Cooperative Diamond Channel With Energy Harvesting Nodes

Berk Gurakan, *Student Member, IEEE*, and Sennur Ulukus, *Fellow, IEEE*

***Abstract*—We consider the energy harvesting diamond channel, where the source and two relays harvest energy from nature and the physical layer is modeled as a concatenation of a broadcast and a multiple access channel. Since the broadcast channel is degraded, one of the relays has the message of the other relay and the multiple access channel can be modeled as a cooperative multiple access channel with common data. We find the optimal offline transmit power and rate allocations that maximize the end-to-end throughput. For the broadcast side, we show that there exists an optimal source power allocation, which is equal to the single-user optimal power allocation for the source energy arrivals. We then show that the fraction of the power spent on each broadcast link depends on the energy arrivals for the relays. For the multiple access side with no co-operation, with fixed source rates, we show that the problem can be cast as a multiple access channel with both data and energy arrivals and can be formulated in terms of data transmission rates only. We use a dual decomposition method to solve the overall problem efficiently. Finally, we focus on the diamond channel with co-operative multiple access capacity region and find the optimal rates and powers using a decomposition into inner and outer maximization problems.**

***Index Terms*—Energy harvesting, cooperative communications, diamond channel, resource allocation.**

I. Introduction

WE CONSIDER the cooperative energy harvesting diamond channel [1], see Fig. 1, where all transmitters harvest energy from nature. We model the physical layer as a concatenation of a Gaussian broadcast channel and a Gaussian multiple access channel. Since the broadcast channel is degraded, one of the relays has the message of the other relay. Therefore, the multiple access channel is an extended multiple access channel with common data [2], which we also call the cooperative multiple access channel. Our aim is to determine the optimum power and rate allocation policies of the users in order to maximize the end-to-end throughput of this system.

There has been a considerable amount of recent work in power control for energy harvesting communications [3]–[26]. In [3], the transmission completion time minimization problem is solved for an unlimited-sized battery. In [4], the throughput maximization problem is solved and its equivalence to the transmission completion time minimization problem is shown for an arbitrarily sized battery. In [5]–[11], the problem is extended to fading, broadcast, multiple access and interference channels. Throughput maximization problem with battery imperfections is considered in [12], [13], processing costs are incorporated in [14]–[17] and decoding costs are studied in [18]. Of particular relevance are [19]–[23], where two-hop communication is considered with energy harvesting nodes for half- or full-duplex relay settings. Recently, in [27], [28], two-hop communication systems with two parallel relays are studied. In [27], two parallel half-duplex relays with various combinations of different transmission modes are considered. Due to the half-duplex nature of the relays, broadcast and multiple access operations are not simultaneously possible. In [28], all four links of the broadcast and multiple access channels are restricted to be orthogonal, and no storage of data is allowed at the relays due to strict delay constraints. The setting in the current paper can be viewed as a generalization of [28] to general broadcast and multiple access channels, and general data storage at the relays.

In the setting of the diamond channel, see Fig. 1, when the transmission rates of the source in the broadcast side are fixed, the problem can be viewed as an energy harvesting multiple access channel where data packets as well as the harvested energies arrive at the transmitters intermittently over time. Of particular relevance to this specific problem, are references [10], [29], [30] where optimal scheduling problems on a multiple access channel are investigated. In [29], minimum energy scheduling problem over a multiple access channel where data packets arrive over time is solved. In [10], a multiple access channel with energy arrivals is considered but it is assumed that the users are infinitely backlogged, i.e., the data packets do not arrive over time. In [30], an energy harvesting multiple access channel with additional maximum power constraints on each user is considered. These previous works either consider data arrivals or energy arrivals but not both; in our current work, we need to consider both constraints due to the two-hop nature of the diamond channel.

In the first part of the paper, in Section III, we focus on the broadcast half of the diamond network. We first show that there exists an optimal source power allocation policy which is equal to the single-user optimal power policy for the source energy arrivals and does not depend on the relay energy arrivals. This is a generalization of [7], [9], which proved the optimality of a single-user power allocation for the capacity region of a broadcast channel; our work shows that the result remains the same even when the broadcast channel is concatenated with a multiple access channel. Our result is also a generalization of the

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The authors are with the Department of Electrical and Computer Engineering, University of Maryland, College Park, MD 20742 USA (e-mail: gurakan@umd.edu; ulukus@umd.edu).

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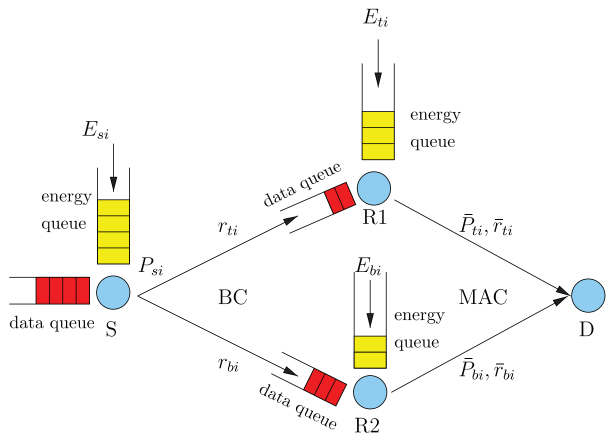

Fig. 1. Cooperative diamond network with energy harvesting nodes.

separation result proved in [19], [20], which showed that, in a single relay channel, the source can optimize its transmit power irrespective of the relay's energy arrivals; our work shows that this result remains the same for the case of two relays forming a multiple access second hop. Next, we show that even though the total power can be selected as the single-user optimal power, the fraction of the power spent on each broadcast link depends on the energy arrivals of the relays. Specifically, we show that the optimal source rate allocation can be found by solving an optimal broadcasting problem with slot-dependent user priorities and these priorities can change only at instants when one of the relay data buffers is empty.

In the second part of the paper, in Section IV, we turn our attention to the multiple access side of the diamond network. As mentioned before, this is a cooperative multiple access channel with common data. To take the full advantage of cooperation arising from common data, the relays need to use commonly generated codebooks. For simplicity of operation, the relays may choose to ignore the constructed common data, and operate the second hop as a regular multiple access channel. Therefore, we first consider a regular Gaussian multiple access channel for the second hop of the diamond channel. In this setting, first we note that when the transmission rates of the source in the broadcast side are fixed, the overall problem becomes a multiple access channel with both data and energy arrivals. Then, we show that this problem can be formulated in terms of data transmission rates only, instead of formulating over both transmission powers and data rates. In the multiple access channel with only energy arrivals, it was observed in [10], that the optimal sum rate is equal to the single-user optimal rate with both user energies merged. This may naturally suggest that, with the presence of the data causality constraints, the optimal sum rate is given by the single-user optimal rate with both data and energy causality constraints merged. In Section IV-A, we show that this suggestion is not entirely valid, but a majorization relationship exists between these two solutions. In Section IV-B, we solve the overall diamond channel problem with non-cooperative multiple access channel, using a dual decomposition method.

In the third part of the paper, in Section V, we recover the original setting of the diamond channel by focusing on the cooperative (extended) multiple access capacity region. With the extended multiple access capacity region, we find the overall solution using a decomposition into inner and outer maximization problems. The outer problem consists of finding the optimal source transmission rates in the broadcast side. The inner problem consists of finding the optimal relay rate and power allocations when the transmission rates of the source in the broadcast side are fixed. We solve the overall problem by iterating between the two sides.

II. SYSTEM MODEL

We consider the energy harvesting diamond channel shown in Fig. 1. The harvested energies are saved in the corresponding batteries. The physical layer is modeled as a concatenation of a broadcast channel and a multiple access channel. In the broadcast channel, relay 1 is the stronger receiver: the channel noises have variances $\sigma_1^2 \le \sigma_2^2$. The Gaussian broadcast channel capacity region with transmitter power p is given by [31]

$$\mathcal{C}_{BC}(p) = \left\{ r_1 \le f\left(\frac{\alpha p}{\sigma_1^2}\right), \quad r_2 \le f\left(\frac{(1-\alpha)p}{\alpha p + \sigma_2^2}\right) \right\} \tag{1}$$

where α is the fraction of power spent for the message of user 1, and $f(x) \triangleq \frac{1}{2}\log(1+x)$. The function $g(r_1, r_2)$ is the minimum energy required to transmit at rates (r_1, r_2):

$$g(r_1, r_2) \triangleq \sigma_1^2 2^{2(r_1+r_2)} + \left(\sigma_2^2 - \sigma_1^2\right) 2^{2r_2} - \sigma_2^2 \tag{2}$$

and is strictly convex in (r_1, r_2). Since relay 2 is degraded with respect to relay 1, relay 1 can decode the messages intended for relay 2. Therefore, the second hop is an extended multiple access channel with common data. The capacity region for this channel with transmitter powers (p_1, p_2) and Gaussian noise power σ_3^2 is given as [1], [2], [32]:

$$\mathcal{C}_{EMAC}(p_1, p_2) = \Big\{ r_1 \le f\left((1-\beta)p_1/\sigma_3^2\right), \quad r_1 + r_2 \le f\left(\left(p_1 + p_2 + 2\sqrt{\beta p_1 p_2}\right)/\sigma_3^2\right) \Big\} \tag{3}$$

If the presence of common data is ignored, the second hop becomes a regular Gaussian multiple access channel whose capacity region is given as [31]:

$$\mathcal{C}_{MAC}(p_1, p_2) = \Big\{ r_1 \le f\left(p_1/\sigma_3^2\right), \; r_2 \le f\left(p_2/\sigma_3^2\right), \; r_1 + r_2 \le f\left((p_1+p_2)/\sigma_3^2\right) \Big\} \tag{4}$$

There are N equal length slots of duration τ seconds and $\tau = 1$ is assumed without loss of generality. We refer to relay 1 as the *top* and relay 2 as the *bottom* relay and use subscripts t and b to denote their parameters; subscript s denotes the source node's parameters. In slot i, the source, top and bottom relays harvest energy with amounts E_{si}, E_{ti}, E_{bi}, respectively. We denote the transmission power of the source as p_{si} and source rates to the top (bottom) relay as r_{ti} (r_{bi}), the transmission power of the top (bottom) relay to the destination as $\bar{p}_{ti}$

($\bar{p}_{bi}$) and data rates of the top (bottom) relays to the destination as $\bar{r}_{ti}$ ($\bar{r}_{bi}$). We denote these power and rate sequences with the vectors $\mathbf{p}_s, \bar{\mathbf{p}}_t, \bar{\mathbf{p}}_b, \mathbf{r}_t, \mathbf{r}_b, \bar{\mathbf{r}}_t, \bar{\mathbf{r}}_b$. The energy that has not yet been harvested cannot be used, leading to the following *energy causality constraints* at all transmitters:

$$\sum_{i=1}^{k} \bar{p}_{ti} \le \sum_{i=1}^{k} E_{ti}, \quad \forall k \tag{5}$$

$$\sum_{i=1}^{k} \bar{p}_{bi} \le \sum_{i=1}^{k} E_{bi}, \quad \forall k \tag{6}$$

$$\sum_{i=1}^{k} p_{si} \le \sum_{i=1}^{k} E_{si}, \quad \forall k \tag{7}$$

The relays cannot forward data that has not yet arrived, leading to the following *data causality constraints* at the relays:

$$\sum_{i=1}^{k} \bar{r}_{ti} \le \sum_{i=1}^{k} r_{ti}, \quad \forall k \tag{8}$$

$$\sum_{i=1}^{k} \bar{r}_{bi} \le \sum_{i=1}^{k} r_{bi}, \quad \forall k \tag{9}$$

The rate allocations must be achievable for each channel:

$$(r_{ti}, r_{bi}) \in \mathcal{C}_{BC}(p_{si}), \quad \forall i \tag{10}$$

$$(\bar{r}_{ti}, \bar{r}_{bi}) \in \mathcal{C}_{EMAC}(\bar{p}_{ti}, \bar{p}_{bi}), \quad \forall i \tag{11}$$

where we will use $\mathcal{C}_{MAC}(\bar{p}_{ti}, \bar{p}_{bi})$ in (11), if we operate the second hop as a regular multiple access channel.

We aim to maximize the end-to-end throughput:

$$\max_{\mathbf{p}_s, \bar{\mathbf{p}}_t, \bar{\mathbf{p}}_b, \mathbf{r}_t, \mathbf{r}_b, \bar{\mathbf{r}}_t, \bar{\mathbf{r}}_b, \boldsymbol{\alpha}} \quad \sum_{i=1}^{N} \bar{r}_{ti} + \sum_{i=1}^{N} \bar{r}_{bi} \quad \text{s.t.} \quad (5)-(11) \tag{12}$$

In this paper we will solve the problem in (12). We will separately focus on the broadcast and the multiple access sides of the problem in the following sections.

III. Broadcast Channel Side

First, we will focus on the broadcast side of the problem. We consider the source which is broadcasting data to the two relays, and focus on the source power (p_{si}) and rate (r_{ti}, r_{bi}) allocations. We first prove some properties of the optimal solution which hold regardless of the existence of the multiple access link.

Lemma 1 *Either the source energy or both of the relay energies must be consumed fully.*

Proof: The proof follows by contradiction. If any excess energy is left, then we can increase the rates, which contradicts optimality. ■

Lemma 2 *There exists an optimal source profile* $(\mathbf{p}_s^*, \mathbf{r}_t^*, \mathbf{r}_b^*)$ *that is on the boundary of the broadcast capacity region in each slot, i.e.,* $r_{ti}^* = f\left(\frac{\alpha_i p_{si}^*}{\sigma_1^2}\right)$, $r_{bi}^* = f\left(\frac{(1-\alpha_i) p_{si}^*}{\alpha p_{si}^* + \sigma_2^2}\right)$, $\forall i$.

Proof: In slots where the constraints $r_{ti}^* \le f\left(\frac{\alpha_i p_{si}^*}{\sigma_1^2}\right)$ and $r_{bi}^* \le f\left(\frac{(1-\alpha_i) p_{si}^*}{\alpha p_{si}^* + \sigma_2^2}\right)$ are satisfied with strict inequality, we can increase r_{ti}^* or r_{bi}^* without violating any feasibility constraints as we can always increase the right hand sides of the data feasibility constraints in (8) and (9). ■

Using Lemma 2 we can remove the broadcast capacity region constraints from the problem and let $p_{si} = g(r_{ti}, r_{bi})$. The corresponding energy causality constraints for the source node can now be written as:

$$\sum_{i=1}^{k} g(r_{ti}, r_{bi}) \le \sum_{i=1}^{k} E_{si}, \quad \forall k \tag{13}$$

The optimization problem can now be written as:

$$\max_{\bar{\mathbf{p}}_t, \bar{\mathbf{p}}_b, \mathbf{r}_t, \mathbf{r}_b, \bar{\mathbf{r}}_t, \bar{\mathbf{r}}_b} \quad \sum_{i=1}^{N} \bar{r}_{ti} + \sum_{i=1}^{N} \bar{r}_{bi} \quad \text{s.t.} \quad (5)-(6), (8)-(9), (11), (13) \tag{14}$$

The following theorem states a key structural property of the optimal policy, and is proved in Appendix A.

Theorem 1 *There exists an optimal total source power sequence* $g(r_{ti}^*, r_{bi}^*)$ *which is the same as the single-user optimal transmit power sequence for the energy arrivals* E_{si}.

Theorem 1 tells us that there exists a solution to the problem in (14) in which $g(r_{ti}^*, r_{bi}^*) = P_i$, where P_is are the single-user optimal transmit powers for the energy arrivals E_{si}. This constraint can always be relaxed to $g(r_{ti}, r_{bi}) \le P_i$. Using Theorem 1, the optimization problem becomes:

$$\max_{\bar{\mathbf{p}}_t, \bar{\mathbf{p}}_b, \mathbf{r}_t, \mathbf{r}_b, \bar{\mathbf{r}}_t, \bar{\mathbf{r}}_b} \quad \sum_{i=1}^{N} \bar{r}_{ti} + \sum_{i=1}^{N} \bar{r}_{bi} \quad \text{s.t.} \quad (5)-(6), (8)-(9), (11), g(r_{ti}, r_{bi}) \le P_i \tag{15}$$

We note that the single-user optimal transmit powers P_is can be found by the directional water filling algorithm in [5] or the staircase water filling algorithm in [6]. Theorem 1 generalizes the results of [7], [9] to the case of concatenated networks, and the results of [19], [20] to the case of multiple relays. While the source power does not depend on the energy arrival profile of the relays, the fraction of the total power spent on each broadcast link depends on the energy arrival profile of the relays. In the following lemmas, we show how to find the distribution of power over the broadcast links.

Lemma 3 *There exists a positive real vector* $\boldsymbol{\mu} \triangleq \{\mu_i\}_{i=1}^N$, $\mu_i \in [0, 1]$ *such that* (r_{ti}^*, r_{bi}^*) *simultaneously solves the problem in (15) and the following optimization problem:*

$$\max_{r_{ti}, r_{bi}} \quad \sum_{i=1}^{N} \mu_i r_{ti} + \sum_{i=1}^{N} r_{bi} \quad \text{s.t.} \quad g(r_{ti}, r_{bi}) \le P_i \tag{16}$$

Lemma 4 μ_i *can increase (decrease) only when the bottom (top) data buffer is empty.*

The proofs of Lemma 3 and Lemma 4 are given in Appendix B.

In a single-hop broadcasting problem as in [7]–[9], the capacity region can be traced by solving the following optimization problem: $\max_{r_{1i},r_{2i}} \mu_1 \sum_{i=1}^{N} r_{1i} + \mu_2 \sum_{i=1}^{N} r_{2i}$ for some $\mu_1, \mu_2 \in \mathbb{R}^+$. Here, μ_1, μ_2 are called *user priorities* and are constant throughout slots. Lemmas 3 and 4 show us that the existence of a multiple access layer affects the broadcast layer by introducing variable user priorities in time. The user priorities can change only when one of the data buffers is empty: the priority of the first user can increase only when the bottom data buffer is empty, and can decrease only when the top data buffer is empty. From [7], the solution to (16) is:

$$r_{ti} = \frac{1}{2}\log(1+\min\{P_{ci}, P_i\}) \tag{17}$$

$$r_{bi} = \frac{1}{2}\log\left(1+\frac{(P_i - P_{ci})^+}{P_{ci}+\sigma_2^2}\right) \tag{18}$$

where, if $\mu_i \geq 1$, all of the power is allocated to the top relay only. If $\mu_i < 1$, we define

$$P_{ci} \triangleq \left(\frac{\mu_i\sigma_2^2 - \sigma_1^2}{1-\mu_i}\right)^+ \tag{19}$$

In other words, given (μ_i, P_i), the rate pairs (r_{ti}, r_{bi}) can uniquely be determined from (17) and (18). We denote the unique rate pairs found from (17) and (18) for fixed (μ_i, P_i) by $r_{ti}(\mu_i, P_i)$ and $r_{bi}(\mu_i, P_i)$. Let us define the function $z(\boldsymbol{\mu})$ which is a maximization over $(\bar{\mathbf{p}}_t, \bar{\mathbf{p}}_b, \bar{\mathbf{r}}_t, \bar{\mathbf{r}}_b)$ for fixed $\boldsymbol{\mu}$:

$$\begin{aligned} z(\boldsymbol{\mu}) = \max_{\bar{\mathbf{p}}_t,\bar{\mathbf{p}}_b,\bar{\mathbf{r}}_t,\bar{\mathbf{r}}_b} \quad & \sum_{i=1}^{N}\bar{r}_{ti} + \sum_{i=1}^{N}\bar{r}_{bi} \\ \text{s.t.} \quad & \sum_{i=1}^{k}\bar{r}_{ti} \leq \sum_{i=1}^{k} r_{ti}(\mu_i, P_i), \quad \forall k \\ & \sum_{i=1}^{k}\bar{r}_{bi} \leq \sum_{i=1}^{k} r_{bi}(\mu_i, P_i), \quad \forall k \\ & (5)-(6), (11) \end{aligned} \tag{20}$$

Then, the original problem in (12) is equivalent to:

$$\max_{\boldsymbol{\mu}\in[0,1]^N} z(\boldsymbol{\mu}) \tag{21}$$

IV. Non-Cooperative Multiple Access Channel Side

In this section, we consider the regular multiple access channel by ignoring the presence of common data. We note that the problem in (20) is a throughput maximization problem in an energy harvesting multiple access channel with data arrivals as shown in Fig. 2. For notational convenience, we denote $d_{ti}i = r_{ti}(\mu_i, P_i)$, $d_{bi} = r_{bi}(\mu_i, P_i)$. When $\boldsymbol{\mu}$ is fixed, the data arrivals to the multiple access side are fixed and the data causality constraints can be written as

$$\sum_{i=1}^{k}\bar{r}_{ti} \leq \sum_{i=1}^{k} d_{ti}, \quad \forall k \tag{22}$$

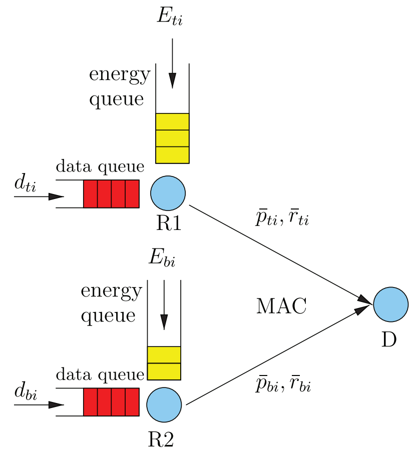

Fig. 2. Multiple access channel with energy and data arrivals.

$$\sum_{i=1}^{k}\bar{r}_{bi} \leq \sum_{i=1}^{k} d_{bi}, \quad \forall k \tag{23}$$

We start this section by reformulating the problem in terms of the rates only. We consider the following energy causality constraints on the rates:

$$\sum_{i=1}^{k}\sigma_3^2(2^{2\bar{r}_{ti}}-1) \leq \sum_{i=1}^{k} E_{ti}, \quad \forall k \tag{24}$$

$$\sum_{i=1}^{k}\sigma_3^2(2^{2\bar{r}_{bi}}-1) \leq \sum_{i=1}^{k} E_{bi}, \quad \forall k \tag{25}$$

$$\sum_{i=1}^{k}\sigma_3^2(2^{2(\bar{r}_{ti}+\bar{r}_{bi})}-1) \leq \sum_{i=1}^{k} E_{ti}+E_{bi}, \quad \forall k \tag{26}$$

and the corresponding throughput maximization problem:

$$\begin{aligned} \max_{\bar{r}_{ti},\bar{r}_{bi}} \quad & \sum_{i=1}^{N}\bar{r}_{ti} + \sum_{i=1}^{N}\bar{r}_{bi} \\ \text{s.t.} \quad & (22)-(26) \end{aligned} \tag{27}$$

The following lemma, proved in Appendix C, shows that this is an equivalent representation for the problem in (20).

Lemma 5 *The problems in (20) and (27) are equivalent.*

We solve the problem in (27) in the remainder of this section. We denote the optimal solution to (27) by $(\bar{r}_{ti}^*, \bar{r}_{bi}^*)$. We have the following lemma.

Lemma 6 *The optimal sum rate for relays is non-decreasing in time, i.e.,* $\bar{r}_{ti}^* + \bar{r}_{bi}^* \leq \bar{r}_{t,i+1}^* + \bar{r}_{b,i+1}^*, \forall i$.

Proof: The proof follows by contradiction. Assume that there is a slot k such that $\bar{r}_{tk}^* + \bar{r}_{bk}^* > \bar{r}_{t,k+1}^* + \bar{r}_{b,k+1}^*$. We will show that this policy cannot be optimal. There can be three cases, case 1: $\bar{r}_{tk}^* > \bar{r}_{t,k+1}^*, \bar{r}_{bk}^* \leq \bar{r}_{b,k+1}^*$, case 2: $\bar{r}_{bk}^* > \bar{r}_{b,k+1}^*, \bar{r}_{tk}^* \leq \bar{r}_{t,k+1}^*$ and case 3: $\bar{r}_{bk}^* > \bar{r}_{b,k+1}^*, \bar{r}_{tk}^* > \bar{r}_{t,k+1}^*$. Assume that the first case happens. Consider the modified policy $\hat{r}_{tk} = \hat{r}_{t,k+1} = \frac{\bar{r}_{tk}^*+\bar{r}_{t,k+1}^*}{2}$. This modified policy is feasible and transmits the same amount of data as $\bar{r}_{ti}^*, \bar{r}_{bi}^*$, but due to the convexity of the functions $2^{2(\bar{r}_{ti}+\bar{r}_{bi})}$ and $2^{2\bar{r}_{ti}}$, consumes less energy. This additional energy can be used to transmit more data and therefore the policy $(\bar{r}_{ti}^*, \bar{r}_{bi}^*)$ cannot be optimal. For the second case, we set $\hat{r}_{bk} = \hat{r}_{b,k+1} = \frac{\bar{r}_{bk}^*+\bar{r}_{b,k+1}^*}{2}$ and for the third case we modify both $\bar{r}_{tk}^*, \bar{r}_{t,k+1}^*$ and $\bar{r}_{bk}^*, \bar{r}_{b,k+1}^*$ to reach a similar contradiction. ■

A. Relaxed Problem and Majorization

Without the data causality constraints of (22) and (23) it was observed in [10], that the optimal sum rate is equal to the single-user optimal rate with the energies merged as $E_{ti} + E_{bi}$. This may naturally suggest that, with the presence of the data causality constraints, the optimal sum rate is given by the single-user optimal rate with both data and energy causality constraints. In this section, we show that this suggestion is not entirely valid, but a majorization relationship exists between these two solutions. Consider the following problem:

$$\begin{aligned} \max_{q_i} \quad & \sum_{i=1}^{N} q_i \\ \text{s.t.} \quad & \sum_{i=1}^{k} \sigma_3^2 (2^{2q_i} - 1) \leq \sum_{i=1}^{k} E_{ti} + E_{bi}, \quad \forall k \\ & \sum_{i=1}^{k} q_i \leq \sum_{i=1}^{k} d_{ti} + d_{bi}, \quad \forall k \end{aligned} \tag{28}$$

This problem can be solved using the geometric approach in [3] or the directional waterfilling with both data and energy arrivals in [5]. We note that the problem in (28) is a relaxed version of (27) where the energy arrivals and data arrivals are merged to a single-user. I.e., we sum up (22) and (23) to obtain a single data arrival constraint and remove (24) and (25). We denote the solution to (28) by q_i^*. Now, we show two weak majorization results whose proofs are provided in Appendix D and E respectively.

Lemma 7 *We must have* $\sum_{i=1}^{k} \bar{r}_{ti}^* + \bar{r}_{bi}^* \leq \sum_{i=1}^{k} q_i^*, \forall k$.

Lemma 8 *If at any slot k, we have* $\sum_{i=1}^{k} \bar{r}_{ti}^* + \bar{r}_{bi}^* = \sum_{i=1}^{k} q_i^*$, *then* $\sum_{i=1}^{k} 2^{2(\bar{r}_{ti}^*+\bar{r}_{bi}^*)} \geq \sum_{i=1}^{k} 2^{2q_i^*}$. *If, in addition, we have* $\sigma_3^2 \left(\sum_{i=1}^{k} 2^{2q_i^*} - 1 \right) = \sum_{i=1}^{k} E_{ti} + E_{bi}$, *then we must have* $\bar{r}_{ti}^* + \bar{r}_{bi}^* = q_i^*$ *for* $i = 1, \ldots, k$.

In some special instances of the problem, Lemmas 7 and 8 can be utilized, by enforcing the constraint $\bar{r}_{ti} + \bar{r}_{bi} = q_i^*, \forall i$, replacing $\bar{r}_{bi} = q_i^* - \bar{r}_{ti}$ and solving a single-user problem.

B. Iterative Solution

In this section, we will solve the overall problem by utilizing a dual decomposition method. After applying Lemma 5, the problem in (15) is equivalent to:

$$\begin{aligned} \max_{\bar{r}_{ti}, \bar{r}_{bi}, r_{ti}, r_{bi}} \quad & \sum_{i=1}^{N} \bar{r}_{ti} + \sum_{i=1}^{N} \bar{r}_{bi} \\ \text{s.t.} \quad & (8), (9), (24) - (26), g(r_{ti}, r_{bi}) \leq P_i \end{aligned} \tag{29}$$

Defining a new variable as $w_i = \bar{r}_{ti} + \bar{r}_{bi}$, we formulate the following equivalent optimization problem:

$$\begin{aligned} \max_{\bar{r}_{ti}, \bar{r}_{bi}, r_{ti}, r_{bi}, w_i} \quad & \sum_{i=1}^{N} w_i \\ \text{s.t.} \quad & (8), (9), (24), (25), g(r_{ti}, r_{bi}) \leq P_i \\ & \sum_{i=1}^{k} \sigma_3^2 (2^{2w_i} - 1) \leq \sum_{i=1}^{k} E_{ti} + E_{bi}, \quad \forall k \\ & w_i = \bar{r}_{ti} + \bar{r}_{bi}, \quad \forall i \end{aligned} \tag{30}$$

which can be relaxed to:

$$\begin{aligned} \max_{\bar{r}_{ti}, \bar{r}_{bi}, r_{ti}, r_{bi}, w_i} \quad & \sum_{i=1}^{N} w_i \\ \text{s.t.} \quad & (8), (9), (24), (25), g(r_{ti}, r_{bi}) \leq P_i \\ & \sum_{i=1}^{k} \sigma_3^2 \left(2^{2w_i} - 1 \right) \leq \sum_{i=1}^{k} E_{ti} + E_{bi}, \quad \forall k \\ & w_i \leq \bar{r}_{ti} + \bar{r}_{bi}, \quad \forall i \end{aligned} \tag{31}$$

since at slots where the last inequality is not satisfied with equality, $\bar{r}_{ti}$ and $\bar{r}_{bi}$ can be decreased until equality is satisfied without changing the throughput. The problem in (31) is convex since the objective function is linear and the constraints are convex. Define the following sets:

$$\mathcal{R}_s = \{(r_{ti}, r_{bi}) \in (\mathbb{R}^+ \times \mathbb{R}^+) : g(r_{ti}, r_{bi}) \leq P_i, \forall i\} \tag{32}$$

$$\mathcal{R}_t = \{\bar{r}_{ti} \in \mathbb{R}^+ : \sum_{i=1}^{k} \sigma_3^2 (2^{2\bar{r}_{ti}} - 1) \leq \sum_{i=1}^{k} E_{ti}, \forall k\} \tag{33}$$

$$\mathcal{R}_b = \{\bar{r}_{bi} \in \mathbb{R}^+ : \sum_{i=1}^{k} \sigma_3^2 (2^{2\bar{r}_{bi}} - 1) \leq \sum_{i=1}^{k} E_{bi}, \forall k\} \tag{34}$$

$$\mathcal{R}_w = \{w_i \in \mathbb{R}^+ : \sum_{i=1}^{k} \sigma_3^2 (2^{2w_i} - 1) \leq \sum_{i=1}^{k} E_{ti} + E_{bi}, \forall k\} \tag{35}$$

Now, we write the partial Lagrangian function for the problem in (31) corresponding to the constraints (8), (9) and $w_i \leq \bar{r}_{ti} + \bar{r}_{bi}$ as follows:

$$\begin{aligned} \mathcal{L} = & \sum_{i=1}^{N} w_i + \sum_{k=1}^{N} \lambda_{1k} \left(\sum_{i=1}^{k} r_{ti} - \sum_{i=1}^{k} \bar{r}_{ti} \right) \\ & + \sum_{k=1}^{N} \lambda_{2k} \left(\sum_{i=1}^{k} r_{bi} - \sum_{i=1}^{k} \bar{r}_{bi} \right) + \sum_{i=1}^{N} \nu_i \left(\bar{r}_{ti} + \bar{r}_{bi} - w_i \right) \end{aligned} \tag{36}$$

Now, the dual function is [33]:

$$\mathcal{K}(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \boldsymbol{\nu}) = \max_{(r_{ti}, r_{bi}) \in \mathcal{R}_s, \bar{r}_{ti} \in \mathcal{R}_t, \bar{r}_{bi} \in \mathcal{R}_b, w_i \in \mathcal{R}_w} \mathcal{L}(\mathbf{r}_t, \mathbf{r}_b, \bar{\mathbf{r}}_t, \bar{\mathbf{r}}_b, \mathbf{w}) \tag{37}$$

$$= \max_{(r_{ti}, r_{bi}) \in \mathcal{R}_s} \left[\sum_{i=1}^{N} r_{ti} \sum_{k=i}^{N} \lambda_{1k} + \sum_{i=1}^{N} r_{bi} \sum_{k=i}^{N} \lambda_{2k} \right]$$

$$+\max_{\bar{r}_{ti}\in\mathcal{R}_t}\left[\sum_{i=1}^{N}\bar{r}_{ti}\left(\nu_i-\sum_{k=i}^{N}\lambda_{1k}\right)\right]$$
$$+\max_{\bar{r}_{bi}\in\mathcal{R}_b}\left[\sum_{i=1}^{N}\bar{r}_{bi}\left(\nu_i-\sum_{k=i}^{N}\lambda_{2k}\right)\right]$$
$$+\max_{w_i\in\mathcal{R}_w}\sum_{i=1}^{N}(1-\nu_i)w_i \tag{38}$$

Denote the collection of Lagrange multiplier vectors as $\boldsymbol{\gamma}\triangleq(\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2,\boldsymbol{\nu})$. For fixed $\boldsymbol{\gamma}$, we define the following subproblems:

$$\mathcal{K}_1(\boldsymbol{\gamma})=\max_{(r_{ti},r_{bi})\in\mathcal{R}_s}\sum_{i=1}^{N}r_{ti}\sum_{k=i}^{N}\lambda_{1k}+\sum_{i=1}^{N}r_{bi}\sum_{k=i}^{N}\lambda_{2k} \tag{39}$$
$$\mathcal{K}_2(\boldsymbol{\gamma})=\max_{\bar{r}_{ti}\in\mathcal{R}_t}\sum_{i=1}^{N}\bar{r}_{ti}\left(\nu_i-\sum_{k=i}^{N}\lambda_{1k}\right) \tag{40}$$
$$\mathcal{K}_3(\boldsymbol{\gamma})=\max_{\bar{r}_{bi}\in\mathcal{R}_b}\sum_{i=1}^{N}\bar{r}_{bi}\left(\nu_i-\sum_{k=i}^{N}\lambda_{2k}\right) \tag{41}$$
$$\mathcal{K}_4(\boldsymbol{\gamma})=\max_{w_i\in\mathcal{R}_w}\sum_{i=1}^{N}(1-\nu_i)w_i \tag{42}$$

Slater's condition holds for the problem in (29), [33]. Therefore, there is no duality gap and the optimal values of the dual problem and the primal problem are the same. This implies that (29) is equivalent to the following problem:

$$\min_{\boldsymbol{\gamma}\geq\mathbf{0}}\quad\mathcal{K}(\boldsymbol{\gamma}) \tag{43}$$

or equivalently:

$$\min_{\boldsymbol{\gamma}\geq\mathbf{0}}\quad\mathcal{H}(\boldsymbol{\gamma}) \tag{44}$$

where $\mathcal{H}\triangleq\mathcal{K}_1+\mathcal{K}_2+\mathcal{K}_3+\mathcal{K}_4$. We observe that for fixed $\boldsymbol{\gamma}$ we can solve the subproblems independently. We solve the problem in (44) by separately solving the outer minimization and inner maximization problems.

1) Inner Maximization: Here, we focus on the inner problems (39)–(42). We start by analyzing (39). We define $a_i=\sum_{k=i}^{N}\lambda_{1k}$ and $b_i=\sum_{k=i}^{N}\lambda_{2k}$. Then (39) becomes:

$$\begin{aligned}\max_{r_{ti},r_{bi}}\quad&\sum_{i=1}^{N}a_ir_{ti}+\sum_{i=1}^{N}b_ir_{bi}\\ \text{s.t.}\quad&g(r_{ti},r_{bi})\leq P_i\end{aligned} \tag{45}$$

Since the constraint set depends only on index i, (45) is solved individually for each i as follows:

$$\begin{aligned}\max_{r_{ti},r_{bi}}\quad&a_ir_{ti}+b_ir_{bi}\\ \text{s.t.}\quad&g(r_{ti},r_{bi})\leq P_i\end{aligned} \tag{46}$$

The problem in (46) is a single-user throughphut maximization problem in a broadcast channel setting as in [7] with user priorities as a_i and b_i. Therefore, the solution to (46) is given by $r_{ti}(a_i/b_i,P_i)$ and $r_{bi}(a_i/b_i,P_i)$ with the definitions as given in (17)–(19).

Now, we examine (40). We define $c_i\triangleq\nu_i-\sum_{k=i}^{N}\lambda_{1k}$ and with this definition (40) becomes:

$$\begin{aligned}\max_{\bar{r}_{ti}}\quad&\sum_{i=1}^{N}c_i\bar{r}_{ti}\\ \text{s.t.}\quad&\sum_{i=1}^{k}\sigma_3^2(2^{2\bar{r}_{ti}}-1)\leq\sum_{i=1}^{k}E_{ti},\quad\forall k\end{aligned} \tag{47}$$

We reformulate the problem in (47) in terms of powers as:

$$\begin{aligned}\max_{\bar{p}_{ti}}\quad&\sum_{i=1}^{N}\frac{c_i}{2}\log\left(1+\frac{\bar{p}_{ti}}{\sigma_3^2}\right)\\ \text{s.t.}\quad&\sum_{i=1}^{k}\bar{p}_{ti}\leq\sum_{i=1}^{k}E_{ti},\quad\forall k\end{aligned} \tag{48}$$

The problem in (47) is a convex optimization problem and by a Lagrangian analysis similar to [5] we obtain:

$$\bar{p}_{ti}=\left(\frac{c_i}{\sum_{k=i}^{N}\pi_k}-1\right)^+=c_i\left(\frac{1}{\sum_{k=i}^{N}\pi_k}-\frac{1}{c_i}\right)^+ \tag{49}$$

where π_k is the Lagrange multiplier corresponding to the energy causality constraint at slot k in (48). The solution to (49) is given by directional waterfilling on rectangles of width c_i and base level $1/c_i$ as explained in [34, Fig. 2]. In slots where $c_i<0$, no power should be allocated and those slots can be treated as if they are not there.

The problems in (41) and (42) have the same structure and are solved similarly. In (41), the fading levels are $d_i\triangleq\nu_i-\sum_{k=i}^{N}\lambda_{2k}$ and energy arrivals are E_{bi} and in (42), the fading levels are $(1-\nu_i)$ and energy arrivals are $E_{ti}+E_{bi}$. If the fading levels are negative in any slot, those slots can be skipped. Denote the solutions to $\mathcal{K}_1(\boldsymbol{\gamma})$ by $(r_{ti}^*(\boldsymbol{\gamma}),r_{bi}^*(\boldsymbol{\gamma}))$ and the solutions to $\mathcal{K}_2(\boldsymbol{\gamma}),\mathcal{K}_3(\boldsymbol{\gamma}),\mathcal{K}_4(\boldsymbol{\gamma})$ by $\bar{r}_{ti}^*(\boldsymbol{\gamma}),\bar{r}_{bi}^*(\boldsymbol{\gamma})$ and $w_i^*(\boldsymbol{\gamma})$, respectively.

2) Outer Minimiziation: The outer minimization problem is the problem of finding optimal $\boldsymbol{\gamma}$ in (44). For this problem we will use the normalized subgradient method, which is defined as

$$\boldsymbol{\gamma}^{l+1}=\left(\boldsymbol{\gamma}^l-\zeta_l\frac{v^l}{\|v^l\|}\right)^+ \tag{50}$$

where $\boldsymbol{\gamma}^{l+1}$ is the lth iterate, v^l is any subgradient of h at $\boldsymbol{\gamma}^l$ and $\zeta_l>0$ is the lth step size. The $(+)$ operator is used to enforce the constraints that $\boldsymbol{\gamma}\geq\mathbf{0}$. For completeness, first we define the subgradient of a function: v is a subgradient of $\mathcal{H}$ at $\boldsymbol{x}$ if [33, Eq. (6.20)]

$$\mathcal{H}(\boldsymbol{y})\geq\mathcal{H}(\boldsymbol{x})+v^{\mathsf{T}}(\boldsymbol{y}-\boldsymbol{x}),\quad\forall\boldsymbol{y} \tag{51}$$

Now, we show that a subgradient for $\mathcal{H}(\boldsymbol{\gamma})$ is readily available once the inner maximization problems are solved. The following lemma is proved in Appendix F.

Lemma 9 *The vector* $\Big[\Big(\sum_{i=1}^{k} r_{ti}^*(\boldsymbol{\gamma}^l) - \sum_{i=1}^{k} \bar{r}_{ti}^*(\boldsymbol{\gamma}^l)\Big), \Big(\sum_{i=1}^{k} r_{bi}^*(\boldsymbol{\gamma}^l) - \sum_{i=1}^{k} \bar{r}_{bi}^*(\boldsymbol{\gamma}^l)\Big), \big(\bar{r}_{ti}^*(\boldsymbol{\gamma}^l) + \bar{r}_{bi}^*(\boldsymbol{\gamma}^l) - w_i^*(\boldsymbol{\gamma}^{(l)})\big)\Big]_{k=1}^{N}$ *is a subgradient for* $\mathcal{H}(\boldsymbol{\gamma})$ *at* $\boldsymbol{\gamma}^l$.

We note that the subgradient method is not a descent method, i.e., the iterations at every step do not necessarily decrease the objective value. Therefore, it is necessary to keep track of the best point found so far. At each step, we set:

$$\mathcal{H}_{\text{best}}^{l} = \min\{\mathcal{H}_{\text{best}}^{l-1}, \mathcal{H}(\boldsymbol{\gamma}^l)\} \tag{52}$$

We denote $\boldsymbol{\gamma}_{\text{best}}^l$ as the argument of $\mathcal{H}_{\text{best}}^l$. It can be shown that for appropriately selected ζ_l, $\mathcal{H}_{\text{best}}^l \to \mathcal{H}^*$ [35, Section 6.3]. Furthermore, if the step size ζ_l is chosen such that $\sum_{l=1}^{\infty} \zeta_l = \infty$, $\sum_{l=1}^{\infty} \zeta_l^2 < \infty$, then $\boldsymbol{\gamma}_{\text{best}}^l \to \boldsymbol{\gamma}^*$ [36, Proposition 5.1]. Once the optimal $\boldsymbol{\gamma}^*$ is found, $w_i^*(\boldsymbol{\gamma}^*)$ is the optimal sum rate and we can find $r_{ti}^*(\boldsymbol{\gamma}^*), r_{bi}^*(\boldsymbol{\gamma}^*)$ as the optimal source rates and $\bar{r}_{ti}^*(\boldsymbol{\gamma}^*), \bar{r}_{bi}^*(\boldsymbol{\gamma}^*)$ as the optimal relay rates. If $\bar{r}_{tk}^*(\boldsymbol{\gamma}^*) + \bar{r}_{bk}^*(\boldsymbol{\gamma}^*) > w_k^*(\boldsymbol{\gamma}^*)$ for some slot k then we can decrease first or second user rates until equality is achieved.

V. Cooperative (Extended) Multiple Access Region

In this section, consider an extended multiple access capacity region for the second hop of the diamond channel. We note that the statement of Theorem 1 still holds when the multiple access region of (3) is used instead of (4). However, the statement of Lemma 5 and the discussions in Section IV do not hold and it is not clear how to formulate the multiple access side using rate expressions only. Therefore, here we keep the expressions in terms of both power and rate allocations. Using the approach followed before, we have that the original problem in (12) is equivalent to:

$$\max_{\boldsymbol{\mu}\in[0,1]^N} z(\boldsymbol{\mu}) \tag{53}$$

where $z(\boldsymbol{\mu})$ is defined as in (20). We solve the problem in (53) in this section.

A. Inner Maximization

In this section, we focus on the inner problem in (20) for fixed $\boldsymbol{\mu}$. We define the new variables $\bar{p}_{1ti} = (1-\beta_i)\bar{p}_{ti}$ and $\bar{p}_{2ti} = \beta_i \bar{p}_{ti}$ and rewrite (20) as:

$$\begin{aligned} \max \quad & \sum_{i=1}^{N} \bar{r}_{ti} + \sum_{i=1}^{N} \bar{r}_{bi} \\ \text{s.t.} \quad & \sum_{i=1}^{k} \bar{r}_{ti} \le \sum_{i=1}^{k} r_{ti}(\mu_i, P_i),\ \sum_{i=1}^{k} \bar{r}_{bi} \le \sum_{i=1}^{k} r_{bi}(\mu_i, P_i) \\ & \sum_{i=1}^{k} \bar{p}_{1ti} + \bar{p}_{2ti} \le \sum_{i=1}^{k} E_{ti},\ \sum_{i=1}^{k} \bar{p}_{bi} \le \sum_{i=1}^{k} E_{bi}, \\ & \bar{r}_{ti} \le f(\bar{p}_{1ti}/\sigma_3^2) \\ & \bar{r}_{ti} + \bar{r}_{bi} \le f\left((\bar{p}_{1ti} + \bar{p}_{2ti} + \bar{p}_{bi} + 2\sqrt{\bar{p}_{2ti}\bar{p}_{bi}})/\sigma_3^2\right) \end{aligned} \tag{54}$$

We denote the vector triple $\mathcal{P} = (\bar{\mathbf{p}}_{1t}, \bar{\mathbf{p}}_{2t}, \bar{\mathbf{p}}_b)$ and define the function $y(\mathcal{P})$ as maximization over $(\bar{\mathbf{r}}_t, \bar{\mathbf{r}}_b)$ for fixed $\mathcal{P}$:

$$\begin{aligned} y(\mathcal{P}) \triangleq \max_{(\bar{\mathbf{r}}_t, \bar{\mathbf{r}}_b)} \quad & \sum_{i=1}^{N} \bar{r}_{ti} + \sum_{i=1}^{N} \bar{r}_{bi} \\ \text{s.t.} \quad & \sum_{i=1}^{k} \bar{r}_{ti} \le \sum_{i=1}^{k} r_{ti}(\mu_i, P_i),\ \sum_{i=1}^{k} \bar{r}_{bi} \le \sum_{i=1}^{k} r_{bi}(\mu_i, P_i) \\ & \bar{r}_{ti} \le f(\bar{p}_{1ti}/\sigma_3^2) \\ & \bar{r}_{ti} + \bar{r}_{bi} \le f\left((\bar{p}_{1ti} + \bar{p}_{2ti} + \bar{p}_{bi} + 2\sqrt{\bar{p}_{2ti}\bar{p}_{bi}})/\sigma_3^2\right) \end{aligned} \tag{55}$$

For fixed $\mathcal{P}$, (55) is a linear program, and $y(\mathcal{P})$ can be determined efficiently. We next note the following fact.

Lemma 10 $y(\mathcal{P})$ *is non-decreasing and concave in* $\mathcal{P}$.

Proof: Since increasing the powers can only expand the feasible region, y is non-decreasing in its arguments. To prove the concavity: Let $\mathcal{P} = (\bar{\mathbf{p}}_{1t}, \bar{\mathbf{p}}_{2t}, \bar{\mathbf{p}}_b)$ and $\mathcal{Q} = (\bar{\mathbf{q}}_{1t}, \bar{\mathbf{q}}_{2t}, \bar{\mathbf{q}}_b)$ be two power vectors. Let $\lambda = 1 - \bar{\lambda} \in [0, 1]$. Let $(\bar{\mathbf{r}}_t, \bar{\mathbf{r}}_b)$ solve $y(\mathcal{P})$ and $(\bar{\mathbf{s}}_t, \bar{\mathbf{s}}_b)$ solve $y(\mathcal{Q})$. Now, we show that $(\lambda\bar{\mathbf{r}}_t + \bar{\lambda}\bar{\mathbf{s}}_t, \lambda\bar{\mathbf{r}}_b + \bar{\lambda}\bar{\mathbf{s}}_b)$ is feasible for the problem $y(\lambda\mathcal{P} + \bar{\lambda}\mathcal{Q})$. The first two constraints in (55) are linear, thus, their linear combinations are feasible. The third constraint is convex because f is concave. The last constraint is convex because f is concave, non-decreasing, and $\sqrt{\bar{p}_{2ti}\bar{p}_{bi}}$ is concave. Thus, $(\lambda\bar{\mathbf{r}}_t + \bar{\lambda}\bar{\mathbf{s}}_t, \lambda\bar{\mathbf{r}}_b + \bar{\lambda}\bar{\mathbf{s}}_b)$ is feasible for $y(\lambda\mathcal{P} + \bar{\lambda}\mathcal{Q})$. Now,

$$y(\lambda\mathcal{P} + \bar{\lambda}\mathcal{Q}) \ge \sum_{i=1}^{N} \lambda\bar{r}_{ti} + \bar{\lambda}\bar{s}_{ti} + \lambda\bar{r}_{bi} + \bar{\lambda}\bar{s}_{bi} \tag{56}$$

$$= \lambda y(\mathcal{P}) + \bar{\lambda} y(\mathcal{Q}) \tag{57}$$

where (56) follows because the maximum value of the problem can be no smaller than the objective value of any feasible point, and (57) follows from the fact that $(\bar{\mathbf{r}}_t, \bar{\mathbf{r}}_b)$ solves $y(\mathcal{P})$ and $(\bar{\mathbf{s}}_t, \bar{\mathbf{s}}_b)$ solves $y(\mathcal{Q})$. ■

The problem in (54) can equivalently be written as:

$$\begin{aligned} \max_{\bar{\mathbf{p}}_{1t}, \bar{\mathbf{p}}_{2t}, \bar{\mathbf{p}}_b} \quad & y(\bar{\mathbf{p}}_{1t}, \bar{\mathbf{p}}_{2t}, \bar{\mathbf{p}}_b) \\ \text{s.t.} \quad & \sum_{i=1}^{k} \bar{p}_{1ti} + \bar{p}_{2ti} \le \sum_{i=1}^{k} E_{ti}, \quad \forall k \\ & \sum_{i=1}^{k} \bar{p}_{bi} \le \sum_{i=1}^{k} E_{bi}, \quad \forall k \end{aligned} \tag{58}$$

The problem in (58) is convex as it involves maximizing a concave function over a feasible set with linear constraints. This can be performed efficiently by iterating over feasible $(\bar{\mathbf{p}}_{1t}, \bar{\mathbf{p}}_{2t}, \bar{\mathbf{p}}_b)$ such that every iteration increases the objective function, for example, using the method described in [28, Section III.B]. Due to convexity, the convergence to an optimal solution is guaranteed. Once $(\bar{\mathbf{p}}_{1t}^*, \bar{\mathbf{p}}_{2t}^*, \bar{\mathbf{p}}_b^*)$ is found, $z(\boldsymbol{\mu}) = y(\bar{\mathbf{p}}_{1t}^*, \bar{\mathbf{p}}_{2t}^*, \bar{\mathbf{p}}_b^*)$.

B. Outer Maximization

The outer maximization problem is the problem of finding the optimal $\boldsymbol{\mu}$ in (53). For this purpose, we use the block coordinate descent method on the vector $\boldsymbol{\mu}$. First, we fix $(\mu_1, \ldots, \mu_{N-1})$ and solve the following problem

$$\max_{\mu_N \in [0,1]} z(\mu_1, \mu_2, \ldots, \mu_{N-1}, \mu_N) \tag{59}$$

which can be done using a one-dimensional search on $\mu_N \in [0, 1]$. Then, using this newly found μ_N, we fix $(\mu_1, \ldots, \mu_{N-2}, \mu_N)$ and maximize over μ_{N-1}. We cyclically iterate through each μ_i, one at a time, maximizing the objective function with respect to that μ_i. By construction, the iterations $z(\boldsymbol{\mu}^{(k)})$ is a monotone increasing sequence and is bounded because the optimal value of problem (12) is bounded, which guarantees convergence. The iterations converge to an optimal point due to the convexity of the original problem. We can utilize Lemma 4 to search over $\boldsymbol{\mu}$ space more efficiently. Using this procedure, we reduced an N dimensional search for $\boldsymbol{\mu}$ to N one dimensional searches for each individual μ_i. For large N, this search can be computationally demanding, however numerically we observed quick convergence.

VI. Numerical Results

In this section, we provide numerical examples and illustrate the resulting optimal policies. We consider band-limited AWGN broadcast and multiple-access channels. The bandwidth is $B_W = 1$ MHz and the noise power spectral density is $N_0 = 10^{-19}$ W/Hz. We assume that the path loss between the source and relay 1 (h_{sr1}) is 123 dB, source and relay 2 (h_{sr2}) is 127 dB and the path loss between relays and destination are assumed to be same ($h_{r1d} = h_{r2d}$) and 130 dB. With these definitions, equations (1) and (2) become:

$$\begin{aligned} r_1 &\le B_W \log_2\left(1 + \frac{\alpha P h_{sr1}}{N_0 B_W}\right) \\ &= \log_2\left(1 + \frac{\alpha P}{0.2}\right) \text{Mbps} && (60) \\ r_2 &\le B_W \log_2\left(1 + \frac{(1-\alpha) P h_{sr2}}{\alpha P h_{sr2} + N_0 B_W}\right) \\ &= \log_2\left(1 + \frac{(1-\alpha)P}{\alpha P + 0.6}\right) \text{Mbps} && (61) \end{aligned}$$

$$g(r_1, r_2) = 0.2 * 2^{(r_1+r_2)} + (0.6 - 0.2) * 2^{r_2} - 0.6\,\text{W} \tag{62}$$

The extended multiple access capacity region described in (3) becomes:

$$\begin{aligned} r_1 &\le B_W \log_2\left(1 + \frac{(1-\beta) h_{r1d} P_1}{N_0 B_W}\right) \\ &= \log_2\left(1 + (1-\beta) P_1\right) \text{Mbps} && (63) \\ r_1 + r_2 &\le B_W \log_2\Big[1 + (N_0 B_W)^{-1} \\ &\quad \left(h_{r1d} P_1 + h_{r2d} P_2 + 2\sqrt{\beta h_{r1d} P_1 h_{r2d} P_2}\right)\Big] \\ &= \log_2\left(1 + P_1 + P_2 + 2\sqrt{\beta P_1 P_2}\right) \text{Mbps} && (64) \end{aligned}$$

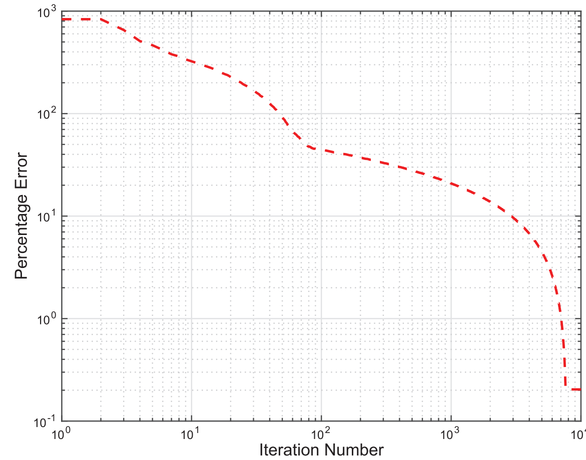

Fig. 3. Percentage error between the best iteration so far and the optimal value vs iteration number k.

Similarly the non-cooperative multiple access capacity region described in (4) becomes

$$\begin{aligned} r_1 &\le B_W \log_2\left(1 + \frac{h_{r1d} P_1}{N_0 B_W}\right) \\ &= \log_2\left(1 + P_1\right) \text{Mbps} && (65) \\ r_2 &\le B_W \log_2\left(1 + \frac{h_{r2d} P_2}{N_0 B_W}\right) \\ &= \log_2\left(1 + P_2\right) \text{Mbps} && (66) \\ r_1 + r_2 &\le B_W \log_2\left(1 + \frac{h_{r1d} P_1 + h_{r2d} P_2}{N_0 B_W}\right) \\ &= \log_2\left(1 + P_1 + P_2\right) \text{Mbps} && (67) \end{aligned}$$

A. Deterministic Energy Arrivals

In this subsection, we consider deterministic energy arrivals, and focus on the offline problem studied in this paper. We study a 3 slot scenario with the following energy arrivals, $\mathbf{E}_s = [5, 20, 9]$ J, $\mathbf{E}_t = [4, 6, 5]$ J, $\mathbf{E}_b = [6, 10, 4]$ J.

First, we investigate the non-cooperative Gaussian multiple access scenario, disregarding the possible cooperation between the top and bottom relays. The evolution of our subgradient descent based algorithm is shown in Fig. 3. The step size is taken as $\zeta_k = \frac{1.3}{k}$ and the initial points are taken as $\boldsymbol{\lambda}_1^0 = [3.4, 1, 1]$, $\boldsymbol{\lambda}_2^0 = [2.8, 1.1, 1.4]$, $\boldsymbol{\nu}^0 = [10, 4, 3]$. The plot shows the percentage error between the best iteration so far and the optimal value of the problem in (12). The algorithm converges after around 10^4 steps to reasonable accuracy. The resulting Lagrange multipliers are found as $\boldsymbol{\lambda}_1 = [3.04, 0.04, 0.4] \times 10^{-3}$, $\boldsymbol{\lambda}_2 = [4.29, 0, 0] \times 10^{-3}$, $\boldsymbol{\nu} = [4.51, 4.29, 4.26] \times 10^{-3}$. The optimal rates are then found as $\mathbf{r}_t = [1.55, 1.18, 1.14]$ Mbits, $\mathbf{r}_b = [0.69, 1.7, 1.72]$ Mbits, $\bar{\mathbf{r}}_t = [1.16, 1.34, 1.35]$ Mbits, $\bar{\mathbf{r}}_b = [1.4, 1.73, 0]$ Mbits, $\mathbf{w} = [1.72, 1.87, 1.87]$ Mbits. We observe that by setting $\bar{\mathbf{r}}_b = \mathbf{w} - \bar{\mathbf{r}}_t = [0.56, 0.53, 0.52]$ Mbits we can get $w_i = \bar{r}_{ti} + \bar{r}_{bi}, \forall i$ and this set of rates is the optimal solution. The feasibility of this solution can be verified. Due to the non-uniqueness of the solution, there may exist multiple $\bar{r}_{ti}^*, \bar{r}_{bi}^*$ pairs that yield the optimal sum rate however the optimal

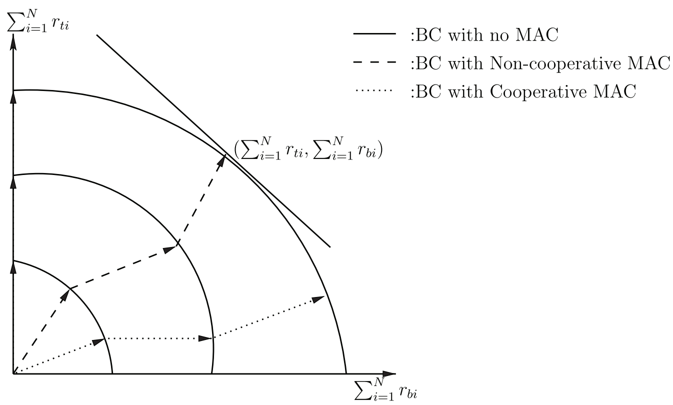

Fig. 4. Maximum departure region and trajectories to reach the optimal point for BC with no MAC, BC with non-cooperative MAC and BC with cooperative MAC.

sum rate $\bar{r}_{ti}^* + \bar{r}_{bi}^*$ is unique. The optimal sum throughput in this case is calculated as $\sum_{i=1}^N \bar{r}_{ti} + \sum_{i=1}^N \bar{r}_{bi} = 5.46$ Mbits. The optimal user priorities for the source are calculated as $\boldsymbol{\mu}_1 = [0.81, 0.72, 0.71]$.

Second, we investigate the extended multiple access scenario. The optimal user priorities for the source are found as $\boldsymbol{\mu}_2 = [0.83, 0.43, 0.43]$. The optimal rates are then found as $\mathbf{r}_t = [0.74, 0.07, 0.07]$ Mbits, $\mathbf{r}_b = [1.27, 2.3, 2.3]$ Mbits, $\bar{\mathbf{r}}_t = [0.74, 0.07, 0.07]$ Mbits, $\bar{\mathbf{r}}_b = [1.27, 2.3, 2.3]$ Mbits, $\bar{\mathbf{p}}_{1t} = [1.8, 0.1, 0.1]$ W, $\bar{\mathbf{p}}_{2t} = [2.2, 5.39, 5.39]$ W, $\bar{\mathbf{p}}_b = [4.82, 7.58, 7.58]$ W. We note that $\bar{\mathbf{p}}_{2t}$ is much larger than $\bar{\mathbf{p}}_{1t}$ which means that relay 1 has spent a significant portion of its power on the cooperative communication rather than forwarding its own data. The optimal sum throughput in this case is calculated as $\sum_{i=1}^N \bar{r}_{ti} + \sum_{i=1}^N \bar{r}_{bi} = 6.76$Mbits which is higher than in the non-cooperative case.

Finally, we examine the maximum departure region and the optimal trajectories for the broadcast side of this diamond channel for the non-cooperative and cooperative Gaussian multiple access channel second hops. Without the existence of relays, for the two user Gaussian broadcast channel, to maximize the sum rate we need to set $\boldsymbol{\mu} = [1, 1, 1]$ and $r_{bi} = 0, \forall i$, i.e., all the power must be allocated to the stronger user [7]. The existence of the multiple access layer changes this structure. We sketch the maximum departure region and trajectories to reach the optimal point in Fig. 4. When there is no multiple access layer, all the power is allocated to the first user. In the case of non-cooperative multiple access layer, the rates to both of the relays follow a balanced pattern. In the case of cooperative multiple access layer, the weaker relay gets more data than the stronger relay due to the possibility of cooperation.

B. Stochastic Energy Arrivals

In this subsection, we consider stochastic energy arrivals and we compare the performance of the offline optimal policy with that of a suboptimal online policy. These policies are inspired by the optimal offline policy while they require partial or no offline knowledge of the energy arrivals. We have shown that a partial separation holds between the broadcast and the multiple access parts of the problem, therefore the online policies we consider will be of separation based. We denote the amount of energy in the batteries of the source, top relay and bottom relay as B_s, B_t, B_b and the data buffers of the top and bottom relays as D_t, D_b. The presented online algorithms are of best-effort type [5], [37], where the transmitters aim to keep a constant power if feasible, or transmit with the currently available power otherwise.

1) Source Power and Rate Allocation: This policy determines the source power p_{si} and rate (r_{ti}, r_{bi}) allocations. We choose a policy that transmits with constant power equal to the average recharge rate of the source battery, if there is enough energy, otherwise it uses all of the battery energy, i.e., $p_s = \min\{\mathbb{E}[E_s], B_s\}$. First, we define a constant C which depends only on the average recharge rates of the top and bottom relays, as follows:

$$C = \begin{cases} \frac{\log_2(1+\mathbb{E}[E_t])}{\log_2(1+\mathbb{E}[E_b])}, & \text{if regular MAC} \\ \frac{\log_2(1+\mathbb{E}[E_b])}{\log_2(1+\mathbb{E}[E_b]+\mathbb{E}[E_t])}, & \text{if cooperative MAC} \end{cases} \tag{68}$$

The reasoning behind the choice of C is as follows. For the regular MAC, the top relay can transmit at most an average rate of $\log_2(1 + \mathbb{E}[E_t])$, considering its own energy arrivals. Similarly, the bottom relay can transmit at most an average rate of $\log_2(1 + \mathbb{E}[E_b])$. Therefore, we have $\frac{\bar{r}_t}{\bar{r}_b} \sim \frac{\log_2(1+\mathbb{E}[E_t])}{\log_2(1+\mathbb{E}[E_b])}$. We choose the source rate division to be exactly equal to this quantity. For the cooperative MAC, we use a constant β policy and set $\beta = 1 - \frac{\mathbb{E}[E_b]}{\mathbb{E}[E_t]}$. Then, from (63) we have $\bar{r}_t \sim \log_2(1 + \mathbb{E}[E_b])$ and from (64) $\bar{r}_b \sim \log_2(1 + \mathbb{E}[E_b] + \mathbb{E}[E_t])$. We choose the source rate division to be exactly equal to the ratio of two rates. From (60) and (61), we choose the power share α^* to satisfy the following equation:

$$C = \frac{\log_2\left(1 + \frac{\alpha \mathbb{E}[E_s]}{0.2}\right)}{\log_2\left(1 + \frac{(1-\alpha)\mathbb{E}[E_s]}{\alpha \mathbb{E}[E_s]+0.6}\right)} \tag{69}$$

2) Top and Bottom Relay Power and Rate Allocation: This policy determines the top and bottom relay power (p_{ti}, p_{bi}) and rate $(\bar{r}_{ti}, \bar{r}_{bi})$ allocations. We note that the policy for the relays must depend on the data arrivals from the source. For the regular MAC, the online policy is determined as follows. We set the top relay power allocation as the average recharge rate of the top relay battery if there is enough energy and data, otherwise it uses either all of the battery energy or transmits at a rate that transmits all of the available data. We set $p_t = \min\{\mathbb{E}[E_t], B_t, 2^{D_t-1}\}$, $r_t = \log_2(1 + p_t)$. Similarly we set $p_b = \min\{\mathbb{E}[E_b], B_b, 2^{D_b-1}\}$, $r_b = \log_2(1 + p_b)$. If the constraint $r_t + r_b \le \log_2(1 + p_t + p_b)$ is not satisfied, then we decrease r_b, p_b until equality is satisfied.

For the cooperative MAC, additional to p_t, p_b we need to determine β given in (64). We set $p_t = \min\{\mathbb{E}[E_t], B_t, 2^{D_t-1}\}$, $r_t = \log_2(1 + p_t)$. We use a constant β policy and set $\beta = 1 - \frac{\mathbb{E}[E_b]}{\mathbb{E}[E_t]}$. Now, we set $p_b = \min\{\mathbb{E}[E_b], B_b\}$ and $r_b = \log_2(1 + p_t + p_b + 2\sqrt{\beta p_t p_b}) - r_t$. If $r_b > D_b$, then r_b, p_b are decreased until equality is satisfied.

3) Simulations: In the simulations, we consider Bernoulli energy arrival processes. The source energy arrivals are $E_{si} = 0$

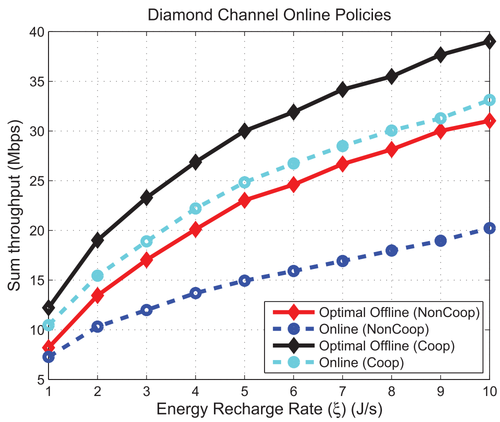

Fig. 5. Average sum throughput versus average recharge rate for offline and online policies.

with probability 0.5 and $E_{si} = 2\xi$ with probability 0.5 where ξ is the average recharge rate, and we denote this process by Ber$(0.5, \xi)$. We assume that $E_{ti} \sim$ Ber$(0.5, 0.5\xi)$ and $E_{bi} \sim$ Ber$(0.5, 0.3\xi)$. We perform simulations for a deadline of 10 slots. The performance metric of the policies is the average sum throughput over 100 realizations of the stochastic energy arrival process. We plot our results in Fig. 5. We observe that the sum throughput increases with increasing energy recharge rate.

VII. Conclusion

We considered the energy harvesting diamond channel where the physical layer is modeled as a concatenation of a broadcast channel and a multiple access channel. In the first part of the paper, we focused on the broadcast half of the diamond network. We first showed that there exists an optimal source power allocation policy which is equal to the single-user optimal power policy for the source energy arrivals and does not depend on the relay energy arrivals. Next, we showed that even though the total power can be selected as the single-user optimal power, the fraction of the power spent on each broadcast link depends on the energy arrivals of the relays. In the second part of the paper, we turned our attention to the multiple access side of the diamond network. This is a cooperative multiple access channel with common data. Initially, we ignored the possible cooperation between the relays and assumed a regular Gaussian multiple access channel with non-cooperating users. In this setting, first we showed that when the transmission rates of the source in the broadcast side are fixed, the overall problem becomes a multiple access channel with both data and energy arrivals. We showed that this problem can be formulated in terms of data transmission rates only, instead of formulating over both transmission powers and data rates. We solved the overall diamond channel problem with non-cooperative multiple access channel using a dual decomposition method. In the last part of the paper, we considered the cooperative (extended) multiple access capacity region for the second hop. With the extended multiple access capacity region, we found the overall solution using a decomposition into inner and outer maximization problems.

Appendix A
Proof of Theorem 1

In this proof, we are only interested in (r_{ti}^*, r_{bi}^*). Therefore, to find the necessary optimality conditions, we write the Lagrangian function of the problem in (14) as:

$$\begin{aligned}\mathcal{L} = &-\sum_{i=1}^{N}\bar{r}_{ti} - \sum_{i=1}^{N}\bar{r}_{bi} + \sum_{k=1}^{N}\lambda_{1k}\left(\sum_{i=1}^{k}\bar{r}_{ti} - \sum_{i=1}^{k} r_{ti}\right) \\ &+ \sum_{k=1}^{N}\lambda_{2k}\left(\sum_{i=1}^{k}\bar{r}_{bi} - \sum_{i=1}^{k} r_{bi}\right) + \sum_{k=1}^{N}\gamma_k\left(\sum_{i=1}^{k} g(r_{ti}, r_{bi}) \right.\\ &\left. - \sum_{i=1}^{k} E_{si}\right) - \sum_{i=1}^{N}\theta_{1i} r_{ti} - \sum_{i=1}^{N}\theta_{2i} r_{bi} + \text{ other terms}\end{aligned} \tag{70}$$

where other terms include the Lagrange multipliers for the other constraints but they are not needed in the proof and are omitted for the sake of brevity. The complementary slackness conditions for these Lagrange multipliers are:

$$\lambda_{1k}\left(\sum_{i=1}^{k}\bar{r}_{ti} - \sum_{i=1}^{k} r_{ti}\right) = \lambda_{2k}\left(\sum_{i=1}^{k}\bar{r}_{bi} - \sum_{i=1}^{k} r_{bi}\right) = 0 \tag{71}$$

$$\gamma_k\left(\sum_{i=1}^{k} g(r_{ti}, r_{bi}) - \sum_{i=1}^{k} E_{si}\right) = 0 \tag{72}$$

$$\theta_{1i} r_{ti} = \theta_{2i} r_{bi} = 0, \qquad \lambda_{1k}, \lambda_{2k}, \gamma_k \geq 0 \tag{73}$$

Taking the derivatives of $\mathcal{L}$ with respect to r_{ti} and r_{bi}:

$$-\sum_{k=i}^{N}\lambda_{1k} + \left(\sum_{k=i}^{N}\gamma_k\right)\sigma_1^2 2^{2(r_{ti}+r_{bi})} - \theta_{1i} = 0 \tag{74}$$

$$-\sum_{k=i}^{N}\lambda_{2k} + \left(\sum_{k=i}^{N}\gamma_k\right)\left(g(r_{ti}, r_{bi}) + \sigma_2^2\right) - \theta_{2i} = 0 \tag{75}$$

From (74) and (75), we get:

$$g(r_{ti}, r_{bi}) = \frac{\theta_{2i} + \sum_{k=i}^{N}\lambda_{2k}}{\sum_{k=i}^{N}\gamma_k} - \sigma_2^2 \tag{76}$$

$$2^{2(r_{ti}+r_{bi})} = \frac{\theta_{1i} + \sum_{k=i}^{N}\lambda_{1k}}{\sigma_1^2 \sum_{k=i}^{N}\gamma_k} \tag{77}$$

Lemma 11 *When the optimal total source power $g(r_{ti}^*, r_{bi}^*)$ increases, the energy buffer must be empty.*

Proof: We will show that if $g(r_{ti}, r_{bi}) < g(r_{t,i+1}, r_{b,i+1})$ then $\gamma_i > 0$. First, assume $r_{b,i+1} > 0$ which implies from (73) that $\theta_{2,i+1} = 0$. Then, from (76), $g(r_{ti}, r_{bi}) < g(r_{t,i+1}, r_{b,i+1})$ is only possible if $\gamma_i > 0$. Next, assume $r_{b,i+1} = 0$ which implies that $r_{t,i+1} > 0$ otherwise $g(r_{t,i+1}, r_{b,i+1}) = 0$ which cannot be optimal. When $r_{b,i+1} = 0$, $g(r_{ti}, r_{bi}) < g(r_{t,i+1}, r_{b,i+1})$ is equivalent to $2^{2(r_{ti}+r_{bi})} < 2^{2(r_{t,i+1}+r_{b,i+1})}$, and from (77) and $\theta_{1,i+1} = 0$, we must have $\gamma_i > 0$. ■

Next, we show that the total source power cannot strictly decrease over the slots.

Lemma 12 *The total source power must be non-decreasing, i.e.,* $g(r_{ti}^*, r_{bi}^*) \leq g(r_{t,i+1}^*, r_{b,i+1}^*)$, $\forall i$.

Proof: We will prove this statement by contradiction. Specifically, we assume a policy in which there is a slot k such that $g(r_{tk}^*, r_{bk}^*) > g(r_{t,k+1}^*, r_{b,k+1}^*)$. We will show that this policy cannot be optimal. ■

We first show that if $g(r_{tk}, r_{bk}) > g(r_{t,k+1}, r_{b,k+1})$ then $\lambda_{1k} = \lambda_{2k} = 0$ cannot happen. First, assume $r_{bk} > 0$ which implies from (73) that $\theta_{2k} = 0$. Then, from (76), $g(r_{tk}, r_{bk}) > g(r_{t,k+1}, r_{b,k+1})$ is only possible if $\lambda_{2k} > 0$. Next, assume $r_{bk} = 0$ which implies that $r_{tk} > 0$ otherwise $g(r_{tk}, r_{bk}) = 0$ which cannot be optimal. When $r_{bk} = 0$, $g(r_{tk}, r_{bk}) > g(r_{t,k+1}, r_{b,k+1})$ is equivalent to $2^{2(r_{tk}+r_{bk})} > 2^{2(r_{t,k+1}+r_{b,k+1})}$, and from (77) and $\theta_{1k} = 0$, we have $\lambda_{1k} > 0$.

Now, for $g(r_{tk}^*, r_{bk}^*) > g(r_{t,k+1}^*, r_{b,k+1}^*)$ to happen, we need to have either $r_{tk}^* > r_{t,k+1}^*, r_{bk}^* \leq r_{b,k+1}^*$ or $r_{bk}^* > r_{b,k+1}^*, r_{tk}^* \leq r_{t,k+1}^*$ or $r_{tk}^* > r_{t,k+1}^*, r_{bk}^* > r_{b,k+1}^*$. We will examine these cases separately.

Case 1: $r_{tk}^* > r_{t,k+1}^*, r_{bk}^* \leq r_{b,k+1}^*$: We must have $r_{tk}^* > 0$ which implies $\theta_{1k} = 0$. In this case, for $g(r_{tk}^*, r_{bk}^*) > g(r_{t,k+1}^*, r_{b,k+1}^*)$, we must also have $r_{tk}^* + r_{bk}^* > r_{t,k+1}^* + r_{b,k+1}^*$. This implies from (77) that $\lambda_{1k} > 0$ and $\sum_{i=1}^k \bar{r}_{ti}^* = \sum_{i=1}^k r_{ti}^*$. From the data causality constraints at the $(k-1)$st slot and $\sum_{i=1}^k \bar{r}_{ti}^* = \sum_{i=1}^k r_{ti}^*$, we must have $\bar{r}_{tk} \geq r_{tk}$. Similarly, from data causality at the $(k+1)$st slot and $\sum_{i=1}^k \bar{r}_{ti}^* = \sum_{i=1}^k r_{ti}^*$, we must have $\bar{r}_{t,k+1} \leq r_{t,k+1}$. This implies that we must have $\bar{r}_{tk} \geq r_{tk} > r_{t,k+1} \geq \bar{r}_{t,k+1}$, thus $\bar{r}_{tk} > \bar{r}_{t,k+1}$. Now, consider the following modified policy for some $\delta > 0$, $\hat{r}_{tk} = r_{tk}^* - \delta$, $\hat{r}_{t,,k+1} = r_{t,k+1}^* + \delta$, $\hat{\bar{r}}_{tk} = \bar{r}_{tk}^* - \delta$, $\hat{\bar{r}}_{t,k+1} = \bar{r}_{t,k+1}^* + \delta$. Data causality constraints are trivially satisfied. Energy causality at the top node can be satisfied by letting $\hat{P}_{tk} = \bar{p}_{tk} - \epsilon$ and $\hat{P}_{t,k+1} = \bar{p}_{t,k+1} + \epsilon$ because there exists $\epsilon > 0$ such that $\bar{r}_{tk}^* \leq f\left((\bar{p}_{tk} - \epsilon)/\sigma_3^2\right)$ and $\bar{r}_{t,k+1}^* \leq f\left((\bar{p}_{t,k+1} + \epsilon)/\sigma_3^2\right)$. Energy causality at the source node is satisfied since at slot k we have $g(r_{tk}^* - \delta, r_{bk}^*) < g(r_{tk}^*, r_{bk}^*)$ and at slot $k+1$ we have $g(r_{tk}^* - \delta, r_{bk}^*) + g(r_{t,k+1}^* + \delta, r_{b,k+1}^*) < g(r_{tk}^*, r_{bk}^*) + g(r_{t,k+1}^*, r_{b,k+1}^*)$ due to joint convexity of $g(\cdot,\cdot)$ and $r_{tk}^* + r_{bk}^* > r_{t,k+1}^* + r_{b,k+1}^*$. This means that the modified policy is feasible, forwards the same amount of data, and consumes strictly less energy than the original one. This additional energy can be used to increase r_{bk}^* and r_{tk}^* which causes the data buffers at the top and bottom relays to be non-empty. This modified policy cannot be optimal because it does not satisfy the fact that if $g(r_{tk}, r_{bk})$ strictly decreases in time, then both λ_{1k} and λ_{2k} cannot be zero, as proved at the beginning above. This also means the original policy cannot be optimal because its throughput is equal to the throughput of a sub-optimal policy.

Case 2: $r_{bk}^* > r_{b,k+1}^*, r_{tk}^* \leq r_{t,k+1}^*$: We must have $r_{bk}^* > 0$, therefore $\theta_{2k} = 0$. From $\lambda_{2k} > 0$, we must have that the bottom data buffer is empty, which implies $\sum_{i=1}^k \bar{r}_{bi}^* = \sum_{i=1}^k r_{bi}^*$. From this point on, the proof follows exactly as in **Case 1** but with modifications to $r_{bk}^*, \bar{r}_{bk}^*, \bar{p}_{bk}$ instead of to $r_{tk}^*, r_{tk}^*, \bar{p}_{tk}$, and we conclude that this case cannot happen.

Case 3: $r_{bk}^* > r_{b,k+1}^*, r_{tk}^* > r_{t,k+1}^*$: This case follows the same line of reasoning as the previous cases and by modifying both r_{tk}^*, r_{bk}^* we reach the same conclusion.

To summarize, since none of the above cases can be true, we have $g(r_{ti}^*, r_{bi}^*) \leq g(r_{t,i+1}^*, r_{b,i+1}^*)$, $\forall i$. ■

We can always impose the constraint $\sum_{i=1}^N g(r_{ti}^*, r_{bi}^*) = \sum_{i=1}^N E_{si}$ on the problem in (14) because this does not change the optimal value. From Lemma 12, the total source power must be non-decreasing, and from Lemma 11, the total source power can only increase when the energy buffer is empty. The source power policy that satisfies these properties is the unique single-user optimal power policy [3], [5].

Appendix B
Proof of Lemma 3

Assume (r_{ti}^*, r_{bi}^*) solves the problem in (15). Carrying out a similar analysis as in Appendix A, the KKT conditions are

$$-\sum_{k=i}^N \lambda_{1k} + \gamma_k \sigma_1^2 2^{2(r_{ti}+r_{bi})} - \theta_{1i} = 0 \tag{78}$$

$$-\sum_{k=i}^N \lambda_{2k} + \gamma_k (g(r_{ti}, r_{bi}) + \sigma_2^2) - \theta_{2i} = 0 \tag{79}$$

where γ_k is the Lagrange multiplier for the constraint $g(r_{ti}, r_{bi}) \leq P_i$. Now, we examine the following optimization problem for some $\boldsymbol{\mu}_1, \boldsymbol{\mu}_2 \in \mathbb{R}^N$.

$$\max_{r_{ti}, r_{bi} \geq 0} \quad \sum_{i=1}^N \mu_{1i} r_{ti} + \sum_{i=1}^N \mu_{2i} r_{bi} \tag{80}$$

$$\text{s.t.} \quad g(r_{ti}, r_{bi}) \leq P_i \tag{81}$$

Since the constraint set depends only on the current slot i, this problem is separable into N local optimization problems which are given as

$$\max_{r_{ti}, r_{bi} \geq 0} \quad \mu_{1i} r_{ti} + \mu_{2i} r_{bi} \tag{82}$$

$$\text{s.t.} \quad g(r_{ti}, r_{bi}) \leq P_i \tag{83}$$

The problem in (83) is convex and is solved in [7]. Following [7, Eqn. (13)] the KKT conditions are

$$-\mu_1 + \eta_k \sigma_1^2 2^{2(r_{ti}+r_{bi})} - \omega_{1i} = 0 \tag{84}$$

$$-\mu_2 + \eta_k (g(r_{ti}, r_{bi}) + \sigma_2^2) - \omega_{2i} = 0 \tag{85}$$

with the complementary slackness conditions as

$$\omega_{1i} r_{ti} = \omega_{2i} r_{bi} = \eta_i \left(g(r_{ti}, r_{bi}) - P_i\right) = 0, \quad \forall i \tag{86}$$

We require the same (r_{ti}^*, r_{bi}^*) pair to solve both of these problems. When $r_{ti}^* = 0$ we set $\mu_{1i} = 0$, otherwise from (78), (84) we have $\mu_{1i} = (\sum_{k=i}^N \lambda_{1k})\eta_k/\gamma_k$. Similarly, when $r_{bi}^* = 0$ we set $\mu_{2i} = 0$ otherwise from (79), (85) we have $\mu_{2i} = (\sum_{k=i}^N \lambda_{2k})\eta_k/\gamma_k$. Note that $\eta_k, \gamma_k > 0$ because the energy causality constraints will always be satisfied with equality at every slot. Now we define

$$\mu_i \triangleq \min\left\{\frac{\mu_{1i}}{\mu_{2i}}, 1\right\} = \min\left\{\frac{\sum_{k=i}^N \lambda_{1k}}{\sum_{k=i}^N \lambda_{2k}}, 1\right\} \tag{87}$$

With this definition, the problems (81) and (16) are equivalent and have the same solution as (15). This proves Lemma 3. We observe from (87) that if $\mu_i > \mu_{i+1}$ then $\lambda_{1k} > 0$ which implies the top data buffer is empty. Similarly, if $\mu_i < \mu_{i+1}$ then $\lambda_{2k} > 0$ which implies the bottom data buffer is empty. This proves Lemma 4.

Appendix C
Proof of Lemma 5

Denote the feasible set and the optimal value of the problem in (20) by $(\mathcal{F}_1, T_1)$ and that of the problem in (27) by $(\mathcal{F}_2, T_2)$. First, we show $T_1 \leq T_2$. For any $(\bar{p}_{ti}, \bar{p}_{bi}, \bar{r}_{ti}, \bar{r}_{bi}) \in \mathcal{F}_1$, from (11) we have

$$\bar{p}_{ti} \geq \sigma_3^2 \left(2^{2\bar{r}_{ti}} - 1\right), \quad \bar{p}_{bi} \geq \sigma_3^2 \left(2^{2\bar{r}_{bi}} - 1\right), \quad (88)$$

$$\bar{p}_{ti} + \bar{p}_{bi} \geq \sigma_3^2 \left(2^{2(\bar{r}_{ti}+\bar{r}_{bi})} - 1\right) \quad (89)$$

These constraints imply

$$\sum_{i=1}^{k} \bar{p}_{ti} \geq \sigma_3^2 \left(\sum_{i=1}^{k} 2^{2\bar{r}_{ti}} - 1\right), \quad \forall k \quad (90)$$

$$\sum_{i=1}^{k} \bar{p}_{bi} \geq \sigma_3^2 \left(\sum_{i=1}^{k} 2^{2\bar{r}_{bi}} - 1\right), \quad \forall k \quad (91)$$

$$\sum_{i=1}^{k} \bar{p}_{ti} + \bar{p}_{bi} \geq \sigma_3^2 \left(\sum_{i=1}^{k} 2^{2(\bar{r}_{ti}+\bar{r}_{bi})} - 1\right), \quad \forall k \quad (92)$$

Together with (5) and (6), (90)–(92) imply

$$\sigma_3^2 \left(\sum_{i=1}^{k} 2^{2\bar{r}_{ti}} - 1\right) \leq \sum_{i=1}^{k} E_{ti}, \quad \forall k \quad (93)$$

$$\sigma_3^2 \left(\sum_{i=1}^{k} 2^{2\bar{r}_{bi}} - 1\right) \leq \sum_{i=1}^{k} E_{bi}, \quad \forall k \quad (94)$$

$$\sigma_3^2 \left(\sum_{i=1}^{k} 2^{2(\bar{r}_{ti}+\bar{r}_{bi})} - 1\right) \leq \sum_{i=1}^{k} E_{ti} + E_{bi}, \quad \forall k \quad (95)$$

This means $(\bar{r}_{ti}, \bar{r}_{bi}) \in \mathcal{F}_2$ and therefore $T_1 \leq T_2$.

Now, we show $T_2 \leq T_1$. For any $(\bar{r}_{ti}, \bar{r}_{bi}) \in \mathcal{F}_2$, we will find $\bar{p}_{ti}, \bar{p}_{bi}$ such that $(\bar{p}_{ti}, \bar{p}_{bi}, \bar{r}_{ti}, \bar{r}_{bi}) \in \mathcal{F}_1$. To accomplish this, we solve the feasibility problem

$$\begin{aligned}
\max_{\bar{p}_{ti}, \bar{p}_{bi}} \quad & 1 \\
\text{s.t.} \quad & \bar{p}_{ti} \geq \sigma_3^2 \left(2^{2\bar{r}_{ti}} - 1\right), \quad \forall i \\
& \bar{p}_{bi} \geq \sigma_3^2 \left(2^{2\bar{r}_{bi}} - 1\right), \quad \forall i \\
& \bar{p}_{ti} + \bar{p}_{bi} \geq \sigma_3^2 \left(2^{2(\bar{r}_{ti}+\bar{r}_{bi})} - 1\right), \quad \forall i \\
& \sum_{i=1}^{k} \bar{p}_{ti} \leq \sum_{i=1}^{k} E_{ti}, \ \sum_{i=1}^{k} \bar{p}_{bi} \leq \sum_{i=1}^{k} E_{bi}, \quad \forall k
\end{aligned} \quad (96)$$

We can let $\bar{p}_{ti} + \bar{p}_{bi} = \sigma_3^2 \left(2^{2(\bar{r}_{ti}+\bar{r}_{bi})} - 1\right), \forall i$ without changing the optimal value of the feasibility problem. Now, we have the following set of inequalities to be satisfied:

$$\bar{p}_{ti} \geq \sigma_3^2 \left(2^{2\bar{r}_{ti}} - 1\right), \quad \forall i \quad (97)$$

$$\bar{p}_{ti} \leq \sigma_3^2 \left(2^{2(\bar{r}_{ti}+\bar{r}_{bi})} - 2^{2\bar{r}_{bi}}\right), \quad \forall i \quad (98)$$

$$\sum_{i=1}^{k} \bar{p}_{ti} \leq \sum_{i=1}^{k} E_{ti}, \quad \forall k \quad (99)$$

$$\sum_{i=1}^{k} \bar{p}_{ti} \geq \sum_{i=1}^{k} \left[\sigma_3^2 \left(2^{2(\bar{r}_{ti}+\bar{r}_{bi})} - 1\right) - E_{bi}\right], \quad \forall k \quad (100)$$

We note that this set of inequalities is consistent by showing every lower bound is no larger than every upper bound. (97) is consistent with (98) since $2^{2(x+y)} - 2^{2y} \geq 2^{2x} - 1, \forall x, y \geq 0$. (97) is consistent with (99) since $\bar{r}_{ti}$ satisfies (93). (98) is consistent with (100) since $\bar{r}_{bi}$ satisfies (94) and finally (99) is consistent with (100) since $\bar{r}_{ti}, \bar{r}_{bi}$ satisfy (95). We also have $\bar{p}_{ti} \geq 0$ which is consistent with both (98) and (99) since these lower bounds are non-negative. This feasibility problem then has a solution and there exists $\bar{p}_{ti}, \bar{p}_{bi}$ that solve (96). This means there exists $(\bar{p}_{ti}, \bar{p}_{bi}, \bar{r}_{ti}, \bar{r}_{bi}) \in \mathcal{F}_1$ and therefore $T_2 \leq T_1$, proving the lemma.

Appendix D
Proof of Lemma 7

The statement is true for $k = N$ because the optimal value of problem (28) is at least as large as that of (27) since any profile that is feasible for (27) is also feasible for (28). We will show that if the statement holds for slot k, i.e., $\sum_{i=1}^{k} \bar{r}_{ti}^* + \bar{r}_{bi}^* \leq \sum_{i=1}^{k} q_i^*$, then it also holds for slot $k-1$. By induction this will imply that it is true for all k. Assume on the contrary that $\sum_{i=1}^{k-1} \bar{r}_{ti}^* + \bar{r}_{bi}^* > \sum_{i=1}^{k-1} q_i^*$. Together with $\sum_{i=1}^{k} \bar{r}_{ti}^* + \bar{r}_{bi}^* \leq \sum_{i=1}^{k} q_i^*$, this implies $\bar{r}_{tk}^* + \bar{r}_{bk}^* < q_k^*$.

Now, we claim that we must have $\sum_{i=1}^{k-1} 2^{2(\bar{r}_{ti}^*+\bar{r}_{bi}^*)} > \sum_{i=1}^{k-1} 2^{2q_i^*}$. This is true because otherwise, up to slot $k-1$, the profile $\bar{r}_{ti}^* + \bar{r}_{bi}^*$ sends more data than q_i^* and in view of the energy constraints in (28) leads to a more relaxed feasible set. This means that the profile q_i^* can be replaced with $\bar{r}_{ti}^* + \bar{r}_{bi}^*$ for slots 1 to $k-1$ and for the remaining slots $k, \ldots, N$ more data can be transmitted because there is more energy left. This contradicts the optimality of q_i^*, therefore we must have $\sum_{i=1}^{k-1} 2^{2(\bar{r}_{ti}^*+\bar{r}_{bi}^*)} > \sum_{i=1}^{k-1} 2^{2q_i^*}$.

Note that this also means $\sigma_3^2 \left(\sum_{i=1}^{k-1} 2^{2q_i^*} - 1\right) < \sum_{i=1}^{k-1} E_{ti} + E_{bi}$ because otherwise $\bar{r}_{ti}^* + \bar{r}_{bi}^*$ cannot be energy feasible. From the assumption, we have $\sum_{i=1}^{k-1} \bar{r}_{ti}^* + \bar{r}_{bi}^* > \sum_{i=1}^{k-1} q_i^*$, which implies $\sum_{i=1}^{k-1} q_i^* < \sum_{i=1}^{k-1} d_{ti}i + d_{bi}$ because otherwise $\bar{r}_{ti}^* + \bar{r}_{bi}^*$ cannot be data feasible. These collectively mean that slot $k-1$ cannot be an energy or data exhausting slot for q_i^* and therefore $q_{k-1}^* = q_k^*$. From this fact and $\bar{r}_{ti}^* + \bar{r}_{bi}^*$ is non-decreasing, we have $\bar{r}_{t,k-1}^* + \bar{r}_{b,k-1}^* \leq \bar{r}_{tk}^* + \bar{r}_{bk}^* < q_k^* = q_{k-1}^*$ which implies $\bar{r}_{t,k-1}^* + \bar{r}_{b,k-1}^* < q_{k-1}^*$. Together with $\sum_{i=1}^{k-1} \bar{r}_{ti}^* + \bar{r}_{bi}^* > \sum_{i=1}^{k-1} q_i^*$, this implies $\sum_{i=1}^{k-2} \bar{r}_{ti}^* + \bar{r}_{bi}^* > \sum_{i=1}^{k-2} q_i^*$. Following the same reasoning as before, we have that $k-2$ is a non energy and data exhausting slot for q_i^* and therefore $q_{k-2}^* = q_{k-1}^*$. We apply the same argument to reach the conclusion that $q_1^* = q_2^* = \cdots = q_k^*$ and $\bar{r}_{ti}^* + \bar{r}_{bi}^* < q_i^*, \forall i \leq k$. This contradicts the assumption $\sum_{i=1}^{k-1} \bar{r}_{ti}^* + \bar{r}_{bi}^* > \sum_{i=1}^{k-1} q_i^*$.

APPENDIX E
PROOF OF LEMMA 8

The proof follows from majorization theory. We know that $\bar{r}_{ti}^* + \bar{r}_{bi}^*$ and q_i^* are non-decreasing in i, so they are ordered vectors. From Lemma 7, we have $\sum_{i=1}^{l} \bar{r}_{ti}^* + \bar{r}_{bi}^* \leq \sum_{i=1}^{l} q_i^*$ $\forall l < k$ and if in addition we have $\sum_{i=1}^{k} \bar{r}_{ti}^* + \bar{r}_{bi}^* = \sum_{i=1}^{k} q_i^*$, then the vector q_i^* is majorized by the vector $\bar{r}_{ti}^* + \bar{r}_{bi}^*$. This means $\sum_{i=1}^{k} g(\bar{r}_{ti}^* + \bar{r}_{bi}^*) \geq \sum_{i=1}^{k} g(q_i^*)$ for any convex, increasing g and in particular for $g = 2^x$ [38, Section I.3.C1B]. Furthermore, if we have $\sigma_3^2 \left(\sum_{i=1}^{k} 2^{2q_i^*} - 1 \right) = \sum_{i=1}^{k} E_{ti} + E_{bi}$, then we have $\sigma_3^2 \left(\sum_{i=1}^{k} 2^{2(\bar{r}_{ti}^* + \bar{r}_{bi}^*)} - 1 \right) \geq \sum_{i=1}^{k} E_{ti} + E_{bi}$. From energy feasibility of $\bar{r}_{ti}^* + \bar{r}_{bi}^*$ we also have $\sigma_3^2 \left(\sum_{i=1}^{k} 2^{2(\bar{r}_{ti}^* + \bar{r}_{bi}^*)} - 1 \right) \leq \sum_{i=1}^{k} E_{ti} + E_{bi}$. These two constraints are feasible if and only if $\sigma_3^2 \left(\sum_{i=1}^{k} 2^{2(\bar{r}_{ti}^* + \bar{r}_{bi}^*)} - 1 \right) = \sum_{i=1}^{k} E_{ti} + E_{bi} = \sigma_3^2 \left(\sum_{i=1}^{k} 2^{2q_i^*} - 1 \right)$. From the strict convexity of 2^x and therefore strict Schur-convexity of $\sum 2^x$ we must have $\bar{r}_{ti}^* + \bar{r}_{bi}^* = q_i^*, \forall i \leq k$.

APPENDIX F
PROOF OF LEMMA 9

Similar to the discussion that follows [35, Section 6.1, Eq. (1.1)] we have:

$$\begin{aligned}
\mathcal{H}(\boldsymbol{\gamma}) \geq & \sum_{i=1}^{N} w_i^*(\boldsymbol{\gamma}^{(l)}) - \sum_{k=1}^{N} \lambda_{1k} \left(\sum_{i=1}^{k} \bar{r}_{ti}^*(\boldsymbol{\gamma}^l) - \sum_{i=1}^{k} r_{ti}^*(\boldsymbol{\gamma}^l) \right) \\
& - \sum_{k=1}^{N} \lambda_{2k} \left(\sum_{i=1}^{k} \bar{r}_{bi}^*(\boldsymbol{\gamma}^l) - \sum_{i=1}^{k} r_{bi}^*(\boldsymbol{\gamma}^l) \right) \\
& - \sum_{i=1}^{N} \nu_i \left(w_i^*(\boldsymbol{\gamma}^{(l)}) - \bar{r}_{ti}^*(\boldsymbol{\gamma}^l) - \bar{r}_{bi}^*(\boldsymbol{\gamma}^l) \right) \qquad (101)
\end{aligned}$$

$$\begin{aligned}
= & \sum_{i=1}^{N} w_i^*(\boldsymbol{\gamma}^{(l)}) - \sum_{k=1}^{N} \lambda_{1k}^l \left(\sum_{i=1}^{k} \bar{r}_{ti}^*(\boldsymbol{\gamma}^l) - \sum_{i=1}^{k} r_{ti}^*(\boldsymbol{\gamma}^l) \right) \\
& - \sum_{k=1}^{N} \lambda_{2k}^l \left(\sum_{i=1}^{k} \bar{r}_{bi}^*(\boldsymbol{\gamma}^l) - \sum_{i=1}^{k} r_{bi}^*(\boldsymbol{\gamma}^l) \right) \\
& - \sum_{i=1}^{N} {\nu_i}^l \left(w_i^*(\boldsymbol{\gamma}^{(l)}) - \bar{r}_{ti}^*(\boldsymbol{\gamma}^l) - \bar{r}_{bi}^*(\boldsymbol{\gamma}^l) \right) \\
& + \sum_{k=1}^{N} (\lambda_{1k}^l - \lambda_{1k}) \left(\sum_{i=1}^{k} \bar{r}_{ti}^*(\boldsymbol{\gamma}^l) - \sum_{i=1}^{k} r_{ti}^*(\boldsymbol{\gamma}^l) \right) \\
& + \sum_{k=1}^{N} (\lambda_{2k}^l - \lambda_{2k}) \left(\sum_{i=1}^{k} \bar{r}_{bi}^*(\boldsymbol{\gamma}^l) - \sum_{i=1}^{k} r_{bi}^*(\boldsymbol{\gamma}^l) \right) \\
& + \sum_{i=1}^{N} ({\nu_i}^l - \nu_i) \left(w_i^*(\boldsymbol{\gamma}^{(l)}) - \bar{r}_{ti}^*(\boldsymbol{\gamma}^l) - \bar{r}_{bi}^*(\boldsymbol{\gamma}^l) \right) \qquad (102)
\end{aligned}$$

$$= \mathcal{H}(\boldsymbol{\gamma}^l) + \nu^{\mathsf{T}}(\boldsymbol{\gamma} - \boldsymbol{\gamma}^l) \qquad (103)$$

where the inequality follows from the fact that $(r_{ti}^*(\boldsymbol{\gamma}^l), r_{bi}^*(\boldsymbol{\gamma}^l)) \in \mathcal{R}_s$ so feasible for $\mathcal{K}_1(\boldsymbol{\gamma})$ but may not solve $\mathcal{K}_1(\boldsymbol{\gamma})$, $\bar{r}_{ti}^*(\boldsymbol{\gamma}^l) \in \mathcal{R}_t$ but may not solve $\mathcal{K}_2(\boldsymbol{\gamma})$, $\bar{r}_{bi}^*(\boldsymbol{\gamma}^l) \in \mathcal{R}_b$ but may not solve $\mathcal{K}_3(\boldsymbol{\gamma})$, and $w_i^*(\boldsymbol{\gamma}^{(l)}) \in \mathcal{R}_w$ but may not solve $\mathcal{K}_4(\boldsymbol{\gamma})$. The expression for ν^{T} is given in the statement of the lemma.

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Berk Gurakan (S’11) received the B.Sc. and M.S. degrees in electrical and electronics engineering from the Middle East Technical University (METU), Ankara, Turkey, in 2008 and 2010, respectively. He is currently pursuing the Ph.D. degree at the University of Maryland, College Park, MD, USA. His research interests include energy harvesting and resource allocation for wireless communication systems.

Sennur Ulukus (S’90–M’98–SM’15–F’16) is a Professor of Electrical and Computer Engineering at the University of Maryland at College Park, where she also holds a joint appointment with the Institute for Systems Research (ISR). Prior to joining UMD, she was a Senior Technical Staff Member at AT&T Labs-Research. She received the B.S. and M.S. degrees in electrical and electronics engineering from Bilkent University, and the Ph.D. degree in electrical and computer engineering from Wireless Information Network Laboratory (WINLAB), Rutgers University. Her research interests are in wireless communications, information theory, signal processing, networking, information theoretic physical layer security, and energy harvesting communications.

Dr. Ulukus is a fellow of the IEEE, and a Distinguished Scholar-Teacher of the University of Maryland. She received the 2003 IEEE Marconi Prize Paper Award in Wireless Communications, an 2005 NSF CAREER Award, the 2010–2011 ISR Outstanding Systems Engineering Faculty Award, and the 2012 ECE George Corcoran Education Award. She is on the Editorial Board of the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS – Series on Green Communications and Networking (2015–). She was an Associate Editor for the IEEE TRANSACTIONS ON INFORMATION THEORY (2007–2010) and IEEE TRANSACTIONS ON COMMUNICATIONS (2003–2007). She was a Guest Editor for the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS (2015 and 2008), *Journal of Communications and Networks* (2012), and IEEE TRANSACTIONS ON INFORMATION THEORY (2011). She is a general TPC cochair of 2017 IEEE ISIT, 2016 IEEE Globecom, 2014 IEEE PIMRC, and 2011 IEEE CTW.