Capacity of a Class of Diamond Channels

Wei Kang, *Member, IEEE*, and Sennur Ulukus, *Member, IEEE*

***Abstract*—We study a special class of diamond channels which was introduced by Schein in 2001. In this special class, each diamond channel consists of a transmitter, a noisy relay, a noiseless relay and a receiver. We prove the capacity of this class of diamond channels by providing an achievability scheme and a converse. The capacity we show is strictly smaller than the cut-set bound. We note that there exists a duality between this diamond channel coding problem and the Kaspi-Berger source coding problem.**

***Index Terms*—Diamond channel, multiple relay networks, cut-set bound, source-channel coding duality.**

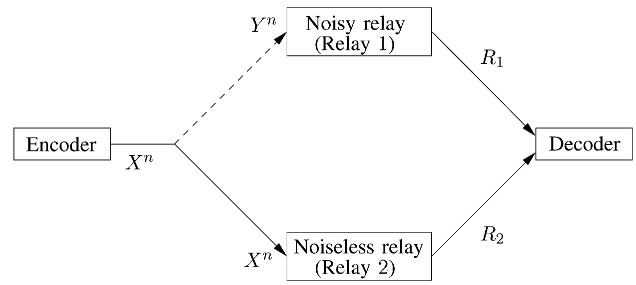

Fig. 1. The diamond channel.

I. Problem Statement and the Result

The diamond channel was first introduced by Schein in 2001 [1]. The diamond channel consists of one transmitter, two relays and a receiver, where the transmitter and the two relays form a broadcast channel as the first stage and the two relays and the receiver form a multiple access channel as the second stage. The diamond channel is a special case of the multiple relay channel [2] with no direct link between the transmitter and the receiver. The capacity of the diamond channel in its most general form is open. Schein explored several special cases of the diamond channel, one of which [1, Sect. 3.5] is specified as follows (see Fig. 1). The multiple access channel consists of two orthogonal links with rate constraints R_1 and R_2, respectively. The broadcast channel contains a noisy branch and a noiseless branch, i.e., with input X and two outputs X and Y. We refer to the relay node receiving Y as the noisy relay and the relay node receiving X as the noiseless relay. Schein provided two achievability schemes for this class of diamond channels. In this paper, we will prove the capacity of this special class of diamond channels.

The formal definition of the problem is as follows. Consider a discrete channel with finite input alphabet $\mathcal{X}$ and finite output alphabet $\mathcal{Y}$, which is characterized by the transition probability $p(y\,|\,x)$. Assume an n-length block code consisting of (f,g,h,φ) where

$$f:\{1,2,\dots,M\}\mapsto\mathcal{X}^n \tag{1}$$
$$g:\mathcal{Y}^n\mapsto\{1,2,\dots,|g|\} \tag{2}$$
$$h:\mathcal{X}^n\mapsto\{1,2,\dots,|h|\} \tag{3}$$
$$\varphi:\{1,2,\dots,|g|\}\times\{1,2,\dots,|h|\}\mapsto\{1,2,\dots,M\} \tag{4}$$

Here f denotes the encoding function at the transmitter, g and h denote the processing functions at the noisy and noiseless relays, respectively, and φ denotes the decoding function at the receiver.

The encoder sends $x^n=f(m)$ into the channel, where $m\in\{1,2,\dots,M\}$. The decoder reconstructs $\hat{m}=\varphi(g(Y^n),h(X^n))$. The average probability of error is defined as

$$P_e\triangleq\frac{1}{M}\sum_{m=1}^{M}Pr(\hat{m}\neq m\,|\,m\text{ is sent}). \tag{5}$$

The rate triple (R,R_1,R_2) is achievable if for every $0<\epsilon<1$, $\eta>0$ and every sufficiently large n, there exists an n-length block code (f,g,h,φ), such that $P_e\le\epsilon$ and

$$\frac{1}{n}\ln M\ge R-\eta \tag{6}$$
$$\frac{1}{n}\ln|g|\le R_1+\eta \tag{7}$$
$$\frac{1}{n}\ln|h|\le R_2+\eta. \tag{8}$$

The following theorem characterizes the capacity of the class of diamond channels considered in this paper.

Theorem 1: The rate triple (R,R_1,R_2) is achievable in the above channel if and only if the following conditions are satisfied:

$$R\le I(U;Y)+H(X\,|\,U) \tag{9}$$

Manuscript received July 03, 2008; revised December 19, 2010; accepted January 14, 2011. Date of current version July 29, 2011. This work was supported by the National Science Foundation by Grants CCR 03-11311, CCF 04-47613, CCF 05-14846, CNS 07-16311, and CCF 07-29127. A portion of the work of W. Kang was supported by the National Natural Science Foundation of China by Grants 60902008 and 60972027. The material in this paper was presented at the Allerton Conference on Communications, Control, and Computing, Monticello, IL, September, 2008.
W. Kang was with the Department of Electrical Engineering, University of Maryland, College Park, MD 20742 USA. He is now with the School of Information Science and Engineering, Southeast University, Nanjing, Jiangsu 210096, China (e-mail: wkang@seu.edu.cn).
S. Ulukus is with the Department of Electrical and Computer Engineering, University of Maryland, College Park, MD 20742 USA (e-mail: ulukus@umd.edu).
Communicated by M. C. Gastpar, Associate Editor for Shannon Theory.
Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.
Digital Object Identifier 10.1109/TIT.2011.2158877

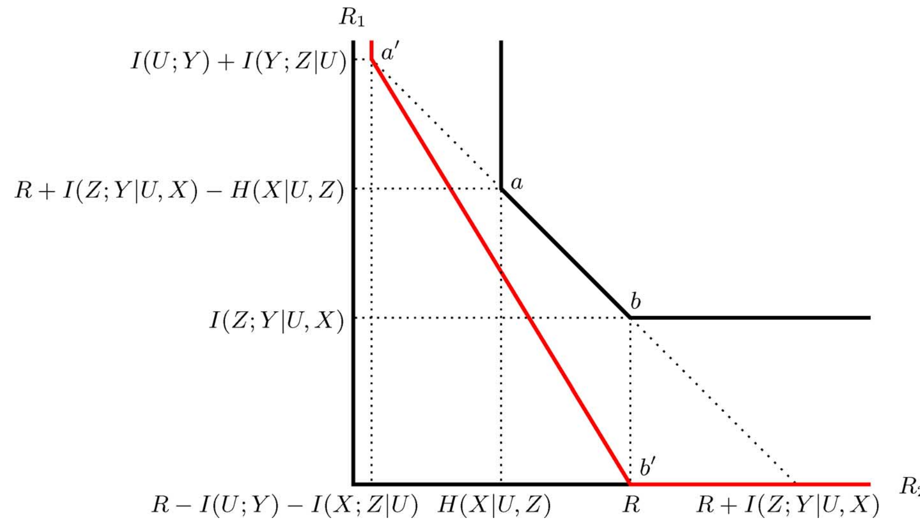

Fig. 2. Rate region of (R_1, R_2) when $H(X\,|\,U,Z) \le R \le I(U;Y) + H(X\,|\,U)$.

$$R_1 \ge I(Z;Y\,|\,U,X) \tag{10}$$
$$R_2 \ge H(X\,|\,Z,U) \tag{11}$$
$$R_1 + R_2 \ge R + I(Z;Y\,|\,U,X.) \tag{12}$$

for some joint distribution

$$p(u,z,x,y) = p(u,x)p(y\,|\,x)p(z\,|\,u,y) \tag{13}$$

with cardinalities of alphabets satisfying

$$|\mathcal{U}| \le |\mathcal{X}| + 4 \tag{14}$$
$$|\mathcal{Z}| \le |\mathcal{U}||\mathcal{Y}| + 3 \le |\mathcal{X}||\mathcal{Y}| + 4|\mathcal{X}| + 3 \tag{15}$$

The above theorem can be restated in the following form:

$$\begin{aligned} R \le \min(I(U;Y) + H(X\,|\,U), R_1 + R_2 - I(Y;Z\,|\,X,U))& \\ \text{subject to } R_1 \ge I(Z;Y\,|\,U,X),\ R_2 \ge H(X\,|\,Z,U)& \end{aligned} \tag{16}$$

where the joint distribution satisfies (13) and cardinalities of alphabets satisfy (14) and (15).

II. The Achievability

Assume a given joint distribution

$$p(u,z,x,y) = p(u,x)p(y\,|\,x)p(z\,|\,u,y) \tag{17}$$

and consider that the information theoretic quantities on the right-hand side (RHS) of (9), (10), (11), and (12) are evaluated with this fixed joint probability distribution.

Consider a message W with rate R. If $R \le H(X\,|\,Z,U)$, reliable transmission can be achieved by letting $g(Y^n)$ be a constant and $h(X^n) = X^n$, i.e., by sending the message through the noiseless relay, because $R_2 \ge H(X\,|\,Z,U) \ge R$. Thus, we will only consider the case where

$$H(X\,|\,Z,U) < R \le I(U;Y) + H(X\,|\,U). \tag{18}$$

We will show that the message can be reliably transmitted with a pair of functions (g,h) such that $(\frac{1}{n}\ln|g|, \frac{1}{n}\ln|h|)$ lies in the inverse pentagon[1] with corners a and b in Fig. 2. However, we instead prove the reliable transmission with $(\frac{1}{n}\ln|g|, \frac{1}{n}\ln|h|)$ lying in the inverse pentagon with corners a' and b', which contains the inverse pentagon with corners a and b and thus imposes a stronger condition to prove.

By enlarging the achievable rate region for a given distribution, we simplify the proof of the achievability, which can be found in the sequel. However, it does not imply that we can achieve any rate larger than the capacity given in Theorem 1. Any achievable rate region, say the union of the inverse pentagon with corners a' and b' over all distributions, is outer bounded by the converse region, which is the union of the inverse pentagon with corners a and b over all distributions. Thus, we conclude that even though for a given distribution the inverse pentagon with corners a' and b' seems to be larger than the inverse pentagon with corners a and b, after the union over all distributions, the two regions coincide.

It is straightforward to have reliable transmission with the rate pair at point b' by letting $g(Y^n)$ be a constant and $h(X^n) = X^n$. Thus, it remains to prove that reliable transmission is possible with the rate pair at point a', i.e.

$$R_1 = I(U;Y) + I(Y;Z\,|\,U) \tag{19}$$
$$R_2 = R - I(U;Y) - I(X;Z\,|\,U). \tag{20}$$

Let us assume that the message W is decomposed as $W = (W_a, W_b, W_c)$. For a positive number ϵ, we have

$$M_a \triangleq |W_a| = \exp(n(I(U;Y) - \epsilon)) \tag{21}$$
$$\begin{aligned} M_b \triangleq |W_b| &= \frac{M}{M_a M_c} \\ &= \exp(\ln M - n(I(U;Y) + I(X;Z\,|\,U) + 2\epsilon)) \end{aligned} \tag{22}$$
$$M_c \triangleq |W_c| = \exp(n(I(X;Z\,|\,U) - \epsilon)). \tag{23}$$

[1]By "inverse pentagon" with corner points a and b, we mean the region in the (R_1, R_2) space that is to the "north-east" of line segment $[a, b]$. More specifically, this is the region described by (10), (11), and (12).

Random codebook generation: We use a superposition code structure. The size of the inner code is M_a. Conditioning on each inner codeword, we independently generate M_b many different outer codebooks. The size of each outer codebook is M_c. In other words, there are M_a cloud centers and $M_b \times M_c$ satellite codewords in each cloud. The detailed generation is as follows.

- Independently generate M_a many sequences, $u^n(1), u^n(2), \dots, u^n(M_a)$, according to $\prod_{i=1}^n p(u_i)$ where $p(u_i) = p(u)$, for $i = 1, 2, \dots, n$.
- For $u^n(j)$, $j = 1, 2, \dots, M_a$, independently generate M_b many different codebooks, $\mathcal{C}(j,1), \mathcal{C}(j,2), \dots, \mathcal{C}(j, M_b)$. Each codebook is generated as follows.
- For the codebook $\mathcal{C}(j,k)$, $j = 1, 2, \dots, M_a$, $k = 1, 2, \dots, M_b$, independently generate M_c many codewords $x^n(j,k,1), x^n(j,k,2), \dots, x^n(j,k,M_c)$ according to $\prod_{i=1}^n p(x_i \,|\, U_i = u_i(j))$, where $p(x_i \,|\, U = u_i(j)) = p(x \,|\, u)$, for $i = 1, 2, \dots, n$.

There will be no overlapping codebooks with high probability when n is sufficiently large, because

$$\frac{1}{n} \ln M_b M_c < H(X \,|\, U) \tag{24}$$

Encoding at the transmitter: Let $W = (W_a, W_b, W_c)$ be the message. We send codeword $X^n = f(W_a, W_b, W_c) \triangleq x^n(W_a, W_b, W_c)$ into the channel.

Processing at the noisy relay: First, from the received signal Y^n, decode U^n using joint typical decoding. The probability of decoding error here will go to zero when (21) is satisfied. Second, according to $\prod_{i=1}^n p(z_i, y_i \,|\, u_i)$, construct a conditional rate distortion code, which maps Y^n into Z^n conditioned on U^n with the codebook rate $L = \exp(n(I(Y; Z \,|\, U) + \tau))$. Finally, send U^n and Z^n to the destination, i.e.

$$g(Y^n) = (U^n, Z^n) \tag{25}$$

where

$$\begin{aligned} |g| &= M_a \times L \\ &\le \exp(n(I(U;Y) + I(Y; Z \,|\, U) + \tau - \epsilon)). \end{aligned} \tag{26}$$

Processing at the noiseless relay: Let $h(f(W_a, W_c, W_b)) = W_b$ where

$$\begin{aligned} |h| &= M_b \\ &= \exp(\ln M - n(I(U;Y) + I(X; Z \,|\, U) + 2\epsilon)). \end{aligned} \tag{27}$$

Decoding: Decoder collects (U^n, Z^n) from the noisy relay. Since $U^n = u^n(W_a)$, the decoder can obtain W_a, the first part of message W, from U^n. The decoder collects W_b, the second part of message W, from the noiseless relay. With (W_a, W_b), the decoder determines that outer code codebook $\mathcal{C}(W_a, W_b)$ is used at the transmitter. The decoder then decodes the codeword $x^n(W_a, W_b, W_c)$ from codebook $\mathcal{C}(W_a, W_b)$ via a joint typical decoder, i.e.

$$(x^n(\hat{W}_a, W_b, W_c), Z^n) \in \mathcal{T}^n_{[XZ \,|\, U]}(U^n). \tag{28}$$

The probability of error goes to zero when (23) is satisfied, which concludes the proof of the achievability part.

III. The Converse

Define $Z_i \triangleq g$ and $U_i \triangleq (Y^{i-1}, X_{i+1}^n)$. We note that

$$p(u_i, x_i, y_i, z_i) = p(u_i, x_i) p(y_i \,|\, x_i) p(z_i \,|\, y_i, u_i). \tag{29}$$

We have (30), shown at the bottom of the next page, where

1) Because of the following equality [3, Lemma 7]:

$$\sum_{i=1}^n I\left(X_{i+1}^n; Y_i \middle| g, h, Y^{i-1}\right) = \sum_{i=1}^n I\left(Y^{i-1}; X_i \middle| g, h, X_{i+1}^n\right). \tag{31}$$

2) Due to Fano's inequality.

3) g is a deterministic function of Y^n. Due to the memoryless property, we have

$$H\left(g \middle| Y_i, h, Y^{i-1}, X_{i+1}^n, X_i\right) = H\left(g \middle| Y_i, h, Y^{i-1}, X_{i+1}^n\right). \tag{32}$$

4) g is a deterministic function of Y^n and h is a deterministic function of X^n. Due to the memoryless property, we have

$$H\left(g \middle| h, Y^{i-1}, X_{i+1}^n, X_i\right) = H\left(g \middle| Y^{i-1}, X_{i+1}^n, X_i\right) \tag{33}$$

$$H\left(g \middle| h, Y^{i-1}, X_{i+1}^n, X_i, Y_i\right) = H\left(g \middle| Y^{i-1}, X_{i+1}^n, X_i, Y_i\right). \tag{34}$$

We have

$$\begin{aligned} \ln |h| &\ge H(h \,|\, g) \\ &\overset{1}{\ge} H(X^n \,|\, g) - n\epsilon \\ &= \sum_{i=1}^n H\left(X_i \middle| X_{i+1}^n, g\right) - \epsilon \\ &\ge \sum_{i=1}^n H\left(X_i \middle| Y^{i-1}, X_{i+1}^n, g\right) - \epsilon \\ &= \sum_{i=1}^n H(X_i \,|\, U_i, Z_i) - \epsilon \end{aligned} \tag{35}$$

where

1) Due to Fano's inequality.

We have

$$\begin{aligned} \ln |g| + \ln |h| &\ge H(g, h) \\ &\ge I(g, h; X^n, Y^n) \\ &= I(X^n; g, h) + I(Y^n; g, h \,|\, X^n) \\ &\ge I(W; g, h) + I(Y^n; g, h \,|\, X^n) \\ &\overset{1}{\ge} \ln M - n\epsilon + I(Y^n; g, h \,|\, X^n) \\ &\overset{2}{=} \ln M - n\epsilon + I(Y^n; g \,|\, X^n) \\ &= \ln M + \sum_{i=1}^n -\epsilon + I(Y_i; g \,|\, X^n, Y^{i-1}) \\ &\overset{3}{=} \ln M + \sum_{i=1}^n -\epsilon + I\left(Y_i; g \middle| X_i, Y^{i-1}, X_{i+1}^n\right) \\ &= \ln M + \sum_{i=1}^n -\epsilon + I(Y_i; Z_i \,|\, X_i, U_i). \end{aligned} \tag{36}$$

1) Due to Fano's inequality.
2) h is a deterministic function of X^n
3) g is a deterministic function of Y^n. Due to the memoryless property, we have

$$H\left(g\left|X_i, Y^{i-1}, X_{i+1}^n, X^{i-1}\right.\right) = H\left(g\left|X_i, Y^{i-1}, X_{i+1}^n\right.\right) \quad (37)$$

$$H\left(g\left|Y_i, X_i, Y^{i-1}, X_{i+1}^n, X^{i-1}\right.\right) = H\left(g\left|Y_i, X_i, Y^{i-1}, X_{i+1}^n\right.\right). \quad (38)$$

We have

$$\begin{aligned}
\ln M &\overset{1}{\leq} H(X^n) + n\epsilon \\
&= \sum_{i=1}^n H\left(X_i\left|X_{i+1}^n\right.\right) + \epsilon \\
&\leq \sum_{i=1}^n I(Y^{i-1};Y_i) + H\left(X_i\left|X_{i+1}^n\right.\right) + \epsilon \\
&= \sum_{i=1}^n I\left(Y^{i-1}, X_{i+1}^n; Y_i\right) - I\left(X_{i+1}^n; Y_i\right|Y^{i-1}\big) + \\
&\quad + H\left(X_i\left|Y^{i-1}, X_{i+1}^n\right.\right) + I\left(Y^{i-1}; X_i\left|X_{i+1}^n\right.\right) + \epsilon \\
&\overset{2}{=} \sum_{i=1}^n I\left(Y^{i-1}, X_{i+1}^n; Y_i\right) + H\left(X_i\left|Y^{i-1}, X_{i+1}^n\right.\right) + \epsilon \\
&= \sum_{i=1}^n I(U_i;Y_i) + H(X_i\,|\,U_i) + \epsilon
\end{aligned} \quad (39)$$

where
1) Due to Fano's inequality.
2) Because of the following equality [3, Lemma 7]:

$$\sum_{i=1}^n I\left(X_{i+1}^n; Y_i\right|Y^{i-1}\big) = \sum_{i=1}^n I\left(Y^{i-1}; X_i\left|X_{i+1}^n\right.\right). \quad (40)$$

Define a time-sharing random variable Q, which is uniformly distributed on $\{1, 2, \ldots, n\}$. Also define a set of random variables $(X, Y, \tilde{U}, \tilde{Z})$ such that

$$\Pr(X = x, Y = y, \tilde{U} = u, \tilde{Z} = z\,|\,Q = i) = p(X_i = x, Y_i = y, U_i = u, Z_i = z), \quad i = 1, 2, \ldots, n. \quad (41)$$

Define $U = (\tilde{U}, Q)$ and $Z = (\tilde{Z}, Q)$. We note that $\frac{1}{n}\ln M \geq R - \eta$, $\frac{1}{n}\ln|g| \leq R_1 + \eta$ and $\frac{1}{n}\ln|h| \leq R_2 + \eta$, for an arbitrary $\eta > 0$. Assume $\epsilon \to 0$, then

$$\begin{aligned}
R &\leq \frac{1}{n}\sum_{i=1}^n I(U_i;Y_i) + H(X_i\,|\,U_i) \\
&= I(\tilde{U};Y\,|\,Q) + H(X\,|\,\tilde{U}, Q) \\
&\leq I(\tilde{U}, Q;Y) + H(X\,|\,\tilde{U}, Q) \\
&= I(U;Y) + H(X\,|\,U)
\end{aligned} \quad (42)$$

$$\begin{aligned}
\ln|g| &\geq H(g|h) \\
&\geq I(g;Y^n|h) \\
&= \sum_{i=1}^n I(g;Y_i\,|\,h, Y^{i-1}) \\
&= \sum_{i=1}^n I\left(g, X_{i+1}^n; Y_i\right|h, Y^{i-1}\big) - I\left(X_{i+1}^n; Y_i\right|g, h, Y^{i-1}\big) \\
&\overset{1}{=} \sum_{i=1}^n I\left(g, X_{i+1}^n; Y_i\right|h, Y^{i-1}\big) - I\left(Y^{i-1}; X_i\left|g, h, X_{i+1}^n\right.\right) \\
&\geq \sum_{i=1}^n I\left(g, X_{i+1}^n; Y_i\right|h, Y^{i-1}\big) - H\left(X_i\left|g, h, X_{i+1}^n\right.\right) \\
&= -H(X^n\,|\,g, h) + \sum_{i=1}^n I\left(g, X_{i+1}^n; Y_i\,|\,h, Y^{i-1}\right) \\
&\overset{2}{\geq} \sum_{i=1}^n I\left(g, X_{i+1}^n; Y_i\right|h, Y^{i-1}\big) - \epsilon \\
&\geq \sum_{i=1}^n I\left(g; Y_i\left|h, Y^{i-1}, X_{i+1}^n\right.\right) - \epsilon \\
&\overset{3}{\geq} \sum_{i=1}^n I\left(g; Y_i\left|h, Y^{i-1}, X_{i+1}^n, X_i\right.\right) - \epsilon \\
&\overset{4}{=} \sum_{i=1}^n I\left(g; Y_i\left|Y^{i-1}, X_{i+1}^n, X_i\right.\right) - \epsilon \\
&= \sum_{i=1}^n I(Z_i; Y_i\,|\,U_i, X_i) - \epsilon
\end{aligned} \quad (30)$$

$$R_1 \geq \frac{1}{n}\sum_{i=1}^{n} I(Z_i;Y_i\,|\,U_i,X_i)$$
$$= I(\tilde{Z};Y\,|\,\tilde{U},Q,X)$$
$$= I(Z;Y\,|\,U,X) \tag{43}$$
$$R_2 \geq \frac{1}{n}\sum_{i=1} H(X_i\,|\,U_i,Z_i)$$
$$= H(X\,|\,\tilde{U},\tilde{Z},Q)$$
$$= H(X\,|\,U,Z) \tag{44}$$
$$R_1+R_2 \geq R + \frac{1}{n}\sum_{i=1}^{n} I(Y_i;Z_i\,|\,X_i,U_i)$$
$$= R + I(\tilde{Z};Y\,|\,\tilde{U},X,Q)$$
$$= R + I(Z;Y\,|\,U,X) \tag{45}$$

where (42), (43), (44), and (45) are the same as (9), (10), (11), and (12), concluding the proof.

Finally, we note that the bounds on the cardinalities of the alphabets in (14) and (15) can be proven in a way similar to [4, Appendix D].

IV. Remarks

We have two remarks regarding this result as follows.

First, the capacity is strictly smaller than the cut-set bound [5]. Consider the following example. Let X and Y be binary and

$$Y = X \oplus W \tag{46}$$

where the sum is a modulo-2 sum and W has a Bernoulli distribution with entropy 0.5 bits. We assume $R_1 = R_2 = 0.5$ bits. The cut-set bound in this example is 1 bit, which is not achievable. The reason is as follows. If we assume that the cut-set bound is achievable, i.e., $R=1$, then we have

$$R = I(U;Y) + H(X\,|\,U) = H(X) = 1. \tag{47}$$

This means that U has to be independent of X and Y. Also, we have

$$R = R_1 + R_2 - I(Y;Z\,|\,U,X) = R_1 + R_2 = 1. \tag{48}$$

This means that Z has to be independent of X and Y if U is independent of X and Y. However, if U and Z are independent of X and Y, we arrive at the following contradiction:

$$0.5 = R_2 \geq H(X\,|\,Z,U) = H(X) = 1 \tag{49}$$

which means that the cut-set bound is not achievable in this example. We note that, even in this binary example where $|\mathcal{X}| = |\mathcal{Y}| = 2$, the cardinalities of the auxiliary random variables U and Z are $|\mathcal{U}| \leq 6$ and $|\mathcal{Z}| \leq 15$. These large cardinality bounds make it practically impossible to evaluate the capacity of this diamond channel. However, we note that, even though we were not able to compute the exact value of the capacity in this example, we were able to conclude that the capacity is strictly less than the cut-set bound, which is 1 bit.

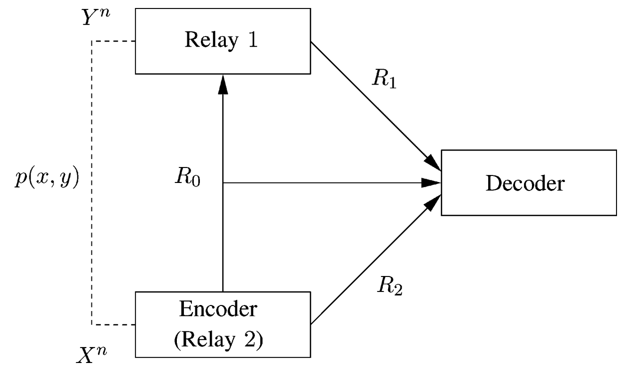

Fig. 3. Kaspi-Berger rate distortion problem.

Second, if we assume $R = H(X) - R_0$, then Theorem 1 can be rewritten as follows:

$$R \leq I(U;Y) + H(X\,|\,U) \longleftrightarrow$$
$$R_0 \geq I(U;X\,|\,Y) \tag{50}$$
$$R_1 \geq I(Z;Y\,|\,U,X) \longleftrightarrow$$
$$R_1 \geq I(Z;Y\,|\,U,X) \tag{51}$$
$$R_2 \geq H(X\,|\,Z,U) \longleftrightarrow$$
$$R_2 \geq I(X;X|Z,U) \tag{52}$$
$$R_1+R_2 \geq R + I(Y;Z|X,U) \longleftrightarrow$$
$$R_0+R_1+R_2 \geq I(X,Y;U,X,Z) \tag{53}$$

for some joint distribution

$$p(u,z,x,y) = p(u,x)p(y\,|\,x)p(z\,|\,u,y). \tag{54}$$

We note that the RHS of (50), (51), (52), and (53) in addition to the distribution constraint in (54) are the same as the rate region of the rate-distortion problem studied by Kaspi and Berger as shown in Fig. 3 [4, Th. 2.1, Case C].

This duality between our diamond channel coding problem and the Kaspi-Berger source coding problem is similar to the duality between the single-user channel coding problem and the Slepian-Wolf source coding problem [6, Sect. 3.1] by viewing the codebook information in the channel coding problem as the information sent to all the terminals in the source coding problem, e.g., the information with rate R_0 in Fig. 3. Thus, the achievability of our diamond channel coding problem can be obtained from the achievability of Kaspi-Berger source coding problem, in the same way that the achievability of the multiple access channel coding problem can be obtained from the achievability of fork network coding problem [6, Sect. 3.2].

References

[1] B. E. Schein, "Distributed coordination in network information theory," Ph.D. dissertation, Mass. Inst. Technol., Cambridge, 2001.
[2] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inf. Theory*, vol. 51, no. 9, pp. 3037–3063, Sep. 2005.
[3] I. Csiszár and J. Körner, "Broadcast channels with confidential messages," *IEEE Trans. Inf. Theory*, vol. 24, no. 3, pp. 339–348, May 1978.

[4] A. H. Kaspi and T. Berger, “Rate-distortion for correlated sources with partially separated encoders,” *IEEE Trans. Inf. Theory*, vol. 28, no. 6, pp. 828–840, Nov. 1982.

[5] T. M. Cover and J. A. Thomas*, Elements of Information Theory*. Hoboken, NJ: Wiley, 1991.

[6] I. Csiszár and J. Körner*, Information Theory: Coding Theorems for Discrete Memoryless Systems*. New York: Academic, 1981.

Wei Kang (S’05–M’08) received the B.Eng. degree in electrical engineering from Beijing University of Posts and Telecommunications, Beijing, China, in 2001, the M.Eng. degree in electrical engineering from McGill University, Montreal, Canada, in 2003, and the Ph.D. degree in electrical engineering from the University of Maryland, College Park, in 2008.

He joined the School of Information Science and Engineering, Southeast University, Nanjing, China, in 2009, where he is currently an Associate Research Professor. His research mainly focuses on network information theory.

Sennur Ulukus (S’90–M’98) received the B.S. and M.S. degrees in electrical and electronics engineering from Bilkent University, Ankara, Turkey, in 1991 and 1993, respectively, and the Ph.D. degree in electrical and computer engineering from Rutgers University, New Brunswick, NJ, in 1998.

During her Ph.D. studies, she was with the Wireless Information Network Laboratory (WINLAB), Rutgers University. From 1998 to 2001, she was a Senior Technical Staff Member with AT&T Labs-Research, NJ. In 2001, she joined the University of Maryland at College Park, where she is currently an Associate Professor with the Department of Electrical and Computer Engineering, with a joint appointment at the Institute for Systems Research (ISR). Her research interests are in wireless communication theory and networking, network information theory for wireless networks, signal processing for wireless communications, and security for multiuser wireless communications.

Dr. Ulukus is a recipient of the 2005 NSF CAREER Award, and a corecipient of the 2003 IEEE Marconi Prize Paper Award in Wireless Communications. She serves/served as an Associate Editor for the IEEE TRANSACTIONS ON INFORMATION THEORY since 2007, as an Associate Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS between 2003–2007, as a Guest Editor for the IEEE TRANSACTIONS ON INFORMATION THEORY for the Special Issue on Interference Networks, as a Guest Editor for the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS for the Special Issue on Multiuser Detection for Advanced Communication Systems and Networks. She has been the Co-Chair of the following conferences: Communication Theory Symposium at the 2007 IEEE Global Telecommunications Conference; Medium Access Control (MAC) Track at the 2008 IEEE Wireless Communications and Networking Conference; Wireless Communications Symposium at the 2010 IEEE International Conference on Communications; 2011 Communication Theory Workshop; Physical-Layer Security Workshop at the 2011 IEEE International Conference on Communications; and was the Secretary of the IEEE Communication Theory Technical Committee (CTTC) in 2007–2009.