Energy Harvesting Two-Way Channels With Decoding and Processing Costs

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***Abstract*—We study the effects of decoding and processing costs in an energy harvesting two-way channel. We design the optimal offline power scheduling policies that maximize the sum throughput by a given deadline, subject to energy causality constraints, decoding causality constraints, and processing costs at both users. In this system, each user spends energy to transmit data to the other user, and also to decode data coming from the other user; that is, each user divides its harvested energy for transmission and reception. Further, each user incurs a processing cost per unit time as long as it communicates. The power needed for decoding the incoming data is modeled as an increasing convex function of the incoming data rate; and the power needed to be *on*, i.e., the processing cost, is modeled to be a constant per unit time. We solve this problem by first considering the cases with decoding costs only and processing costs only individually. In each case, we solve the single energy arrival scenario, and then use the solution's insights to provide an iterative algorithm that solves the multiple energy arrivals scenario. Then, we consider the general case with *both* decoding and processing costs in a single setting, and solve it for the most general scenario of multiple energy arrivals.**

***Index Terms*—Energy harvesting transmitters, energy harvesting receivers, twoway channels, decoding costs, processing costs.**

I. Introduction

IN THIS paper, we consider an energy harvesting two-way channel, see Fig. 1, where each user relies solely on energy harvested from nature. We design optimal offline power scheduling policies that maximize the sum throughput by a given deadline, subject to energy and decoding causality constraints at both users, with processing costs. We divide our development into three main parts. We first discuss the case with only decoding costs at both users, followed by the case with only processing costs. Then, we solve the general case with both decoding and processing costs using ideas developed for the solution of the first two cases.

Energy harvesting communication systems have been studied extensively in recent literature. References [3]–[22] focus on energy harvesting at the transmitter side, and consider the single-user setting [3]–[6], broadcast, multiple access, and interference channels [7]–[12], two-hop and relay channels [13]–[15], two-way channels [16], [17], energy sharing and energy cooperation concepts [18]–[20], battery imperfections [21], [22], sensor networks [23]–[25], MIMO systems [26], and so on. Most of these references optimize the transmit power schedules of the users over time, using concave rate-power relationships, to minimize the transmission completion time or maximize the throughput by a deadline.

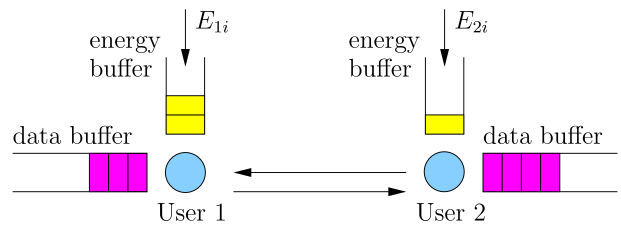

Fig. 1. Two-way channel with energy harvesting transceivers.

References [27]–[32] focus on energy harvesting at the receivers. In these references, the energy needed for receiving incoming data is modeled as a monotone increasing convex function of the incoming rate (see also [33], [34]). In this case, the receivers need to optimally allocate their harvested energy for decoding, and the transmitters need to optimize their transmit powers (and therefore rates) such that the receivers can handle, i.e., decode, the incoming data with their available energies. In the above references, each energy harvesting node is either a transmitter or a receiver, i.e., each node either needs to optimize its transmit power over time slots or needs to optimize its decoding power over time slots.

In the two-way energy harvesting channel we consider in this paper, each node transmits data to the other user, and receives data from the other user in a full duplex manner. Therefore, each node is simultaneously an energy harvesting transmitter and an energy harvesting receiver, and needs to optimize its power schedule over time slots by optimally dividing its energy for transmission and decoding. The power used for transmission is modeled through a concave rate-power relationship as in the Shannon formula; and the power used for decoding is modeled as a convex increasing function of the incoming rate. In particular, throughout this paper, we focus on decoding costs that are exponential in the incoming rate [30], [33].

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Even in the case of energy harvesting transmitters only and energy harvesting receivers only, the energy availability of one side limits the transmission and reception abilities of the other side; energy harvesting introduces coupling between transmitters and receivers. In the energy harvesting two-way channel, this coupling is even stronger. In addition, we assume that power consumption at a user includes power spent for processing as well, i.e., power spent for the circuitry. This is the power spent for the user to be *on* and communicating. Depending on the energy availability and the communication distance, processing costs at the transmitter could be a significant system factor. References [35]–[39] study the impact of processing costs on energy harvesting communications. As discussed, decoding power at the receiver could be a significant system factor as well [27]–[32]. The differentiating aspect regarding processing costs and decoding costs is as follows: the processing cost is modeled as a constant power spent per unit time whenever the transmitter is on [40], whereas the decoding cost at a receiver is modeled as an increasing convex function of the incoming rate to be decoded [28], [32]. In this paper, we consider *both* decoding and processing costs in a single setting.

In the first part of this work, we focus on the case with only decoding costs. We first consider the case with a single energy arrival at each user. We show that the transmission is limited by the user with smaller energy; the user with larger energy may not consume all of its energy. We next consider the case with multiple energy arrivals at both users. We show that the optimal power allocations are non-decreasing over time, and they increase synchronously at both users. We develop an iterative algorithm based on two-slot updates to obtain the optimal power allocations for both users that converges to the optimal solution.

Next, we focus on the case with only processing costs. We assume that both users incur processing costs per unit time as long as they are communicating. We first consider the formulation for a single energy arrival. In this case, we show that transmission can be bursty [40]; users may opt to communicate for only a portion of the time. We also show that it is optimal for the two users to be fully synchronized; the two users should be switched on for the same portion of the time during which they both exchange data, and then they switch off together. Then, we generalize this to the case of multiple energy arrivals, and show that any throughput optimal policy can be transformed into a *deferred* policy, in which users postpone their energy consumption to fill out later slots first. We find the optimal deferred policy by iteratively applying a modified version of the single energy arrival result in a backward manner.

Finally, we study the general case with *both* decoding and processing costs in a single setting. We formulate a sum throughput optimization problem that it is a generalization of the setting with only decoding costs or only processing costs. We solve this general problem in the single energy arrival scenario, and then present an iterative algorithm to solve the multiple energy arrival case that is a combination of the algorithms used to solve the cases with only decoding and only processing costs.

II. The Case With Only Decoding Costs

A. Single Energy Arrival

In this section, we consider the case where both users have a single energy arrival each. Users 1 and 2 have E_1 and E_2 amounts of energy available at the beginning of communication, respectively. Without loss of generality, the communication takes place over a time slot of unit length. The physical layer is Gaussian with unit-variance noise at both users. In the full-duplex Gaussian two-way channel, the sum rate is given by the sum of the single-user rates [41]. Therefore, the rate per user is the single-user Shannon rate of $\frac{1}{2}\log(1+p)$, where p is the transmit power. Throughout this paper, log is the natural logarithm. A receiver decodes a message of rate r by spending a decoding power $\phi(r)$ that is exponential in the incoming rate, i.e., $\phi(r) = a(e^{br}+c)$ for some $a, b > 0$ and $c \geq -1$. Throughout this paper, we take $b = 2$ and $c = -1$ for convenience and mathematical tractability. Without loss of generality, any other such exponential decoding power can be handled by appropriately modifying the incoming energy. Therefore, if the first user transmits with power p, the incoming rate is $\frac{1}{2}\log(1+p)$, and the second user spends a power of ap to decode the incoming data. Thus, the throughput maximization problem is

$$\begin{aligned} \max_{p_1,p_2} \quad & \frac{1}{2}\log(1+p_1) + \frac{1}{2}\log(1+p_2) \\ \text{s.t.} \quad & p_1 + ap_2 \leq E_1 \\ & p_2 + ap_1 \leq E_2 \end{aligned} \tag{1}$$

where p_1 and p_2 are the powers of users 1 and 2, respectively. We assume $a \neq 1$, for if $a = 1$, by concavity of the log, the optimal solution will be given by $p_1^* = p_2^* = \min\{E_1, E_2\}/2$. We have the following lemma regarding this problem.

Lemma 1: In the optimal policy, at least one user consumes all of its energy in transmission and decoding. This is the user with the smaller energy.

Proof: The first part of the lemma follows directly by noting that if neither of the constraints holds with equality, then we can increase the power (and therefore rate) of one of the users until one of the constraints becomes tight. Now assume that $E_1 \leq E_2$, but only the second user consumes all of its energy, i.e., $p_2^* + ap_1^* = E_2 \geq E_1 > p_1^* + ap_2^*$, which further leads to having

$$p_1^* < p_2^*, \quad \text{if } a < 1 \tag{2}$$

$$p_1^* > p_2^*, \quad \text{if } a > 1 \tag{3}$$

Let us consider the case in (2) (similar arguments follow for the case in (3)), choose some $\epsilon > 0$, and define the following new policy: $\tilde{p}_1 = p_1^* + \epsilon$, $\tilde{p}_2 = p_2^* - \epsilon$. Since the first user did not consume all of its energy, we can choose ϵ small enough such that the new policy consumes the following amounts of energy

$$\tilde{p}_2 + a\tilde{p}_1 = p_2^* + ap_1^* - (1-a)\epsilon < E_2 \tag{4}$$

$$\tilde{p}_1 + a\tilde{p}_2 = p_1^* + ap_2^* + (1-a)\epsilon \leq E_1 \tag{5}$$

By concavity of the log, this new policy strictly increases the sum rate, and therefore, the original policy cannot be optimal, i.e., the first user has to consume all of its energy. ■

The above lemma states that, in the presence of decoding costs, one user may not be able to use up all of its energy. This is because each user now needs to adapt its power (and rate) to both its own energy and to the energy of the other user, in order to guarantee decodability. This makes the user with smaller energy be a bottleneck for the system.

Without loss of generality, we continue assuming $E_1 \le E_2$. Therefore, by Lemma 1, we have $p_1^* + ap_2^* = E_1$. Substituting this condition in (1), we get the following problem for $a < 1$

$$\begin{aligned} \max_{p_2} \quad & \frac{1}{2}\log(1+E_1-ap_2)+\frac{1}{2}\log(1+p_2) \\ \text{s.t.} \quad & 0 \le p_2 \le \frac{E_2-aE_1}{1-a^2} \end{aligned} \tag{6}$$

Alternatively, we get the following problem for $a > 1$

$$\begin{aligned} \max_{p_1} \quad & \frac{1}{2}\log(1+p_1)+\frac{1}{2}\log\left(1+\frac{E_1-p_1}{a}\right) \\ \text{s.t.} \quad & 0 \le p_1 \le \frac{aE_2-E_1}{a^2-1} \end{aligned} \tag{7}$$

In both problems, the objective function is concave and the feasible set is an interval. It then follows that the optimal power can be found via equating the derivative of the objective function to 0, and projecting the solution onto the feasible set. For instance, the optimal second user power in problem (6) is given by

$$p_2^* = \min\left\{\left[\frac{1+E_1-a}{2a}\right]^+, \frac{E_2-aE_1}{1-a^2}\right\} \tag{8}$$

where $[x]^+ = \max(x, 0)$.

B. Multiple Energy Arrivals

We now consider the case of multiple energy arrivals. Energies arrive at the beginning of time slot i with amounts E_{1i} and E_{2i} at the first and the second user, respectively, ready to be used in the same slot. Unused energies are saved in batteries for later slots. The goal is to maximize the sum throughput by a given deadline N. The problem becomes

$$\begin{aligned} \max_{\boldsymbol{p}_1,\boldsymbol{p}_2} \quad & \sum_{i=1}^{N}\frac{1}{2}\log(1+p_{1i})+\frac{1}{2}\log(1+p_{2i}) \\ \text{s.t.} \quad & \sum_{i=1}^{k}p_{1i}+ap_{2i} \le \sum_{i=1}^{k}E_{1i}, \quad \forall k \\ & \sum_{i=1}^{k}p_{2i}+ap_{1i} \le \sum_{i=1}^{k}E_{2i}, \quad \forall k \end{aligned} \tag{9}$$

which is a convex optimization problem [42]. The Lagrangian is

$$\begin{aligned} \mathcal{L} = & -\sum_{i=1}^{N}\frac{1}{2}\log(1+p_{1i}) - \sum_{i=1}^{N}\frac{1}{2}\log(1+p_{2i}) \\ & + \sum_{k=1}^{N}\lambda_{1k}\left(\sum_{i=1}^{k}p_{1i}+ap_{2i}-\sum_{i=1}^{k}E_{1i}\right) \\ & + \sum_{k=1}^{N}\lambda_{2k}\left(\sum_{i=1}^{k}p_{2i}+ap_{1i}-\sum_{i=1}^{k}E_{2i}\right) \end{aligned} \tag{10}$$

where $\{\lambda_{1k}\}$ and $\{\lambda_{2k}\}$ are non-negative Lagrange multipliers associated with the energy causality constraints of the first and the second user, respectively. KKT optimality conditions [42] are

$$p_{1i} = \frac{1}{\sum_{k=i}^{N}(\lambda_{1k}+a\lambda_{2k})} - 1, \quad \forall i \tag{11}$$

$$p_{2i} = \frac{1}{\sum_{k=i}^{N}(\lambda_{2k}+a\lambda_{1k})} - 1, \quad \forall i \tag{12}$$

along with the complementary slackness conditions

$$\lambda_{1k}\left(\sum_{i=1}^{k}p_{1i}+ap_{2i}-\sum_{i=1}^{k}E_{1i}\right) = 0, \quad \forall k \tag{13}$$

$$\lambda_{2k}\left(\sum_{i=1}^{k}p_{2i}+ap_{1i}-\sum_{i=1}^{k}E_{2i}\right) = 0, \quad \forall k \tag{14}$$

In the following lemmas, we characterize the properties of the optimal solution of this problem.

Lemma 2: In the optimal policy, both users' powers are non-decreasing in time, i.e., $p_{1(i+1)} \ge p_{1i}$ *and* $p_{2(i+1)} \ge p_{2i}$, $\forall i$.

Proof: The proof follows from (11)-(12) since the denominators are non-negative and non-increasing as $\lambda_{1k}, \lambda_{2k} \ge 0$, $\forall k$. ∎

Lemma 3: In the optimal policy, the power of user $j \in \{1, 2\}$ *increases in a time slot only if at least one of the two users consumes all of its available energy in transmission/decoding in the previous time slot.*

Proof: From (11)-(12), we see that powers can only increase from slot i to slot $i+1$ if at least λ_{1i} or λ_{2i} is strictly positive, or else powers will stay the same. By complementary slackness conditions in (13)-(14), we see that the first (resp., second) user's energies must all be consumed by slot i if $\lambda_{1i} > 0$ (resp., $\lambda_{2i} > 0$). ∎

Lemma 4: In the optimal policy, powers of both users increase synchronously.

Proof: Let us assume that we have $p_{1i} < p_{1(i+1)}$. By Lemma 3, we must have at least $\lambda_{1i} > 0$ or $\lambda_{2i} > 0$. This in turn makes $p_{2i} < p_{2(i+1)}$ from (12). Similarly, if we have $p_{2i} < p_{2(i+1)}$, then we must also have $p_{1i} < p_{1(i+1)}$ from (11). This concludes the proof. ∎

1) The Case of Two Arrivals: We now solve the case of two energy arrivals at each user explicitly. We will provide an iterative algorithm to solve the general multiple energy arrivals case by utilizing the two-slot solution. In a two-slot setting, it is optimal to have at least one user consume all of its energy in the second slot. It is not clear, however, if this is the case in the first slot. Towards that, we check the feasible energy consumption strategies and choose the one that gives the maximum sum rate. For each strategy, we find the optimal residual energy transferred from the first to the second slot for a given user. We begin by checking a constant-power strategy which, by concavity of the objective function, is optimal if it is feasible [3]. This occurs when neither user consumes all of its energy in the first slot, and hence, by Lemma 3, the powers of each user in the two slots are equal, i.e., $p_{11} = p_{12} \triangleq p_1$, and $p_{21} = p_{22} \triangleq p_2$. This leaves us with solving a *single-arrival* problem, as discussed in Section II-A, with the average energy

$E_1 = \frac{E_{11}+E_{12}}{2}$ and $E_2 = \frac{E_{21}+E_{22}}{2}$, at the first and the second user, respectively. There can be four more energy consumption strategies to check if the above is infeasible. We highlight one of them in the following analysis. The remaining ones follow similarly.

We consider the strategy in which the first user consumes all of its energy in the first slot, and the second user consumes all of its energy in the second slot. The second user may have some residual energy left from the first slot to be used in the second slot. Denoting this energy residual by r, we have: $p_{11} + ap_{21} = E_{11}$, and $p_{21} + ap_{11} = E_{21} - r$. Solving these two equations for p_{11} and p_{21}, we obtain: $p_{11} = \frac{E_{11}-a(E_{21}-r)}{1-a^2}$, and $p_{21} = \frac{E_{21}-r-aE_{11}}{1-a^2}$. Since the second user consumes all of its energy in the second slot we have: $p_{22} + ap_{12} = E_{22} + r$. Next, we divide the energy consumption in the second slot between the two users as: $p_{12} = \frac{\delta}{a}$ and $p_{22} = E_{22} + r - \delta$, for some $\delta \geq 0$. Finding the optimal sum rate in this strategy is tantamount to solving for the optimal values of r and δ. Thus, problem (9) for $N = 2$ in this case can be rewritten as

$$\begin{aligned}\max_{r,\delta} \quad & \frac{1}{2}\log\left(1 + \frac{E_{11} - a(E_{21} - r)}{1 - a^2}\right) \\ & + \frac{1}{2}\log\left(1 + \frac{E_{21} - r - aE_{11}}{1 - a^2}\right) \\ & + \frac{1}{2}\log\left(1 + \frac{\delta}{a}\right) + \frac{1}{2}\log(1 + E_{22} + r - \delta) \\ \text{s.t.} \quad & 0 \leq \delta \leq E_{22} + r \\ & \left(E_{21} - \frac{E_{11}}{a}\right)^+ \leq r \leq E_{21} - aE_{11} \\ & \delta \leq \frac{a}{1 - a^2}(E_{12} - a(E_{22} + r)) \end{aligned} \tag{15}$$

which is a convex optimization problem in (r, δ) [42]. Note that for the above problem to be feasible, we need to have: $E_{21} \geq aE_{11}$, and $E_{12} \geq aE_{22}$. Other consumption strategies will have similar necessary conditions.

To solve the above problem, we first assume that the Lagrange multiplier associated with the last constraint is zero, i.e., the constraint is not binding (this is the energy causality constraint of the first user in the second time slot), and obtain a solution. The solution is optimal if it satisfies that constraint with strict inequality. Otherwise, the constraint is binding, and needs to be satisfied with equality. In the latter case, we substitute $\delta = \frac{1}{1-a^2}(E_{12} - a(E_{22} + r))$ in the objective function and solve a problem of only one variable, r, which can be solved by direct first derivative analysis over the feasible region of r. We now characterize the solution after removing that last constraint. We define $r_1 \triangleq \left(E_{21} - \frac{E_{11}}{a}\right)^+$ and $r_2 \triangleq E_{21} - aE_{11}$ for convenience, and introduce the following Lagrangian

$$\begin{aligned}\mathcal{L} = & -\frac{1}{2}\log\left(1 + \frac{E_{11} - a(E_{21} - r)}{1 - a^2}\right) \\ & - \frac{1}{2}\log\left(1 + \frac{E_{21} - r - aE_{11}}{1 - a^2}\right) \\ & - \frac{1}{2}\log\left(1 + \frac{\delta}{a}\right) - \frac{1}{2}\log(1 + E_{22} + r - \delta) \\ & + \lambda_\delta(\delta - E_{22} - r) - \eta_\delta\delta + \lambda_r(r - r_1) + \eta_r(r_2 - r) \end{aligned} \tag{16}$$

where λ_δ, η_δ, λ_r, and η_r are the non-negative Lagrange multipliers. Taking the derivatives with respect to δ, r, and equating to 0, we get the following

$$\frac{1}{a + \delta} + \eta_\delta = \frac{1}{1 + E_{22} + r - \delta} + \lambda_\delta \tag{17}$$

$$\begin{aligned}\frac{1}{1 + E_{22} + r - \delta} + \frac{a}{1 - a^2 + E_{11} - a(E_{21} - r)} + \eta_r \\ = \frac{1}{1 - a^2 + E_{21} - r - aE_{11}} + \lambda_r \end{aligned} \tag{18}$$

From (17), we solve for δ in terms of r as follows

$$\delta(r) = \begin{cases} 0, & a > 1 + E_{22} + r \\ \frac{1+E_{22}+r-a}{2}, & 1 - (E_{22} + r) \leq a \leq 1 + E_{22} + r \\ E_{22} + r, & a < 1 - (E_{22} + r) \end{cases} \tag{19}$$

Next, we find the optimal value of r. For that, we substitute by $\delta(r)$ in (18). Assuming that the middle expression in (19) holds, we have

$$\eta_r + f_1(r) = \lambda_r + f_2(r) \tag{20}$$

where f_1 and f_2 are given by

$$f_1(r) = \frac{2}{1 + E_{22} + a + r} + \frac{a}{1 - a^2 + E_{11} - aE_{21} + ar} \tag{21}$$

$$f_2(r) = \frac{1}{1 - a^2 + E_{21} - aE_{11} - r} \tag{22}$$

To solve this, we first assume $\lambda_r = \eta_r = 0$, and equate both sides of (20). The existence of a feasible solution of r in this case depends on the extreme values of f_1 and f_2. In particular, since $f_1(r)$ is decreasing in r, while $f_2(r)$ is increasing in r, the solution exists if and only if $f_1(r_2) \leq f_2(r_2)$ and $f_1(r_1) \geq f_2(r_1)$. Note that such solution can be found, for example, by a bisection search. If this condition is not satisfied, then one of the Lagrange multipliers (λ_r, η_r) needs to be strictly positive in order to equate both sides in (20). In particular, if $f_1(r_2) > f_2(r_2)$, then we need $\lambda_r > 0$, which implies by complementary slackness that $r = r_2$. On the other hand, if $f_1(r_1) < f_2(r_1)$, then we need $\eta_r > 0$, which implies by complementary slackness that $r = r_1$. After solving for r, we check if it is consistent with the chosen expression of $\delta(r)$ by checking the conditions in (19). If not, then we check the other two cases: $\delta(r) = 0$ and $\delta(r) = E_{22} + r$, and re-solve for r. The analysis in these cases follows similarly as above. This concludes the solution of the two-slot case. In the next section, we use the above analysis to find the optimal solution in the general case of multiple energy arrivals.

2) Iterative Solution for the General Case: We solve problem (9) iteratively in a two-slot by two-slot manner, starting from the last two slots and going backwards. Once we reach the first two slots, we re-iterate starting from the last two slots, and go backwards again. Iterations stop if the powers do not change after we reach the first two slots. The details are as follows.

We first initialize the energy status of each slot of both users by $\boldsymbol{S}_1 = \boldsymbol{E}_1$ and $\boldsymbol{S}_2 = \boldsymbol{E}_2$, where $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ are vectors of energy arrivals at user 1 and 2, respectively, and solve each slot independently, as discussed in Section II-A, to get an initial feasible power policy $\{\boldsymbol{p}_1^{(0)}, \boldsymbol{p}_2^{(0)}\}$. We then start by examining slots $N-1$ and N. We solve the throughput maximization problem for these two slots with energies $\{S_{1(N-1)}, S_{1N}\}$ and $\{S_{2(N-1)}, S_{2N}\}$ at the first and second user, respectively, as discussed in Section II-B1. After we solve this problem, we update the energy status vectors $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$, and move back one slot to examine slots $N-2$ and $N-1$. We solve the throughput maximization problem for these two slots using the updated energy status $\{S_{1(N-2)}, S_{1(N-1)}\}$ and $\{S_{2(N-2)}, S_{2(N-1)}\}$ at the first and second user, respectively. We update the energy status vector after solving this problem, and continue moving backwards until we solve for slots 1 and 2. After that, we get another feasible power policy $\{\boldsymbol{p}_1^{(1)}, \boldsymbol{p}_2^{(1)}\}$, where the superscript stands for the iteration index. We then compare this power policy with the initial one. If they are the same, we stop. If not, we perform this process again starting from the last two slots, going backwards, until we get an updated power policy $\{\boldsymbol{p}_1^{(2)}, \boldsymbol{p}_2^{(2)}\}$. We stop after the kth iteration if $\boldsymbol{p}_1^{(k-1)} = \boldsymbol{p}_1^k$ and $\boldsymbol{p}_2^{(k-1)} = \boldsymbol{p}_2^k$. Since the sum throughput can only increase with the iterations, and since it is also upper bounded due to the energy constraints, the convergence of the above two-slot iterations is guaranteed.

Next, we check whether the limit point satisfies the KKT optimality conditions. Namely, we solve for the Lagrange multipliers in (11) and (12). If they are all non-negative, then the KKT conditions are satisfied and, by the convexity of the problem, the limit point is optimal [42]. If not, then the energy status vectors need to be updated. This might be the case for instance if while updating some given two slots, more than necessary amount of energy is transferred forward. While this may be optimal with respect to these two slots, it does not take into consideration the energy arrival vectors in the entire N slots. Therefore, in such cases, we perform another round of iterations where we take some of the energy back if this increases the objective function. Taking energy back without violating causality can be done, e.g., via putting measuring meters in between the slots during the two-slot update phase to record the amount of energy moving forward [18]. Since the problem feasibility is maintained with each update, and by the convexity of the problem, cycling through all the slots infinitely often converges to the optimal policy.

This concludes the discussion of the problem with only decoding costs. In the next section, we discuss the case with only processing costs.

III. The Case With Only Processing Costs

A. Single Energy Arrival

In this section, we study the case where each user has only one energy arrival. In this two-way setting, we incorporate the processing costs into our problem as follows: each user incurs a processing cost when it is *on* for either transmitting or receiving or both. We note that due to the processing costs, it might be optimal for the users to be turned on for only a portion of the time. In this case, the transmission scheme becomes *bursty* [40]. At this point, it is not clear whether it is optimal for the two users to be fully synchronized, i.e., switch on/off simultaneously. For instance, it might be the case that the second user's energy is higher, and therefore it uses the channel for a larger portion of the time $\theta_2 > \theta_1$. In this case, the first user stops transmitting after θ_1 amount of the time, but stays on for an extra $\theta_2 - \theta_1$ amount of time to receive the rest of the second user's data. The same argument could hold for the second user if the first user's energy is larger. Therefore, for the general case of $\theta_1 \neq \theta_2$, each user stays on for a $\max\{\theta_1, \theta_2\}$ amount of time. We formulate the problem as

$$\begin{aligned} \max_{\theta_1,\theta_2,p_1,p_2} \quad & \frac{\theta_1}{2}\log(1+p_1) + \frac{\theta_2}{2}\log(1+p_2) \\ \text{s.t.} \quad & \theta_1 p_1 + \max\{\theta_1,\theta_2\}\epsilon_1 \leq E_1 \\ & \theta_2 p_2 + \max\{\theta_1,\theta_2\}\epsilon_2 \leq E_2 \\ & 0 \leq \theta_1, \theta_2 \leq 1 \end{aligned} \tag{23}$$

where ϵ_j is the processing cost per unit time for user j, $j = 1, 2$.

We have the following two lemmas regarding this problem: Lemma 5 states that both users need to use up all of their available energies. Lemma 6 states that both users need to be fully synchronized, i.e., they need to turn on for exactly the same duration of time, and turn off together. Hence, whenever a user is turned on, it both sends and receives data.

Lemma 5: In the optimal solution of problem (23)*, both users exhaust their available energies.*

Proof: This follows by directly noting that if one user does not use all its energy, then we can increase its power until it does. This strictly increases the objective function. ■

Lemma 6: In the optimal solution of problem (23)*, we have* $\theta_1^* = \theta_2^*$.

Proof: We show this by contradiction. Assume without loss of generality that it is optimal to have $\theta_1 < \theta_2$. By Lemma 5, we have the powers given by

$$p_1 = \frac{E_1 - \theta_2\epsilon_1}{\theta_1}, \quad p_2 = \frac{E_2}{\theta_2} - \epsilon_2 \tag{24}$$

Therefore, we rewrite problem (23) as

$$\begin{aligned} \max_{\theta_1,\theta_2} \quad & \frac{\theta_1}{2}\log\left(1 + \frac{E_1 - \theta_2\epsilon_1}{\theta_1}\right) + \frac{\theta_2}{2}\log\left(1 + \frac{E_2}{\theta_2} - \epsilon_2\right) \\ \text{s.t.} \quad & 0 \leq \theta_1 \leq \theta_2 \leq \theta_m \end{aligned} \tag{25}$$

where $\theta_m \triangleq \min\{1, \frac{E_1}{\epsilon_1}, \frac{E_2}{\epsilon_2}\}$ assures positivity of powers. Next, we note that the first term in the objective function above is monotonically increasing in θ_1, and therefore its value is maximized at the boundary of the feasible set, i.e., at $\theta_1 = \theta_2$, which gives a contradiction. ■

By Lemma 6, problem (23) now reduces to having only one time variable $\theta \triangleq \theta_1 = \theta_2$

$$\begin{aligned} \max_{\theta,p_1,p_2} \quad & \frac{\theta}{2}\log(1+p_1) + \frac{\theta}{2}\log(1+p_2) \\ \text{s.t.} \quad & \theta(p_1+\epsilon_1) \leq E_1 \\ & \theta(p_2+\epsilon_2) \leq E_2 \\ & 0 \leq \theta \leq 1 \end{aligned} \tag{26}$$

We will solve (26), and its most general multiple energy arrival version, in the rest of this section. We first note that the problem is non-convex. Applying the change of variables: $\bar{p}_1 \triangleq \theta p_1$, $\bar{p}_2 \triangleq \theta p_2$, we get the following equivalent problem

$$\begin{aligned} \max_{\theta,\bar{p}_1,\bar{p}_2} \quad & \frac{\theta}{2}\log\left(1+\frac{\bar{p}_1}{\theta}\right)+\frac{\theta}{2}\log\left(1+\frac{\bar{p}_2}{\theta}\right) \\ \text{s.t.} \quad & \bar{p}_1+\theta\epsilon_1 \le E_1 \\ & \bar{p}_2+\theta\epsilon_2 \le E_2 \\ & 0 \le \theta \le 1 \end{aligned} \tag{27}$$

which is convex, as the objective function is now concave because it is the perspective of a concave function [42], and the constraints are affine in both variables. Using Lemma 5, we equate the energy constraints and substitute them back in the objective function to get

$$\max_{0\le\theta\le\theta_m} \quad \frac{\theta}{2}\log\left(1+\frac{E_1-\theta\epsilon_1}{\theta}\right)+\frac{\theta}{2}\log\left(1+\frac{E_2-\theta\epsilon_2}{\theta}\right) \tag{28}$$

where θ_m is as in Lemma 6. Note that the objective function in the above problem is concave since the function $x\log(b+c/x)$ is concave in x, for $x>0$, and for any real-valued constants b and c. Since the feasible set is an interval, it then follows that the optimal solution is given by projecting stationary points of the objective function onto the feasible set. Differentiating, we obtain the following equation in θ

$$f_1(\theta)\cdot f_2(\theta)=e^{-2} \tag{29}$$

where the function $f_j(\theta)$, for $j=1,2$, is defined as

$$f_j(\theta) \triangleq \frac{e^{(\epsilon_j-1)/((E_j/\theta)-(\epsilon_j-1))}}{(E_j/\theta)-(\epsilon_j-1)} \tag{30}$$

One can show that $f_j(\theta)$ is monotonically increasing in θ, for all θ feasible. Therefore, (29) has a unique solution in θ, which we denote by $\bar{\theta}$. Finally, the optimal (burstiness factor) θ^* is given by $\theta^*=\min\{\bar{\theta},1\}$.

We note that the value of θ^* can be strictly less than 1, which leads to *bursty transmission* from the two users. The amount of burstiness depends on the available energies at both users and their processing costs, the relation among which is captured by the functions f_1 and f_2 in (29). The two users' energies and processing costs affect each other; one user having relatively low energy or relatively high processing cost can decrease the value of θ^*, i.e., increase the amount of burstiness in the channel. Finally, once the optimal θ^* is found, the optimal powers of the users are found by substituting θ^* in the energy constraints.

B. Multiple Energy Arrivals

We now extend our results to the case of multiple energy arrivals. During slot i, the two users can be turned on for a θ_i portion of the time. We argue that the users have to be synchronized. For if they were not, then given the optimal energy distribution among the slots, we can synchronize both users in each slot independently, which gives higher throughput, as discussed in the single energy arrival scenario. Then, the problem becomes

$$\begin{aligned} \max_{\boldsymbol{\theta},\boldsymbol{p}_1,\boldsymbol{p}_2} \quad & \sum_{i=1}^{N}\frac{\theta_i}{2}\log(1+p_{1i})+\frac{\theta_i}{2}\log(1+p_{2i}) \\ \text{s.t.} \quad & \sum_{i=1}^{k}\theta_i(p_{1i}+\epsilon_1)\le\sum_{i=1}^{k}E_{1i}, \quad \forall k \\ & \sum_{i=1}^{k}\theta_i(p_{2i}+\epsilon_2)\le\sum_{i=1}^{k}E_{2i}, \quad \forall k \\ & 0\le\theta_i\le 1, \quad \forall i \end{aligned} \tag{31}$$

As we did in the single energy arrival case, we apply the change of variables $\bar{p}_{1i}=\theta_i p_{1i}$ and $\bar{p}_{2i}=\theta_i p_{2i}$, $\forall i$, to get the following equivalent convex optimization problem

$$\begin{aligned} \max_{\boldsymbol{\theta},\bar{\boldsymbol{p}}_1,\bar{\boldsymbol{p}}_2} \quad & \sum_{i=1}^{N}\frac{\theta_i}{2}\log\left(1+\frac{\bar{p}_{1i}}{\theta_i}\right)+\frac{\theta_i}{2}\log\left(1+\frac{\bar{p}_{2i}}{\theta_i}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{k}\bar{p}_{1i}+\theta_i\epsilon_1\le\sum_{i=1}^{k}E_{1i}, \quad \forall k \\ & \sum_{i=1}^{k}\bar{p}_{2i}+\theta_i\epsilon_2\le\sum_{i=1}^{k}E_{2i}, \quad \forall k \\ & \bar{p}_{1i}\ge 0, \quad \bar{p}_{2i}\ge 0, \quad 0\le\theta_i\le 1, \quad \forall i \end{aligned} \tag{32}$$

The Lagrangian for this problem is

$$\begin{aligned} \mathcal{L} = & -\left(\sum_{i=1}^{N}\frac{\theta_i}{2}\log\left(1+\frac{\bar{p}_{1i}}{\theta_i}\right)+\frac{\theta_i}{2}\log\left(1+\frac{\bar{p}_{2i}}{\theta_i}\right)\right) \\ & +\sum_{j=1}^{N}\lambda_{1j}\left(\sum_{i=1}^{j}\bar{p}_{1i}+\theta_i\epsilon_1-\sum_{i=1}^{j}E_{1i}\right)-\sum_{i=1}^{N}\eta_{1i}\bar{p}_{1i} \\ & +\sum_{j=1}^{N}\lambda_{2j}\left(\sum_{i=1}^{j}\bar{p}_{2i}+\theta_i\epsilon_2-\sum_{i=1}^{j}E_{2i}\right)-\sum_{i=1}^{N}\eta_{2i}\bar{p}_{2i} \\ & +\sum_{i=1}^{N}\omega_i(\theta_i-1)-\sum_{i=1}^{N}\nu_i\theta_i \end{aligned} \tag{33}$$

where λ_{1i}, η_{1i}, λ_{2i}, η_{2i}, ω_i, ν_i are non-negative Lagrange multipliers. Differentiating with respect to $\bar{p}_{1i}$ and $\bar{p}_{2i}$, we obtain the following KKT optimality conditions

$$\frac{\bar{p}_{1i}}{\theta_i}=\left(\frac{1}{\sum_{j=i}^{N}\lambda_{1j}}-1\right)^+, \quad \frac{\bar{p}_{2i}}{\theta_i}=\left(\frac{1}{\sum_{j=i}^{N}\lambda_{2j}}-1\right)^+ \tag{34}$$

along with the usual complementary slackness conditions [42]. The following two lemmas characterize the optimal power policy for problem (32). The proofs follow as in Lemmas 2 and 3, and are omitted for brevity.

Lemma 7: In the optimal solution of problem (32)*, powers of both users are non-decreasing over time.*

Lemma 8: In the optimal solution of problem (32)*, if a user's energy is saved from one time slot to the next, then the powers spent by this user in the two slots have to be equal.*

Next, we note that the optimal solution of problem (32) is not unique. For instance, assume that one solution of the problem required some energy to be transferred from the ith to the $(i+1)$st slot at both users, and that the optimal values of θ_i and θ_{i+1} are both less than 1. By Lemma 8, since we transferred some energy between the two slots, we must have equal powers in both slots. Now, if we transfer an extra amount of energy between the two slots, this allows us to do the following: 1) decrease the value of θ_i and increase that of θ_{i+1}, and 2) change the value of $\bar{p}_{ji}$ and $\bar{p}_{j(i+1)}$, $j = 1, 2$, correspondingly so that we obtain the same values of powers at the two slots as before. This leaves us with the same value for the objective function, as what we did is that we changed the values of the pre-log factors in a feasible manner while keeping the values inside the logs as they were. We can keep doing this until either slot $i+1$ is completely filled, i.e., $\theta_{i+1} = 1$, or all of the energy is transferred from slot i, i.e., $\theta_i = 0$.

We coin this type of policies as *deferred* policies; no new time slots are opened unless all time slots in the future are completely filled, i.e., $0 < \theta_i \leq 1$ iff $\theta_k = 1, \forall k = i+1, \ldots, N$. Consequently, $\{\theta_i\}_{i=1}^N$ will be non-decreasing. There can only be one unique optimal deferred policy for problem (32). In the sequel, we determine that policy.

1) Optimal Deferred Policy: Finding the optimal deferred policy relies on the fact that, by energy causality, energies can only be used after they have been harvested. To this end, we begin from the last slot, and make sure that it is completely filled, i.e., it has no burstiness, before opening up a previous slot. We apply a modified version of the single energy arrival result iteratively in a backward manner through two main phases: 1) deferring, and 2) refinement. These are illustrated as follows.

We first start by the deferring phase. The goal of this phase is to determine an initial feasible deferred policy. In the refinement phase, the optimality of such policy is investigated. We first initialize the energy status of each slot of both users by $\boldsymbol{S}_1 = \boldsymbol{E}_1$ and $\boldsymbol{S}_2 = \boldsymbol{E}_2$, and start from the last slot and move backwards. In the kth slot, we start by examining the use of the kth slot energies in the kth slot only. This is done using the results of the single energy arrival (29). If the resulting $\theta_k < 1$, then we transfer some energy from previous slots forward to the kth slot until either it is completely filled, i.e., $\theta_k = 1$, or all previous slots' energies are exhausted. We test the possibility of the former condition by moving all energy from a previous slot $l < k$, and re-solving for θ_k. If the result is unity, then the energies of slot l can for sure fill out slot k. Next, we show how much energy is actually needed to do so.

We have two conditions to satisfy: 1) $\theta_k = 1$, and 2) powers of user j in slots l and k are equal, $p_{jl} = p_{jk} \triangleq p'_j$, if user j transfers energy from slot l to k (according to Lemma 8). Let us denote the burstiness in slot l by θ'. Hence, if both users transfer energy, the optimal policy is found by solving the following problem

$$\begin{aligned} \max_{\theta', p'_1, p'_2} \quad & \frac{1+\theta'}{2} \log(1+p'_1) + \frac{1+\theta'}{2} \log(1+p'_2) \\ \text{s.t.} \quad & \left(1+\theta'\right)\left(p'_1 + \epsilon_1\right) = S_{1l} + S_{1k} \\ & \left(1+\theta'\right)\left(p'_2 + \epsilon_2\right) = S_{2l} + S_{2k} \\ & 0 \leq \theta' \leq 1 \end{aligned} \tag{35}$$

Following the same analysis as in the single energy arrival case, we solve

$$f_1\left(1+\theta'\right) \cdot f_2\left(1+\theta'\right) = e^{-2} \tag{36}$$

On the other hand, if only the first user transfers energy, the optimal policy is found by replacing the second constraint in problem (35) by $\theta'(p_{2l} + \epsilon_2) = S_{2l}$, where $p_{2k} = S_{2k} - \epsilon_2$ in this case. This gives the following to solve for θ'

$$f_1\left(1+\theta'\right) \cdot f_2\left(\theta'\right) = e^{-2} \tag{37}$$

Similarly, if the transfer is done only from the second user we solve

$$f_1\left(\theta'\right) \cdot f_2\left(1+\theta'\right) = e^{-2} \tag{38}$$

In all the three cases of energy transfer above, the equations to solve have an increasing left hand side, and hence a unique solution. Finally, the optimal policy is the one that gives the maximum sum throughput among the feasible ones. It is worth noting that, by the concavity of the objective function, transferring energy from both users is optimal if feasible, since it equalizes arguments (powers) of a concave objective function [3].

If the initially resulting $\theta_k = 1$ in the kth slot, we do directional water-filling over the future slots, which gives the optimal sum rate [5]. Next, we check if energy should be transferred from a previous slot l from the first, second, or both users, in exactly the same way as above, i.e., by solving (36)-(38). If energy transfer (from either or both users) is feasible and gives a higher objective function, we do directional water-filling again from slot k over future slots, followed by repeating the above energy transfer checks once more. These inner iterations stop if either no energy transfer occurs, or no directional water-filling occurs. The deferring phase ends after examining the first slot. During this phase, we record how much energy is being moved forward to fill up future slots. Meters are put in between slots for that purpose.

In the refinement phase, the goal is to check whether the currently reached energy distribution is optimal. One reason it might not be optimal is that during the deferring phase, some excess amounts of energy can be transferred from, e.g., slot k forward unnecessarily without taking into account the energies available before slot k. We check the optimality of the deferring phase policy by performing two-slot updates starting from the last two slots going backwards. During the updates, energy can be drawn back from future slots if this increases the objective function as long as it does not violate causality. This can be done by checking the values stored in the meters in between the slots. See [1] for details on how to update a given two slots. We summarize the steps of finding the optimal solution discussed in this section in Algorithm 1.

Algorithm 1 Optimal Deferred Policy

Phase 1: Deferring

Set $\boldsymbol{S}_1 = \boldsymbol{E}_1$, $\boldsymbol{S}_2 = \boldsymbol{E}_2$, $\boldsymbol{m}_1 = \boldsymbol{m}_2 = \boldsymbol{0}$, and $k = N$
while $k \geq 1$ **do**
  Using energies $\{S_{1k}, S_{2k}\}$, solve for θ_k using (29)
  if $\theta_k < 1$ **then**
    repeat
      Transfer all energy from slot $k - l$ to slot k
      Re-solve for θ_k using (29)
      if Slot k is completely filled **then**
        Find energy needed to fill it using (36)-(38)
      else $l \leftarrow \min\{l+1, k-1\}$
      end if
    until $\theta_k = 1$, or all previous energies are exhausted
  else
    repeat
      Directional water-filling over slots $\{k, \ldots, N\}$
      Check for energy transfer using (36)-(38)
    until No water-filling or energy transfer occur
  end if
  Update the energy status values $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$
  Update the meters' values $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$
  $k \leftarrow k - 1$
end while

Phase 2: Refinement

repeat
  for $k = 0 : N - 2$ **do**
    Update the energy status of slots $(N-k-1, N-k)$ taking energy back if needed
  end for
until Meters' values $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$ do not change
$\boldsymbol{p}_1^* = \boldsymbol{S}_1$, and $\boldsymbol{p}_2^* = \boldsymbol{S}_2$.

IV. Decoding and Processing Costs Combined

We have thus far considered throughput maximizing policies for two-way channels with either decoding or processing costs. In this section, we study the general setting with *both* decoding and processing costs. In this setup, user j spends a decoding cost whenever it is receiving the other user's message, and in addition to that, it incurs a processing cost per unit time ϵ_j whenever it is operating. We allow user j to transmit for a θ_j portion of the time, and formulate the general problem where θ_1 can be different than θ_2 as follows

$$\begin{aligned} \max_{\boldsymbol{\theta},\boldsymbol{p}} \quad & \sum_{i=1}^{N} \frac{\theta_{1i}}{2}\log(1+p_{1i}) + \frac{\theta_{2i}}{2}\log(1+p_{2i}) \\ \text{s.t.} \quad & \sum_{i=1}^{k} \theta_{1i}p_{1i} + \theta_{2i}ap_{2i} + \max(\theta_{1i},\theta_{2i})\epsilon_1 \leq \sum_{i=1}^{k} E_{1i}, \ \ \forall k \\ & \sum_{i=1}^{k} \theta_{2i}p_{2i} + \theta_{1i}ap_{1i} + \max(\theta_{1i},\theta_{2i})\epsilon_2 \leq \sum_{i=1}^{k} E_{2i}, \ \ \forall k \\ & 0 \leq \theta_{1i}, \theta_{2i} \leq 1, \quad \forall i \end{aligned} \tag{39}$$

Note that the above problem is a generalization of the problems considered in Sections II and III. On one hand, if we set $a = 0$, i.e., do not consider decoding costs, we get back to problem (31), after applying the synchronization argument to get $\theta_{1i} = \theta_{2i}$, $\forall i$. On the other hand, setting $\epsilon_1 = \epsilon_2 = 0$, i.e., not considering processing costs, and applying the change of variables $\bar{p}_j \triangleq \theta_j p_j$, $j = 1, 2$, we get

$$\begin{aligned} \max_{\boldsymbol{\theta},\bar{\boldsymbol{p}}} \quad & \sum_{i=1}^{N} \frac{\theta_{1i}}{2}\log\left(1+\frac{\bar{p}_{1i}}{\theta_{1i}}\right) + \frac{\theta_{2i}}{2}\log\left(1+\frac{\bar{p}_{2i}}{\theta_{2i}}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{k} \bar{p}_{1i} + a\bar{p}_{2i} \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k \\ & \sum_{i=1}^{k} \bar{p}_{2i} + a\bar{p}_{1i} \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k \\ & 0 \leq \theta_{1i}, \theta_{2i} \leq 1, \quad \forall i \end{aligned} \tag{40}$$

It is direct to see that the objective function is increasing in $\boldsymbol{\theta}_1, \boldsymbol{\theta}_2$, and therefore the maximum is attained at $\boldsymbol{\theta}_1^* = \boldsymbol{\theta}_2{}^* = 1$, i.e., we get back to problem (9). We solve problem (39) in the remainder of this paper.

A. Single Energy Arrival

We first consider the case where each user harvests only one energy packet. Note that (39) is not a convex optimization problem. We apply the change of variables $\bar{p}_j \triangleq \theta_j p_j$, $j = 1, 2$, to get

$$\begin{aligned} \max_{\theta_1,\theta_2,\bar{p}_1,\bar{p}_2} \quad & \frac{\theta_1}{2}\log\left(1+\frac{\bar{p}_1}{\theta_1}\right) + \frac{\theta_2}{2}\log\left(1+\frac{\bar{p}_2}{\theta_2}\right) \\ \text{s.t.} \quad & \bar{p}_1 + a\bar{p}_2 + \max(\theta_1,\theta_2)\epsilon_1 \leq E_1 \\ & \bar{p}_2 + a\bar{p}_1 + \max(\theta_1,\theta_2)\epsilon_2 \leq E_2 \\ & 0 \leq \theta_1, \theta_2 \leq 1 \end{aligned} \tag{41}$$

which is now a convex optimization problem [42]. Next, we have the following lemma.

Lemma 9: In the optimal solution of problem (41), $\theta_1^* = \theta_2^*$.

Proof: Assume, e.g., $\theta_1^* < \theta_2^*$. Setting $\theta_1 = \theta_2^*$ is always feasible since the feasible set is only affected by the maximum of the θ_1 and θ_2. This strictly increases the objective function since it is monotonically increasing in θ_1. ■

Lemma 9 shows that it is optimal for the two users to be fully synchronized; they turn on, exchange information, and then turn off simultaneously, similar to what Lemma 6 states in the scenario with no decoding costs. This reduces the problem to the following

$$\begin{aligned} \max_{\theta,\bar{p}_1,\bar{p}_2} \quad & \frac{\theta}{2}\log\left(1+\frac{\bar{p}_1}{\theta}\right) + \frac{\theta}{2}\log\left(1+\frac{\bar{p}_2}{\theta}\right) \\ \text{s.t.} \quad & \bar{p}_1 + a\bar{p}_2 + \theta\epsilon_1 \leq E_1 \\ & \bar{p}_2 + a\bar{p}_1 + \theta\epsilon_2 \leq E_2 \\ & 0 \leq \theta \leq 1 \end{aligned} \tag{42}$$

We have the following lemma regarding this problem, whose proof is similar to that of Lemma 1.

Lemma 10: In the optimal solution of problem (42)*, at least one user consumes all its energy.*

Next, we solve (42) for the case $a = 1$. By the previous lemma, we have $\bar{p}_1^* + \bar{p}_2^* = \min\{E_1 - \theta^*\epsilon_1, E_2 - \theta^*\epsilon_2\}$, and by concavity of the objective function, we further have $\bar{p}_1^* = \bar{p}_2^*$. Substituting the powers back in the objective function, we get a reduced problem in only one variable θ

$$\max_{0 \le \theta \le \theta_m} \quad \theta \log\left(1 + \frac{\min\{E_1 - \theta\epsilon_1, E_2 - \theta\epsilon_2\}}{2\theta}\right) \tag{43}$$

where $\theta_m \triangleq \min\{1, \frac{E_1}{\epsilon_1}, \frac{E_2}{\epsilon_2}\}$ assures the positivity of the powers. Note that by monotonicity of the log, and non-negativity of θ, we have

$$\theta \log\left(1 + \frac{\min\{E_1 - \theta\epsilon_1, E_2 - \theta\epsilon_2\}}{2\theta}\right) = \min\left\{\theta \log\left(1 + \frac{E_1 - \theta\epsilon_1}{2\theta}\right), \theta \log\left(1 + \frac{E_2 - \theta\epsilon_2}{2\theta}\right)\right\} \tag{44}$$

It is direct to show that each of the terms inside the minimum expression on the right hand side of the above equation is concave in θ, and therefore the minimum of the two is also concave in θ [42]. Hence, problem (43) is a convex optimization problem [42]. Let us define $\bar{\theta} \triangleq \frac{E_1 - E_2}{\epsilon_1 - \epsilon_2}$ as the value of θ at which $E_1 - \theta\epsilon_1 = E_2 - \theta\epsilon_2$. We now consider two different cases.

The first case is when $\bar{\theta} \notin [0, \theta_m]$, then the minimum expression in the objective function reduces to only one of its two terms for all θ feasible. Let us assume without loss of generality that it is equal to $E_1 - \theta\epsilon_1$. Hence, taking the derivative of the objective function and setting it to 0, we solve the following for θ

$$\log\left(1 - \frac{\epsilon_1}{2} - \frac{E_1}{2\theta}\right) = \frac{E_1/2\theta}{1 - \epsilon_1/2 + E_1/2\theta} \tag{45}$$

The above equation has a unique solution since both sides are monotone in θ; the term on the left is higher than the term on the right as θ approaches 0; and is lower than the term on the right as θ approaches $\frac{E_1}{\epsilon_1}$. We denote this unique solution by $\hat{\theta}$. We note that in this problem, we always have $\theta^* > 0$; we also have $\theta^* = \theta_m$ only if $\theta_m = 1$, or else the throughput is zero. Thus, if $\theta_m < 1$, then $\hat{\theta}$ is always feasible and $\theta^* = \hat{\theta}$. While if $\theta_m = 1$, then $\hat{\theta}$ might not be feasible, and therefore in general we have $\theta^* = \min\{\hat{\theta}, 1\}$. This concludes the first case.

The second case is when $\bar{\theta} \in [0, \theta_m]$. In this case, depending on the sign of $\epsilon_1 - \epsilon_2$, the minimum expression in the objective function is given by one term in the interval $[0, \bar{\theta}]$ (let us assume it to be $E_1 - \theta\epsilon_1$ without loss of generality), and is given by the other term ($E_2 - \theta\epsilon_2$) in the interval $[\bar{\theta}, \theta_m]$. We solve the problem in this case sequentially as follows: We solve (45) for $\hat{\theta}_1$ and compute $\theta_1^* = \min\{\hat{\theta}_1, 1\}$. If θ_1^* is less than $\bar{\theta}$ then, by concavity of the objective function, it is the optimal solution. Else, if $\theta_1^* \ge \bar{\theta}$, we solve the following equation

$$\log\left(1 - \frac{\epsilon_2}{2} - \frac{E_2}{2\theta}\right) = \frac{E_2/2\theta}{1 - \epsilon_2/2 + E_2/2\theta} \tag{46}$$

for $\hat{\theta}_2$ and compute $\theta_2^* = \min\{\hat{\theta}_2, 1\}$, which will now be no less than $\bar{\theta}$, and is equal to the optimal solution. We finally note that $\theta^* = \bar{\theta}$ iff $\theta_1^* = \theta_2^* = \bar{\theta}$. This concludes the second case.

Next, we discuss the case $a < 1$ (similar arguments follow for the case $a > 1$, and are omitted for brevity). We have the following lemma in this case, whose proof is similar to that of Lemma 1.

Lemma 11: If the energies and processing costs are such that $E_1 - \theta\epsilon_1$ is less (resp., larger) than $E_2 - \theta\epsilon_2$ for all θ feasible, then the first (resp., second) user consumes all its energy.

We solve the problem by assuming the situation of the above lemma is true, i.e., one user is energy tight for all θ feasible. If this is not the case, then as we did in the $a = 1$ case above, we solve the problem twice assuming one user is tight at each time, and check which is feasible (or equivalently pick the solution with higher sum throughput). Thus, without loss of generality, we assume the first user consumes all its energy, i.e., we have $\bar{p}_1 = E_1 - \theta\epsilon_1 - a\bar{p}_2$. Substituting this in problem (42), we get the following

$$\begin{aligned} \max_{\theta, \bar{p}_2} \quad & \frac{\theta}{2} \log\left(1 + \frac{E_1 - \theta\epsilon_1 - a\bar{p}_2}{\theta}\right) + \frac{\theta}{2} \log\left(1 + \frac{\bar{p}_2}{\theta}\right) \\ \text{s.t.} \quad & 0 \le \bar{p}_2 \le \frac{E_1 - \theta\epsilon_1}{a} \\ & \bar{p}_2 \le \frac{E_2 - aE_1 - \theta(\epsilon_2 - a\epsilon_1)}{1 - a^2} \\ & 0 \le \theta \le \theta_m \end{aligned} \tag{47}$$

where the upper bound in the first constraint assures the non-negativity of the first user's power. We note that if $\bar{p}_2^* \in \{0, \frac{E_1 - \theta^*\epsilon_1}{a}\}$, i.e., if either of the two users is not transmitting, the problem reduces to the following in terms of only one variable θ

$$\max_{0 \le \theta \le \theta_m} \quad \frac{\theta}{2} \log\left(1 + \frac{E_1 - \theta\epsilon_1}{\theta}\right) \tag{48}$$

which can be solved in a similar manner as we solved problem (43). On the other hand, if the third constraint is tight, i.e., if the second user also consumes all its energy, the problem becomes

$$\begin{aligned} \max_{\tilde{\theta}_l \le \theta \le \tilde{\theta}_m} \quad & \frac{\theta}{2} \log\left(1 + \frac{E_2 - aE_1 - \theta(\epsilon_2 - a\epsilon_1)}{(1 - a^2)\theta}\right) \\ & + \frac{\theta}{2} \log\left(1 + \frac{E_1 - aE_2 - \theta(\epsilon_1 - a\epsilon_2)}{(1 - a^2)\theta}\right) \end{aligned} \tag{49}$$

where $\tilde{\theta}_l$ and $\tilde{\theta}_m$ are such that $E_1 - aE_2 \ge \theta(\epsilon_1 - a\epsilon_2)$ and $E_2 - aE_1 \ge \theta(\epsilon_2 - a\epsilon_1)$, i.e., to assure non-negativity of powers. Note that the objective function in the above problem is concave. Hence, following a Lagrangian approach [42], we solve the following for θ

$$\tilde{f}_1(\theta) \cdot \tilde{f}_2(\theta) = e^{-2} \tag{50}$$

where $\tilde{f}_j(\theta)$, $j = 1, 2$ is defined as

$$\tilde{f}_j(\theta) \triangleq \frac{e^{(\tilde{\epsilon}_j - 1)/(\tilde{E}_j/\theta) - (\tilde{\epsilon}_j - 1)}}{\left(\tilde{E}_j/\theta\right) - \left(\tilde{\epsilon}_j - 1\right)} \tag{51}$$

with $\tilde{E}_j \triangleq E_j - aE_k$ and $\tilde{\epsilon}_j \triangleq \epsilon_j - a\epsilon_k$, $j \ne k$. We note that the above equation is similar to (29), in the case with only processing costs. It can be shown by simple first derivative analysis that $\tilde{f}_1$ and $\tilde{f}_2$ are both increasing in θ, and therefore (50) has a unique solution. Let us denote such solution by $\tilde{\theta}$. Finally, by concavity of the objective function, the optimal θ^* in this case is given by projecting $\tilde{\theta}$ onto the feasible set $\{\theta{:}\tilde{\theta}_l \le \theta \le \tilde{\theta}_m\}$ [42].

Now that we know how to solve problem (47) when either of the first two constraints is tight, we proceed to solve the problem in general as follows. We first solve the problem assuming $\bar{p}_2^*$ is an interior point, i.e., neither of the first two constraints is tight. If the solution in this case is feasible, then it is optimal. Else, by concavity of the objective function, we project the solution onto the feasible set $\{\bar{p}_2 : 0 \le \bar{p}_2 \le \min\{\frac{E_1-\theta\epsilon_1}{a}, \frac{E_2-aE_1-\theta(\epsilon_2-a\epsilon_1)}{1-a^2}\}\}$. In case $\bar{p}_2$ is given by the upper limit in this feasible set, we solve the problem twice assuming the minimum expression is given by one of its terms in each, and pick the one with higher throughput.

Finally, it remains to present the interior point solution. We introduce the following Lagrangian for the problem in this case

$$\mathcal{L} = -\frac{\theta}{2}\log\left(1+\frac{E_1-\theta\epsilon_1-a\bar{p}_2}{\theta}\right) - \frac{\theta}{2}\log\left(1+\frac{\bar{p}_2}{\theta}\right) + \omega(\theta-\theta_m) \tag{52}$$

Taking the derivative with respect to $\bar{p}_2$ and θ and equating to 0, we get the following

$$a\left(1+\frac{\bar{p}_2}{\theta}\right) = 1-\epsilon_1+\frac{E_1-a\bar{p}_2}{\theta} \tag{53}$$

$$\log\left(1+\frac{\bar{p}_2}{\theta}\right)+\log\left(1-\epsilon_1+\frac{E_1-a\bar{p}_2}{\theta}\right) = \frac{\bar{p}_2/\theta}{1+\bar{p}_2/\theta}+\frac{(E_1-a\bar{p}_2)/\theta}{1-\epsilon_1+(E_1-a\bar{p}_2)/\theta}+\omega \tag{54}$$

substituting the first equation in the second, and denoting $y \triangleq 1+\bar{p}_2/\theta$, we further get

$$\log(y) = 1-\frac{1}{2}\log(a)-\frac{\frac{1}{2}(1+(1-\epsilon_1)/a)}{y}+\omega/2 \tag{55}$$

which has a unique solution, y^*, for $y \ge 1$. If $\omega^* > 0$, then by complementary slackness, $\theta^* = \theta_m$, and $\bar{p}_2^*$ is found by substituting in (53), else if $\omega^* = 0$, then θ^* is found by substituting y^* also in (53). By that, we conclude our analysis of the single arrival case.

B. Multiple Energy Arrivals

In this section, we study the multiple energy arrival problem. Following the same synchronization argument as in Section III-B, problem (39) reduces to

$$\begin{aligned} \max_{\boldsymbol{\theta},\bar{\boldsymbol{p}}_1,\bar{\boldsymbol{p}}_2} \quad & \sum_{i=1}^{N}\frac{\theta_i}{2}\log\left(1+\frac{\bar{p}_{1i}}{\theta_i}\right)+\sum_{i=1}^{N}\frac{\theta_i}{2}\log\left(1+\frac{\bar{p}_{2i}}{\theta_i}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{k}\bar{p}_{1i}+a\bar{p}_{2i}+\theta_i\epsilon_1 \le \sum_{i=1}^{k}E_{1i}, \quad \forall k \\ & \sum_{i=1}^{k}\bar{p}_{2i}+a\bar{p}_{1i}+\theta_i\epsilon_2 \le \sum_{i=1}^{k}E_{2i}, \quad \forall k \\ & 0 \le \theta_i \le 1, \quad \forall i \end{aligned} \tag{56}$$

which is a convex optimization problem [42]. The Lagrangian is

$$\begin{aligned} \mathcal{L} = & -\sum_{i=1}^{N}\frac{\theta_i}{2}\log\left(1+\frac{\bar{p}_{1i}}{\theta_i}\right)-\sum_{i=1}^{N}\frac{\theta_i}{2}\log\left(1+\frac{\bar{p}_{2i}}{\theta_i}\right) \\ & +\sum_{k=1}^{N}\lambda_{1k}\left(\sum_{i=1}^{k}\bar{p}_{1i}+a\bar{p}_{2i}+\theta_i\epsilon_1-\sum_{i=1}^{k}E_{1i}\right) \\ & +\sum_{k=1}^{N}\lambda_{2k}\left(\sum_{i=1}^{k}\bar{p}_{2i}+a\bar{p}_{1i}+\theta_i\epsilon_2-\sum_{i=1}^{k}E_{2i}\right) \\ & +\sum_{i=1}^{N}\omega_i(\theta_i-1)-\sum_{i=1}^{N}\eta_i\theta_i \end{aligned} \tag{57}$$

Taking the derivative with respect to $\bar{p}_{1i}$ and $\bar{p}_{2i}$ and equating to 0 we get

$$\frac{\bar{p}_{1i}}{\theta_i} = \left(\frac{1}{\sum_{k=i}^{N}\lambda_{1k}+a\lambda_{2k}}-1\right)^+ \tag{58}$$

$$\frac{\bar{p}_{2i}}{\theta_i} = \left(\frac{1}{\sum_{k=i}^{N}\lambda_{2k}+a\lambda_{1k}}-1\right)^+ \tag{59}$$

along with the complementary slackness conditions [42]. Therefore, we have the following lemma for this problem. The proof follows using similar arguments as in Lemmas 2, 3, and 4.

Lemma 12: In the optimal policy of problem (56)*, the powers of both users are non-decreasing; increase only if at least one user consumes all energy; and increase synchronously.*

We note that, as discussed in Section III-B, the optimal policy for problem (56) is not unique. Using similar arguments, any optimal policy can be transferred into a (unique) deferred policy. Hence, in the remainder of this paper, we find the optimal deferred policy for problem (56). We present an algorithm that is a combination of the ideas used in Sections II and III as follows.

We start by a deferring phase similar to the one discussed in Section III-B1. We highlight the main differences in the following. First, to determine how much energy is needed to be transferred to fill a given slot k from a previous slot l, we assume that both users transfer energy, and similar to problem (35), we solve the following single energy arrival problem

$$\begin{aligned} \max_{\theta,\bar{p}_1,\bar{p}_2} \quad & \frac{1+\theta}{2}\log\left(1+\frac{\bar{p}_1}{1+\theta}\right)+\frac{1+\theta}{2}\log\left(1+\frac{\bar{p}_2}{1+\theta}\right) \\ \text{s.t.} \quad & \bar{p}_1+a\bar{p}_2+(1+\theta)\epsilon_1 \le S_{1l}+S_{1k} \\ & \bar{p}_2+a\bar{p}_1+(1+\theta)\epsilon_2 \le S_{2l}+S_{2k} \\ & 0 \le \theta \le 1 \end{aligned} \tag{60}$$

After solving this problem, we set $\theta_{k-1} = \theta^*$, and $p_{j(k-1)} = p_{jk} = (1+\theta^*)\bar{p}_j^*$, $j = 1, 2$. The resulting policy is optimal if feasible since it equalizes powers [3]. If not, then we need to check the other ways of transfer, namely, transferring from the first user only, or from the second user only. We also need to assume an energy consumption strategy in slot k, i.e., which user consumes all its energy. We solve for all possible

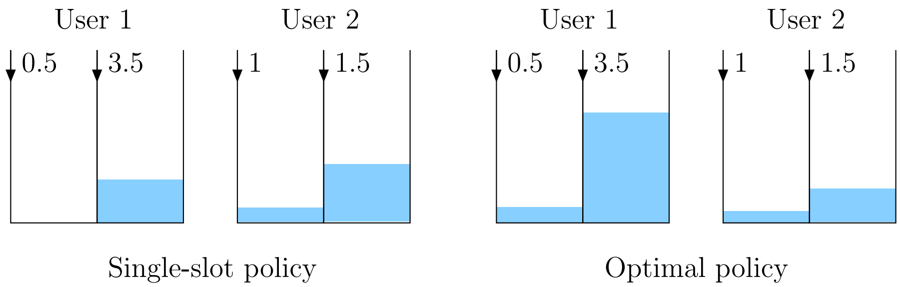

Fig. 2. Two-slot system with only decoding costs.

strategies, and pick the one with maximum sum throughput among the feasible ones. We highlight the solution of one energy consumption strategy in the following discussion. The rest follows similarly.

We discuss the strategy of transferring energy only from the second user in slot l, and that the second user consumes all its energy in slot k. Towards that end, we first fix $\theta_l = \theta$, and then, as discussed in Section II-B, we solve the following equivalent problem in (r, δ)

$$\begin{aligned}
\max_{r,\delta} \quad & \frac{1}{2}\log\left(1+\frac{S_{1l}-aS_{2l}-\theta(\epsilon_1-a\epsilon_2)+ar}{\theta\left(1-a^2\right)}\right) \\
& +\frac{1}{2}\log\left(1+\frac{S_{2l}-aS_{1l}-\theta(\epsilon_2-a\epsilon_1)-r}{\theta\left(1-a^2\right)}\right) \\
& +\frac{1}{2}\log\left(1+\frac{\delta}{a}\right)+\frac{1}{2}\log(1+S_{2k}+r-\delta) \\
\text{s.t.} \quad & 0 \le \delta \le S_{2k}-\epsilon_2+r \\
& r \ge \left(\frac{aS_{2l}-S_{1l}+\theta(\epsilon_1-a\epsilon_2)}{a}\right)^+ \\
& r \le \min\{S_{2l}, S_{2l}-aS_{1l}-\theta(\epsilon_2-a\epsilon_1)\} \\
& \delta \le \frac{a}{1-a^2}(S_{1k}-aS_{2k}-(\epsilon_1-a\epsilon_2)-ar) \qquad (61)
\end{aligned}$$

We note that the above problem is exactly the same as problem (15) if we set $\theta = 1$, and $\epsilon_1 = \epsilon_2 = 0$. With processing costs, the problem can be solved similarly. We solve the above problem for all given θ and do a one dimensional line search to find the optimal θ_l^*.

By the end of the deferring phase above, there will exist a time slot k^*, after which all time slots are completely filled, and before which all time slots are empty, i.e., we will have $\theta_l = 1, \forall l > k^*$; $\theta_l = 0, \forall l < k^*$; and $\theta_{k^*} \le 1$. We can now focus on the non-empty time slots $k^*, \dots, N$. Each will have a certain energy distribution $\{S_{ji}\}_{i=k^*}^N$, $j = 1, 2$, from the deferring phase. We also record the amount of energy transferred to future slots in meters as we did in Section II-B. Next, we check if such energy distributions need improvement. We note that if $\theta_{k^*} = 1$, then the problem becomes a decoding cost problem that can be solved iteratively as discussed in Section II-B with equivalent energies: $\{S_{ji} - \epsilon_j\}_{i=k^*}^N$, $j = 1, 2$. If $\theta_{k^*} < 1$, however, then as we reach slots $\{k^*, k^* + 1\}$ in the two-slot updates, we update the distributions by finding the best energy transfer strategy, i.e., transfer from only one or both users, as discussed in problems (60) and (61). Iterations converge to the optimal solution.

V. Numerical Results

A. Deterministic Arrivals

In this section we present numerical examples to further illustrate our results. We begin by the building blocks of the proposed algorithms; two-slot systems. We start with the case with only decoding costs and consider a system with energies $\boldsymbol{E}_1 = [0.5, 3.5]$ and $\boldsymbol{E}_2 = [1, 1.5]$. The decoding power factor is equal to $a = 0.5$. We first solve for each slot independently using the single arrival result to get $\boldsymbol{p}_1 = [0, 1]$ and $\boldsymbol{p}_2 = [0.33, 1.33]$. Then, we find the optimal solution as discussed in Section II-B1. First, we check the constant-power strategy, where neither user consumes its energy in the first slot, and solve a single arrival problem with average energy arrivals $\bar{E}_1 = 2$ and $\bar{E}_2 = 1.25$ to get $\bar{p}_1 = 1.75$ and $\bar{p}_2 = 0.375$, which are found infeasible. Thus, we move to check the second consumption strategy: the first user consumes all energy in the first slot while the second user consumes all energy in the second slot, i.e., we solve problem (15). We first remove the last constraint, and take $\delta(r) = \frac{1+E_{22}+r-a}{2}$, the middle term of (19), and solve for r using (20). This gives $r = 0.55$, which satisfies the middle constraint in (19), thus the assumed $\delta(r)$ is correct, and gives $\delta = 1.27$. Finally, we check the relaxed (last) constraint of (15); we find that it is satisfied with strict inequality. Therefore, $(r^* = 0.55, \delta^* = 1.27)$ is the optimal solution for this consumption strategy. The corresponding powers are given by $\boldsymbol{p}_1 = [0.36, 2.55]$ and $\boldsymbol{p}_2 = [0.26, 0.77]$. Next, we check the other strategies. Among the feasible ones, we find that the maximum throughput is given by that of the second strategy above, and is therefore the optimal solution of this two-slot system. In Fig. 2, we show the single-slot solution on the left and the optimal solution on the right of the figure. The height of the water in blue represents the power level of a user in a given slot. We note that the first user's optimal power in the first slot is larger than the corresponding single-slot power allocation. That is because the second user's optimal power is smaller than the single-slot power allocation, which gives more room for the first user to transmit. This shows how decoding costs closely couple the performance of the two users.

Next, we consider the case with only processing costs, with energies $\boldsymbol{E}_1 = [0.5, 1]$ and $\boldsymbol{E}_2 = [1, 1]$, and processing costs $\epsilon_1 = 0.5$ and $\epsilon_2 = 0.4$. In Fig. 3, we present one feasible, and two optimal, power policies. The height of the water levels in blue represents the actual transmit powers $\{p_{1i}, p_{2i}\}$, while the width represents the burstiness $\{\theta_i\}$, for $i = 1, 2$. On the left,

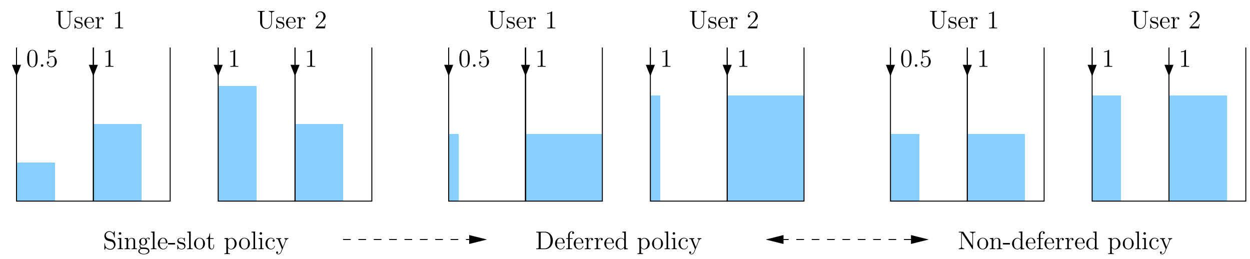

Fig. 3. Optimal deferred policy in a two-slot system with only processing costs.

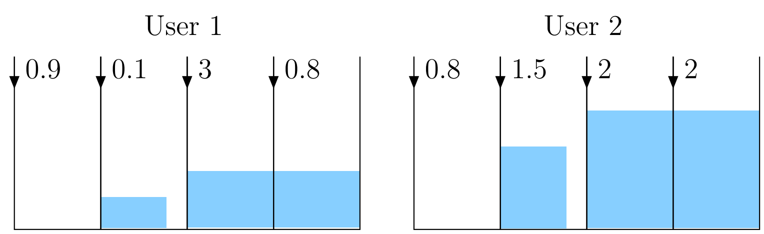

Fig. 4. Optimal policy in a four-slot system with both decoding and processing costs.

we solve for each slot independently using the single arrival result. This gives a non-deferred policy with $\boldsymbol{\theta} = [0.47, 0.65]$, $\boldsymbol{p}_1 = [0.57, 1.04]$, $\boldsymbol{p}_2 = [1.75, 1.14]$, and a sum throughput equal to 0.541. We then transfer all the energy from the 1st to the 2nd slot and re-solve for θ_2 using (29). The result is $\theta_2 = 1$, which means that the 1st slot's energies are capable of totally filling the 2nd slot. We therefore compute the exact amount needed to do so by setting $\theta_2 = 1$ and solving for $\theta_1 = \theta'$ assuming both users transfer energy, i.e., using (36). This gives $\theta_1 = 0.122$, $\boldsymbol{p}_1^* = [0.84, 0.84]$, $\boldsymbol{p}_2^* = [1.39, 1.39]$, and a sum throughput equal to 1.656. This transfer strategy is found feasible, and hence optimal. We show the optimal deferred policy at the middle of Fig. 3. Finally, on the right of Fig. 3, we show another optimal, yet non-deferred, power policy. This is simply done by shifting some of the water back, in a feasible manner, from slot 2 to slot 1. Namely, we increase the value of θ_1 to 0.35 and decrease that of θ_2 to 0.772, with the same transmit powers. This is a feasible non-deferred policy, and gives the same objective function of 1.656. This shows the non-uniqueness of the solution of problem (32).

We now solve a more involved four-slot system with energies $\boldsymbol{E}_1 = [0.9, 0.1, 3, 0.8]$ and $\boldsymbol{E}_2 = [0.8, 1.5, 2, 2]$. Here we consider both decoding and processing costs with parameters $a = 0.7$, $\epsilon_1 = 0.3$, and $\epsilon_2 = 0.6$. We begin by the initialization step; filling up later slots first in a backward manner. This leaves us with an energy distribution of $\boldsymbol{S}_1 = [0, 1, 1.7788, 2.021]$ and $\boldsymbol{S}_2 = [0, 0.936, 3.236, 2.128]$ at the first and the second user, respectively. We then begin the two-slot updates to check whether the given distributions need improvement. With the possibility of drawing back energy as feasible as imposed by the meters put between slots, our algorithm converges to the optimal solution in 8 iterations. The optimal powers are given by $\boldsymbol{p}_1^* = [0, 0.3585, 0.65, 0.65]$, $\boldsymbol{p}_2^* = [0, 0.9407, 1.357, 1.357]$, and the deferred burstiness is given by $\boldsymbol{\theta}^* = [0, 0.76, 1, 1]$. We see that the optimal powers are non-decreasing, and increase synchronously, as stated in Lemma 12, and that $\{\theta_i^*\}$ is non-decreasing, which is an attribute of a deferred policy. The optimal policy is shown in Fig. 4. Next, we remove the decoding costs and solve the same problem with only processing costs as discussed in Section III-B. We reach the optimal deferred policy after 5 iterations, which is given by $\boldsymbol{p}_1^* = [0.67, 0.67, 1.6, 1.6]$, $\boldsymbol{p}_2^* = [1.47, 1.47, 1.47, 1.47]$, and $\boldsymbol{\theta}^* = [0.033, 1, 1, 1]$. We notice that the first time slot is utilized in this case, when the decoding costs are removed. Finally, we remove the processing costs and solve the same problem with only decoding costs as discussed in Section II-B. After 7 iterations, we get the optimal $\boldsymbol{p}_1^* = [0.1, 0.1, 0.8, 0.8]$ and $\boldsymbol{p}_2^* = [0.57, 0.57, 1.57, 1.57]$.

In Fig. 5, we show the effect of decoding and processing costs on the sum rate. We consider a five-slot system with $\boldsymbol{E}_1 = [2, 3, 1, 1, 5]$ and $\boldsymbol{E}_2 = [4, 2, 2, 3, 3]$. Initially we set $a = 0.7$, $\epsilon_1 = 0.8$, and $\epsilon_2 = 0.5$. We then vary one parameter and fix the rest, and observe how it affects the sum rate. As expected, adding costs decreases the achievable throughput as we see from the figure. We also note that the sum rate is almost constant for initial small values of ϵ_2. That is due to the fact that the second user's processing costs are not the bottleneck to the system in this range. In fact, the first user is the bottleneck in this range. This shows how the two users are strongly coupled in this two-way setting with decoding and processing costs.

B. Stochastic Arrivals

We now discuss online scenarios where energy is known causally after being harvested, while only its statistics is known a priori. We present a best effort online scheme to compare with our optimal offline solution. Namely, we assume that the energy harvesting process is i.i.d. with mean μ, and that in time slot i, the jth user energy consumption is bounded by $\min\{b_{ji}, \mu\}$, where b_{ji} is the battery state of user j in slot i, capturing the energy arrival at slot i, E_{ji}, and the residual from previous slots, if any. This scheme decouples the multiple

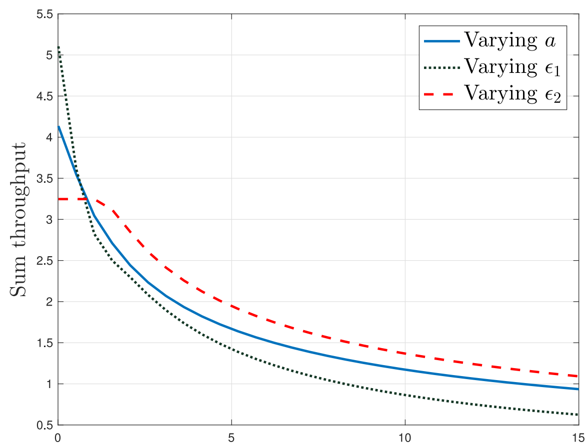

Fig. 5. Effect of processing and decoding costs on the sum rate in a five-slot system.

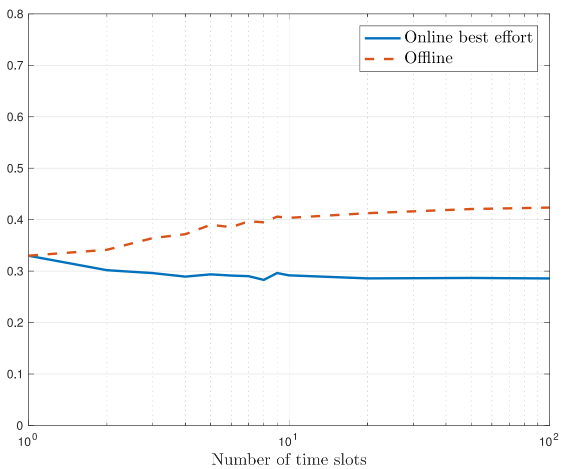

Fig. 6. Comparison of an online best effort scheme and the optimal offline scheme.

arrival problem into N single arrival problems that can be solved as discussed in Section IV-A, without violating the causal knowledge of the energy arrival information. In Fig. 6, we plot the average throughput of this online policy for different time slots, and compare it with the optimal offline policy discussed in Section IV-B. Energies follow a uniform distribution on $[0, 3]$, processing costs are $\epsilon_1 = 0.8$ and $\epsilon_2 = 0.5$, and the decoding cost factor is $a = 0.7$. We run the simulations multiple times for every time slot and take the average, and then plot the sum rate divided by the number of time slots. We see from the figure that as the number of time slots increases, the gap between the online and the offline throughputs increases, and then converges to a constant value. This is due to the fact that in this best effort policy the problem is decoupled as discussed above, and the optimal energy distribution among the slots is no longer achieved, and therefore, the loss of optimality increases with the increase in the number of slots. However, as N grows large, and since we are using i.i.d. arrivals, the best effort policy's loss with respect to the optimal offline one converges to a constant value.

VI. Conclusion

We designed throughput-optimal offline power scheduling policies in an energy harvesting two-way channel where users incur decoding and processing costs. Each user spends a decoding power that is an exponential function of the incoming rate, and in addition, incurs a constant processing power as long is it is communicating. We first studied the case with only decoding costs, followed by that with only processing costs. We then formulated the general problem with both decoding and processing costs in a single setting, and provided an iterative algorithm to find the optimal power policy in this case using insights from the solutions of the case with only decoding and only processing costs.

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