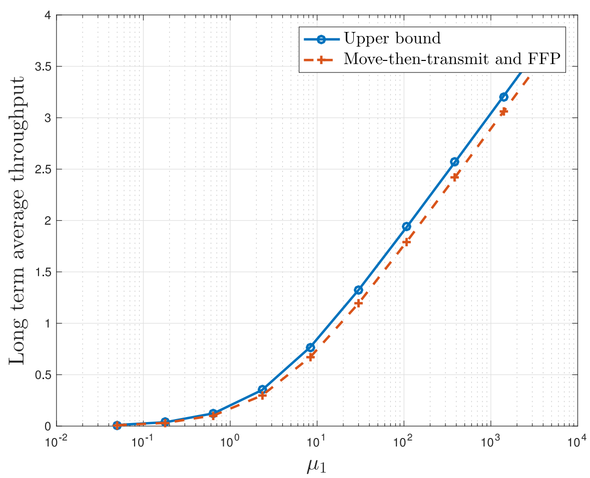

Fig. 7. Long term average rate achieved by the proposed move-then-transmit and fixed fraction policy, and the theoretical upper bound, versus the average harvesting rate of the first source. In this example we set $\mu_2 = 1.2\mu_1$.

VI. Discussion and Possible Extensions

In this section, we discuss some extensions to the problems and the model of this paper. Regarding the movement path, we considered a one-dimensional straight line movement profile in this paper as a first step to characterize the movement-throughput tradeoff. It would be of interest to extend the movement path to other two-dimensional or three-dimensional geometric shapes and understand the movement-throughput tradeoff in more general settings.

Regarding the energy sources, we considered the case where the sources emit energy in each time slot according to some random phenomenon. One way to extend this model is to optimize the amounts of the sources' emitted energy by introducing storage devices at the energy source side, i.e., transform the random energy source to a controlled energy sharing entity. In this case, the energy harvested at the transmitter at time slot i is given by

$$E(i, x_i) = \frac{\beta_{1i}}{(x_i + \ell)^\alpha} + \frac{\beta_{2i}}{(L - x_i + \ell)^\alpha} \tag{44}$$

where $\{\beta_{1i}\}$ and $\{\beta_{1i}\}$ satisfy the usual energy causality constraints

$$\sum_{i=1}^{k} \beta_{1i} \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k \tag{45}$$

$$\sum_{i=1}^{k} \beta_{2i} \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k \tag{46}$$

In other words, the two sources now generate energy with amounts $\{E_{1i}\}$ and $\{E_{2i}\}$ but only share $\{\beta_{1i}\}$ and $\{\beta_{2i}\}$ portion of them with the transmitter. We note that in this case, procrastinating policies [19], where energy sources share energy in a time slot only if it will be used in the same time slot, need not be optimal since the energy sharing efficiency is changing with the position of the transmitter. We also note that even with a single energy arrival at the two sources the problem now does not admit a closed form solution as shown in Section III-A; this would be the case only if we consider one time slot $N = 1$. Thus, even with a single energy arrival, one has to optimize the amounts of shared energy over multiple time slots.

VII. Conclusion

We considered mobility effects on energy harvesting nodes. Energy arrivals at a node depend on the node's relative position to energy emitting sources, and therefore movement is motivated by finding better energy harvesting locations. However, nodes incur a moving cost per unit distance travelled. We considered movement along a straight line, where two energy sources are located towards the opposite ends of the line. We characterized the optimal tradeoff between staying at the same spot so as to spend all available energy in transmission, and spending some energy to move to a potentially better energy location so as to achieve higher throughput. We studied this problem in both offline and online settings. In the offline setting, we designed movement and transmission policies that maximize the sum throughput by a given deadline. We first solved the case with a single energy arrival at each source, and then generalized that to the case of multiple energy arrivals. In the online setting, we proposed an optimal *move-then-transmit* scheme that maximizes the long term average throughput, where the transmitter first moves towards the energy source with higher energy harvesting mean, and then starts transmission. We analyzed the performance of this scheme under both infinite and finite battery capacities at the transmitter.

References

[1] J. Yang and S. Ulukus, "Optimal packet scheduling in an energy harvesting communication system," *IEEE Trans. Commun.*, vol. 60, no. 1, pp. 220–230, Jan. 2012.

[2] K. Tutuncuoglu and A. Yener, "Optimum transmission policies for battery limited energy harvesting nodes," *IEEE Trans. Wireless Commun.*, vol. 11, no. 3, pp. 1180–1189, Mar. 2012.

[3] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, "Transmission with energy harvesting nodes in fading wireless channels: Optimal policies," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 8, pp. 1732–1743, Sep. 2011.

[4] C. K. Ho and R. Zhang, "Optimal energy allocation for wireless communications with energy harvesting constraints," *IEEE Trans. Signal Process.*, vol. 60, no. 9, pp. 4808–4818, Sep. 2012.

[5] J. Yang, O. Ozel, and S. Ulukus, "Broadcasting with an energy harvesting rechargeable transmitter," *IEEE Trans. Wireless Commun.*, vol. 11, no. 2, pp. 571–583, Feb. 2012.

[6] M. A. Antepli, E. Uysal-Biyikoglu, and H. Erkal, "Optimal packet scheduling on an energy harvesting broadcast link," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 8, pp. 1721–1731, Sep. 2011.

[7] O. Ozel, J. Yang, and S. Ulukus, "Optimal broadcast scheduling for an energy harvesting rechargeable transmitter with a finite capacity battery," *IEEE Trans. Wireless Commun.*, vol. 11, no. 6, pp. 2193–2203, Jun. 2012.

[8] J. Yang and S. Ulukus, "Optimal packet scheduling in a multiple access channel with energy harvesting transmitters," *J. Commun. Netw.*, vol. 14, no. 2, pp. 140–150, Apr. 2012.

[9] Z. Wang, V. Aggarwal, and X. Wang, "Iterative dynamic water-filling for fading multiple-access channels with energy harvesting," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 3, pp. 382–395, Mar. 2015.

[10] N. Su, O. Kaya, S. Ulukus, and M. Koca, "Cooperative multiple access under energy harvesting constraints," in *Proc. IEEE Globecom*, San Diego, CA, USA, Dec. 2015, pp. 1–6.

[11] K. Tutuncuoglu and A. Yener, "Sum-rate optimal power policies for energy harvesting transmitters in an interference channel," *J. Commun. Netw.*, vol. 14, no. 2, pp. 151–161, Apr. 2012.

[12] C. Huang, R. Zhang, and S. Cui, "Throughput maximization for the Gaussian relay channel with energy harvesting constraints," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 8, pp. 1469–1479, Aug. 2013.

[13] D. Gündüz and B. Devillers, "Two-hop communication with energy harvesting," in *Proc. IEEE CAMSAP*, Dec. 2011, pp. 201–204.

[14] Y. Luo, J. Zhang, and K. B. Letaief, "Optimal scheduling and power allocation for two-hop energy harvesting communication systems," *IEEE Trans. Wireless Commun.*, vol. 12, no. 9, pp. 4729–4741, Sep. 2013.

[15] B. Gurakan and S. Ulukus, "Cooperative diamond channel with energy harvesting nodes," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 5, pp. 1604–1617, May 2016.

[16] K. Tutuncuoglu, B. Varan, and A. Yener, "Optimum transmission policies for energy harvesting two-way relay channels," in *Proc. IEEE ICC*, Budapest, Hungary, Jun. 2013, pp. 586–590.

[17] K. Tutuncuoglu and A. Yener, "The energy harvesting and energy cooperating two-way channel with finite-sized batteries," in *Proc. IEEE Globecom*, Austin, TX, USA, Dec. 2014, pp. 1424–1429.

[18] B. Gurakan, O. Ozel, J. Yang, and S. Ulukus, "Energy cooperation in energy harvesting communications," *IEEE Trans. Commun.*, vol. 61, no. 12, pp. 4884–4898, Dec. 2013.

[19] K. Tutuncuoglu and A. Yener, "Energy harvesting networks with energy cooperation: Procrastinating policies," *IEEE Trans. Commun.*, vol. 63, no. 11, pp. 4525–4538, Nov. 2015.

[20] B. Gurakan and S. Ulukus, "Energy harvesting diamond channel with energy cooperation," in *Proc. IEEE ISIT*, Honolulu, HI, USA, Jul. 2014, pp. 986–990.

[21] K. Tutuncuoglu and A. Yener, "Communicating with energy harvesting transmitters and receivers," in *Proc. UCSD ITA*, Feb. 2012, pp. 240–245.

[22] H. Mahdavi-Doost and R. D. Yates, "Energy harvesting receivers: Finite battery capacity," in *Proc. IEEE ISIT*, Istanbul, Turkey, Jul. 2013, pp. 1799–1803.

[23] P. Grover, K. Woyach, and A. Sahai, "Towards a communication-theoretic understanding of system-level power consumption," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 8, pp. 1744–1755, Sep. 2011.

[24] J. Rubio, A. Pascual-Iserte, and M. Payaró, "Energy-efficient resource allocation techniques for battery management with energy harvesting nodes: A practical approach," in *Proc. Eur. Wireless Conf.*, Apr. 2013, pp. 1–6.

[25] R. Nagda, S. Satpathi, and R. Vaze, "Optimal offline and competitive online strategies for transmitter-receiver energy harvesting," in *Proc. IEEE ICC*, London, U.K., Jun. 2015, pp. 74–79.

[26] A. Arafa and S. Ulukus, "Optimal policies for wireless networks with energy harvesting transmitters and receivers: Effects of decoding costs," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 12, pp. 2611–2625, Dec. 2015.

[27] A. Arafa, A. Baknina, and S. Ulukus, "Energy harvesting two-way channels with decoding and processing costs," *IEEE Trans. Green Commun. Netw.*, vol. 1, no. 1, pp. 3–16, Mar. 2017.

[28] J. Xu and R. Zhang, "Throughput optimal policies for energy harvesting wireless transmitters with non-ideal circuit power," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 2, pp. 322–332, Feb. 2014.

[29] O. Orhan, D. Gündüz, and E. Erkip, "Energy harvesting broadband communication systems with processing energy cost," *IEEE Trans. Wireless Commun.*, vol. 13, no. 11, pp. 6095–6107, Nov. 2014.

[30] O. Ozel, K. Shahzad, and S. Ulukus, "Optimal energy allocation for energy harvesting transmitters with hybrid energy storage and processing cost," *IEEE Trans. Signal Process.*, vol. 62, no. 12, pp. 3232–3245, Jun. 2014.

[31] M. Gregori and M. Payaró, "Throughput maximization for a wireless energy harvesting node considering the circuitry power consumption," in *Proc. IEEE VTC*, Quebec City, QC, Canada, Sep. 2012, pp. 1–5.

[32] Q. Bai, J. Li, and J. A. Nossek, "Throughput maximizing transmission strategy of energy harvesting nodes," in *Proc. IEEE IWCLD*, Rennes, France, Nov. 2011, pp. 1–5.

[33] A. Arafa, A. Baknina, and S. Ulukus, "Optimal policies in energy harvesting two-way channels with processing costs," in *Proc. IEEE WiOpt*, Tempe, AZ, USA, May 2016, pp. 1–8.

[34] D. Shaviv and A. Özgür, "Universally near optimal online power control for energy harvesting nodes," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 12, pp. 3620–3631, Dec. 2016.

[35] A. Baknina and S. Ulukus, "Optimal and near-optimal online strategies for energy harvesting broadcast channels," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 12, pp. 3696–3708, Dec. 2016.

[36] H. A. Inan and A. Ozgur, "Online power control for the energy harvesting multiple access channel," in *Proc. IEEE WiOpt*, Tempe, AZ, USA, May 2016, pp. 1–6.

[37] A. Baknina and S. Ulukus, "Online policies for multiple access channel with common energy harvesting source," in *Proc. IEEE ISIT*, Barcelona, Spain, Jul. 2016, pp. 2739–2743.

[38] A. Baknina and S. Ulukus, "Online scheduling for energy harvesting channels with processing costs," *IEEE Trans. Green Commun. Netw.*, vol. 1, no. 3, pp. 281–293, Sep. 2017.

[39] A. Arafa, A. Baknina, and S. Ulukus, "Energy harvesting networks with general utility functions: Near optimal online policies," in *Proc. IEEE ISIT*, Aachen, Germany, Jun. 2017, pp. 809–813.

[40] A. Arafa and S. Ulukus, "Near optimal online distortion minimization for energy harvesting nodes," in *Proc. IEEE ISIT*, Aachen, Germany, Jun. 2017, pp. 1117–1121.

[41] M. B. Khuzani and P. Mitran, "On online energy harvesting in multiple access communication systems," *IEEE Trans. Inf. Theory*, vol. 60, no. 3, pp. 1883–1898, Mar. 2014.

[42] F. Amirnavaei and M. Dong, "Online power control optimization for wireless transmission with energy harvesting and storage," *IEEE Trans. Wireless Commun.*, vol. 15, no. 7, pp. 4888–4901, Jul. 2016.

[43] B. T. Bacinoglu and E. Uysal-Biyikoglu, "Finite horizon online lazy scheduling with energy harvesting transmitters over fading channels," in *Proc. IEEE ISIT*, Honolulu, HI, USA, Jun. 2014, pp. 1176–1180.

[44] Y. Shi, L. Xie, Y. T. Hou, and H. D. Sheali, "On renewable sensor networks with wireless energy transfer," in *Proc. IEEE Infocom*, Shanghai, China, Apr. 2011, pp. 1350–1358.

[45] A. H. Coarasa, P. Nintanavongsa, S. Sanyal, and K. R. Chowdhury, "Impact of mobile transmitter sources on radio frequency wireless energy harvesting," in *Proc. IEEE ICNC*, San Diego, CA, USA, Jan. 2013, pp. 573–577.

[46] O. Ozel and S. Ulukus, "Achieving AWGN capacity under stochastic energy harvesting," *IEEE Trans. Inf. Theory*, vol. 58, no. 10, pp. 6471–6483, Oct. 2012.

[47] S. He *et al.*, "Energy provisioning in wireless rechargeable sensor networks," *IEEE Trans. Mobile Comput.*, vol. 12, no. 10, pp. 1931–1942, Oct. 2013.

[48] L. Fu, P. Cheng, Y. Gu, J. Chen, and T. He, "Minimizing charging delay in wireless rechargeable sensor networks," in *Proc. IEEE Infocom*, Turin, Italy, Apr. 2013, pp. 2922–2930.

[49] K. Huang and V. K. N. Lau, "Enabling wireless power transfer in cellular networks: Architecture, modeling and deployment," *IEEE Trans. Wireless Commun.*, vol. 13, no. 2, pp. 902–912, Feb. 2014.

[50] S. P. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.

[51] B. R. Marks and G. P. Wright, "A general inner approximation algorithm for nonconvex mathematical programs," *Oper. Res.*, vol. 26, no. 4, pp. 681–683, Jul./Aug. 1978.

[52] M. Chiang, C. W. Tan, D. P. Palomar, D. O'Neill, and D. Julian, "Power control by geometric programming," *IEEE Trans. Wireless Commun.*, vol. 6, no. 7, pp. 2640–2651, Jul. 2007.

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