Online Scheduling for Energy Harvesting Channels With Processing Costs

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***Abstract*—We consider scheduling for energy harvesting channels in which the users incur processing cost per unit time that they are on. The presence of processing costs forces the users to operate in a bursty mode. We consider *online* transmission scheduling where the users know the energy harvests only causally as they happen, and need to determine the optimum transmit powers and the optimum burst durations on the fly. We first consider the single-user channel. For this case, we first consider the case of independent and identically distributed (i.i.d.) Bernoulli energy arrivals, and then extend our analysis to the case of general i.i.d. energy arrivals. We determine the *exactly optimum* online policy for Bernoulli arrivals and propose a *nearly optimum* online policy for general arrivals. The proposed policy is near-optimum in that it performs within a constant gap from the optimum policy for all energy arrivals and battery-sizes. We then extend our analysis to the case of two-way energy harvesting channels with processing costs; in this case, the users incur processing costs for being on for transmitting or receiving data. We consider the case where the users harvest energy from fully correlated energy sources. We determine the *exactly optimum* online power schedule for i.i.d. Bernoulli energy arrivals and develop a *nearly optimum* online power schedule for general i.i.d. energy arrivals. Our proposed policy is distributed, which users can apply independently with no need for cooperation or coordination between them.**

***Index Terms*—Energy harvesting communications, online scheduling, processing costs, two-way channels, near-optimal policy, fully-correlated.**

I. Introduction

WE FIRST consider a single-user energy harvesting channel, see Fig. 1, where the transmitter incurs a processing cost per unit time that it is on. The processing cost is the power consumed by the transmitter to be on and transmitting. This cost forces the transmitter to transmit in bursts instead of transmitting continually. The transmitter has a finite-sized battery, which is recharged by an exogenous i.i.d. energy harvesting process. We consider the problem of *online* scheduling, where the transmitter knows the energy arrivals only causally, and needs to determine a power allocation and burst length policy with only a causal knowledge of the energy arrivals. We then extend our analysis to the case of a two-way energy harvesting channel, Fig. 2, where users harvest energy from a fully correlated energy source. The users have finite but arbitrary-sized batteries to save unused energy for future use. Each user is subject to an arbitrary processing cost which accounts for power used per unit time by the user for being on to transmit or receive data. The processing costs force users to operate in bursty modes, where they do not utilize the entire duration available for communication. The users need to determine their power allocation and burst length policies based only on causal knowledge of energy arrivals.

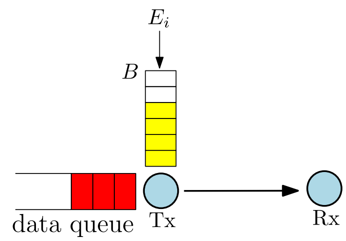

Fig. 1. Single-user energy harvesting channel.

Offline scheduling, where the transmitter knows the energy harvesting profile non-causally ahead of time, has been considered extensively in [1]–[21], starting with the single-user channel [1]–[4], extending to multi-user and multi-hop settings [5]–[16], and incorporating processing costs [17]–[21] which lead to bursty communication as in glue-pouring in [22]. Early work in *online* scheduling, where the transmitter gets to know the energy harvesting profile only causally [3], [4], [23]–[31] has formulated the problem using dynamic programming and Markov decision process techniques.

In this paper, we follow the unique approach developed in [32]–[34] for *online* scheduling in a single-user energy harvesting system with a finite-sized battery. This approach has recently been extended to multiple access [35], [36] and broadcast [37], [38] settings. In this paper, we extend this approach for the single-user and two-way channels with processing costs. For the single-user case, our paper may be viewed as an extension of the online setting in [33] to incorporate processing costs at the transmitter, or equivalently, as an extension of the offline setting in [17] and [18] which consider processing costs to an online setting. In addition, we further extend our

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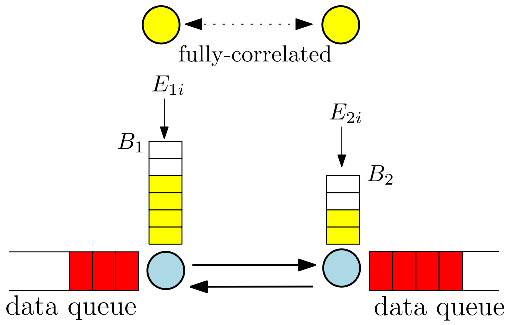

Fig. 2. Two-way energy harvesting channel with fully-correlated energy arrivals.

setting from consideration of a single-user channel to the case of two-way channels.

For the single-user case, as is common in this approach [32]–[38], we first consider the case of i.i.d. Bernoulli energy arrivals, where the energy arrival amount is either zero or equals the size of the battery, i.e., either no energy arrives or the energy arrival fills the battery resetting the system. For this case, we determine the exactly optimal online transmission policy. We show that the optimum transmit power decreases exponentially between energy arrivals. Due to the presence of processing costs, there may exist bursts in the transmission, i.e., slots may not be fully utilized. We show that the bursty transmission can only occur in the last slot. We also show that the total transmission duration decreases with the processing cost. Next, we consider the case of general i.i.d. energy arrivals, and propose a sub-optimal policy. We develop multiplicative and additive lower bounds on the performance of the proposed policy, and a universal upper bound for the performance of any online policy with processing costs. We show that the developed lower and upper bounds are within a constant gap for all energy arrivals and battery sizes; hence, the proposed sub-optimal policy performs within a constant gap from the optimal policy.

We then consider the two-way channel with fully-correlated energy arrivals, see Fig. 2. This may happen in practice if the users harvest energy from a common source, which may occur, for instance, if the users are within a close proximity of each other and are exposed to the same energy harvesting source. We note that even though the energy arrivals are fully-correlated, the energy intakes of the users are different due to their different battery sizes, see Fig. 2. As in the single-user case, we first consider i.i.d. Bernoulli arrivals where each energy arrival amount is either zero or larger than the sizes of both batteries so it fills both batteries simultaneously resetting the system. We show that the optimum powers of the users decrease over time, and the on-off times of the users are fully synchronized. We show that a burst may occur only in the last slot. Next, we consider the case of general i.i.d. energy arrivals. For this case, we propose a distributed sub-optimal policy for power and burst duration selection. The policy is fully distributed and can be applied by each user independently without a need for cooperation or coordination. We develop multiplicative and additive lower bounds on the performance of the proposed policy. We show that the proposed sub-optimal policy is near-optimal in that it performs within a constant gap of the optimal policy for all energy arrival processes and sizes of the batteries at the users.

II. Single-User Channel

We first consider a single-user energy harvesting channel, see Fig. 1. The transmitter has a battery of size B. Time is slotted. The amount of energy in the battery, b_i, evolves as:

$$b_{i+1} = \min\{B, b_i - \theta_i(P_i + \epsilon) + E_{i+1}\} \tag{1}$$

where E_i is the energy harvested in slot i, ϵ is the processing cost (power) per unit time, and θ_i is the duration in slot i that the transmitter is on and transmitting. In (1), $\theta_i P_i$ is the energy spent for transmission, and $\theta_i \epsilon$ is the energy spent for being on.

The physical layer is Gaussian with rate transmitted in slot i [39],

$$r_i = \frac{\theta_i}{2}\log(1 + P_i) \tag{2}$$

where P_i is the allocated power and θ_i is the transmission duration in slot i. These two variables satisfy $\theta_i(P_i + \epsilon) \leq b_i$, which means that the total energy used is less than the energy available in the battery in this slot.

We first consider the case where E_i are i.i.d. Bernoulli random variables with a particular support: $\mathbb{P}[E_i = B] = p$ and $\mathbb{P}[E_i = 0] = 1 - p$, that is, when energy arrives it fills the battery completely. For this case, we determine the optimal online policy in the next sub-section. We then consider the general i.i.d. energy arrivals and propose a near-optimal policy in the following sub-section, and prove optimality guarantees on it.

A. Optimal Strategy: Case of Bernoulli Arrivals

Due to the special i.i.d. Bernoulli energy arrival structure, when an energy arrives, it fills the battery, and resets the system. This constitutes a *renewal*. Then, from [40, Th. 3.6.1] (see also [32]–[38]), the long-term average rate can be found as:

$$\lim_{n\to\infty} \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} r_i\right] = \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} r_i\right] \tag{3}$$

$$= p\sum_{k=1}^{\infty} p(1-p)^{k-1}\sum_{i=1}^{k} r_i \tag{4}$$

$$= \sum_{i=1}^{\infty}\sum_{k=i}^{\infty} p^2(1-p)^{k-1} r_i \tag{5}$$

$$= \sum_{i=1}^{\infty} p(1-p)^{i-1} r_i \tag{6}$$

where L is the inter-arrival time between energy harvests, which is geometric with parameter p, and $\mathbb{E}[L] = 1/p$. Note that, via renewal reward theory, (3) reduces the infinite horizon problem into a finite horizon problem; instead of calculating the average reward over time, it is calculated over a single renewal event. The renewal event here is an energy arrival.

Then, (4) follows by substituting a geometric distribution with parameter p for random variable L, (5) follows by interchanging the order of summations, and (6) follows by evaluating the inner sum.

Inserting (2) in (6), the online power allocation problem is:

$$\begin{aligned} \max_{\{P_i,\theta_i\}} \quad & \sum_{i=1}^{\infty} p(1-p)^{i-1}\frac{\theta_i}{2}\log(1+P_i) \\ \text{s.t.} \quad & \sum_{i=1}^{\infty} \theta_i(P_i+\epsilon) \le B \\ & 0 \le \theta_i \le 1, \quad P_i \ge 0, \quad \forall i \end{aligned} \tag{7}$$

This optimization problem can be viewed as maximizing the *expected* transmitted rate before the next energy arrival.

The problem in (7) is non-convex. We transform it to an equivalent convex problem by defining new variables $\bar{P}_i = P_i\theta_i$,

$$\begin{aligned} \max_{\{\bar{P}_i,\theta_i\}} \quad & \sum_{i=1}^{\infty} p(1-p)^{i-1}\frac{\theta_i}{2}\log\left(1+\frac{\bar{P}_i}{\theta_i}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{\infty} \bar{P}_i + \theta_i\epsilon \le B \\ & 0 \le \theta_i \le 1, \quad \bar{P}_i \ge 0, \quad \forall i. \end{aligned} \tag{8}$$

Here, $\bar{P}_i$ can be interpreted as the transmit energy allocated to the ith slot, and θ_i is the duration during which this energy is transmitted. The optimum online scheduling problem is to find the sequence of $\{\bar{P}_i, \theta_i\}_{i=1}^{\infty}$.

The Lagrangian for the problem in (8) is:

$$\begin{aligned} \mathcal{L} = & -\sum_{i=1}^{\infty} p(1-p)^{i-1}\frac{\theta_i}{2}\log\left(1+\frac{\bar{P}_i}{\theta_i}\right) - \sum_{i=1}^{\infty}\gamma_i\bar{P}_i \\ & + \lambda\left(\sum_{i=1}^{\infty}\bar{P}_i + \theta_i\epsilon - B\right) - \sum_{i=1}^{\infty}\mu_i\theta_i - \sum_{i=1}^{\infty}\nu_i(1-\theta_i) \end{aligned} \tag{9}$$

where $\lambda, \gamma_i, \mu_i, \nu_i$ are non-negative Lagrange multipliers.

First, we note that, in the optimum solution of (7), $P_i = 0$ if and only if $\theta_i = 0$. This follows because, when P_i or θ_i is zero, the objective function is zero, and choosing the other variable non-zero wastes resources. While by definition $\bar{P}_i = 0$ when either $P_i = 0$ or $\theta_i = 0$, from the preceding argument, in the optimum solution of (8), $\bar{P}_i = 0$ if and only if $P_i = 0$ and $\theta_i = 0$. Since the problem in (8) is convex, the optimal solution is found by the KKT optimality conditions. Taking the derivative of (9) with respect to $\bar{P}_i$, equating it to zero, and using the corresponding complementary slackness condition:

$$\frac{\bar{P}_i}{\theta_i} = \frac{p(1-p)^{i-1}}{\lambda} - 1 \tag{10}$$

for slots where $\theta_i > 0$. When $\theta_i = 0$, from the preceding discussion $\bar{P}_i = 0$. Noting that $P_i = \frac{\bar{P}_i}{\theta_i}$, from (10), we conclude that the optimal power is decreasing over time. Therefore, there exists a time slot when it hits zero. Hence, we define $\tilde{N}$ for which we have $\bar{P}_i, P_i, \theta_i > 0, \forall i \in \{1, \dots, \tilde{N}\}$, and $\bar{P}_i = P_i = \theta_i = 0, \forall i \in \{\tilde{N}+1, \dots\}$. Note that the transmission duration of the single-user problem with no processing costs in [33] (let us denote it as $\tilde{N}_{npc}$) forms an upper bound for the transmission duration here, i.e., $\tilde{N} \le \tilde{N}_{npc}$. This is because, any processing costs use up energy for being on and reduce the effective battery size, and the transmission duration is an increasing function of the battery size [37].

Next, taking the derivative of (9) with respect to θ_i, we have

$$-p(1-p)^{i-1}\log\left(1+\frac{\bar{P}_i}{\theta_i}\right) + \frac{\bar{P}_i}{\theta_i}\frac{p(1-p)^{i-1}}{1+\frac{\bar{P}_i}{\theta_i}} + \lambda\epsilon - \mu_i + \nu_i = 0 \tag{11}$$

The optimal θ_i can be 0, 1, or $0 < \theta_i < 1$. When $0 < \theta_i < 1$, we have bursty transmission. In this case, from complementary slackness, we have $\mu_i = \nu_i = 0$. Then, from (10)-(11),

$$p(1-p)^{i-1}\left(\log\left(\frac{p(1-p)^{i-1}}{\lambda}\right) - 1\right) = \lambda(\epsilon - 1) \tag{12}$$

Hence, (12) should be satisfied in any slot i where $0 < \theta_i < 1$, i.e., where there is burstiness. Next, we note that, since the left hand side of (12) is monotonically decreasing in i, (12) can be satisfied in at most one slot. Moreover, this slot can only be the last slot. This follows from the presence of factor $p(1-p)^{i-1}$ in front of the log in (8). Hence, it is always better to fill-up (i.e., $\theta_i = 1$) earlier slots first; fractional θ_i should come later.

Next, we discuss how to solve for the optimum online policy. We just showed above that for all slots we have $\theta_i = 1$, except for possibly the last slot where $\theta_{\tilde{N}} \le 1$. From the total energy constraint and (10), we have:

$$\sum_{i=1}^{\tilde{N}-1}\left(\frac{p(1-p)^{i-1}}{\lambda} - 1 + \epsilon\right) + \theta_{\tilde{N}}\left(\frac{p(1-p)^{\tilde{N}-1}}{\lambda} - 1 + \epsilon\right) = B \tag{13}$$

In addition, for $i \in \{1, \dots, \tilde{N}\}$, we need to satisfy:

$$p(1-p)^{i-1} \ge \lambda \tag{14}$$

$$p(1-p)^{i-1}\left(1 - \log\left(\frac{p(1-p)^{i-1}}{\lambda}\right)\right) + \lambda(\epsilon - 1) \le 0 \tag{15}$$

where (14) ensures the non-negativity of power in (10), and (15) ensures the existence of non-negative Lagrange multipliers $\{\nu_i\}$ satisfying (11). Hence, we need to find the optimal $\tilde{N}, \lambda, \theta_{\tilde{N}}$ that satisfy (12), (13), (14) and (15). Towards this end, we consider the following approach: We first fix $\tilde{N}$ to be the single-user transmission duration with no processing costs in [33], i.e., $\tilde{N} = \tilde{N}_{npc}$, and solve for λ in (12) with $i = \tilde{N}$. Then, we check whether (14) and (15) are satisfied. If they are, then, we solve for $\theta_{\tilde{N}}$ from (13). If there does not exist a solution, then we reduce $\tilde{N}$ and repeat until we reach $\tilde{N} = 1$. If we do not have a solution when we reach $\tilde{N} = 1$, then this means that (12) cannot be satisfied, and we must have $\theta_{\tilde{N}} = 1$. In this case, (13) becomes:

$$\sum_{i=1}^{\tilde{N}}\left(\frac{p(1-p)^{i-1}}{\lambda} - 1 + \epsilon\right) = B \tag{16}$$

For this case, we solve (16) along with (14)-(15) for the largest $\tilde{N}$ and the corresponding λ.

B. Near-Optimal Strategy: General Arrivals

Now, we consider a general i.i.d. energy arrival process E_i with recharge rate $\mathbb{E}[E_i] = \mu$. In this case, we no longer have a *renewal* structure, and finding the *exactly optimal* online policy is analytically intractable. Instead, we propose a sub-optimal online policy and prove that it performs close to optimal.

1) Sub-Optimal Policy: We first define the proposed sub-optimal online policy for Bernoulli energy arrivals and then extend it to general energy arrivals. We note from (10) that, for Bernoulli energy arrivals, the optimal total transmit power allocated decreases exponentially over time. As in [32]–[38], this motivates us to construct a fractional power policy over time, in particular, we use a total allocated energy of $Bp(1-p)^{i-1}$ in slot i. That is, we allocate a fixed p fraction of available energy in the battery to use in slot i. We then decide on the duration of the burst θ_i by solving a single-slot problem as:

$$\begin{aligned} \max_{\bar{P}_i,\theta_i} \quad & \frac{\theta_i}{2}\log\left(1+\frac{\bar{P}_i}{\theta_i}\right) \\ \text{s.t.} \quad & \bar{P}_i + \theta_i\epsilon \le Bp(1-p)^{i-1} \\ & 0 \le \theta_i \le 1, \quad \bar{P}_i \ge 0 \end{aligned} \tag{17}$$

In the optimal policy, the first constraint is satisfied with equality, hence $\bar{P}_i = Bp(1-p)^{i-1} - \theta_i\epsilon$, and the problem can be written only in terms of θ_i as:

$$\max_{\theta_i\in[0,1]} \frac{\theta_i}{2}\log\left(1+\frac{Bp(1-p)^{i-1}}{\theta_i}-\epsilon\right) \tag{18}$$

For general energy arrivals, we allocate a fraction $q = \mu/B$ of the available energy in the battery for slot i, i.e., qb_i. Then, solve for the optimum burst θ_i in each slot as in (18):

$$\max_{\theta_i\in[0,1]} \frac{\theta_i}{2}\log\left(1+\frac{qb_i}{\theta_i}-\epsilon\right). \tag{19}$$

2) A Lower Bound on the Proposed Online Policy: In Lemma 1 and Lemma 2 below, we develop multiplicative and additive lower bounds for the performance of the proposed sub-optimal algorithm for Bernoulli arrivals. In the following, we denote the solution of maximization problems in (18) and (19) for available power P as $\theta^*(P,\epsilon)$, i.e., the solution of (18) is $\theta^*(Bp(1-p)^{i-1},\epsilon)$ and the solution of (19) is $\theta^*(qb_i,\epsilon)$.

Lemma 1: The achievable rate with the proposed sub-optimal policy for any i.i.d. Bernoulli energy arrival process with average recharge rate of $\mu = \mathbb{E}[E_i]$ is lower bounded as,

$$\begin{aligned} r &\ge \frac{1}{2-\frac{\mu}{B}} \max_{\theta\in[0,1]} \frac{\theta}{2}\log\left(1+\frac{\mu}{\theta}-\epsilon\right) && (20) \\ &\ge \frac{1}{2} \max_{\theta\in[0,1]} \frac{\theta}{2}\log\left(1+\frac{\mu}{\theta}-\epsilon\right) && (21) \end{aligned}$$

We provide the proof of Lemma 1 in Appendix A. The multiplicative bound in Lemma 1 performs well when the achievable rates are small, whereas the additive bound in Lemma 2 performs well when the achievable rates are large. We provide the proof of Lemma 2 in Appendix B.

Lemma 2: The achievable rate with the proposed sub-optimal policy for any i.i.d. Bernoulli energy arrival process with average recharge rate of $\mu = \mathbb{E}[E_i]$ is lower bounded as,

$$r \ge \max_{\theta\in[0,1]} \frac{\theta}{2}\log\left(1+\frac{\mu}{\theta}-\epsilon\right) - 0.72 - \frac{1}{2}\log^+(\epsilon) \tag{22}$$

where $\log^+(x) = \max\{\log(x), 0\}$.

We next show that i.i.d. Bernoulli energy arrivals yield the lowest rate over all i.i.d. energy arrivals with the same mean. The proof follows by the approach in [33, Proposition 4] as,

$$f(x) = \max_{\theta_i\in[0,1]} \frac{\theta_i}{2}\log\left(1+\frac{qx}{\theta_i}-\epsilon\right) \tag{23}$$

is concave in x. The concavity of $f(x)$ follows since it is equivalent to the maximization of $\frac{\theta_i}{2}\log\left(1+\frac{\bar{P}_i}{\theta_i}\right)$ over the feasible set $\bar{P}_i + \theta_i\epsilon \le qx$, $0 \le \theta_i \le 1$, $\bar{P}_i \ge 0$. The objective of this equivalent problem is jointly concave in $\theta_i, \bar{P}_i$, and the constraint set is affine in x, θ_i and $\bar{P}_i$. Then, it follows that $f(x)$ is concave in x; see also [41, Sec. 3.2.5].

Lemma 3: The rate of the proposed sub-optimal policy with any i.i.d. energy arrival process is no smaller than that with an i.i.d. Bernoulli energy arrival process of the same mean.

Combining Lemmas 1, 2, and 3, we have the following general theorem for arbitrary i.i.d. energy arrival processes.

Theorem 1: The achievable rate with the proposed sub-optimal policy for any arbitrary i.i.d. energy arrival process with average recharge rate $\mu = \mathbb{E}[E_i]$ is lower bounded as in (20) and (22).

3) An Upper Bound for Online Policies: In Theorem 2 below, we develop a universal upper bound for the performance of any online policy in terms of $\mathbb{E}[E_i] = \mu$.

Theorem 2: For a recharge rate of $\mathbb{E}[E_i] = \mu$, the achievable rate of any online algorithm is upper bounded as,

$$r \le \max_{\theta\in[0,1]} \frac{\theta}{2}\log\left(1+\frac{\mu}{\theta}-\epsilon\right). \tag{24}$$

Proof: We consider the rate of the optimum offline algorithm which upper bounds the rates achievable by any online algorithm. We consider the following larger than actual feasible region for the offline policy by neglecting the no-energy-overflow constraints due to the finite-sized battery [2], [3]:

$$\mathcal{F}^n \triangleq \left\{\{\bar{P}_i,\theta_i\}_{i=1}^n : \frac{1}{m}\sum_{i=1}^m \bar{P}_i + \theta_i\epsilon \le \frac{1}{m}\sum_{i=1}^m E_i,\ \forall m\right\} \tag{25}$$

Then, we consider the further larger feasible set by keeping only the bound for $m = n$, and starting with a full battery B,

$$\mathcal{G}^n \triangleq \left\{\{\bar{P}_i,\theta_i\}_{i=1}^n : \frac{1}{n}\sum_{i=1}^n \bar{P}_i + \theta_i\epsilon \le \frac{1}{n}\left(\sum_{i=1}^n E_i + B\right)\right\} \tag{26}$$

Then, we have:

$$r \le \lim_{n\to\infty} \max_{\{\bar{P}_i,\theta_i\}_{i=1}^n\in\mathcal{G}^n} \frac{1}{n}\sum_{i=1}^n \frac{\theta_i}{2}\log\left(1+\frac{\bar{P}_i}{\theta_i}\right) \tag{27}$$

Since the energies E_i are i.i.d., from strong law of large numbers, for all $\delta > 0$, there exists an integer N such that, for all $n \ge N$, we have $\frac{1}{n}(\sum_{i=1}^n E_i + B) \le \mu + \delta$. Hence, for large enough n, i.e., $n \ge N$, we have $\mathcal{G}^n$ to be

$$\mathcal{G}^n \triangleq \left\{\{\bar{P}_i,\theta_i\}_{i=1}^n : \frac{1}{n}\sum_{i=1}^n \bar{P}_i + \theta_i\epsilon \le \mu + \delta\right\} \tag{28}$$

Then, from the joint concavity of the objective function, it is maximized when all $\theta_i = \theta$ and all $\bar{P}_i = \bar{P}$. Hence, we have $\bar{P} + \theta\epsilon \le \mu + \delta$. Since this is valid for all $\delta > 0$, we take its limit to zero, which gives the desired result in (24). ■

4) Putting the Bounds Together: The additive lower bound in Theorem 1 (i.e., (22)) together with the general upper bound in Theorem 2 (i.e., (24)) imply that there is a constant gap between the bounds. Both the proposed sub-optimal policy and the optimal policy live between these bounds which are separated by a finite gap. Hence, the proposed online policy performs within a constant gap of the optimal online policy for all system parameters.

III. Two-Way Channel

We next consider a two-way energy harvesting channel with a common energy harvesting source, see Fig. 2. Transmitter j has a battery of finite size B_j. The energy available in the jth user battery in slot i, b_{ji}, evolves as:

$$b_{j(i+1)} = \min\left\{B_j, b_{ji} - \theta_{ji}P_{ji} - \max\{\theta_{1i}, \theta_{2i}\}\epsilon_j + E_{j(i+1)}\right\} \tag{29}$$

where P_{ji} is the power transmitted by user j in slot i, E_{ji} is the energy harvested at the jth user in slot i, ϵ_j is the processing cost incurred per unit time for being on,[1] and θ_{ji} is the duration for which user j is on, either transmitting or receiving, in slot i.

The physical layer is Gaussian with sum rate in slot i [39],

$$r_{1i} + r_{2i} = \frac{\theta_{1i}}{2}\log(1 + P_{1i}) + \frac{\theta_{2i}}{2}\log(1 + P_{2i}) \tag{30}$$

where r_{ji} is the rate of user j in slot i. The power and burst of user j, θ_{ji}, P_{ji}, are constrained by the current battery state as $\theta_{ji}P_{ji} + \max\{\theta_{1i}, \theta_{2i}\}\epsilon_j \le b_{ji}$. The objective of the online scheduling is to obtain the optimal policy which consists of $\{\theta_{1i}, \theta_{2i}, P_{1i}, P_{2i}\}$ to maximize the expected rate. In (30), the $\frac{1}{2}$ factors in front of logs are due to Shannon capacity formula (see [39, eq. (9.17)]), not due to time-sharing. There is no time-sharing; the system is full-duplex, and hence, the sum of two single-user rates is achievable (see [39, Sec. 15.1.6]).

In the following, we first consider the case where the energy arrivals, $E_{1i} = E_{2i} = E_i$, are i.i.d. Bernoulli random variables with support $\{0, B\}$, and with $\mathbb{P}[E_{1i} = E_{2i} = B] = p$, where $B \ge \max\{B_1, B_2\}$, i.e., when an energy comes it fills both batteries completely. For this case, we determine the optimal online policy. Subsequently, we consider the case of general i.i.d. energy arrivals, and propose a distributed near-optimal policy.

A. Optimal Strategy: Case of Bernoulli Arrivals

With Bernoulli energy arrivals, each energy arrival resets the system state; energy arrivals form a *renewal process*. From [40, Th. 3.6.1], the long-term average throughput is,

$$\lim_{n\to\infty} \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n}(r_{1i} + r_{2i})\right] = \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}(r_{1i} + r_{2i})\right] \tag{31}$$

$$= p\sum_{k=1}^{\infty} p(1-p)^{k-1}\sum_{i=1}^{k}(r_{1i} + r_{2i}) \tag{32}$$

$$= \sum_{i=1}^{\infty}\sum_{k=i}^{\infty} p^2(1-p)^{k-1}(r_{1i} + r_{2i}) \tag{33}$$

$$= \sum_{i=1}^{\infty} p(1-p)^{i-1}(r_{1i} + r_{2i}) \tag{34}$$

where L is the geometric inter-arrival time with $\mathbb{E}[L] = 1/p$.

Hence, the online power allocation problem becomes:

$$\begin{aligned} \max_{\{P_{ji}\},\{\theta_{ji}\}} \quad & \sum_{i=1}^{\infty} p(1-p)^{i-1}(r_{1i} + r_{2i}) \\ \text{s.t.} \quad & \sum_{i=1}^{\infty}(\theta_{1i}P_{1i} + \max\{\theta_{1i}, \theta_{2i}\}\epsilon_1) \le B_1 \\ & \sum_{i=1}^{\infty}(\theta_{2i}P_{2i} + \max\{\theta_{1i}, \theta_{2i}\}\epsilon_2) \le B_2 \\ & P_{1i}, P_{2i} \ge 0, \quad 0 \le \theta_{1i}, \theta_{2i} \le 1, \quad \forall i \end{aligned} \tag{35}$$

This optimization problem can be viewed as maximizing the *expected* transmitted sum rate before the next energy arrival.

Problem (35) is non-convex. We transform it to an equivalent convex problem by defining new variables $\bar{P}_{ji} = \theta_{ji}P_{ji}$,

$$\begin{aligned} \max \quad & \sum_{i=1}^{\infty} p(1-p)^{i-1}\left(\frac{\theta_{1i}}{2}\log\left(1 + \frac{\bar{P}_{1i}}{\theta_{1i}}\right) + \frac{\theta_{2i}}{2}\log\left(1 + \frac{\bar{P}_{2i}}{\theta_{2i}}\right)\right) \\ \text{s.t.} \quad & \sum_{i=1}^{\infty}\left(\bar{P}_{1i} + \max\{\theta_{1i}, \theta_{2i}\}\epsilon_1\right) \le B_1 \\ & \sum_{i=1}^{\infty}\left(\bar{P}_{2i} + \max\{\theta_{1i}, \theta_{2i}\}\epsilon_2\right) \le B_2 \\ & \bar{P}_{1i}, \bar{P}_{2i} \ge 0, \quad 0 \le \theta_{1i}, \theta_{2i} \le 1, \quad \forall i \end{aligned} \tag{36}$$

where the maximization is over $\{\bar{P}_{ji}\}, \{\theta_{ji}\}$.

Before proceeding with finding the optimal policy, we state two important observations: First, both users should consume all of their energies in their batteries. If a user does not consume all of its energy, then we can increase its power until all of its energy is used, increasing the objective function. Second, the two users' transmissions should be fully synchronized, i.e., $\theta_{1i} = \theta_{2i}$, for all i. If for a slot i users are not synchronized, say, e.g., $\theta_{1i} < \theta_{2i}$, then we can always increase θ_{1i} to be equal to θ_{2i} without violating the constraints of the problem, while increasing the objective function. Hence, hereafter, we will assume that $\theta_{1i} = \theta_{2i} = \theta_i$ for all i, so that $\max\{\theta_{1i}, \theta_{2i}\} = \theta_i$. In this case, the Lagrangian for (36) is:

$$\begin{aligned} \mathcal{L} = & -\sum_{i=1}^{\infty} p(1-p)^{i-1}\left(\frac{\theta_i}{2}\log\left(1 + \frac{\bar{P}_{1i}}{\theta_i}\right) + \frac{\theta_i}{2}\log\left(1 + \frac{\bar{P}_{2i}}{\theta_i}\right)\right) \\ & + \lambda_1\left(\sum_{i=1}^{\infty}\left(\bar{P}_{1i} + \theta_i\epsilon_1\right) - B_1\right) - \sum_{i=1}^{\infty}\nu_{1i}P_{1i} \\ & + \lambda_2\left(\sum_{i=1}^{\infty}\left(\bar{P}_{2i} + \theta_i\epsilon_2\right) - B_2\right) - \sum_{i=1}^{\infty}\nu_{2i}P_{2i} \\ & - \sum_{i=1}^{\infty}\mu_i^l\theta_i - \sum_{i=1}^{\infty}\mu_i^u(1 - \theta_i) \end{aligned} \tag{37}$$

[1] In this paper, we assume that the cost of being on while transmitting is the same as cost of being on while receiving. A more general model could be to consider different energy costs for being on for transmission and reception.

From the KKTs, the optimal powers for the slots with $\theta_i > 0$:

$$\frac{\bar{P}_{1i}}{\theta_i} = \left(\frac{p(1-p)^{i-1}}{\lambda_1} - 1\right)^+, \quad \frac{\bar{P}_{2i}}{\theta_i} = \left(\frac{p(1-p)^{i-1}}{\lambda_2} - 1\right)^+ \tag{38}$$

For the slots with $\theta_i = 0$, both powers are zero, i.e., $\bar{P}_{1i} = \bar{P}_{2i} = 0$, as otherwise, any assigned positive power is wasted, since the objective function is zero when $\theta_i = 0$.

From (38), we observe that for slots with $\theta_i > 0$, the powers P_{1i} and P_{2i} are monotonically decreasing in time. In addition, due to the decreasing $p(1-p)^{i-1}$ factors before the log, we can divide the slots into $\{1, \ldots, \tilde{N}\}$ where $\theta_i > 0$, and $\{\tilde{N}+1, \ldots\}$ where $\theta_i = 0$. Furthermore, the transmission duration $\tilde{N}$ is bounded above by the maximum of the user transmission durations without any processing costs (define them as $\tilde{N}_{npc1}$ and $\tilde{N}_{npc2}$), i.e., $\tilde{N} \leq \max\{\tilde{N}_{npc1}, \tilde{N}_{npc2}\}$. This follows as the processing costs reduce the energy available in the battery dedicated for transmission, and hence reduce the effective battery size at both users; it is shown in [37] that the transmission duration is monotone increasing in the battery size.

Similarly, from the optimality conditions, the bursts satisfy:

$$\sum_{j=1}^{2} \log\left(1 + \frac{\bar{P}_{ji}}{\theta_i}\right) - \frac{\frac{\bar{P}_{ji}}{\theta_i}}{1 + \frac{\bar{P}_{ji}}{\theta_i}} = \frac{\sum_{j=1}^{2} \lambda_j \epsilon_j + \mu_i^u - \mu_i^l}{p(1-p)^{i-1}} \tag{39}$$

substituting (38), we obtain,

$$\sum_{j=1}^{2} \log\left(\frac{p(1-p)^{i-1}}{\lambda_j}\right) = \frac{\sum_{j=1}^{2} \lambda_j(\epsilon_j - 1) + \mu_i^u - \mu_i^l}{p(1-p)^{i-1}} + 2 \tag{40}$$

From complementary slackness, if $\theta_i \in (0, 1)$, then we have $\mu_i^u = \mu_i^l = 0$. Thus, in this case, (40) becomes:

$$\sum_{j=1}^{2} \left(\log\left(\frac{p(1-p)^{i-1}}{\lambda_j}\right) - 1\right) = \frac{\sum_{j=1}^{2} \lambda_j\left(\epsilon_j - 1\right)}{p(1-p)^{i-1}} \tag{41}$$

The left and right hand sides of (41) are monotone decreasing and increasing, respectively. Hence, (41) can be satisfied at most for one time index, thus the bursty transmission can occur at most in one slot. Due to decreasing $p(1-p)^{i-1}$ multiplying the rate, this bursty transmission can occur only in the last slot.

From the above, the optimal solution is characterized by $\lambda_1, \lambda_2, \tilde{N}, \theta_{\tilde{N}}$. Next, we solve for them. For the complete solution we need to solve (40) along with the total power constraints, which using (38) become:

$$\sum_{i=1}^{\tilde{N}-1} \left[\frac{p(1-p)^{i-1}}{\lambda_1} - 1 + \epsilon_1\right] + \theta_{\tilde{N}} \left[\frac{p(1-p)^{\tilde{N}-1}}{\lambda_1} - 1 + \epsilon_1\right] = B_1 \tag{42}$$

$$\sum_{i=1}^{\tilde{N}-1} \left[\frac{p(1-p)^{i-1}}{\lambda_2} - 1 + \epsilon_2\right] + \theta_{\tilde{N}} \left[\frac{p(1-p)^{\tilde{N}-1}}{\lambda_2} - 1 + \epsilon_2\right] = B_2 \tag{43}$$

Solving (42) and (43) for $\theta_{\tilde{N}}$ we have:

$$\theta_{\tilde{N}} = \frac{B_1 - \sum_{i=1}^{\tilde{N}-1}\left(\frac{p(1-p)^{i-1}}{\lambda_1} - 1 + \epsilon_1\right)}{\frac{p(1-p)^{\tilde{N}-1}}{\lambda_1} - 1 + \epsilon_1} \tag{44}$$

$$= \frac{B_2 - \sum_{i=1}^{\tilde{N}-1}\left(\frac{p(1-p)^{i-1}}{\lambda_2} - 1 + \epsilon_2\right)}{\frac{p(1-p)^{\tilde{N}-1}}{\lambda_2} - 1 + \epsilon_2} \tag{45}$$

We note that (44) and (45) are strictly increasing in λ_1 and λ_2 when the numerators and denominators are non-negative. Hence, for each fixed λ_1 which makes $\theta_{\tilde{N}} \in (0, 1)$ there corresponds a unique λ_2 which makes (44) and (45) equal. This in effect makes it easy to search over the pairs $\{\lambda_1, \lambda_2\}$ which equate (44) and (45), using a one dimensional search on either λ_1 or λ_2. We also need to satisfy for $i \in \{1, \ldots, \tilde{N}\}$:

$$\lambda_1 \leq p(1-p)^{i-1} \tag{46}$$

$$\lambda_2 \leq p(1-p)^{i-1} \tag{47}$$

$$0 \leq \sum_{j=1}^{2} \left(\log\left(\frac{p(1-p)^{i-1}}{\lambda_j}\right) - 1\right) + \frac{\sum_{j=1}^{2} \lambda_j\left(1 - \epsilon_j\right)}{p(1-p)^{i-1}} \tag{48}$$

where (46) and (47) ensure the non-negativity of the power, and (48) guarantees the existence of a non-negative Lagrange multiplier μ_i^u satisfying (40).

Towards this end, next, we present a method to obtain the optimal solution. We first initialize $\tilde{N} = \max\{\tilde{N}_{npc1}, \tilde{N}_{npc2}\}$, where $\tilde{N}_{npcj}$ can be found by solving a single-user problem with no processing costs for user j as in [33]. From this, we obtain $\{\lambda_1, \lambda_2\}$ pairs which equate equations (44) and (45) and make $\theta_{\tilde{N}} \in (0, 1)$. Then, we check if any of the obtained pairs satisfies (41), (46), (47) and (48). If yes, then this is the optimal solution. Otherwise, we decrease $\tilde{N}$ by one and repeat this again. If we reach $\tilde{N} = 1$ and no solution is found, then, this implies that $\theta_{\tilde{N}} = 1$. Hence, we solve similarly for the largest integer $\tilde{N}$ and that corresponding λ_1, λ_2 that satisfy:

$$\sum_{i=1}^{\tilde{N}} \left(\frac{p(1-p)^{i-1}}{\lambda_1} - 1 + \epsilon_1\right) = B_1 \tag{49}$$

$$\sum_{i=1}^{\tilde{N}} \left(\frac{p(1-p)^{i-1}}{\lambda_2} - 1 + \epsilon_2\right) = B_2 \tag{50}$$

along with the conditions (46), (47) and (48).

B. Near-Optimal Strategy: General Arrivals

Now, we consider an arbitrary i.i.d. energy arrival process E_i with average admitted recharge rate $\mu_j = \mathbb{E}[\max\{B_j, E_i\}]$ at user j. Although finding the *exactly optimal* policy in this case may not be tractable, we propose a *distributed* sub-optimal policy which we show is near-optimal.

1) Sub-Optimal Policy: We first present our proposed sub-optimal policy for the Bernoulli case and then extend it to the case of general energy arrivals. For Bernoulli energy arrivals, motivated by (38), we assign exponentially decaying total power for each user. In each slot, the users allocate a fraction p of the available energy in the battery, and then optimize the transmit power and burst duration. Hence, in slot i, the energy allocated for transmission by user j is $B_j p(1-p)^{i-1}$. Then, the users solve the following single-slot optimization problem:

$$\begin{aligned} \max_{\bar{P}_{ji},\theta_i} \quad & \frac{\theta_i}{2}\log\left(1+\frac{\bar{P}_{1i}}{\theta_i}\right)+\frac{\theta_i}{2}\log\left(1+\frac{\bar{P}_{2i}}{\theta_i}\right) \\ \text{s.t.} \quad & \bar{P}_{1i}+\theta_i\epsilon_1 \leq B_1 p(1-p)^{i-1} \\ & \bar{P}_{2i}+\theta_i\epsilon_2 \leq B_2 p(1-p)^{i-1} \\ & \bar{P}_{1i},\bar{P}_{2i}\geq 0, \quad 0\leq\theta_i\leq 1 \end{aligned} \tag{51}$$

Since, the first two constraints will be satisfied with equality we have $\bar{P}_{ji}=B_j p(1-p)^{i-1}-\theta_i\epsilon_j$, which reduces (51) to:

$$\begin{aligned} \max_{\theta_i\in[0,1]} \frac{\theta_i}{2} \; & \log\left(1+\frac{B_1 p(1-p)^{i-1}}{\theta_i}-\epsilon_1\right) \\ & +\frac{\theta_i}{2}\log\left(1+\frac{B_2 p(1-p)^{i-1}}{\theta_i}-\epsilon_2\right) \end{aligned} \tag{52}$$

Similarly, for the case of general energy arrivals, we allocate a fraction $q_j=\frac{\mu_j}{B_j}$ of the battery energy, i.e., $q_j b_{ji}$, and solve:

$$\max_{\theta_i\in[0,1]} \frac{\theta_i}{2}\log\left(1+\frac{q_1 b_{1i}}{\theta_i}-\epsilon_1\right)+\frac{\theta_i}{2}\log\left(1+\frac{q_2 b_{2i}}{\theta_i}-\epsilon_2\right) \tag{53}$$

Problems (52) and (53) can be solved by both users independently, because both users know the energy arrival E_i, and they are consuming the power in a deterministic fractional way, hence, both users can track the state of both batteries.

2) An Upper Bound for Online Policies: In the following theorem, we develop an upper bound for all online policies in terms of the average admitted energy.

Theorem 3: For an average admitted energy μ_j at user j, the achievable rate for any online policy is upper bounded by:

$$r_{ub}=\max_{\theta\in[0,1]}\frac{\theta}{2}\left(\log\left(1+\frac{\mu_1}{\theta}-\epsilon_1\right)+\log\left(1+\frac{\mu_2}{\theta}-\epsilon_2\right)\right). \tag{54}$$

Proof: We denote the admitted energy arrivals as $\tilde{E}_{ji}=\min\{B_j,E_{ji}\}$. We use the offline achievable rate as an upper bound for the online achievable rate. We consider the following set which is larger than the feasible set of the offline case:

$$\begin{aligned} \mathcal{F}^n \triangleq \Bigg\{ & \{\bar{P}_{1i},\bar{P}_{2i},\theta_i\}_{i=1}^n : \frac{1}{m}\sum_{i=1}^m \bar{P}_{1i}+\theta_i\epsilon_1 \leq \frac{1}{m}\left(\sum_{i=1}^m \tilde{E}_{1i}\right), \\ & \frac{1}{m}\sum_{i=1}^m \bar{P}_{2i}+\theta_i\epsilon_2 \leq \frac{1}{m}\left(\sum_{i=1}^m \tilde{E}_{2i}\right), \forall m=1,\ldots,n\Bigg\} \end{aligned} \tag{55}$$

which neglects the overflow constraints due to the finite battery [2], [3]. We then consider a bigger feasible set by considering the constraints only when $m=n$ to get:

$$\begin{aligned} \mathcal{G}^n \triangleq \Bigg\{ & \{\bar{P}_{1i},\bar{P}_{2i},\theta_i\}_{i=1}^n : \frac{1}{n}\sum_{i=1}^n \bar{P}_{1i}+\theta_i\epsilon_1 \leq \frac{1}{n}\left(\sum_{i=1}^n \tilde{E}_{1i}\right), \\ & \frac{1}{n}\sum_{i=1}^n \bar{P}_{2i}+\theta_i\epsilon_2 \leq \frac{1}{n}\left(\sum_{i=1}^n \tilde{E}_{2i}\right)\Bigg\} \end{aligned} \tag{56}$$

Hence, the online achievable rate is upper bounded by:

$$\lim_{n\to\infty}\max_{\mathcal{G}^n}\frac{1}{n}\sum_{i=1}^n\frac{\theta_i}{2}\left(\log\left(1+\frac{\bar{P}_{1i}}{\theta_i}\right)+\log\left(1+\frac{\bar{P}_{2i}}{\theta_i}\right)\right) \tag{57}$$

Since the energies $\tilde{E}_{1i},\tilde{E}_{2i}$ are i.i.d., from strong law of large numbers, for all $\delta>0$ there exists an integer N such that for all $n\geq N$, we have $\frac{1}{n}\sum_{i=1}^n\tilde{E}_{1i}\leq\mu_1+\delta$ and $\frac{1}{n}\sum_{i=1}^n\tilde{E}_{2i}\leq\mu_2+\delta$. For large enough n, i.e., $n\geq N$, the constraints in (56) will be:

$$\frac{1}{n}\sum_{i=1}^n\bar{P}_{1i}+\theta_i\epsilon_1\leq\mu_1+\delta, \quad \frac{1}{n}\sum_{i=1}^n\bar{P}_{2i}+\theta_i\epsilon_2\leq\mu_2+\delta \tag{58}$$

Then, from the joint concavity of the objective function, it is maximizes when all $\theta_i=\theta$ and $\bar{P}_{ji}=\bar{P}_j$. Since this is valid for all $\delta>0$, we can the take δ to zero, which gives (54). ■

3) A Lower Bound on the Proposed Online Policy: In this section, we derive multiplicative and additive bounds for the performance of the proposed sub-optimal policy. In what follows, we denote the solution of the problems in (52) and (53) for available powers P_1,P_2 as $\theta^*(P_1,P_2)$, i.e., the solutions of (52) and (53) are denoted as $\theta^*(B_1p(1-p)^{i-1},B_2p(1-p)^{i-1})$ and $\theta^*(q_1b_{1i},q_2b_{2i})$, respectively.

Lemma 4: The achievable rate with the proposed fractional policy for any i.i.d. Bernoulli energy arrival process with average admitted energy μ_j at user j is lower bounded as:

$$r\geq\frac{1}{2}r_{ub} \tag{59}$$

We provide the proof of Lemma 4 in Appendix C.

Lemma 5: The achievable rate under the proposed fractional policy for any i.i.d. Bernoulli energy arrival process with average admitted energy μ_j at user j is lower bounded as:

$$r\geq r_{ub}-1.44-\frac{1}{2}\log^+(\epsilon_1)-\frac{1}{2}\log^+(\epsilon_2) \tag{60}$$

where $\log^+(x)=\max\{0,\log(x)\}$.

We provide the proof of Lemma 5 in Appendix D.

We next show that i.i.d. Bernoulli energy arrivals give the lowest rate over all i.i.d. energy arrivals with the same mean. The proof follows by the approach in [33, Proposition 4] as

$$\begin{aligned} f(x_1,x_2)\triangleq\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\Bigg(&\log\left(1+\frac{q_1x_1}{\theta_i}-\epsilon_1\right) \\ &+\log\left(1+\frac{q_2x_2}{\theta_i}-\epsilon_2\right)\Bigg) \end{aligned} \tag{61}$$

is jointly concave in x_1,x_2. The concavity of $f(x_1,x_2)$ follows since it is equivalent to maximizing $\frac{\theta_i}{2}\log(1+\frac{\bar{P}_{1i}}{\theta_i})+\frac{\theta_i}{2}\log(1+\frac{\bar{P}_{2i}}{\theta_i})$ over the feasible set $\bar{P}_{1i}+\theta_i\epsilon_1\leq q_1x_1$, $\bar{P}_{2i}+\theta_i\epsilon_2\leq q_2x_2$, $0\leq\theta_i\leq 1$, $\bar{P}_{1i},\bar{P}_{2i}\geq 0$. The objective function here is jointly concave $\theta_i,\bar{P}_{1i},\bar{P}_{2i}$ and the constraint set is affine in $x_1,x_2,\theta_i,\bar{P}_{1i},\bar{P}_{2i}$. Then, it follows that $f(x_1,x_2)$ is concave in x_1,x_2; [41, Sec. 3.2.5]. In addition, [33, Lemma 2] can be used as we have a single random variable representing the common energy arrival.

Lemma 6: For the proposed fractional policy, any i.i.d. energy arrival process yields an achievable sum rate no less than that of the Bernoulli energy arrivals with the same mean.

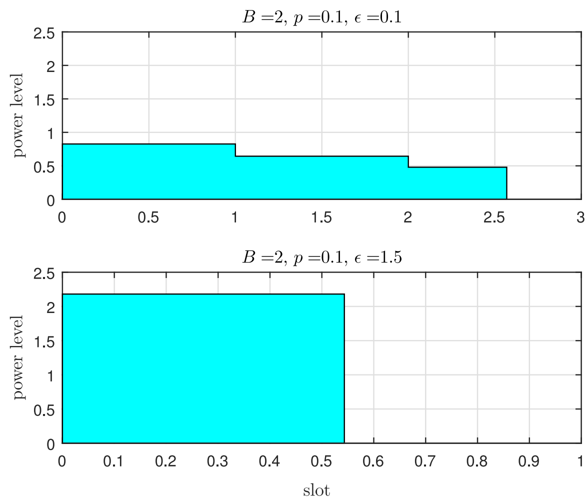

Fig. 3. Optimum online power allocation for i.i.d. Bernoulli arrivals.

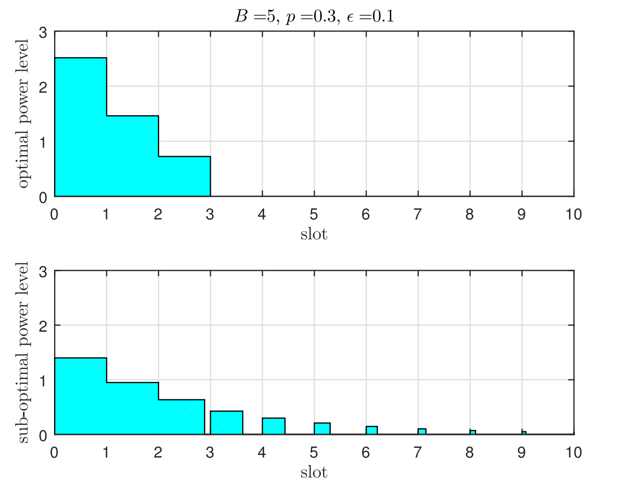

Fig. 4. Optimum online power allocation versus sub-optimal power allocation for i.i.d. Bernoulli arrivals.

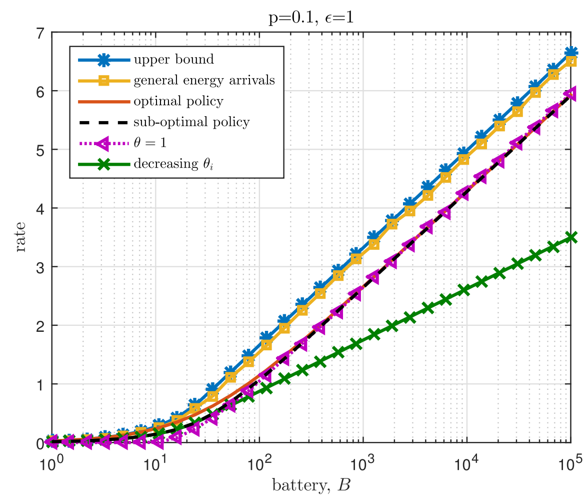

Fig. 5. Optimum online policy versus proposed sub-optimum online policy.

Combining Lemmas 4, 5, and 6, we have the following general theorem for arbitrary i.i.d. energy arrival processes.

Theorem 4: The achievable sum rate with the proposed sub-optimal policy for any arbitrary i.i.d. energy arrival process with average admitted energy of μ_j at user j and with $\frac{\mu_1}{B_1} = \frac{\mu_2}{B_2}$ is lower bounded by (59) *and* (60).

4) Putting the Bounds Together: The additive lower bound in Theorem 4 (i.e., (60)) together with the general upper bound in Theorem 3 (i.e., (54)) imply that there is a constant gap between the bounds. Both the proposed sub-optimal policy and the optimal policy live between these bounds which are separated by a finite gap. Hence, the proposed online policy performs within a constant gap of the optimal online policy for all system parameters.

IV. Numerical Results

In this section, we illustrate our results using several numerical examples. We begin with the single-user setting. We first show the optimal policy for Bernoulli energy arrivals. We fix the battery size to $B = 2$ and the probability of energy arrival to $p = 0.1$. We show the optimal policy in Fig. 3 for ϵ values of 0.1 and 1.5. As the processing cost increases, the transmission time decreases. When $\epsilon = 0.1$, the optimal power is decreasing and is non-zero for a total duration of 2.6 slots. However, when the processing cost is 1.5, the transmission duration decreases to 0.55 slots. Next, in Fig. 4, for the case of Bernoulli energy arrivals, we show the optimal policy versus the proposed sub-optimal policy. Here, we have $B = 3$, $p = 0.3$, $\epsilon = 0.1$. In the sub-optimal policy the energy is spread over more (infinite) slots.

In Figs. 5 and 6, we show the performance of the proposed sub-optimal policy and the optimal policy in terms of the expected rate versus the battery size. We fix $p = 0.1$ and show the performance for processing costs of $\epsilon = 1$ and $\epsilon = 10$ in Figs. 5 and 6. We note that, for the case of Bernoulli arrivals, the performance of the proposed sub-optimal policy is quite close to the performance of the optimal policy, in fact, much closer than the derived theoretical bounds show. In Figs. 5 and 6, we further plot two other sub-optimal schemes. The first scheme uses the same total fractional power as our proposed policy but fixes $\theta_i = 1$ for all i (i.e., neglects the processing cost effect) and transmits whenever it is feasible to transmit. The second scheme also uses the same total fractional power as our proposed policy but uses a fractional decreasing burstiness as $\theta_i = (1-p)^{i-1}\theta^*$. We observe that both of these policies perform worse than our proposed policy. We observe that the policy with $\theta = 1$ performs close to the optimal when the value of processing cost is negligible with respect to the battery size, i.e., for large battery sizes. However, for small battery sizes, e.g., B in [1, 10] when $\epsilon = 1$ and B in [1, 100] when $\epsilon = 10$, this algorithm performs poorly.

In Figs. 5 and 6, we also plot the performance of the proposed sub-optimal policy when the energy arrivals come from a continuous uniform distribution (non-Bernoulli) with the same mean as the Bernoulli energy arrivals. As expected, the rate is higher for the case of general energy arrivals compared to Bernoulli energy arrivals with the same mean. Finally, we show the performance of our scheme versus the processing cost in Fig. 7. The gap between the optimal and the sub-optimal decreases for high processing costs.

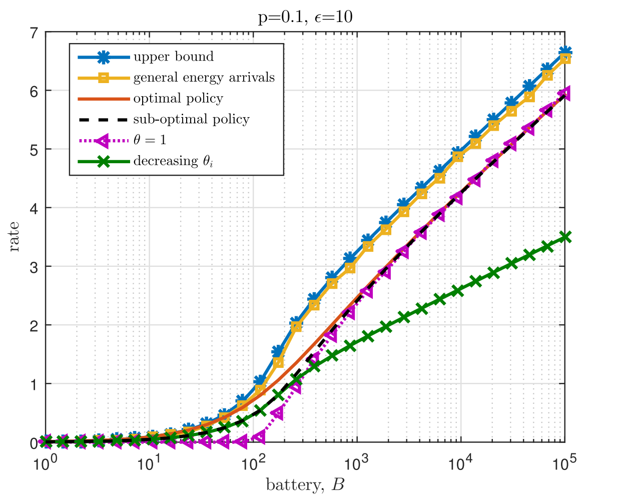

Fig. 6. Optimum online policy versus proposed sub-optimum online policy.

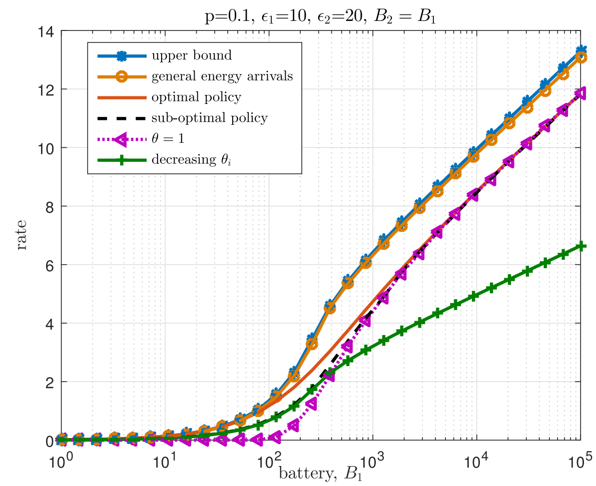

Fig. 9. Performance of Bernoulli and general energy arrivals.

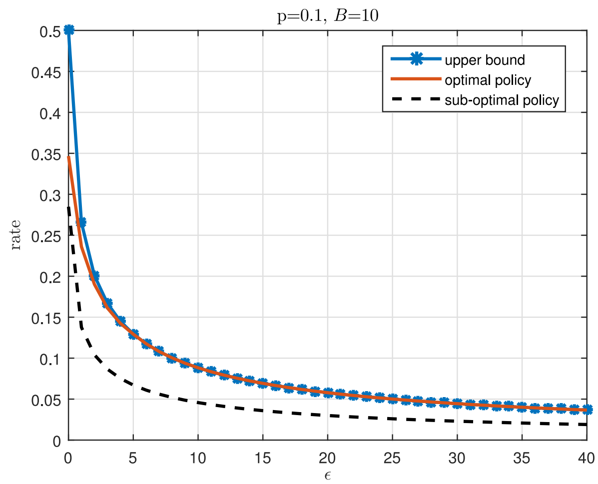

Fig. 7. Performance versus processing cost for i.i.d. Bernoulli arrivals.

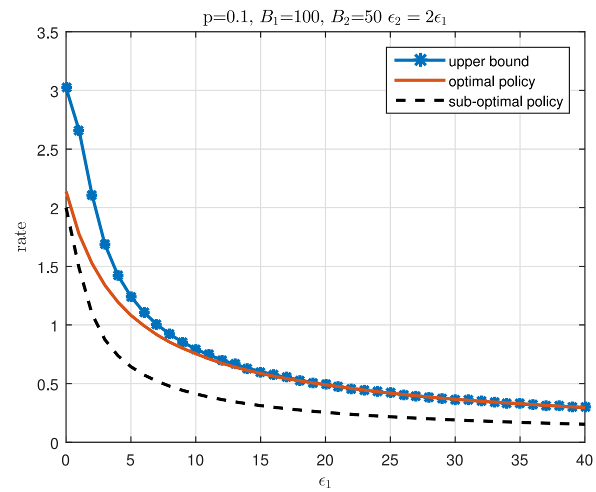

Fig. 10. Performance of Bernoulli energy arrivals versus the processing cost.

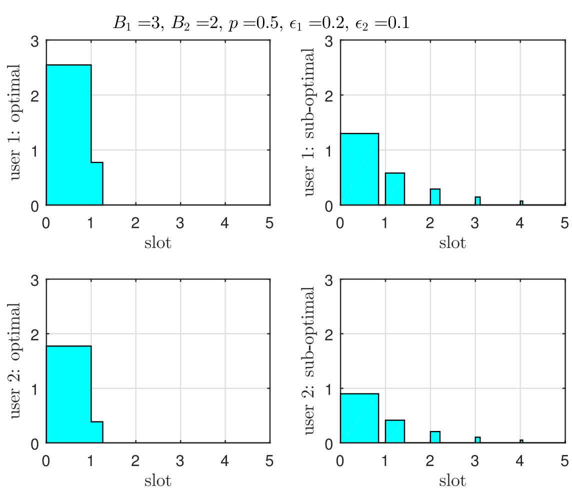

Fig. 8. The optimal and sub-optimal power allocations for Bernoulli.

Next, we consider the two-way channel. We first show the optimal versus proposed sub-optimal power allocation for Bernoulli arrivals in Fig. 8. As we showed, in the optimal power allocation, bursty transmission takes place only in the last slot. We then compare the performance of the proposed sub-optimal scheme and the optimal policy in Fig. 9. The performance of our proposed policy is close to the optimal. We also show the performance of the sub-optimal policy on a general energy arrival with a continuous uniform distribution with the same mean as Bernoulli. In Fig. 9 we also show the performance of the fractional θ_i scheme which is used in the proof of Lemma 4, and a scheme which always uses $\theta_i = 1$ whenever feasible, i.e., neglects the processing costs. Both perform worse than our proposed policy. Finally, we show the performance of our scheme versus the processing cost in Fig. 10. We observe that for high processing costs the performance gap is small.

V. Conclusion

We considered energy harvesting channels where users incur processing costs (power spent to run the circuitry) for being *on* to transmit or receive data, in addition to the power spent for communication. Such processing costs may result in *bursty* transmissions, where users may not be on all the time. In such channels, the users need to determine the optimal burst duration (duration to be on) and the optimal transmit power. In this paper, we considered the design of *online* power control algorithms which use only the causal knowledge of energy arrivals. First, we studied the single-user channel. In this channel, we characterized the optimal online policy for the case of Bernoulli energy arrivals. We showed that the optimal power policy is decreasing and can be bursty (i.e., the user may not utilize the entire slot). However, the bursty transmission can occur only in the last slot of transmission. We then considered the case of general energy arrivals. For this case, we proposed a sub-optimal online power control scheme, and proved that it performs within a constant gap of the optimal. The sub-optimal policy allocates powers fractionally over time and solves a single-slot optimization problem to determine the burst duration in each slot. We then extended our analysis to the two-way channel model. We considered the special case of fully-correlated energy arrivals at the users. In this channel, we first characterized the optimal policy for the case of Bernoulli energy arrivals. We showed that the powers of both users decrease and the transmission of both users need to be synchronized, i.e., both users turn on or off simultaneously. We then proposed a sub-optimal distributed policy for the case of general fully-correlated energy arrivals. The proposed policy allocates powers fractionally in a distributed manner, and each user solves a single-slot problem distributedly. We proved that the proposed distributed scheme performs within a constant gap of the optimal.

In the two-way channel, we assumed that the energy cost for being on is the same for transmitting and receiving data. As a future work, different energy costs for transmission and reception can be considered. In addition, we assumed that the energy arrivals at the two users are fully-correlated. Arbitrarily correlated energy arrivals at the users can be considered in future work. Further research directions are to consider energy cooperation between the users in an online setting, and finite-sized data buffers at both users.

APPENDIX A
PROOF OF LEMMA 1

We lower bound the performance as follows. The first lower bounding step in (63) is obtained by choosing all θ_i as $\theta_i = (1-p)^{i-1}\theta^*$, where θ^* denotes $\theta^*(Bp, \epsilon)$ in short:

$$r = \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \max_{\theta_i \in [0,1]} \frac{\theta_i}{2} \log\left(1 + \frac{Bp(1-p)^{i-1}}{\theta_i} - \epsilon\right)\right] \quad (62)$$

$$\geq \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \frac{\theta^*(1-p)^{i-1}}{2} \log\left(1 + \frac{Bp}{\theta^*} - \epsilon\right)\right] \quad (63)$$

$$= \frac{\theta^*}{2} \log\left(1 + \frac{Bp}{\theta^*} - \epsilon\right) \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}(1-p)^{i-1}\right] \quad (64)$$

$$= \frac{\theta^*}{2} \log\left(1 + \frac{Bp}{\theta^*} - \epsilon\right) p\left[\sum_{L=1}^{\infty} p(1-p)^{L-1} \sum_{i=1}^{L}(1-p)^{i-1}\right] \quad (65)$$

$$= \frac{\theta^*}{2} \log\left(1 + \frac{Bp}{\theta^*} - \epsilon\right)\left[\sum_{L=1}^{\infty} p^2(1-p)^{L-1} \frac{1-(1-p)^L}{p}\right] \quad (66)$$

$$= \frac{\theta^*}{2} \log\left(1 + \frac{Bp}{\theta^*} - \epsilon\right)\left[\sum_{L=1}^{\infty} p(1-p)^{L-1}\left(1-(1-p)^L\right)\right] \quad (67)$$

$$= \frac{\theta^*}{2} \log\left(1 + \frac{Bp}{\theta^*} - \epsilon\right)\left[\sum_{L=1}^{\infty} p\left((1-p)^{L-1} - (1-p)^{2L-1}\right)\right] \quad (68)$$

$$= \frac{\theta^*}{2} \log\left(1 + \frac{Bp}{\theta^*} - \epsilon\right)\left[p\left(\frac{1}{p} - \frac{(1-p)}{2p-p^2}\right)\right] \quad (69)$$

$$= \frac{\theta^*}{2} \log\left(1 + \frac{Bp}{\theta^*} - \epsilon\right)\left(\frac{1}{2-p}\right) \quad (70)$$

$$= \frac{1}{2 - \frac{\mu}{B}} \max_{\theta \in [0,1]} \frac{\theta}{2} \log\left(1 + \frac{\mu}{\theta} - \epsilon\right) \quad (71)$$

which is (20). Here, (71) follows since $\mathbb{E}[E_i] = \mu = Bp$ and $\theta^* = \theta^*(Bp, \epsilon) = \theta^*(\mu, \epsilon)$. Finally, (21) follows as $\frac{\mu}{B} \geq 0$. ■

APPENDIX B
PROOF OF LEMMA 2

We first prove for the case $\epsilon < 1$. The first lower bounding step in (73) is obtained by choosing all θ_i as $\theta_i = \theta^*$, where θ^* denotes $\theta^*(Bp, \epsilon)$ in short:

$$r = \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \max_{\theta_i \in [0,1]} \frac{\theta_i}{2} \log\left(1 + \frac{Bp(1-p)^{i-1}}{\theta_i} - \epsilon\right)\right] \quad (72)$$

$$\geq \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \frac{\theta^*}{2} \log\left(1 + \frac{Bp(1-p)^{i-1}}{\theta^*} - \epsilon\right)\right] \quad (73)$$

$$= \frac{\theta^*}{2}\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \log\left((1-\epsilon)\left(1 + \frac{Bp(1-p)^{i-1}}{(1-\epsilon)\theta^*}\right)\right)\right] \quad (74)$$

$$= \frac{\theta^*}{2}\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \log(1-\epsilon) + \log\left(1 + \frac{Bp(1-p)^{i-1}}{(1-\epsilon)\theta^*}\right)\right] \quad (75)$$

$$\geq \frac{\theta^*}{2}\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \log(1-\epsilon) + \log\left(1 + \frac{Bp}{(1-\epsilon)\theta^*}\right) + \log\left((1-p)^{i-1}\right)\right] \quad (76)$$

$$\geq \frac{\theta^*}{2}\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \log(1-\epsilon) + \log\left(1 + \frac{Bp}{(1-\epsilon)\theta^*}\right)\right] - 0.72 \quad (77)$$

$$= \frac{\theta^*}{2} \log\left(1 + \frac{Bp}{\theta^*} - \epsilon\right) - 0.72 \quad (78)$$

which is (22), since $\mathbb{E}[E_i] = \mu = Bp$, $\theta^* = \theta^*(Bp, \epsilon)$, and $\log^+(\epsilon) = 0$ in this case.

Next, we prove for the case $\epsilon \geq 1$:

$$r = \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \max_{\theta_i \in [0,1]} \frac{\theta_i}{2} \log\left(1 + \frac{Bp(1-p)^{i-1}}{\theta_i} - \epsilon\right)\right] \quad (79)$$

$$\geq \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \max_{\theta_i \in [0,1]} \frac{\theta_i}{2} \log\left(1 + \frac{\frac{Bp(1-p)^{i-1}}{\theta_i} - \epsilon}{\epsilon}\right)\right] \quad (80)$$

$$= \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \max_{\theta_i \in [0,1]} \frac{\theta_i}{2} \log\left(1 + \frac{Bp(1-p)^{i-1}}{\theta_i \epsilon} - 1\right)\right] \quad (81)$$

$$\geq \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\log\left(\frac{Bp}{\theta_i}\right)\right] - \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\log(\epsilon)\right] - \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\log\left(\frac{1}{(1-p)^{i-1}}\right)\right] \tag{82}$$

$$= \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\log\left(\frac{Bp}{\theta_i}\right)\right] - \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\frac{1}{2}\log(\epsilon)\right] - \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\frac{1}{2}\log\left((1-p)^{i-1}\right)\right] \tag{83}$$

$$= \max_{\theta\in[0,1]}\frac{\theta}{2}\log\left(\frac{Bp}{\theta}\right) - \frac{1}{2}\log(\epsilon) - 0.72 \tag{84}$$

$$\geq \max_{\theta\in[0,1]}\frac{\theta}{2}\log\left(1+\frac{Bp}{\theta}-\epsilon\right) - \frac{1}{2}\log(\epsilon) - 0.72 \tag{85}$$

which is (22), since $\log^+(\epsilon) = \log(\epsilon)$ in this case. Here, (80) follows since at the maximum $\frac{Bp(1-p)^{i-1}}{\theta_i} - \epsilon$ is non-negative and $\epsilon \geq 1$, (82) follows since for any three positive functions $a(x), b(x), c(x)$, we have: $\max_x [a(x) - b(x) - c(x)] \geq \max_x a(x) - \max_x b(x) - \max_x c(x)$, and (85) follows since we added a negative term $(1-\epsilon)$ inside the log. ■

APPENDIX C
PROOF OF LEMMA 4

The first step, (87), for the lower bound follows by using a sub-optimal decreasing burst as $\theta_i = \theta^*(1-p)^{i-1}$, where θ^* is a short notation for $\theta^*(B_1p, B_2p)$:

$$r = \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\left(\log\left(1+\frac{B_1p(1-p)^{i-1}}{\theta_i}-\epsilon_1\right) + \log\left(1+\frac{B_2p(1-p)^{i-1}}{\theta_i}-\epsilon_2\right)\right)\right] \tag{86}$$

$$\geq \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}(1-p)^{i-1}r_{ub}\right] \tag{87}$$

$$= r_{ub}\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}(1-p)^{i-1}\right] \tag{88}$$

$$= r_{ub}\frac{1}{\mathbb{E}[L]}\left[\sum_{L=1}^{\infty}p(1-p)^{L-1}\sum_{i=1}^{L}(1-p)^{i-1}\right] \tag{89}$$

$$= r_{ub}\left[\sum_{L=1}^{\infty}p^2(1-p)^{L-1}\frac{1-(1-p)^L}{p}\right] \tag{90}$$

$$= r_{ub}\left[p\left(\frac{1}{p}-\frac{(1-p)}{2p-p^2}\right)\right] \tag{91}$$

$$= r_{ub}\left(\frac{1}{2-p}\right) \tag{92}$$

$$\geq \frac{1}{2}r_{ub} \tag{93}$$

where (93) follows since $p \geq 0$. ■

APPENDIX D
PROOF OF LEMMA 5

The proof technique we use for the case $\epsilon_j \leq 1$ is different than $\epsilon_j > 1$. In what follows, we assume that $\epsilon_1 > 1$ while $\epsilon_2 \leq 1$, however, all other combinations follow similarly. We bound the performance of (86) as follows:

$$r \geq \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\left(\log\left(1+\frac{\frac{B_1p(1-p)^{i-1}}{\theta_i}-\epsilon_1}{\epsilon_1}\right) + \log\left(1+\frac{B_2p(1-p)^{i-1}}{\theta_i}-\epsilon_2\right)\right)\right] \tag{94}$$

$$= \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\left(\log\left(1+\frac{B_1p(1-p)^{i-1}}{\epsilon_1\theta_i}-1\right) + \log\left(1+\frac{B_2p(1-p)^{i-1}}{\theta_i}-\epsilon_2\right)\right)\right] \tag{95}$$

$$\geq \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\left(\log\left(\frac{B_1p}{\theta_i}\right) + \log\left(1+\frac{B_2p(1-p)^{i-1}}{\theta_i}-\epsilon_2\right)\right)\right] - \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\log(\epsilon_1)\right] - \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\log\left(\frac{1}{(1-p)^{i-1}}\right)\right] \tag{96}$$

$$\geq \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\left(\log\left(\frac{B_1p}{\theta_i}\right) + \log\left(1+\frac{B_2p(1-p)^{i-1}}{\theta_i}-\epsilon_2\right)\right)\right] - \frac{1}{2}\log(\epsilon_1) - 0.72 \tag{97}$$

$$\geq \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\left(\log\left(1+\frac{B_1p}{\theta_i}-\epsilon_1\right) + \log\left(1+\frac{B_2p(1-p)^{i-1}}{\theta_i}-\epsilon_2\right)\right)\right] - \frac{1}{2}\log(\epsilon_1) - 0.72 \tag{98}$$

$$\geq \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\frac{\theta^*}{2}\left(\log\left(1+\frac{B_1p}{\theta^*}-\epsilon_1\right) + \log\left(1+\frac{B_2p(1-p)^{i-1}}{\theta^*}-\epsilon_2\right)\right)\right] - \frac{1}{2}\log(\epsilon_1) - 0.72 \tag{99}$$

$$= \frac{\theta^*}{2}\log\left(1+\frac{B_1p}{\theta^*}-\epsilon_1\right) - \frac{1}{2}\log(\epsilon_1) - 0.72 + \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\frac{\theta^*}{2}\log\left((1-p)^{i-1}\left(\frac{1-\epsilon_2}{(1-p)^{i-1}}+\frac{B_2p}{\theta^*}\right)\right)\right] \tag{100}$$

$$\geq \frac{\theta^*}{2}\log\left(1+\frac{B_1 p}{\theta^*}-\epsilon_1\right)-\frac{1}{2}\log(\epsilon_1)-0.72 + \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\frac{\theta^*}{2}\log\left((1-p)^{i-1}\left(1-\epsilon_2+\frac{B_2 p}{\theta^*}\right)\right)\right] \quad (101)$$

$$\geq \frac{\theta^*}{2}\log\left(1+\frac{B_1 p}{\theta^*}-\epsilon_1\right)-\frac{1}{2}\log(\epsilon_1)-1.44 + \frac{\theta^*}{2}\log\left(1+\frac{B_2 p}{\theta^*}-\epsilon_2\right) \quad (102)$$

where (94) follows as the maximum $\frac{B_1 p(1-p)^{i-1}}{\theta_i}-\epsilon_1$ is non-negative, and $\epsilon_1 > 1$, (96) follows since for any three positive functions $a(x), b(x), c(x)$ we have: $\max_x[a(x)-b(x)-c(x)] \geq \max_x a(x) - \max_x b(x) - \max_x c(x)$, (97) follows by bounding the last term numerically by 0.72, (98) follows since we added $1-\epsilon_1$ which is negative, (102) follows again by numerically bounding $\frac{1}{\mathbb{E}[L]}\mathbb{E}[\sum_{i=1}^{L}\frac{1}{2}\log(\frac{1}{(1-p)^{i-1}})]$ by 0.72. The θ^* used here is a shorthand for $\theta^*(B_1 p, B_2 p)$. ■

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