Energy and Data Cooperative Multiple Access Channel With Intermittent Data Arrivals

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***Abstract*— We consider an energy harvesting two user cooperative Gaussian multiple access channel, where both of the users harvest energy from nature. The users cooperate at the physical layer (data cooperation) by establishing common messages through overheard signals and then cooperatively sending them. We study two scenarios within this model. In the first scenario, the data packets arrive intermittently over time. We find the optimal offline transmit power and rate allocation policy that maximize the departure region. We first show that there exists an optimal policy, in which the single user rate constraints in each time slot are tight, yielding a one-to-one relation between the powers and rates. Then, we formulate the departure region maximization problem as a weighted sum departure maximization in terms of rates only. Next, we propose a sequential convex approximation method to approximate the problem at each step and show that it converges to the optimal solution. We solve the approximate problems using an inner–outer decomposition method. In the second scenario, the data packets are available at the beginning of the transmission, but the users now have the ability to cooperate at the battery level (energy cooperation), in addition to data cooperation. The energy cooperation is facilitated by wireless energy transfer and is bidirectional. For this scenario, we find the jointly optimal offline transmit power and rate allocation policy together with the energy transfer policy that maximize the departure region. We provide necessary conditions for energy transfer and prove some properties of the optimal transmit policy, thereby shedding some light on the interplay between energy and data cooperation.**

***Index Terms*— Energy cooperation, energy harvesting, wireless energy transfer, multiple access channel, intermittent data arrivals.**

I. Introduction

WE CONSIDER the two user cooperative Gaussian multiple access channel (MAC), see [1], where both of the users harvest energy from the nature. The users cooperate at the physical layer (data cooperation), by establishing common messages through overheard signals and then cooperatively sending them. We consider two extensions to this channel model. In the first extension, see Fig. 1, we investigate the effect of intermittent data arrivals to the users. We find the optimal offline transmit power and rate allocation policy that maximize the departure region. In the second extension, see Fig. 2, we investigate the scenario where the users can cooperate at the battery level (energy cooperation), in addition to data cooperation, via wireless energy transfer. We find the jointly optimal offline transmit power and rate allocation policy together with the energy transfer policy that maximizes the departure region.

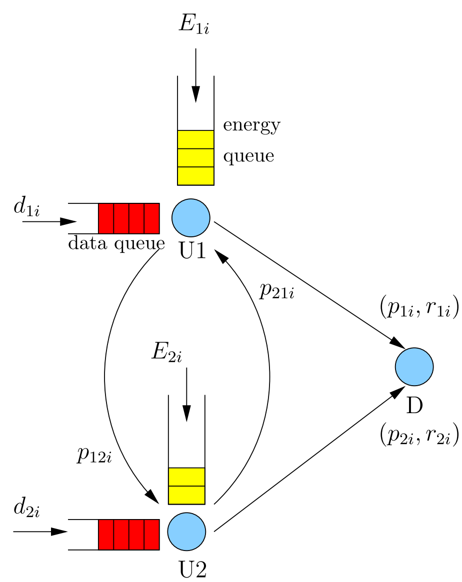

Fig. 1. Data cooperation under intermittent data arrivals.

There has been a considerable amount of recent work in power control for energy harvesting communications [1]–[25]. In [2], the transmission completion time minimization problem is solved for an unlimited-sized battery. In [3], the throughput maximization problem is solved and its equivalence to the transmission completion time minimization problem is shown for an arbitrary-sized battery. In [4] and [6]–[11] the problem is extended to fading, broadcast, multiple access and interference channels. Throughput maximization problem with battery imperfections is considered in [12] and [13] and processing costs are incorporated in [14]–[16]. Two-hop communication is considered with energy harvesting nodes for half- or full-duplex relay settings in [17]–[22]. In [26] online

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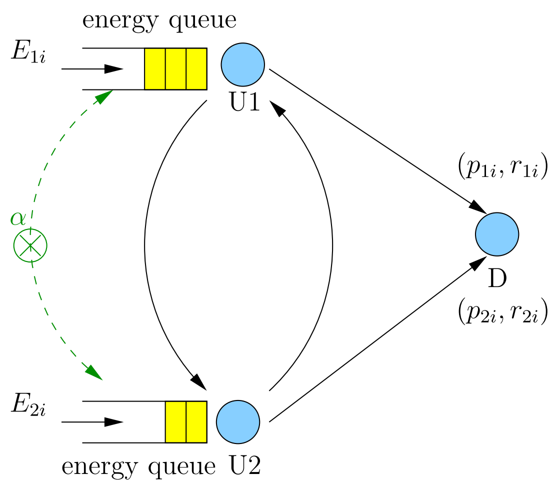


Fig. 2. Joint data and energy cooperation.

algorithms for throughput maximization in energy harvesting MAC are investigated. While online algorithms are an interesting research direction, they are beyond the scope of this paper. Of particular relevance to us are [1], [9], [10], [24], [27], and [28] where optimal scheduling problems on a MAC are investigated. In [27], minimum energy scheduling problem over a MAC where data packets arrive over time is solved. In [9], a MAC with energy arrivals is considered with infinitely backlogged users, i.e., the data packets do not arrive over time. In [10], an energy harvesting MAC with additional maximum power constraints on each user is considered. In [28], a MAC with conferencing encoders which share a common energy harvester is considered. Recently, in [24], a MAC with both energy and data arrivals is considered and in [1], a cooperative MAC with only energy arrivals is considered.

While most of the offline optimization literature on cooperative systems has focused on throughput maximization under the assumption of infinitely back-logged data queues, in many applications, data may arrive intermittently at the nodes just like energy. Two prominent examples of such scenarios are multi-hop networks and sensor networks. In multi-hop networks, each hop forwards the data that has arrived from the previous hop, therefore, data is not always available and arrives intermittently depending on the energy arrivals and achievable rates of the previous hops; an example of such scenario is investigated in a diamond network in [29]. In sensor networks, sensor nodes make measurements of an event of interest. Therefore, data to send becomes available as the event occurs intermittently. The transmission of this data may often require cooperation, depending on the energy state of the sensors. Motivated by these, in the first part of this paper, in Section III, we consider a cooperative MAC with both energy and data arrivals as shown in Fig. 1. We consider a system where data arrives during transmissions. For the intermittent data arrival scenario, we first show that there exists an optimal rate and power allocation which is on the achievable rate region boundary of the cooperative MAC at every slot, instead of being strictly inside the achievable rate region. Then, we formulate the problem in terms of data rates only, rather than both transmission powers and data rates. Although this new problem is non-convex, we show that strong duality holds. As a result, we are able to employ a successive convex approximation technique in which non-convex constraints are approximated by suitable convex functions. Using this approximation, we solve the problem using an iterative algorithm which iterates between inner and outer maximization problems.

The concept of cooperation in energy harvesting networks is not limited to data cooperation, and can be extended to energy cooperation. In the energy harvesting framework, energy cooperation and energy sharing is introduced in [23] and communication systems with energy exchange are investigated in [23], [25], and [30]–[34]. In the second part of this paper, in Section IV, we consider a cooperative MAC with both energy and data cooperation as shown in Fig. 2. We use this system model to investigate interactions of data and energy cooperation, and study their joint optimization. In this case, in addition to data cooperation being implemented at the physical layer by decoding and forwarding the overheard data, energy cooperation is implemented at the battery level by forwarding energy between users by using wireless energy transfer. By such a formulation, we investigate the interaction between data and energy cooperation, their relative effectiveness, and the direction, timing and amount of energy exchange in coordination with data cooperation. We first show that in this scenario, the cooperative powers in all slots must be non-zero for both users. Then, we derive a one-to-one relation between the optimal transmission rates and the optimal transmission powers. Next, we show that data cooperation always precedes energy cooperation. In other words, excess energy must first be used to increase cooperative powers and then to further assist the other user by means of direct energy transfer. We determine necessary conditions for energy transfer to take place. We then propose an algorithm which solves the offline energy transfer and power allocation problem iteratively based on these conditions. We demonstrate that, unlike MAC with energy cooperation only [25], optimal energy transfer policy among the users may change direction from one slot to the next. That is, the sum-power loss caused by lossy energy transfer may be compensated by the gain from user cooperation.

II. System Model and Problem Formulation

We consider two energy harvesting cooperative MAC models: (i) cooperative MAC with intermittent data arrivals, as shown in Fig. 1 and (ii) cooperative MAC with bidirectional energy cooperation, as shown in Fig 2. The harvested energies are saved in the corresponding batteries. There are N equal length slots and without loss of generality slot durations are assumed to be 1 second. We use subscripts 1 and 2 to denote the parameters of users 1 and 2.

In both models, the physical layer is a cooperative Gaussian MAC with unit-variance Gaussian noises at the users and $\sigma^2 > 1$ variance Gaussian noise at the receiver. We employ the delay constrained cooperation model proposed in [1]. The users cooperate in a slot by slot basis, by first exchanging information and then beamforming, to send the established common information, in each given slot. We assume full duplex operation at the transmitters with perfect self-interference cancellation. While this is an idealized model, it serves as a benchmark for the performance of practical systems as the employed block Markov superposition encoding is known to achieve the largest known rate region in cooperative multiple access channels. The full duplex operation is further motivated by some promising recent practical works on full duplex cooperation [35], [36]. The specifics of the encoding and decoding policy can be found in [1, Sec. II]. The transmission rates in slot i, denoted as r_{1i}, r_{2i}, are achievable with arbitrarily low probability of error if they belong to the set [1], [37]:

$$\mathcal{C}(p_{12i}, p_{21i}, p_{U1i}, p_{U2i}) = \left\{ \begin{array}{l} r_{1i} \leq f(1+p_{12i}), \\ r_{2i} \leq f(1+p_{21i}), \\ r_{1i}+r_{2i} \leq f(s_i/\sigma^2) \end{array} \right\} \quad (1)$$

where $f(x) = \frac{1}{2}\log(x)$, p_{12i}, p_{21i}, p_{U1i}, p_{U2i} are transmitter sub-powers as defined in [1], $p_{1i} = p_{12i} + p_{U1i}$, $p_{2i} = p_{21i} + p_{U2i}$ and

$$s_i = \sigma^2 + p_{1i} + p_{2i} + 2\sqrt{p_{U1i}p_{U2i}} \quad (2)$$

The operational meaning of the sub-powers will be important to us: p_{12i} and p_{21i} denote the powers used in slot i to build up common information at the cooperative partner, while p_{U1i} and p_{U2i} are cooperative powers used for jointly conveying the common information to the receiver.

In the first half of this paper, we investigate data cooperation under intermittent data arrivals. The system model is given in Fig. 1. In slot i, there are energy and data arrivals to both users with amounts E_{1i}, E_{2i} and d_{1i}, d_{2i}, respectively. Data arrivals are assumed to occur at the beginning of each slot, as data arriving in slot $i-1$ can always be accumulated until the beginning of the next slot, and sent in slot i onwards. The data that has not arrived yet cannot be transmitted, leading to the following *data causality constraints*:

$$\sum_{i=1}^{k} r_{1i} \leq \sum_{i=1}^{k} d_{1i}, \quad \forall k, \quad (3)$$

$$\sum_{i=1}^{k} r_{2i} \leq \sum_{i=1}^{k} d_{2i}, \quad \forall k. \quad (4)$$

In general, energy arrivals to the users may occur at any point in time. For simplicity, we assume a slotted model for energy arrivals as well, where energy arrivals are accumulated until the beginning of the next time slot, at which point they become available for use for all upcoming transmissions. Therefore, it is possible to replace any energy arrival process by one in which the energy arrivals simply occur in bulk at the beginning of each slot. Furthermore, we also assume that the time slots are long enough so that a non-zero amount of energy is collected at each time slot for both users, i.e., $E_{1i} > 0, E_{2i} > 0, \forall i$. The energy that has not arrived yet cannot be used for data transmission, leading to the following *energy causality constraints*:

$$\sum_{i=1}^{k} p_{1i} \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k, \quad (5)$$

$$\sum_{i=1}^{k} p_{2i} \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k. \quad (6)$$

The rate allocations must be achievable for the cooperative MAC in each slot:

$$(r_{1i}, r_{2i}) \in \mathcal{C}(p_{12i}, p_{21i}, p_{U1i}, p_{U2i}), \quad \forall i. \quad (7)$$

For notational convenience, we denote the sub-power and rate sequences by the vectors $\mathbf{p}_{12}, \mathbf{p}_{21}, \mathbf{p}_{U1}, \mathbf{p}_{U2}, \mathbf{r}_1, \mathbf{r}_2$. The departure region maximization problem is stated as a weighted sum departure maximization for given priorities $0 \leq \mu_1$, $\mu_2 \leq 1$, due to the convexity of the departure region:

$$(\boldsymbol{\mathcal{A}}) \quad \max_{\mathbf{p}_{12},\mathbf{p}_{21},\mathbf{p}_{U1},\mathbf{p}_{U2},\mathbf{r}_1,\mathbf{r}_2 \geq \mathbf{0}} \mu_1 \sum_{i=1}^{N} r_{1i} + \mu_2 \sum_{i=1}^{N} r_{2i} \quad (8)$$

$$\text{s.t. (3)-(7)} \quad (9)$$

In the second half of the paper, we investigate joint energy and data cooperation. The system model is given in Fig. 2. We assume that, at the beginning of slot i, there are energy arrivals to both users with amounts E_{1i}, E_{2i} respectively. Energy transfers from user 1 (2) to user 2 (1) are denoted by δ_{1i} (δ_{2i}). Energy transfer efficiency is $0 \leq \alpha < 1$: when user 1 (2) transfers δ_{1i} (δ_{2i}) Joules of energy to user 2 (1), $\alpha\delta_{1i}$ ($\alpha\delta_{2i}$) Joules of energy enters the energy queue of user 2 (1). When there is wireless energy transfer, this is done by two separate orthogonal energy transfer units whose coupling frequencies are set differently [38]. Finally, data transmission and energy transfer channels are orthogonal, i.e., energy transfer does not create interference to data communication and takes place over dedicated frequency bands not intended for data transmission. One example of energy transfer units are coils that work based on the principle of inductive coupling, operating at lower frequencies and narrower bandwidths, not suitable for high data rate transmission.

The net energy available for user $\ell \in \{1, 2\}$ in each slot $k \in \{1, \ldots, N\}$ is given by $\sum_{i=1}^{k}(E_{\ell i} - \delta_{\ell i} + \alpha\delta_{mi})$ where m is the other user. The energy causality constraints become:

$$\sum_{i=1}^{k} p_{1i} \leq \sum_{i=1}^{k}(E_{1i} - \delta_{1i} + \alpha\delta_{2i}), \quad \forall k \quad (10)$$

$$\sum_{i=1}^{k} p_{2i} \leq \sum_{i=1}^{k}(E_{2i} - \delta_{2i} + \alpha\delta_{1i}), \quad \forall k \quad (11)$$

We denote the energy transfer sequences by the vectors $\boldsymbol{\delta}_1, \boldsymbol{\delta}_2$. Then, the departure region maximization problem can be stated as:

$$(\boldsymbol{\mathcal{B}}) \quad \max_{\mathbf{p}_{12},\mathbf{p}_{21},\mathbf{p}_{U1},\mathbf{p}_{U2},\mathbf{r}_1,\mathbf{r}_2,\boldsymbol{\delta}_1,\boldsymbol{\delta}_2 \geq \mathbf{0}} \mu_1 \sum_{i=1}^{N} r_{1i} + \mu_2 \sum_{i=1}^{N} r_{2i} \quad (12)$$

$$\text{s.t. (7), (10), (11)} \quad (13)$$

III. Data Cooperation Under Intermittent Data Arrivals

In this section, we focus on the scenario with intermittent data arrivals which is shown in Fig. 1. We solve the problem in ($\mathcal{A}$). First, we prove some properties of the optimal solution.

Lemma 1: There exists an optimal profile that satisfies the following property,

$$r_{1i} = f(1 + p_{12i}), \quad r_{2i} = f(1 + p_{21i}), \quad \forall i. \tag{14}$$

Proof: We will prove this lemma by showing that for any policy that does not satisfy the above property, there exists another policy that satisfies it and achieves the same weighted sum departure. Assume there exists an optimal policy and slot i such that $r_{1i} < f(1 + p_{12i})$. Now consider the modified policy, $q_{12i} = p_{12i} - \epsilon$, $q_{U1i} = p_{U1i} + \epsilon$ while keeping the remaining variables fixed. In this modified policy, $q_{1i} = q_{12i} + q_{U1i} = p_{12i} + p_{U1i} = p_{1i}$, therefore the new policy spends the same amount of energy as the previous one and is energy feasible. It is easy to check that this modification increases s_i and (r_{1i}, r_{2i}) still belongs to the set $\mathcal{C}(q_{12i}, p_{21i}, q_{U1i}, p_{U2i})$. Since we have not changed the rates, the data causality constraints are still feasible. By repeating this process we will reach a profile where $r_{1i} = f(1 + p_{12i})$. By using similar arguments for r_{2i} and modifying p_{21i} and p_{U2i} we will reach a profile where $r_{2i} = f(1 + p_{21i})$. Since we have not changed the rates, the weighted sum departure is the same and the policy is still optimal. ■

With Lemma 1 and enforcing the constraints in (14) the sum rate constraints in (1) become:

$$f(1 + p_{12i}) + f(1 + p_{21i}) \le f(s_i/\sigma^2), \quad \forall i \tag{15}$$

In addition to the rate-power relationships dictated by Lemma 1, we further introduce the auxiliary rate variables, r_{U1i}, r_{U2i}, and perform the variable changes, $r_{U1i} = f(1 + p_{U1i}), r_{U2i} = f(1 + p_{U2i})$. Then $s_i = \sigma^2 + 2^{2r_{1i}} + 2^{2r_{U1i}} + 2^{2r_{2i}} + 2^{2r_{U2i}} + 2\sqrt{(2^{2r_{U1i}} - 1)(2^{2r_{U2i}} - 1)} - 4$. We formulate the problem only in terms of rates as,

$$(\mathcal{A}_1) \quad \max_{\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_{U1}, \mathbf{r}_{U2}} \sum_{i=1}^{N} \mu_1 r_{1i} + \mu_2 r_{2i} \tag{16}$$

$$\text{s.t.} \sum_{i=1}^{k} 2^{2r_{1i}} + 2^{2r_{U1i}} \le \sum_{i=1}^{k} (E_{1i} + 2), \ \forall k, \tag{17}$$

$$\sum_{i=1}^{k} 2^{2r_{2i}} + 2^{2r_{U2i}} \le \sum_{i=1}^{k} (E_{2i} + 2), \ \forall k, \tag{18}$$

$$\sum_{i=1}^{k} r_{1i} \le \sum_{i=1}^{k} d_{1i}, \quad \forall k, \tag{19}$$

$$\sum_{i=1}^{k} r_{2i} \le \sum_{i=1}^{k} d_{2i}, \ \forall k, \tag{20}$$

$$r_{1i} + r_{2i} \le f(s_i/\sigma^2), \ \forall i. \tag{21}$$

The problem in ($\mathcal{A}_1$) is a non-convex optimization problem due to the last set of constraints $r_{1i} + r_{2i} \le f(s_i/\sigma^2)$, $\forall i$. We use the successive convex approximation technique to approximate the constraints in (21) as explained in [39]. We use the first order Taylor expansion to the function $f(s_i/\sigma^2)$ around the point $\mathbf{R}^n \triangleq (\mathbf{r}_1^n, \mathbf{r}_2^n, \mathbf{r}_{U1}^n, \mathbf{r}_{U2}^n)$ for iteration $n + 1$, by

$$\begin{aligned} f^{n+1}(s_i/\sigma^2) \simeq C_i^n &+ a_{1i}^n(r_{1i} - r_{1i}^n) + a_{2i}^n(r_{2i} - r_{2i}^n) \\ &+ b_{1i}^n(r_{U1i} - r_{U1i}^n) + b_{2i}^n(r_{U2i} - r_{U2i}^n), \end{aligned} \tag{22}$$

where the values of the coefficients are given in Appendix A and depend only on the solution of the previous iteration n. With this approximation the problem in ($\mathcal{A}_1$) becomes

$$(\mathcal{A}_2) \quad \max_{\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_{U1}, \mathbf{r}_{U2}} \sum_{i=1}^{N} \mu_1 r_{1i} + \mu_2 r_{2i} \tag{23}$$

$$\text{s.t. (17)-(20)} \tag{24}$$

$$\begin{aligned} &(1 - a_{1i}^n) r_{1i} + (1 - a_{2i}^n) r_{2i} \\ &- b_{1i}^n r_{U1i} - b_{2i}^n r_{U2i} \le D_i^n, \ \forall i, \end{aligned} \tag{25}$$

where $D_i^n \triangleq C_i^n - a_{1i}^n r_{1i}^n - a_{2i}^n r_{2i}^n - b_{1i}^n r_{U1i}^n - b_{2i}^n r_{U2i}^n$ and is a constant for this optimization problem. At iteration $n + 1$, we evaluate the coefficients in (22) using the optimal rate allocations at iteration n, we solve the problem in ($\mathcal{A}_2$) using these coefficients and we update the initial point as $\mathbf{R}^{n+1} = \mathbf{R}^{*n}$ where $\mathbf{R}^{*n}$ denotes the optimal values of the variables when ($\mathcal{A}_2$) is solved. Now we show that this procedure stops at an optimal solution to ($\mathcal{A}_1$). To achieve this, we first show that strong duality holds for ($\mathcal{A}_1$). The proof is given in Appendix B.

Lemma 2: Strong duality holds for the problem in ($\mathcal{A}_1$).

Now we show that the procedure converges to an optimal solution. The proof is in Appendix C.

Lemma 3: $\mathbf{R}^n \to \mathbf{R}^*$ *where* $\mathbf{R}^*$ *solves* ($\mathcal{A}_1$).

In the next section, we solve the problem in ($\mathcal{A}_2$) for fixed n.

A. Solution for Approximate Problems

In this sub-section, we solve the approximate problems for iteration $n + 1$. For notational convenience we drop the superscript n from the last constraints in (25) noting that they depend only on the solution of the problem at the previous iteration n. Therefore the coefficients $a_{1i}^n, a_{2i}^n, b_{1i}^n, b_{2i}^n$ are essentially constants for the problem at step $n + 1$.

Lemma 4: There exists an optimal solution in which $(1 - a_{1i})r_{1i} + (1 - a_{2i})r_{2i} - b_{1i}r_{U1i} - b_{2i}r_{U2i} = D_i, \ \forall i$.

Proof: Assume there exists a profile where $(1 - a_{1i})r_{1i} + (1 - a_{2i})r_{2i} - b_{1i}r_{U1i} - b_{2i}r_{U2i} < D_i$ for some slot i. Then we can decrease, r_{U1i} or r_{U2i} to achieve equality. ■

Invoking Lemma 4, the problem becomes,

$$(\mathcal{A}_3) \quad \max_{\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_{U1}, \mathbf{r}_{U2}} \sum_{i=1}^{N} \mu_1 r_{1i} + \mu_2 r_{2i} \tag{26}$$

$$\text{s.t. (17)-(20)} \tag{27}$$

$$\begin{aligned} &(1 - a_{1i}) r_{1i} + (1 - a_{2i}) r_{2i} \\ &- b_{1i} r_{U1i} - b_{2i} r_{U2i} = D_i, \ \forall i. \end{aligned} \tag{28}$$

We solve the problem in ($\mathcal{A}_3$) using a primal decomposition. Since this procedure is fairly involved, we move this discussion to Appendix D and we refer interested readers there.

Algorithm 1 Algorithm to Solve ($\mathcal{A}_1$)

Initialize

1: Find initial feasible $\mathbf{R}^0 \triangleq (\mathbf{r}_1^0, \mathbf{r}_2^0, \mathbf{r}_{U1}^0, \mathbf{r}_{U2}^0)$

Define function to find z(t)

2: **function** SOLVEZ($a_{1i}^n, a_{2i}^n, b_{1i}^n, b_{2i}^n, D_i^n$) ▷ Solves z
3: Set $\mathbf{u} \leftarrow \mathbf{0}, \mathbf{t}_1 \leftarrow \mathbf{u}, \mathbf{t}_2 \leftarrow \mathbf{u}$
4: Solve $z_1(\mathbf{u}), z_2(\mathbf{u})$ as explained after (103) and (105)
5: $z(\mathbf{u}) \leftarrow z_1(\mathbf{u}) + z_2(\mathbf{u})$
6: **for** $i = 1 : N$ **do**
7: $t_{1i} \leftarrow u_i + \epsilon, t_{2i} \leftarrow u_i - \epsilon$
8: Solve $z_1(\mathbf{t}_1), z_2(\mathbf{t}_1), z_1(\mathbf{t}_2), z_2(\mathbf{t}_2)$
9: $z(\mathbf{t}_1) \leftarrow z_1(\mathbf{t}_1) + z_2(\mathbf{t}_1), z(\mathbf{t}_2) \leftarrow z_1(\mathbf{t}_2) + z_2(\mathbf{t}_2)$
10: **if** $[z(\mathbf{t}_1) > z(\mathbf{u})]$ **then** $\mathbf{u} \leftarrow \mathbf{t}_1$
11: **else if** $[z(\mathbf{t}_2) > z(\mathbf{u})]$ **then** $\mathbf{u} \leftarrow \mathbf{t}_2$
12: **end if**
13: **end for**
14: Go to (6) until convergence
15: **return** last found optimal $(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_{U1}, \mathbf{r}_{U2})$
16: **end function**

Main Algorithm

17: **repeat**
18: Find $A_i^n, a_{1i}^n, a_{2i}^n, b_{1i}^n, b_{2i}^n, C_i^n$ from (67) - (72)
19: $D_i^n \leftarrow C_i^n - a_{1i}^n r_{1i}^n - a_{2i}^n r_{2i}^n - b_{1i}^n r_{U1i}^n - b_{2i}^n r_{U2i}^n$
20: $\mathbf{R}^{n+1} \leftarrow$ SOLVEZ($a_{1i}^n, a_{2i}^n, b_{1i}^n, b_{2i}^n, D_i^n$)
21: $n \leftarrow n + 1$
22: **until** convergence

Once ($\mathcal{A}_3$) is solved, we update the variables as discussed in the previous section. The overall solution algorithm is given in Algorithm 1, given at the top of the this page.

IV. Joint Data and Energy Cooperation

In this section, we focus on the scenario with joint energy and data cooperation. The system model is shown in Fig. 2. We solve the problem in ($\mathcal{B}$). First, we state the necessary conditions for the optimal profile. These conditions lead to interesting interpretations regarding the nature of energy exchange, including its direction, timing and physical relation to data cooperation. We relax the equality in (2) to reformulate ($\mathcal{B}$) as follows.[1]

$$(\mathcal{B}_1) \quad \max_{\mathbf{p}_{12},\mathbf{p}_{21},\mathbf{p}_{U1},\mathbf{p}_{U2},\mathbf{r}_1,\mathbf{r}_2,\boldsymbol{\delta}_1,\boldsymbol{\delta}_2,\mathbf{s}'\geq\mathbf{0}} \mu_1 \sum_{i=1}^N r_{1i} + \mu_2 \sum_{i=1}^N r_{2i}$$

$$\text{s.t.} \sum_{i=1}^k (p_{12i} + p_{U1i}) \leq \sum_{i=1}^k (E_{1i} - \delta_{1i} + \alpha\delta_{2i}), \ \forall k \quad (29)$$

$$\sum_{i=1}^k (p_{21i} + p_{U2i}) \leq \sum_{i=1}^k (E_{2i} - \delta_{2i} + \alpha\delta_{1i}), \ \forall k \quad (30)$$

$$r_{1i} \leq f(1 + p_{12i}), \quad \forall i \quad (31)$$

$$r_{2i} \leq f(1 + p_{21i}), \quad \forall i \quad (32)$$

$$r_{1i} + r_{2i} \leq f(s_i'/\sigma^2), \quad \forall i \quad (33)$$

$$s_i' \leq \sigma^2 + p_{12i} + p_{U1i} + p_{21i} + p_{U2i} + 2\sqrt{p_{U1i}p_{U2i}}, \quad \forall i \quad (34)$$

The problem ($\mathcal{B}_1$) is convex. This can be easily verified by noting that the objective in ($\mathcal{B}_1$) and constraints (29)-(30) are linear and the logarithms on the right hand sides of (31)-(33), as well as $\sqrt{p_{U1i}p_{U2i}}$ on the right hand side of (34) are all concave in the vector of all variables. However, (34) is non-differentiable due to the term $\sqrt{p_{U1i}p_{U2i}}$ when $p_{U1i} = 0$ or $p_{U2i} = 0$. Now, we show that in the optimal solution, the cooperative powers p_{U1i}, p_{U2i} are non-zero at all slots. The proof is given in Appendix E.

Lemma 5: The cooperative powers are strictly positive in all slots, i.e., $p_{U1i} > 0, p_{U2i} > 0, \forall i$.

Utilizing Lemma 5, the functions $\sqrt{p_{U1i}p_{U2i}}$ are now differentiable. We write the Lagrangian of problem ($\mathcal{B}_1$) as [40]

$$\begin{aligned}\mathcal{L} = &-\mu_1 \sum_{i=1}^N r_{1i} - \mu_2 \sum_{i=1}^N r_{2i} \\ &+ \sum_{k=1}^N \lambda_{1k}\Big[\sum_{i=1}^k (p_{12i} + p_{U1i} - E_{1i} + \delta_{1i} - \alpha\delta_{2i})\Big] \\ &+ \sum_{k=1}^N \lambda_{2k}\Big[\sum_{i=1}^k (p_{21i} + p_{U2i} - E_{2i} + \delta_{2i} - \alpha\delta_{1i})\Big] \\ &+ \sum_{i=1}^N \theta_{1i}\left[r_{1i} - f(1 + p_{12i})\right] \\ &+ \sum_{i=1}^N \theta_{2i}\left[r_{2i} - f(1 + p_{21i})\right] \\ &+ \sum_{i=1}^N \theta_{3i}\Big[r_{1i} + r_{2i} - f(s_i'/\sigma^2)\Big] \\ &+ \sum_{i=1}^N \phi_i\big[s_i' - \sigma^2 - p_{12i} - p_{U1i} - p_{21i} - p_{U2i} \\ &\qquad - 2\sqrt{p_{U1i}p_{U2i}}\big] \\ &- \sum_{i=1}^N \big[\gamma_{1i}p_{12i} + \gamma_{2i}p_{21i} + \gamma_{3i}p_{U1i} + \gamma_{4i}p_{U2i} + \gamma_{5i}r_{1i} \\ &\qquad + \gamma_{6i}r_{2i} + \gamma_{7i}\delta_{1i} + \gamma_{8i}\delta_{2i} + \gamma_{9i}s_i'\big]\end{aligned} \quad (35)$$

Then, the KKT optimality conditions are found as:

$$\frac{\partial \mathcal{L}}{\partial r_{1i}} = -\mu_1 + \theta_{1i} + \theta_{3i} - \gamma_{5i} = 0, \forall i \quad (36)$$

$$\frac{\partial \mathcal{L}}{\partial r_{2i}} = -\mu_2 + \theta_{2i} + \theta_{3i} - \gamma_{6i} = 0, \forall i \quad (37)$$

$$\frac{\partial \mathcal{L}}{\partial p_{12i}} = \sum_{k=i}^N \lambda_{1k} - \frac{\theta_{1i}}{(1 + p_{12i})} - \phi_i - \gamma_{1i} = 0, \forall i \quad (38)$$

$$\frac{\partial \mathcal{L}}{\partial p_{21i}} = \sum_{k=i}^N \lambda_{2k} - \frac{\theta_{2i}}{(1 + p_{21i})} - \phi_i - \gamma_{2i} = 0, \forall i \quad (39)$$

[1] Lemma 8 proves that problems ($\mathcal{B}_1$) and ($\mathcal{B}$) are equivalent.

$$\frac{\partial \mathcal{L}}{\partial p_{U1i}} = \sum_{k=i}^{N} \lambda_{1k} - \phi_i \left(1 + \frac{\sqrt{p_{U2i}}}{\sqrt{p_{U1i}}}\right) - \gamma_{3i} = 0, \forall i \quad (40)$$

$$\frac{\partial \mathcal{L}}{\partial p_{U2i}} = \sum_{k=i}^{N} \lambda_{2k} - \phi_i \left(1 + \frac{\sqrt{p_{U1i}}}{\sqrt{p_{U2i}}}\right) - \gamma_{4i} = 0, \forall i \quad (41)$$

$$\frac{\partial \mathcal{L}}{\partial \delta_{1i}} = \sum_{k=i}^{N} \lambda_{1k} - \alpha \sum_{k=i}^{N} \lambda_{2k} - \gamma_{7i} = 0, \forall i \quad (42)$$

$$\frac{\partial \mathcal{L}}{\partial \delta_{2i}} = \sum_{k=i}^{N} \lambda_{2k} - \alpha \sum_{k=i}^{N} \lambda_{1k} - \gamma_{8i} = 0, \forall i \quad (43)$$

$$\frac{\partial \mathcal{L}}{\partial s_i'} = -\frac{\theta_{3i}}{s_i'} + \phi_i - \gamma_{9i} = 0, \forall i \quad (44)$$

with complementary slackness conditions:

$$\lambda_{1k} \sum_{i=1}^{k} (p_{12i} + p_{U1i} - E_{1i} + \delta_{1i} - \alpha\delta_{2i}) = 0, \forall k \quad (45)$$

$$\lambda_{2k} \sum_{i=1}^{k} (p_{21i} + p_{U2i} - E_{2i} + \delta_{2i} - \alpha\delta_{1i}) = 0, \forall k \quad (46)$$

$$\theta_{1i} (r_{1i} - f(1 + p_{12i})) = 0, \forall i \quad (47)$$

$$\theta_{2i} (r_{2i} - f(1 + p_{21i})) = 0, \forall i \quad (48)$$

$$\theta_{3i} \left(r_{1i} + r_{2i} - f(s_i'/\sigma^2)\right) = 0, \forall i \quad (49)$$

$$\phi_i (s_i' - \sigma^2 - p_{12i} - p_{U1i} - p_{21i} - p_{U2i} - 2\sqrt{p_{U1i} p_{U2i}}) = 0 \quad (50)$$

$$\gamma_{1i} p_{12i} = \gamma_{2i} p_{21i} = \gamma_{3i} p_{U1i} = \gamma_{4i} p_{U2i} = 0 \quad (51)$$

$$\gamma_{5i} r_{1i} = \gamma_{6i} r_{2i} = \gamma_{7i} \delta_{1i} = \gamma_{8i} \delta_{2i} = \gamma_{9i} s_i' = 0 \quad (52)$$

From Lemma 5, $\gamma_{3i} = \gamma_{4i} = 0, \forall i$. We must also have $s_i' > 0$ since otherwise from (33) we have $r_{1i} = r_{2i} = 0$ which cannot be optimal. Now, we investigate the optimal Lagrange multipliers in the following two lemmas.

Lemma 6: We have $\phi_i > 0, \forall i$.

Proof: We will prove by contradiction. Assume $\phi_i = 0$. From (44), $\theta_{3i} = 0$. From (38) and (40):

$$\frac{\theta_{1i}}{(1 + p_{12i})} + \phi_i + \gamma_{1i} = \phi_i \left(1 + \frac{\sqrt{p_{U2i}}}{\sqrt{p_{U1i}}}\right) + \gamma_{3i} \quad (53)$$

From Lemma 5, we have $\gamma_{3i} = 0$. Then, (53) becomes:

$$\frac{\theta_{1i}}{(1 + p_{12i})} + \gamma_{1i} = 0 \quad (54)$$

which implies $\theta_{1i} = 0$. Together with $\theta_{3i} = 0$ and (36) we have: $\mu_1 + \gamma_{5i} = 0$ which is only possible when $\mu_1 = 0$ and $\gamma_{5i} = 0$. Using a similar line of reasoning and using eqns. (39), (41) and (37) we will have $\mu_2 + \gamma_{6i} = 0$ which is only possible when $\mu_2 = 0$ and $\gamma_{6i} = 0$. Since $\mu_1 = \mu_2 = 0$ yields a trivial case with the objective function being 0, $\phi_i = 0$ can not correspond to a point on the rate region boundary. ■

We note that Lemma 6 further means, from (40) and (41), that $\sum_{k=i}^{N} \lambda_{1k} > 0, \sum_{k=i}^{N} \lambda_{2k} > 0, \forall i$; which in turn means that all available energy must be depleted in the last slot, by both users.

Lemma 7: We have $\gamma_{9i} = 0, \forall i$.

Proof: Assume $\gamma_{9i} > 0$ for some i. This implies $s_i' = 0$ and from (33), $r_{1i} = r_{2i} = 0$, which cannot be optimal. ■

Using the structure of the optimal Lagrange multipliers, the following lemma states some properties of the optimal solution.

Lemma 8: The optimal profile must satisfy:

1) $s_i' = \sigma^2 + p_{12i} + p_{U1i} + p_{21i} + p_{U2i} + 2\sqrt{p_{U1i} p_{U2i}}, \forall i$.
2) $r_{1i} + r_{2i} = f(s_i'/\sigma^2), \forall i$
3) $r_{1i} = f(1 + p_{12i}), r_{2i} = f(1 + p_{12i}), \forall i$

Proof: We prove the lemma as follows:
1) Follows from Lemma 6 and (50).
2) From Lemma 7 and (44), we have $\theta_{3i} = \phi_i s_i'$. Since $\phi_i > 0$ from Lemma 6, $\theta_{3i} > 0$ which implies $r_{1i} + r_{2i} = f(s_i'/\sigma^2)$ from (49).
3) If $p_{12i} = 0$, then we must have $r_{1i} = 0$ and $r_{1i} = f(1 + p_{12i})$ is satisfied. If $p_{12i} > 0$, then $\gamma_{1i} = 0$ from (51). From (38) and (40), $\theta_{1i} = \phi_i \sqrt{p_{U2i}/p_{U1i}}(1 + p_{12i}) > 0$. From (47), $r_{1i} = f(1 + p_{12i})$. Similarly, if $p_{21i} = 0$, then we must have $r_{2i} = 0$ and $r_{2i} = f(1 + p_{21i})$. If $p_{21i} > 0$, then $\gamma_{2i} = 0$ from (51). From (39) and (41), $\theta_{2i} = \phi_i \sqrt{p_{U1i}/p_{U2i}}(1 + p_{21i}) > 0$. From (48), $r_{2i} = f(1 + p_{21i})$. ■

Lemma 8 shows that there is a one-to-one correspondence between the transmission rates and transmission powers. Furthermore, the transmission powers should satisfy

$$f(1 + p_{12i}) + f(1 + p_{21i}) = f(s_i'/\sigma^2), \ \forall i. \quad (55)$$

This means that, like ($\mathcal{A}_1$), ($\mathcal{B}_1$) can also be restated over a reduced set of variables; this time powers only. Moreover, in this case, the problem retains its convexity. Now, we show that, data cooperation always precedes energy cooperation. In other words, a user with excess energy to be invested in cooperation in a given slot, must first invest more energy for data cooperation than its partner; only then can it invest energy for direct energy cooperation.

Lemma 9: The optimal profile satisfies the following:

1) *If* $\delta_{2i} > 0$ *then* $p_{U2i} > p_{U1i}$.
2) *If* $\delta_{1i} > 0$ *then* $p_{U1i} > p_{U2i}$.

Proof: We start with the first item. If $\delta_{2i} > 0$, then from (52), we have $\gamma_{8i} = 0$. From (43), we have $\sum_{k=i}^{N} \lambda_{2k} = \alpha \sum_{k=i}^{N} \lambda_{1k}$. This implies from (40) and (41),

$$\phi_i \left(1 + \frac{\sqrt{p_{U1i}}}{\sqrt{p_{U2i}}}\right) = \alpha\phi_i \left(1 + \frac{\sqrt{p_{U2i}}}{\sqrt{p_{U1i}}}\right) \quad (56)$$

Since $\phi_i > 0$ and $\alpha < 1$, (56) implies:

$$\left(1 + \frac{\sqrt{p_{U1i}}}{\sqrt{p_{U2i}}}\right) < \left(1 + \frac{\sqrt{p_{U2i}}}{\sqrt{p_{U1i}}}\right) \quad (57)$$

which implies $p_{U2i} > p_{U1i}$. The second item is proved similarly. ■

Now, we show that if, in a given slot, a user with high priority transfers energy to a user with lower priority, the user with higher priority must already be transmitting at a higher data rate in that slot than the user with lower priority.

Lemma 10: The optimal profile satisfies the following:

1) *For* $\mu_2 \geq \mu_1$, *if* $\delta_{2i} > 0$, *then* $r_{2i} \geq r_{1i}$.
2) *For* $\mu_1 \geq \mu_2$, *if* $\delta_{1i} > 0$, *then* $r_{1i} \geq r_{2i}$.

Proof: We start with the first item. Assume $\mu_1 \geq \mu_2$ and $\delta_{2i} > 0$. If $p_{12i} = 0$, then $r_{1i} = 0$ and the statement holds trivially. We will assume $p_{12i} > 0$. From (52), $\gamma_{8i} = 0$.

From (43), $\sum_{k=i}^{N} \lambda_{2k} < \sum_{k=i}^{N} \lambda_{1k}$. From (38) and (39), this implies

$$\frac{\theta_{2i}}{(1+p_{21i})} + \phi_i + \gamma_{2i} < \frac{\theta_{1i}}{(1+p_{12i})} + \phi_i + \gamma_{1i} = \frac{\theta_{1i}}{(1+p_{12i})} + \phi_i \quad (58)$$

where the equality follows since $p_{12i} > 0$ implies $\gamma_{1i} = 0$. Then we have,

$$\frac{\theta_{2i}}{(1+p_{21i})} < \frac{\theta_{1i}}{(1+p_{12i})} \quad (59)$$

From (36) and (37) we have,

$$\theta_{1i} = \mu_1 + \gamma_{5i} - \theta_{3i} = \mu_1 - \theta_{3i} \quad (60)$$

$$\theta_{2i} = \mu_2 + \gamma_{6i} - \theta_{3i} \geq \mu_2 - \theta_{3i} \quad (61)$$

where (60) follows from $r_{1i} = f(1+p_{12i}) > 0$, therefore $\gamma_{5i} = 0$. Since $\mu_2 \geq \mu_1$, we have $\theta_{2i} \geq \theta_{1i}$. Together with (59), this implies we have $p_{21i} > p_{12i}$ and therefore $r_{2i} > r_{1i}$. The second item is proved similarly. ∎

A. Procrastinating Policies

In this sub-section, we show the existence of *procrastinating policies* that solve the optimal data and energy cooperation problem. Procrastinating policies are introduced in [25]. They are formally defined in [25, Definition 1] as policies which spend more energy than energy received by cooperation in each slot. Here, we provide an alternative and stricter definition: we call a policy *strictly procrastinating* if it has the property that any energy transferred in slot i, is immediately consumed entirely by the receiving party within the same slot. We formalize this definition below.

Definition 1: A policy is called strictly procrastinating if, at the end of any slot k for which $\delta_{jk} > 0$, the battery of the user receiving the energy is depleted, i.e.,

$$\sum_{i=1}^{k} \left(p_{mji} + p_{Umi}\right) = \sum_{i=1}^{k} \left(E_{mi} - \delta_{mi} + \alpha\delta_{ji}\right) \quad (62)$$

where m and j denote distinct users.

Lemma 11: There exists a strictly procrastinating policy that solves the problem in ($\mathcal{B}$).

Proof: Let $\mathbf{p}_{12}^*, \mathbf{p}_{21}^*, \mathbf{p}_{U1}^*, \mathbf{p}_{U2}^*, \boldsymbol{\delta}_1^*, \boldsymbol{\delta}_2^*$ be an optimal policy which is not strictly procrastinating, with

$$\alpha\delta_{jk}^* > \sum_{i=1}^{k-1} \left(p_{mji}^* + p_{Umi}^* - E_{mi} + \delta_{mi}^* - \alpha\delta_{ji}^*\right) + p_{mjk}^* + p_{Umk}^* - E_{mk} \triangleq \kappa > 0 \quad (63)$$

for some k. Define $\varepsilon = \delta_{jk}^* - \kappa/\alpha$. Now, reduce the energy transferred in slot k by ε to obtain $\delta_{jk}' = \delta_{jk}^* - \varepsilon$, and add this saved amount to the energy transfer in the next slot: $\delta_{j(k+1)}' = \delta_{j(k+1)}^* + \varepsilon$. The resulting policy is still optimal, since the value of the objective function remains unchanged, and the energy causality is still satisfied. By repeating this procedure for all slots, we reach a strictly procrastinating policy, which yields the same optimum objective function value. ∎

As a result of Lemma 11, we can limit our attention to only policies which transfer just the right amount of energy required in that slot, and assume that all power is used up whenever energy is transferred. In light of this finding, in the next subsection, we develop an algorithmic solution based on the KKT conditions.

B. Algorithmic Solution

While we have shown several important properties of the optimal solution, we still need to solve the problem to obtain the transmit scheduling and energy transfer policy. We do this using an algorithmic approach based on the KKT conditions given earlier. We determine the conditions under which energy transfer occurs. Then, we develop an algorithm to compute the optimal energy transfer and power allocation policy. Now, we show that energy transfers are never bidirectional, i.e., in any slot energy transfer happens only in a single direction.

Lemma 12: In the optimal profile if $\delta_{1i} > 0$ then $\delta_{2i} = 0$ and if $\delta_{2i} > 0$ then $\delta_{1i} = 0$, i.e. $\delta_{1i}\delta_{2i} = 0$, $\forall i$.

Proof: Assume for some slot i, $\delta_{1i} > 0, \delta_{2i} > 0$. Then, from (52), $\gamma_{7i} = \gamma_{8i} = 0$, and from (42) and (43), $\sum_{k=i}^{N} \lambda_{1k} = \alpha \sum_{k=i}^{N} \lambda_{2k} = \alpha(\alpha \sum_{k=i}^{N} \lambda_{1k})$ which cannot happen unless $\alpha = 1$. ∎

Lemma 13: If $\alpha < \frac{\sum_{k=i}^{N} \lambda_{1k}}{\sum_{k=i}^{N} \lambda_{2k}} < \frac{1}{\alpha}$, there is no energy transfer in either direction at slot i, i.e., $\delta_{1i} = \delta_{2i} = 0$.

Proof: Let $\alpha < \frac{\sum_{k=i}^{N} \lambda_{1k}}{\sum_{k=i}^{N} \lambda_{2k}} < \frac{1}{\alpha}$, or equivalently $\sum_{k=i}^{N} \lambda_{1k} > \alpha \sum_{k=i}^{N} \lambda_{2k}$ and $\sum_{k=i}^{N} \lambda_{2k} > \alpha \sum_{k=i}^{N} \lambda_{1k}$. From (42) and (43), $\gamma_{7i} > 0, \gamma_{8i} > 0$ and from (52) $\delta_{1i} = \delta_{2i} = 0$. ∎

Lemma 14: A power allocation policy which yields $\frac{\sum_{k=i}^{N} \lambda_{1k}}{\sum_{k=i}^{N} \lambda_{2k}} < \alpha$ or $\frac{\sum_{k=i}^{N} \lambda_{1k}}{\sum_{k=i}^{N} \lambda_{2k}} > \frac{1}{\alpha}$ is strictly suboptimal.

Proof: Follows from (42), (43) and $\gamma_{7i} \geq 0, \gamma_{8i} \geq 0$. ∎

Lemma 15: In a given slot $i \in 1, \ldots, N$, user $\ell \in \{1, 2\}$ transfers energy to user $m \in \{1, 2\}, m \neq \ell$, i.e., $\delta_{li} > 0$, if $\frac{\sum_{k=i}^{N} \lambda_{\ell k}}{\sum_{k=i}^{N} \lambda_{mk}} = \alpha$.

Proof: If $\delta_{1i} > 0$ then from (52) we have $\gamma_{7i} = 0$. If $\delta_{2i} > 0$ then from (52) we have $\gamma_{8i} = 0$. The result then follows from (42) and (43). ∎

Lemmas 13, 14 and 15 have the following physical interpretation: the ratio of the generalized water levels $\upsilon_{\ell i} \triangleq \left(\sum_{k=i}^{N} \lambda_{\ell k}\right)^{-1}$, $\ell \in \{1, 2\}$, determines whether or not there should be energy cooperation in each given slot. In particular, for slot i in which the generalized water level ratio $\upsilon_{\ell i}/\upsilon_{mi}$ without energy transfer is below the energy transfer efficiency α, energy should be transferred from user m to user ℓ, until the ratio is exactly equal to α. If there is not much discrepancy between the water levels, i.e., the ratio is between α and $1/\alpha$, then there should be no energy transfer.

Note that, the KKT conditions pertaining to the energy transfer policy do not explicitly depend on the powers, and the KKT conditions pertaining to the optimal power distribution policy do not explicitly depend on the energy transfer variables. Since these two sets of conditions are coupled only through the generalized water levels, it is possible to develop an iterative algorithm that iterates over power distribution and energy transfer steps, updating the generalized water levels in each energy transfer step based on Lemmas 13, 14 and 15. Such an algorithm, that provably converges, is given in Algorithm 2.

Algorithm 2 Optimal Energy and Data Cooperation Algorithm

Initialize

```
for i = 1 : N do
    p_{1i} ← E_{1i}, p_{2i} ← E_{2i}
    Determine subpowers p_{12i}, p_{U1i}, p_{21i}, p_{U2i}
    Determine water levels Σ_{k=i}^{N} λ_{1k}, Σ_{k=i}^{N} λ_{2k} from (38)-(41)
end for
```

Main Algorithm

```
repeat
    for i = 1 : N do
If Σ_{k=i}^{N} λ_{1k} < α Σ_{k=i}^{N} λ_{2k}, transfer energy from user 1 to user 2
If Σ_{k=i}^{N} λ_{2k} < α Σ_{k=i}^{N} λ_{1k}, transfer energy from user 2 to user 1
        Determine new subpowers p_{12i}, p_{U1i}, p_{21i}, p_{U2i}
        Determine new water levels
    end for
until
Σ_{k=i}^{N} λ_{1k} = α Σ_{k=i}^{N} λ_{2k} or Σ_{k=i}^{N} λ_{2k} = α Σ_{k=i}^{N} λ_{1k}
```

V. Numerical Results

In this section, we provide numerical examples and illustrate the resulting optimal policies. We consider band-limited AWGN broadcast and multiple-access channels. The bandwidth is $B_W = 1$ MHz and the noise power spectral density is $N_0 = 10^{-19}$ W/Hz. We assume that the path loss from user 1 to user 2 (h_{12}) and user 2 to user 1 (h_{21}) are the same and are selected as $h_{12} = h_{21} = -130$dB. The path losses, h_{1d} and h_{2d}, on the user to destination links are also same and are selected as $h_{1d} = h_{2d} = -133$dB. With these definitions, equations (1) and (2) become:

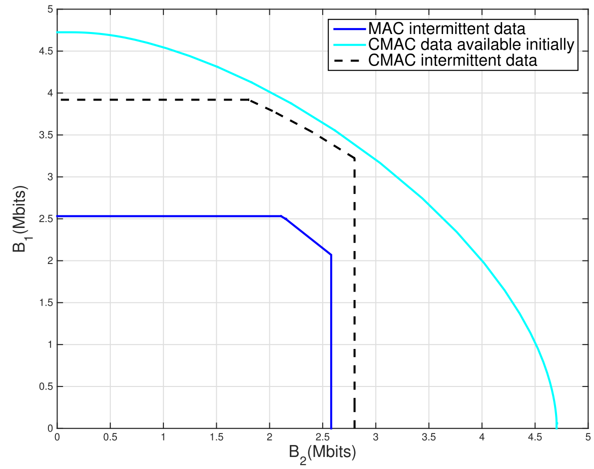
$$r_{1i} \le B_W \log_2\left(1+\frac{h_{12}p_{12i}}{N_0 B_W}\right) = \log_2(1+p_{12i})\,\text{Mbps} \tag{64}$$

$$r_{2i} \le B_W \log_2\left(1+\frac{h_{21}p_{21i}}{N_0 B_W}\right) = \log_2(1+p_{21i})\,\text{Mbps} \tag{65}$$

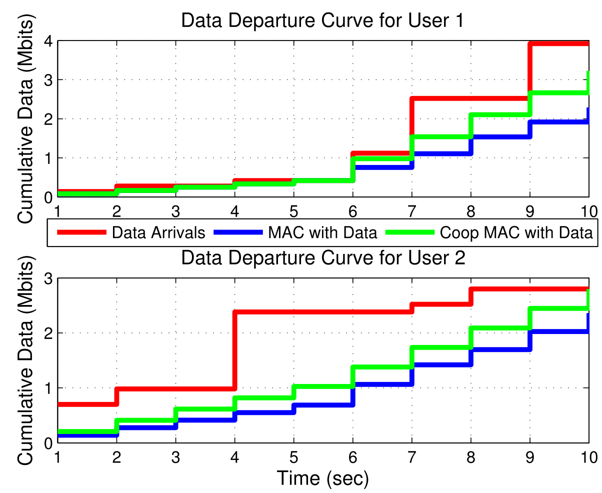
$$\begin{aligned} r_{1i}+r_{2i} &\le B_W \log_2\left[1+(N_0B_W)^{-1}\left(h_{1d}p_{1i}+h_{2d}p_{2i} + 2\sqrt{h_{1d}p_{U1i}h_{2d}p_{U2i}}\right)\right] \\ &= \log_2[1+(p_{12i}+p_{21i}+2\sqrt{p_{U1i}p_{U2i}})/2]\text{Mbps} \end{aligned} \tag{66}$$

A. Intermittent Data Arrivals Scenario

In this subsection, we demonstrate numerically that user cooperation improves the achievable departure region of a MAC, under data and energy arrival constraints. In Fig. 3a we plot the achievable departure region of the proposed cooperative MAC model with energy and data arrival constraints. The vertical and horizontal axes are given by the total number of departed bits, i.e., $B_1 = \sum_{i=1}^{N} r_{1i}$, $B_2 = \sum_{i=1}^{N} r_{2i}$. The energy and data arrivals are chosen as $\mathbf{E}_1 = [5, 0, 5, 0, 0, 0, 10, 0, 0]$ mJ, $\mathbf{E}_2 = [5, 0, 0, 0, 0, 10, 0, 0, 5, 0]$ mJ, $\mathbf{d}_1 = [1.4, 1.4, 0, 1.4, 0, 7, 14, 0, 14, 0] \times 10^{-1}$ Mbits, $\mathbf{d}_2 = [7, 2.8, 0, 14, 0, 0, 1.4, 2.8, 0, 0] \times 10^{-1}$ Mbits. The transmission deadline is chosen as 10 seconds. We observe that, a cooperative MAC in which data arrives intermittently during the transmission achieves a better departure region than its non-cooperative counterpart, and the improvement is more significant near the sum-departure point. In fact, a comparison to a benchmark system in which data arrives in bulk prior to the transmission, shows that there is little loss due to intermittent data arrivals near the sum-departure, as opposed to the significant losses near single user departure points. This is due to the cap put on the departing number of bits due to data causality: the potential from cooperation cannot be sufficiently capitalized on for one sided cooperation as there is not enough data to be transmitted; while near the sum departure, flexibility of cooperatively transmitting either user's data leads to a better use of cooperation.



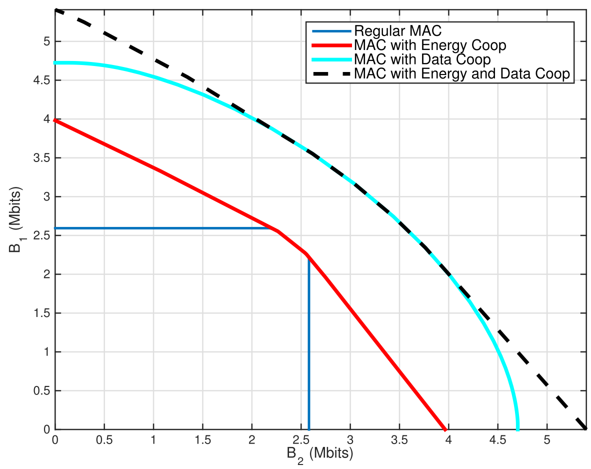
(a) Departure regions regular vs. cooperative MAC.



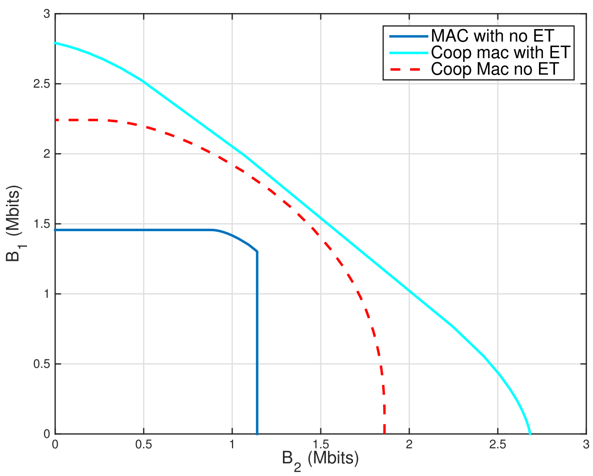
(b) Data departure curves for the users when $\mu_1 = \mu_2 = 1$.

Fig. 3. Simulation results for intermittent data arrivals.

Additionally, we plot the data departure curves for both users in Fig. 3b in the case of sum departure maximization,



(a) Energy profile 1 and $\alpha = 0.6$



(b) Energy profile 2 and $\alpha = 0.8$

Fig. 4. Departure regions of regular MAC, MAC with energy cooperation, MAC with data cooperation and MAC with energy and data cooperation.

i.e., $\mu_1 = \mu_2 = 1$. We see that the possibility of user cooperation allows for higher data rates to be sustained using the same amount of energy.

B. Energy Cooperation Scenario

We now demonstrate that joint energy and data cooperation improves the achievable departure region of a MAC. In Fig. 4 we plot the achievable departure regions of the proposed cooperative MAC model with energy and data cooperation. For comparison, we also plot the departure region of a cooperative MAC channel with only data cooperation which was studied in [1]. We use the channel parameters as described before. We investigate two energy arrival profiles.

1) Energy Arrival Profile 1: The energy arrivals are $\mathbf{E}_1 = [5, 0, 5, 0, 0, 0, 0, 10, 0, 0]$ mJ, $\mathbf{E}_2 = [5, 0, 0, 0, 0, 10, 0, 0, 5, 0]$ mJ, with energy transfer efficiency of $\alpha = 0.6$ and the transmission deadline is chosen as 10 seconds. In Fig. 4a, we observe that energy cooperation together with data cooperation has enhanced the departure region of the MAC. It is interesting that this effect is more pronounced in single user optimal points rather than sum departure optimal point. At the sum departure optimal point, $\sum_{i=1}^{N} r_{1i} + r_{2i}$ is optimized and the discrepancies in the energy arrival patterns are negated due to the powers appearing as a summation term.

2) Energy Arrival Profile 2: In Fig. 4b we plot the achievable departure regions with $\mathbf{E}_1 = [5, 7, 0]$ mJ, $\mathbf{E}_2 = [1, 0, 10]$ mJ, and $\alpha = 0.8$. Note that, at the sum departure optimal point ($\mu_1 = \mu_2 = 1$), the achievable departure region of the cooperative MAC with energy and data cooperation is strictly larger than that of the cooperative MAC with only data cooperation. This is unlike the energy cooperative data non-cooperative MAC, where it is known that energy transfer cannot improve the sum capacity [25]. Furthermore, when $\mu_1 = \mu_2 = 1$, the optimal energy transfers are found to be $\boldsymbol{\delta}_1^* = [1.22, 2.76, 0]$ mJ and $\boldsymbol{\delta}_2^* = [0, 0, 4.18]$ mJ. This optimal energy transfer is particularly interesting because at slots 1 and 2, user 1 has transferred energy to user 2 but at the last slot, user 2 has transferred energy to user 1. It was shown in [25] that in an energy cooperating MAC, the optimal energy transfer direction remains the same throughout the transmission. In the data and energy cooperative MAC scenario, we numerically observe that this is not the case: not only can the sum departure be improved by joint data and energy cooperation, but also the energy transfer direction may change throughout the transmission.

VI. Conclusion

In the first part of the paper, we considered a cooperative MAC with intermittent data and energy arrivals. We found the optimal offline power and rate allocation policies that maximize the departure region. We first showed that there exists an optimal policy, in which the single user rate constraints in each time slot are tight. Then, we formulated the departure region maximization problem as a weighted sum departure maximization in terms of rates only. Next, we proposed a sequential convex approximation method and showed that it converges to the optimal solution. Finally, we solved the approximate problems with an inner outer decomposition method. Numerically, we observed that higher data rates can be sustained using the same amount of energy.

In the second part of the paper, we considered a cooperative MAC with data and energy cooperation. We found the optimal offline transmit power and rate allocation policy that maximizes the departure region. We first showed that, the cooperative powers in each slot must be non-zero for both users. Next, we showed that, data cooperation always precedes energy cooperation. In other words, excess energy must first be used to increase cooperative powers and then to assist the other user via energy cooperation. Then, we showed that if a high priority user transfers energy to a low priority user, the higher priority user must already be transmitting at a higher data rate than the other user. Then, we showed the existence of procrastinating policies, which have the property that energy transferred in a slot must be consumed in that slot immediately. Finally, we derived necessary conditions for the existence of energy transfer and we provided an algorithmic solution to obtain the maximum departure region.

APPENDIX A
COEFFICIENTS OF (22)

By differentiating $f(s_i/\sigma^2)$ the coefficients are,

$$A_i^n = 2^{2r_{1i}^n} + 2^{2r_{U1i}^n} + 2^{2r_{2i}^n} + 2^{2r_{U2i}^n} + 2\sqrt{(2^{2r_{U1i}^n}-1)(2^{2r_{U2i}^n}-1)} - 4, \quad (67)$$

$$a_{1i}^n \triangleq \frac{\partial g}{\partial r_{1i}}\Big|_{r_{1i}^n} = \frac{0.5}{1+A_i^n/\sigma^2} 2^{2r_{1i}^n}, \quad (68)$$

$$a_{2i}^n \triangleq \frac{\partial g}{\partial r_{2i}}\Big|_{r_{2i}^n} = \frac{0.5}{1+A_i^n/\sigma^2} 2^{2r_{2i}^n}, \quad (69)$$

$$b_{1i}^n \triangleq \frac{\partial g}{\partial r_{U1i}}\Big|_{r_{U1i}^n} = \frac{0.5}{1+A_i^n/\sigma^2} 2^{2r_{U1i}^n}\left(1+\frac{\sqrt{2^{2r_{U2i}^n}-1}}{\sqrt{2^{2r_{U1i}^n}-1}}\right), \quad (70)$$

$$b_{2i}^n \triangleq \frac{\partial g}{\partial r_{U2i}}\Big|_{r_{U2i}^n} = \frac{0.5}{1+A_i^n/\sigma^2} 2^{2r_{U2i}^n}\left(1+\frac{\sqrt{2^{2r_{U1i}^n}-1}}{\sqrt{2^{2r_{U2i}^n}-1}}\right), \quad (71)$$

$$C_i^n = \frac{1}{2}\log_2\left(1+\frac{A_i^n}{\sigma^2}\right). \quad (72)$$

APPENDIX B
PROOF OF LEMMA 2

We will prove a more general result. Consider the two problems (P1) and (P2) given below.

$$\textbf{(P1):} \ \min_{\mathbf{x}} \ f_0(\mathbf{x}) \quad \text{s.t. } f_i(\mathbf{x}) \le 0, \ i=1,\dots,m. \quad (73)$$

$$\textbf{(P2):} \ \min_{\mathbf{y}} \ f_0(h(\mathbf{y})) \quad \text{s.t. } f_i(h(\mathbf{y})) \le 0, \ i=1,\dots,k, \quad f_i(h(\mathbf{y})) = 0, \ i=k+1,\dots,m. \quad (74)$$

Here $\{f_i\}_{i=1}^m$ are convex, differentiable functions and $h(\mathbf{y})$ is a collection of one-to-one, invertible functions. (P2) is obtained from (P1) by enforcing some inequality constraints with equality and by a change of variables, $\mathbf{x} = h(\mathbf{y})$. Since (P1) is a convex optimization problem, strong duality holds [40]. We denote the primal optimal values of problems (P1) and (P2) as p_1^*, p_2^* respectively. We show the following lemma.

Lemma 16: If $p_1^* = p_2^*$, then strong duality also holds for (P2).

Proof: The dual function and the Lagrange dual problem for (P1) are,

$$g_1(\boldsymbol{\lambda}) = \min_{\mathbf{x}} [f_0(\mathbf{x}) + \sum_{i=1}^m \lambda_i f_i(\mathbf{x})], \quad (75)$$

$$q_1^* = \max_{\boldsymbol{\lambda}\ge \mathbf{0}} \ g_1(\boldsymbol{\lambda}), \quad (76)$$

where $\boldsymbol{\lambda}$ are the Lagrange multipliers corresponding to the inequality constraints in (73) and q_1^* denotes the optimal dual value. Similarly for (P2),

$$g_2(\boldsymbol{\beta},\boldsymbol{\gamma}) = \min_{\mathbf{y}} \left[f_0(h(\mathbf{y})) + \sum_{i=1}^k \phi_i f_i(h(\mathbf{y})) + \sum_{i=k+1}^m \gamma_i f_i(h(\mathbf{y}))\right], \quad (77)$$

$$q_2^* = \max_{\boldsymbol{\beta}\ge \mathbf{0},\boldsymbol{\gamma}} \ g_2(\boldsymbol{\beta},\boldsymbol{\gamma}), \quad (78)$$

where ϕ_i and γ_i correspond to the inequality and equality constraints in (74), respectively. We do not have the constraints $\boldsymbol{\gamma} \ge \mathbf{0}$ since $\boldsymbol{\gamma}$ corresponds to equality constraints. Since h is invertible, we let $\mathbf{x} = h^{-1}(\mathbf{y})$ and rewrite (77) as,

$$g_2(\boldsymbol{\beta},\boldsymbol{\gamma}) = \min_{\mathbf{x}} [f_0(\mathbf{x}) + \sum_{i=1}^k \phi_i f_i(\mathbf{x}) + \sum_{i=k+1}^m \gamma_i f_i(\mathbf{x})]. \quad (79)$$

Now we have,

$$q_2^* \ge \max_{(\boldsymbol{\beta},\boldsymbol{\gamma})\ge \mathbf{0}} \ g_2(\boldsymbol{\beta},\boldsymbol{\gamma}) = \max_{\boldsymbol{\lambda}\ge\mathbf{0}} \ g_2(\boldsymbol{\lambda}) = \max_{\boldsymbol{\lambda}\ge\mathbf{0}} \ g_1(\boldsymbol{\lambda}) = q_1^*, \quad (80)$$

where the first inequality follows from the fact that $\boldsymbol{\gamma} \ge \mathbf{0}$ yields to a more restricted feasible set, the first equality is a rewriting of the problem in terms of variable $\boldsymbol{\lambda}$, the second equality follows from comparing (77) to (75). Furthermore,

$$q_2^* \ge q_1^* = p_1^* = p_2^*, \quad q_2^* \le p_2^*, \quad (81)$$

where $q_1^* = p_1^*$ follows from strong duality of (P1) and $p_1^* = p_2^*$ from assumption and $q_2^* \le p_2^*$ follows from weak duality of (P2) which always holds irrespective of convexity of the problem. Then we have $q_2^* = p_2^*$ and strong duality holds. ■

The problem in $(\boldsymbol{\mathcal{A}}_1)$ is obtained from $(\boldsymbol{\mathcal{A}})$ similar to how (P2) is obtained from (P1) without changing the primal objective value and the problem in $(\boldsymbol{\mathcal{A}})$ is a convex problem. Therefore the problem in $(\boldsymbol{\mathcal{A}}_1)$ has strong duality.

APPENDIX C
PROOF OF LEMMA 3

In [39] and [41] a non-convex problem is solved by a convex approximation method, in which non-convex constraints $g(\mathbf{x})$ are approximated around point $\mathbf{x}^n$ by a differentiable convex function $\bar{g}(\mathbf{x},\mathbf{x}^n)$. This is called as Sequential Parametric Convex Approximation (SCPA) Method. Each function $\bar{g}(\mathbf{x},\mathbf{x}^n)$ must satisfy:

- $g(\mathbf{x}) \le \bar{g}(\mathbf{x},\mathbf{x}^n)$ for all feasible $\mathbf{x}$,
- $g(\mathbf{x}) = g(\mathbf{x}^n,\mathbf{x}^n)$,
- $\partial g(\mathbf{x}^n)/\partial\mathbf{x}^n = \partial\bar{g}(\mathbf{x}^n,\mathbf{x}^n)/\partial\mathbf{x}^n$.

In our problem, the non-convex constraint function g is given as $r_{1i}+r_{2i}-f(s_i/\sigma^2) \le 0$. The last two properties are satisfied when $\bar{g}$ is taken as the Taylor expansion of the function g. The function $f(s_i/\sigma^2)$ is a convex function since it is of the form $\log(\sum 2^x)$. Then, g is concave. The first property is satisfied since linear approximations are over-estimators for concave functions. Now we prove convergence to a KKT point as follows. First, we apply a one-to-one transformation to the objective function in $(\boldsymbol{\mathcal{A}}_1)$ while keeping all the constraints the same:

$$(\boldsymbol{\mathcal{Q}}_1) \quad \max_{\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_{U1},\mathbf{r}_{U2}} \ \log\left(1+\sum_{i=1}^N \mu_1 r_{1i} + \mu_2 r_{2i}\right) \quad (82)$$

$$\text{s.t. } \sum_{i=1}^k 2^{2r_{1i}} + 2^{2r_{U1i}} \le \sum_{i=1}^k (E_{1i}+2), \ \forall k, \quad (83)$$

$$\sum_{i=1}^k 2^{2r_{2i}} + 2^{2r_{U2i}} \le \sum_{i=1}^k (E_{2i}+2), \ \forall k, \quad (84)$$

$$\sum_{i=1}^{k} r_{1i} \le \sum_{i=1}^{k} d_{1i},\ \forall k, \tag{85}$$
$$\sum_{i=1}^{k} r_{2i} \le \sum_{i=1}^{k} d_{2i},\ \forall k, \tag{86}$$
$$r_{1i} + r_{2i} \le f(s_i/\sigma^2),\ \forall i. \tag{87}$$

The function $\log(1+x)$ is strictly concave, and the feasible set in ($\mathcal{Q}_1$) is closed and bounded, therefore compact. From [41, Lemma 3.1], we have that the optimization function in ($\mathcal{Q}_1$) is strongly concave on this feasible set. Then, from [41, Proposition 3.2(i)], we have that the sequence of points generated by the SPCA Method converges to the KKT point of ($\mathcal{Q}_1$). We note that SCPA on ($\mathcal{Q}_1$) and ($\mathcal{A}_1$) are essentially the same because of $\log(1+x)$ being one-to-one and strictly increasing. Now we show below in Claim 1 that, because of the linearity of our objective function, there exists a one-to-one transformation between the KKT points of ($\mathcal{Q}_1$) and ($\mathcal{A}_1$). This implies, SCPA generates points that converge to the KKT point of ($\mathcal{A}_1$) also. Then, from Lemma 2, strong duality holds and therefore Kuhn-Tucker conditions are both necessary and sufficient for global optimality. Therefore $\mathbf{R}^*$ is a global optimal solution to ($\mathcal{A}_1$).

Claim 1: Assume we have two problems (P1) and (P2) given below:

$$\textbf{(P1):}\ \min_{\mathbf{x}\in\mathbb{R}^n}\ \mathbf{a}^T\mathbf{x} \quad s.t.\ e_i(\mathbf{x}) \le 0,\quad i=1,\ldots,m. \tag{88}$$
$$\textbf{(P2):}\ \min_{\mathbf{x}\in\mathbb{R}^n}\ \log(1+\mathbf{a}^T\mathbf{x}) \quad s.t.\ e_i(\mathbf{x}) \le 0,\quad i=1,\ldots,m.$$

where $e_i : \mathbb{R}^n \to \mathbb{R}$ are continuously differentiable functions and $\mathbf{a} \in \mathbb{R}^n$. If $(\mathbf{x}^*, \boldsymbol{\xi}^*)$ is a KKT point for (P2), where $\boldsymbol{\xi}^*$ is a set of optimal Lagrange multipliers, then $\left(\mathbf{x}^*, \boldsymbol{\xi}^*(1+\mathbf{a}^T\mathbf{x}^*)\right)$ is a KKT point for (P1).

Proof: Proof follows after writing KKT conditions for (P1) and (P2). ■

In our setting, (P1) corresponds to ($\mathcal{A}_1$) and (P2) corresponds to ($\mathcal{Q}_1$).

Appendix D
Algorithmic Solution to Problem ($\mathcal{A}_3$)

We add a new optimization variable $\mathbf{t} \in \mathbb{R}^N$ and equivalently formulate ($\mathcal{A}_3$) as follows:

$$\max_{\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_{U1},\mathbf{r}_{U2},\mathbf{t}} \sum_{i=1}^{N} \mu_1 r_{1i} + \mu_2 r_{2i} \tag{89}$$
$$\text{s.t. (17)-(20)} \tag{90}$$
$$(1-a_{1i})r_{1i} - b_{1i}r_{U1i} = D_i + t_i, \tag{91}$$
$$(1-a_{2i})r_{2i} - b_{2i}r_{U2i} = -t_i,\ \forall i. \tag{92}$$

Let us define the function $z(\mathbf{t})$ which is a maximization over $(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_{U1}, \mathbf{r}_{U2})$ for fixed $\mathbf{t}$:

$$z(\mathbf{t}) = \max_{\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_{U1},\mathbf{r}_{U2}} \sum_{i=1}^{N} \mu_1 r_{1i} + \mu_2 r_{2i} \tag{93}$$
$$\text{s.t. (17)-(20), (91), (92).} \tag{94}$$

Then the original problem in ($\mathcal{A}_3$) is equivalent to

$$\max_{\mathbf{t}} z(\mathbf{t}). \tag{95}$$

We solve (95) by separately solving the outer and inner maximization problems.

A. Inner Maximization

In this section, we focus on the inner problem in (94) for fixed $\mathbf{t}$. Note that when $\mathbf{t}$ is fixed, the variables $(\mathbf{r}_1, \mathbf{r}_{U1})$ and $(\mathbf{r}_2, \mathbf{r}_{U2})$ are decoupled and (94) can be separated into two sub-problems. We define $z_1(\mathbf{t})$ and $z_2(\mathbf{t})$ as

$$z_1(\mathbf{t}) = \max_{\mathbf{r}_1,\mathbf{r}_{U1}} \sum_{i=1}^{N} r_{1i}$$
$$\text{s.t.} \sum_{i=1}^{k} 2^{2r_{1i}} + 2^{2r_{U1i}} \le \sum_{i=1}^{k}(E_{1i}+2), \tag{96}$$
$$\sum_{i=1}^{k} r_{1i} \le \sum_{i=1}^{k} d_{1i},\ \forall k, \tag{97}$$
$$(1-a_{1i})r_{1i} - b_{1i}r_{U1i} = D_i + t_i,\ \forall i. \tag{98}$$
$$z_2(\mathbf{t}) = \max_{\mathbf{r}_2,\mathbf{r}_{U2}} \sum_{i=1}^{N} r_{2i}$$
$$\text{s.t.} \sum_{i=1}^{k} 2^{2r_{2i}} + 2^{2r_{U2i}} \le \sum_{i=1}^{k}(E_{2i}+2), \tag{99}$$
$$\sum_{i=1}^{k} r_{2i} \le \sum_{i=1}^{k} d_{2i},\ \forall k, \tag{100}$$
$$(1-a_{2i})r_{2i} - b_{2i}r_{U2i} = -t_i,\ \forall i. \tag{101}$$

and note that $z(\mathbf{t}) = \mu_1\ z_1(\mathbf{t}) + \mu_2\ z_2(\mathbf{t})$. First we concentrate on solving z_1. Let $w_{1i} = (1-a_{1i})/b_{1i}$, $v_{1i} = 2^{-2(D_i+t_i)/b_{1i}}$. Using the equality constraints in (98) we get,

$$\max_{\mathbf{r}_1} \sum_{i=1}^{N} r_{1i}$$
$$\text{s.t.} \sum_{i=1}^{k} 2^{2r_{1i}} + v_{1i}2^{2w_{1i}r_{1i}} \le \sum_{i=1}^{k}(E_{1i}+2), \tag{102}$$
$$\sum_{i=1}^{k} r_{1i} \le \sum_{i=1}^{k} d_{1i},\ \forall k. \tag{103}$$

This is a single-user problem with data arrivals d_{1i}, energy arrivals E_{1i} and a modified energy consumption function $m(r_{1i}) = 2^{2r_{1i}} + v_{1i}2^{2\ w_{1i}r_{1i}}$. In order to solve it, first, we perform directional waterfilling, [4], on the data arrivals d_{1i}. Second, we perform directional waterfilling on the energy arrivals E_{1i} with the understanding that $\frac{\partial m}{\partial r_{1i}} = 2 \cdot 2^{2r_{1i}} + 2\ v_{1i}w_{1i}2^{2\ w_{1i}r_{1i}}$ is the quantity to be kept constant over the slots. Then, we take the minimum of the two solutions ensuring that any unused data or energy must be carried over to the future slots.

Now we solve z_2. Let $w_{2i} = (1-a_{2i})/b_{2i}$ and $v_{2i} = 2^{t_i/b_{2i}}$. From the constraints (101),

$$\max_{\mathbf{r}_2} \sum_{i=1}^{N} r_{2i}$$
$$\text{s.t.} \sum_{i=1}^{k} 2^{2r_{2i}} + v_{2i} 2^{2w_{2i}r_{2i}} \leq \sum_{i=1}^{k} (E_{2i}+2), \quad (104)$$
$$\sum_{i=1}^{k} r_{2i} \leq \sum_{i=1}^{k} d_{2i}, \ \forall k. \quad (105)$$

This problem is solved similarly as in the case of z_1.

B. Outer Maximization

The outer maximization problem is that of finding optimal $\mathbf{t}$ in (95). The equality constraints in (91) and (92) impose some feasibility constraints on $\mathbf{t}$. Then the problem is equivalent to

$$\max_{\mathbf{t}} \ z(\mathbf{t})$$
$$\text{s.t. } z_1(\mathbf{t}), \ z_2(\mathbf{t}) \text{ are feasible.} \quad (106)$$

It can be shown that $z(\mathbf{t})$ is concave in $\mathbf{t}$. Solving this problem can be performed efficiently by iterating over feasible $\mathbf{t}$ such that every iteration increases the objective function, for example, using the method described in [32, Sec. III.B]. Due to convexity, the convergence to an optimal solution is guaranteed. The overall solution algorithm is given in Algorithm 1. The solution to outer maximization problem is in lines 2 to 16.

Appendix E
Proof of Lemma 5

We discuss three cases to reach a contradiction in each case.

Case 1: Let $\exists k$ such that $p_{U1k} = 0, p_{U2k} > 0$. Then, $s_k = \sigma^2 + p_{12k} + p_{21k} + p_{U2k}$. We define a new power allocation vector as $\tilde{p}_{U2k} = p_{U2k} - \epsilon_1 - \epsilon_2$, $\tilde{p}_{21k} = p_{21k} + \epsilon_1$, $\tilde{p}_{U1k} = \alpha\epsilon_2$, $\tilde{p}_{12k} = p_{12k}$, for some $\epsilon_1 > 0, \epsilon_2 > 0$. Here, we have transferred ϵ_2 amount of energy from user 2 to user 1 and consumed it in the cooperative power of user 1. Additionally, we decreased p_{U2k} by ϵ_1 and increased p_{21k} by ϵ_1. The energy causality constraints are satisfied for the new power allocation. Rate region constraints (31) and (32) become:

$$r_{1k} \leq f(1+\tilde{p}_{12k}) = f(1+p_{12k}) \quad (107)$$
$$r_{2k} < f(1+\tilde{p}_{21k}) = f(1+p_{21k}+\epsilon_1) \quad (108)$$

For constraint (33), we have

$$\tilde{s}_k = \sigma^2 + \tilde{p}_{12k} + \tilde{p}_{U1k} + \tilde{p}_{21k} + \tilde{p}_{U2k} + 2\sqrt{\tilde{p}_{U1k}\tilde{p}_{U2k}} \quad (109)$$
$$= \sigma^2 + p_{12k} + \alpha\epsilon_2 + p_{21k} + \epsilon_1 + p_{U2k} - \epsilon_1 - \epsilon_2$$
$$+ 2\sqrt{\alpha\epsilon_2(p_{U2k}-\epsilon_1-\epsilon_2)} \quad (110)$$
$$= s_k + (\alpha-1)\epsilon_2 + 2\sqrt{\alpha\epsilon_2(p_{U2k}-\epsilon_1-\epsilon_2)} > s_k \quad (111)$$

where last inequality holds since $2\sqrt{\alpha\epsilon_2(p_{U2k}-\epsilon_1-\epsilon_2)} > (1-\alpha)\epsilon_2$ for small ϵ_1, ϵ_2. Therefore,

$$r_{1k} + r_{2k} < f(\tilde{s}_k/\sigma^2) \quad (112)$$

The constraints (108), (112) are loose and we can increase r_{2k} to get a larger optimal value which contradicts the optimality of the original profile. Therefore, case 1 cannot happen.

Case 2: Let $\exists k$ such that $p_{U1k} > 0, p_{U2k} = 0$. Similar to case 1, we will reach a contradiction.

Case 3: Let $\exists k$ such that $p_{U1k} = 0, p_{U2k} = 0$. Then, $s_k = \sigma^2 + p_{12k} + p_{21k}$. We cannot have $r_{1k} = f(1+p_{12k}), r_{2k} = f(1+p_{21k})$ because $f(1+p_{12k}) + f(1+p_{21k}) > f(s_k/\sigma^2)$ so this is not feasible. Without loss of generality, assume $r_{1k} < f(1+p_{12k})$. We define a new power allocation vector as $\tilde{p}_{12k} = p_{12k} - \epsilon_1 - \epsilon_2$, $\tilde{p}_{U1k} = \epsilon_1$, $\tilde{p}_{21k} = p_{21k}$, $\tilde{p}_{U2k} = \alpha\epsilon_2$. Here, we have transferred ϵ_2 amount of energy from user 1 to user 2 and consumed it in the cooperative power of user 2. Additionally, we decreased p_{12k} by ϵ_1 and increased p_{U1k} by ϵ_1.

For small ϵ_1, ϵ_2 we still have $r_{1k} < f(1+\tilde{p}_{12k})$ which implies (31) is satisfied. Since p_{21k} has not been changed, (32) is satisfied. For constraint (33) we have,

$$\tilde{s}_k = \sigma^2 + \tilde{p}_{12k} + \tilde{p}_{U1k} + \tilde{p}_{21k} + \tilde{p}_{U2k} + 2\sqrt{\tilde{p}_{U1k}\tilde{p}_{U2k}} \quad (113)$$
$$= \sigma^2 + p_{12k} - \epsilon_1 - \epsilon_2 + \epsilon_1 + \alpha\epsilon_2 + p_{21k} + 2\sqrt{\epsilon_1\alpha\epsilon_2} \quad (114)$$
$$= s_k + (\alpha-1)\epsilon_2 + 2\sqrt{\epsilon_1\alpha\epsilon_2} > s_k \quad (115)$$

where last inequality holds for $\epsilon_1 > \epsilon_2(1-\alpha)^2/(4\alpha)$ which we enforce. Then, $r_{1k} + r_{2k} < f(\tilde{s}_k/\sigma^2)$. Now, we increase r_{1k} which is a contradiction. Therefore, case 3 cannot happen.

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References

[1] N. Su, O. Kaya, S. Ulukus, and M. Koca, "Cooperative multiple access under energy harvesting constraints," in *Proc. IEEE Globecom*, Dec. 2015, pp. 1–6.

[2] J. Yang and S. Ulukus, "Optimal packet scheduling in an energy harvesting communication system," *IEEE Trans. Commun.*, vol. 60, no. 1, pp. 220–230, Jan. 2012.

[3] K. Tutuncuoglu and A. Yener, "Optimum transmission policies for battery limited energy harvesting nodes," *IEEE Trans. Wireless Commun.*, vol. 11, no. 3, pp. 1180–1189, Mar. 2012.

[4] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, "Transmission with energy harvesting nodes in fading wireless channels: Optimal policies," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 8, pp. 1732–1743, Sep. 2011.

[5] C. K. Ho and R. Zhang, "Optimal energy allocation for wireless communications with energy harvesting constraints," *IEEE Trans. Signal Process.*, vol. 60, no. 9, pp. 4808–4818, Sep. 2012.

[6] J. Yang, O. Ozel, and S. Ulukus, "Broadcasting with an energy harvesting rechargeable transmitter," *IEEE Trans. Wireless Commun.*, vol. 11, no. 2, pp. 571–583, Feb. 2012.

[7] M. A. Antepli, E. Uysal-Biyikoglu, and H. Erkal, "Optimal packet scheduling on an energy harvesting broadcast link," *IEEE J. Sel. Areas Commun.*, vol. 29, no. 8, pp. 1721–1731, Sep. 2011.

[8] O. Ozel, J. Yang, and S. Ulukus, "Optimal broadcast scheduling for an energy harvesting rechargeable transmitter with a finite capacity battery," *IEEE Trans. Wireless Commun.*, vol. 11, no. 6, pp. 2193–2203, Jun. 2012.

[9] J. Yang and S. Ulukus, "Optimal packet scheduling in a multiple access channel with energy harvesting transmitters," *J. Commun. Netw.*, vol. 14, no. 2, pp. 140–150, 2012.

[10] Z. Wang, V. Aggarwal, and X. Wang, "Iterative dynamic water-filling for fading multiple-access channels with energy harvesting," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 3, pp. 382–395, Mar. 2015.

[11] K. Tutuncuoglu and A. Yener, "Sum-rate optimal power policies for energy harvesting transmitters in an interference channel," *J. Commun. Netw.*, vol. 14, no. 2, pp. 151–161, 2012.

[12] B. Devillers and D. Gunduz, "A general framework for the optimization of energy harvesting communication systems with battery imperfections," *J. Commun. Netw.*, vol. 14, no. 2, pp. 130–139, 2012.

[13] K. Tutuncuoglu, A. Yener, and S. Ulukus, "Optimum policies for an energy harvesting transmitter under energy storage losses," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 3, pp. 467–481, Mar. 2015.

[14] O. Orhan, D. Gündüz, and E. Erkip, "Energy harvesting broadband communication systems with processing energy cost," *IEEE Trans. Wireless Commun.*, vol. 13, no. 11, pp. 6095–6107, Nov. 2014.

[15] J. Xu and R. Zhang, "Throughput optimal policies for energy harvesting wireless transmitters with non-ideal circuit power," *IEEE J. Sel. Areas Commun.*, vol. 32, no. 2, pp. 322–332, Feb. 2014.

[16] O. Ozel, K. Shahzad, and S. Ulukus, "Optimal energy allocation for energy harvesting transmitters with hybrid energy storage and processing cost," *IEEE Trans. Signal Process.*, vol. 62, no. 12, pp. 3232–3245, Jun. 2014.

[17] C. Huang, R. Zhang, and S. Cui, "Throughput maximization for the Gaussian relay channel with energy harvesting constraints," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 8, pp. 1469–1479, Aug. 2013.

[18] D. Gündüz and B. Devillers, "Two-hop communication with energy harvesting," in *Proc. IEEE CAMSAP*, Dec. 2011, pp. 201–204.

[19] Y. Luo, J. Zhang, and K. B. Letaief, "Optimal scheduling and power allocation for two-hop energy harvesting communication systems," *IEEE Trans. Wireless Commun.*, vol. 12, no. 9, pp. 4729–4741, Sep. 2013.

[20] O. Orhan and E. Erkip, "Throughput maximization for energy harvesting two-hop networks," in *Proc. IEEE ISIT*, Jul. 2013, pp. 1596–1600.

[21] B. Varan and A. Yener, "Energy harvesting two-way communications with limited energy and data storage," in *Proc. Asilomar Conf. Signals, Syst. Comput.*, Nov. 2014, pp. 1671–1675.

[22] B. Gurakan and S. Ulukus, "Energy harvesting cooperative diamond channel," in *Proc. IEEE ISIT*, Jun. 2015, pp. 2732–2736.

[23] B. Gurakan, O. Ozel, J. Yang, and S. Ulukus, "Energy cooperation in energy harvesting communications," *IEEE Trans. Commun.*, vol. 61, no. 12, pp. 4884–4898, Dec. 2013.

[24] B. Gurakan and S. Ulukus, "Energy harvesting multiple access channel with data arrivals," in *Proc. IEEE Globecom*, Dec. 2015, pp. 1–6.

[25] K. Tutuncuoglu and A. Yener, "Energy harvesting networks with energy cooperation: Procrastinating policies," *IEEE Trans. Commun.*, vol. 63, no. 11, pp. 4525–4538, Nov. 2015.

[26] M. B. Khuzani and P. Mitran, "On Online energy harvesting in multiple access communication systems," *IEEE Trans. Inf. Theory*, vol. 60, no. 3, pp. 1883–1898, Mar. 2014.

[27] E. Uysal-Biyikoglu and A. El Gamal, "On adaptive transmission for energy efficiency in wireless data networks," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3081–3094, Dec. 2004.

[28] D. Zhao, C. Huang, Y. Chen, F. Alsaadi, and S. Cui, "Resource allocation for multiple access channel with conferencing links and shared renewable energy sources," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 3, pp. 423–437, Mar. 2015.

[29] B. Gurakan and S. Ulukus, "Cooperative diamond channel with energy harvesting nodes," *IEEE J. Sel. Areas Commun.*, vol. 34, no. 5, pp. 1604–1617, May 2016.

[30] P. Grover and A. Sahai, "Shannon meets Tesla: Wireless information and power transfer," in *Proc. IEEE ISIT*, Jul. 2010, pp. 2363–2367.

[31] L. R. Varshney, "Transporting information and energy simultaneously," in *Proc. IEEE ISIT*, Jul. 2008, pp. 1612–1616.

[32] B. Gurakan and S. Ulukus, "Energy harvesting diamond channel with energy cooperation," in *Proc. IEEE ISIT*, Jun./Jul. 2014, pp. 986–990.

[33] Z. Guo, T. J. Lim, and M. Motani, "Base station energy cooperation in green cellular networks," in *Proc. IEEE GlobalSIP*, Dec. 2013, pp. 349–352.

[34] P. Popovski, A. M. Fouladgar, and O. Simeone, "Interactive joint transfer of energy and information," *IEEE Trans. Commun.*, vol. 61, no. 5, pp. 2086–2097, May 2013.

[35] J. I. Choi, M. Jain, K. Srinivasan, P. Levis, and S. Katti, "Achieving single channel, full duplex wireless communication," in *Proc. MOBICOM*, 2010, pp. 1–12.

[36] M. Jain *et al.*, "Practical, real-time, full duplex wireless," in *Proc. MOBICOM*, 2011, pp. 301–312.

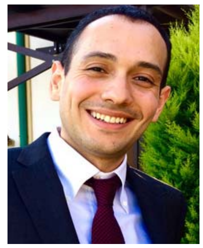
[37] O. Kaya and S. Ulukus, "Power control for fading cooperative multiple access channels," *IEEE Trans. Wireless Commun.*, vol. 6, no. 8, pp. 2915–2923, Aug. 2007.

[38] A. Kurs, A. Karalis, R. Moffatt, J. D. Joannopoulos, P. Fisher, and M. Soljačić, "Wireless power transfer via strongly coupled magnetic resonances," *Science*, vol. 317, no. 5834, pp. 83–86, 2007.

[39] B. R. Marks and G. P. Wright, "A general inner approximation algorithm for nonconvex mathematical programs," *Oper. Res.*, vol. 26, no. 4, pp. 681–683, 1978.

[40] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.

[41] A. Beck, A. Ben-Tal, and L. Tetruashvili, "A sequential parametric convex approximation method with applications to nonconvex truss topology design problems," *J. Global Optim.*, vol. 47, no. 1, pp. 29–51, 2010.



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