It is natural to consider extending our proof techniques to clients with different degrees of adaptivity ω. The major difficulty in extending our techniques to a wider range of values for ω lies in the expressions that arise when computing the value of the expected waiting time. For smaller values of ω, these expressions do not differ significantly from those appearing in this work and an analysis of similar nature may be performed. For larger values of ω, one needs to take into consideration that several packets may be transmitted in the time interval $[t-\omega, t]$. This work is a first step toward the design of worst case efficient schedules. As a future research, it would be interesting to establish tight bounds on the worst case expected waiting time for every value of ω.

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The Capacity Region of a Class of Discrete Degraded Interference Channels

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***Abstract*—We provide a single-letter characterization for the capacity region of a class of discrete degraded interference channels (DDICs). The class of DDICs considered includes the DADIC studied by Benzel in 1979. We show that for the class of DDICs studied, encoder cooperation does not enlarge the capacity region, and therefore, the capacity region of the class of DDICs is the same as the capacity region of the corresponding degraded broadcast channel.**

***Index Terms*—Capacity region, degradedness, interference channel.**

I. Introduction

In wireless communications, where multiple transmitter and receiver pairs share the same communication medium, interference is unavoidable. How to best manage interference coming from other users and how not to cause too much interference to other users while maintaining the quality of communication is a challenging question and of a great deal of practical interest.

To be able to understand the effect of interference on communications better, an interference channel (IC) has been introduced in [2]. The IC is a simple network consisting of two pairs of transmitters and receivers. Each pair wishes to communicate at a certain rate with negligible probability of error. However, the two communications interfere with each other. To best understand the management of interference, we need to find the capacity region of the IC. However, the problem of finding the capacity region of the IC is essentially open except in

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some special cases, e.g., a class of deterministic ICs [3], discrete additive degraded interference channels (DADICs) [1], strong ICs [4], [5], ICs with statistically equivalent outputs [6]–[8].

In this correspondence, we consider a class of DDICs. In a DDIC, only the "bad" receiver faces interference, while the "good" receiver has the ability to decode both messages and thus, behaves like the receiver of a multiple-access channel. It is this fact that makes the DDIC easier to analyze as compared to the IC, where both receivers are faced with interference.

We provide a single-letter characterization for the capacity region of a class of DDICs. This class of DDICs includes the DADICs studied by Benzel [1]. We show that for the class of DDICs studied here, encoder cooperation does not enlarge the capacity region, and therefore, the capacity region of this class of DDICs is the same as the capacity region of the corresponding degraded broadcast channel, which is known.

II. System Model

A discrete memoryless IC consists of two transmitters and two receivers. Transmitter 1 has message W_1 to send to receiver 1. Transmitter 2 has message W_2 to send to receiver 2. Messages W_1 and W_2 are independent. The channel consists of two input alphabets, $\mathcal{X}_1$ and $\mathcal{X}_2$, and two output alphabets, $\mathcal{Y}_1$ and $\mathcal{Y}_2$. The channel transition probability is $p(y_1, y_2|x_1, x_2)$.

In this correspondence, our definition of degradedness is in the stochastic sense, i.e., we say that an IC is DDIC if there exists a probability distribution $p'(y_2|y_1)$ such that

$$p(y_2|x_1, x_2) = \sum_{y_1 \in \mathcal{Y}_1} p(y_1|x_1, x_2) p'(y_2|y_1) \tag{1}$$

for all $x_1 \in \mathcal{X}_1$, $x_2 \in \mathcal{X}_2$, and $y_2 \in \mathcal{Y}_2$. However, we note that for any DDIC, we can form another DDIC (physically degraded) by

$$p(y_1, y_2|x_1, x_2) = p(y_1|x_1, x_2) p'(y_2|y_1) \tag{2}$$

which has the same marginals $p(y_1|x_1, x_2)$ and $p(y_2|x_1, x_2)$ as the original DDIC. Since the receivers do not cooperate in an IC, similar to the case of the broadcast channel [9, Problem 14.10], the capacity region is only a function of the marginals $p(y_1|x_1, x_2)$ and $p(y_2|x_1, x_2)$, and the rate pairs in the capacity region can be achieved by the same achievability scheme for different ICs with the same marginals. Hence, the capacity results that we obtain for DDICs which satisfy (2) will be valid for any DDIC that has the same marginals, $p(y_1|x_1, x_2)$ and $p(y_2|x_1, x_2)$. Thus, without loss of generality, from now on, we may restrict ourselves to studying DDICs that satisfy (2).

A DDIC is characterized by two transition probabilities: $p'(y_2|y_1)$ and $p(y_1|x_1, x_2)$. For notational convenience, let T' denote the $|\mathcal{Y}_2| \times |\mathcal{Y}_1|$ matrix of transition probabilities $p'(y_2|y_1)$, and $T_{\bar{x}_2}$ denote the $|\mathcal{Y}_1| \times |\mathcal{X}_1|$ matrix of transition probabilities $p(y_1|x_1, \bar{x}_2)$, for each $\bar{x}_2 \in \mathcal{X}_2$.

Throughout the correspondence, Δ_n will denote the probability simplex

$$\left\{ (p_1, p_2, \ldots, p_n) \middle| \sum_{i=1}^{n} p_i = 1, \quad p_i \geq 0, i = 1, 2, \ldots, n \right\} \tag{3}$$

and $\mathcal{J}_n$ will denote the representation of the symmetric group of permutations of n objects by the $n \times n$ permutation matrices.

The definition of an input symmetric channel, which will be used later on, is given in [10, Sec. II-D]. For completeness, we repeat it here. For an $m \times n$ stochastic matrix T' (an n input, m output channel), the input symmetry group $\mathcal{G}$ is defined as

$$\mathcal{G} = \{ G \in \mathcal{J}_n : \exists \Pi \in \mathcal{J}_m, \quad T'G = \Pi T' \} \tag{4}$$

i.e., $\mathcal{G}$ is the set of permutation matrices G such that the column permutations of T' with G may be achieved with corresponding row permutations. T' is input symmetric, if $\mathcal{G}$ is transitive, i.e., any element of $\{1, 2, \ldots, n\}$ can be mapped to every other element of $\{1, 2, \ldots, n\}$ by some member of $\mathcal{G}$.

The class of DDICs we consider in this paper satisfies the following conditions.

1. T' is input symmetric. Let the input symmetry group be $\mathcal{G}$.
2. For any $x_2', x_2'' \in \mathcal{X}_2$, there exists a permutation matrix $G \in \mathcal{G}$, such that
$$T_{x_2'} = G T_{x_2''} \tag{5}$$
3. $H(Y_1|X_1 = x_1, X_2 = x_2) = \eta$, independent of x_1, x_2.
4. $p(y_1|x_1, x_2)$ satisfies
$$\sum_{x_2} p(y_1|x_1, x_2) = \frac{|\mathcal{X}_2|}{|\mathcal{Y}_1|}, \quad x_1 \in \mathcal{X}_1, y_1 \in \mathcal{Y}_1. \tag{6}$$
5. Let $\boldsymbol{p}_{x_1,x_2}$ be the $|\mathcal{Y}_1|$-dimensional vector of probabilities $p(y_1|x_1, x_2)$ for a given x_1, x_2. Then, there exists an $\tilde{x}_2 \in \mathcal{X}_2$, such that
$$\left\{ \sum_{x_1,x_2} a_{x_1,x_2} \boldsymbol{p}_{x_1,x_2} : \sum_{x_1,x_2} a_{x_1,x_2} = 1, a_{x_1,x_2} \geq 0 \right\} \subseteq \left\{ G \left(\sum_{x_1} b_{x_1} \boldsymbol{p}_{x_1,\tilde{x}_2} \right) : \sum_{x_1} b_{x_1} = 1, b_{x_1} \geq 0, G \in \mathcal{G} \right\} \tag{7}$$

Examples of DDICs that satisfy Conditions 1–5 are given in Section VI.

Channel T' being input symmetric means that the output entropy of channel T' is maximized when the input distribution is chosen to be the uniform distribution, i.e.,

$$\max_{\boldsymbol{p} \in \Delta_n} \quad H(T'\boldsymbol{p}) = H(T'\boldsymbol{u}) \tag{8}$$

where $\boldsymbol{u}$ denotes the uniform distribution in Δ_n. This is because, for any $\boldsymbol{p} \in \Delta_n$, if we let $\boldsymbol{q} = |\mathcal{G}|^{-1} \sum_{G \in \mathcal{G}} G\boldsymbol{p}$, then we have

$$H(T'\boldsymbol{q}) = H\left(|\mathcal{G}|^{-1} \sum_{G \in \mathcal{G}} T'G\boldsymbol{p} \right) \tag{9}$$
$$= H\left(|\mathcal{G}|^{-1} \sum_{G \in \mathcal{G}} \Pi_G T'\boldsymbol{p} \right) \tag{10}$$
$$\geq |\mathcal{G}|^{-1} \sum_{G \in \mathcal{G}} H(\Pi_G T'\boldsymbol{p}) \tag{11}$$
$$= H(T'\boldsymbol{p}) \tag{12}$$

where (10) follows from the fact that $G \in \mathcal{G}$, and (11) follows from the concavity of the entropy function. Note that for any $G' \in \mathcal{G}$

$$G'\boldsymbol{q} = \boldsymbol{q} \tag{13}$$

by the fact that $\mathcal{G}$ is a group. Since $\mathcal{G}$ is also transitive, $\boldsymbol{q} = \boldsymbol{u}$.

Condition 2 implies that for any $p(x_1)$, $H(Y_1|X_2 = x_2)$ does not depend on x_2. Combined with Condition 1, Condition 2 further implies that $H(Y_2|X_2 = x_2)$ does not depend on x_2 either. These two facts will be proved and utilized later.

A sufficient condition for Condition 3 to hold is that the vectors $p(y_1|X_1 = x_1, X_2 = x_2)$ for all $(x_1, x_2) \in \mathcal{X}_1 \times \mathcal{X}_2$ are permutations of each other. This is true for instance when the channel from X_1 and X_2 to Y_1 is additive [1].

By Condition 4, we can show that when X_2 takes the uniform distribution, Y_1 will also be uniformly distributed. Combined with Condition 1, Condition 4 implies that when X_2 takes the uniform distribution, $H(Y_2)$ is maximized, irrespective of $p(x_1)$.

In Condition 5, the first line of (7) denotes the set of all convex combinations of vectors $\boldsymbol{p}_{x_1,x_2}$ for all $(x_1,x_2) \in \mathcal{X}_1 \times \mathcal{X}_2$, while the second line denotes all convex combinations, and their permutations with $G \in \mathcal{G}$, of vectors $\boldsymbol{p}_{x_1,\tilde{x}_2}$ for all $x_1 \in \mathcal{X}_1$, but for a fixed $\tilde{x}_2 \in \mathcal{X}_2$. Therefore, this condition means that all convex combinations of $\boldsymbol{p}_{x_1,x_2}$ may be obtained by a combination of convex combinations of $\boldsymbol{p}_{x_1,\tilde{x}_2}$ for a fixed $\tilde{x}_2$, and permutations in $\mathcal{G}$.

The DADICs considered in [1] satisfy Conditions 1–5, as we will show in Section VI-A.

The aim of this correspondence is to provide a single-letter characterization for the capacity region of DDICs that satisfy Conditions 1–5. Rather than following the proof technique of [1], we use a time-sharing random variable in the achievable region in place of the convex-hull operation and obtain a somewhat simpler proof.

III. An Outer Bound

When we assume that the encoders are able to fully cooperate, i.e., both encoders know both messages W_1 and W_2, we get a corresponding degraded broadcast channel with input $x = (x_1, x_2)$. The capacity region of the corresponding degraded broadcast channel serves as an outer bound on the capacity region of the DDIC. The capacity region of the degraded broadcast channel is known [9], [11], [12], and thus, a single-letter outer bound on the capacity region of the DDIC is

$$\bigcup_{p(u),p(x_1,x_2|u)} \{(R_1,R_2) : R_1 \le I(X_1,X_2;Y_1|U), R_2 \le I(U;Y_2)\} \tag{14}$$

where the auxiliary random variable U satisfies the Markov chain $U \longrightarrow (X_1,X_2) \longrightarrow (Y_1,Y_2)$, and its cardinality is bounded by $|\mathcal{U}| \le \min(|\mathcal{Y}_1|, |\mathcal{Y}_2|, |\mathcal{X}_1||\mathcal{X}_2|)$. More specifically, for DDICs that satisfy Condition 3, (14) can be written as

$$\bigcup_{p(u),p(x_1,x_2|u)} \left\{(R_1,R_2) : R_1 \le H(Y_1|U) - \eta, R_2 \le I(U;Y_2)\right\}. \tag{15}$$

Let us define τ as

$$\tau = \max_{\boldsymbol{p}\in\Delta_{|\mathcal{Y}_1|}} H(T'\boldsymbol{p}) \tag{16}$$

which is the maximum entropy of Y_2 over all possible distributions of Y_1. Then, the following region is a further upper bound on the capacity region of the DDIC:

$$\bigcup_{p(u),p(x_1,x_2|u)} \Big\{(R_1,R_2) : R_1 \le H(Y_1|U) - \eta \qquad R_2 \le \tau - H(Y_2|U)\Big\} \tag{17}$$

which, for later purposes, is rewritten as

$$\bigcup_{p_U(u),p(x_1,x_2|u)} \Big\{(R_1,R_2) : R_1 \le \sum_{u\in\mathcal{U}} p_U(u)H(Y_1|U=u) - \eta \qquad R_2 \le \tau - \sum_{u\in\mathcal{U}} p_U(u)H(Y_2|U=u)\Big\}. \tag{18}$$

IV. An Achievable Region

Based on [7, Theorem 4], the following region is achievable:

$$\overline{\text{co}}\left[\bigcup_{p(x_1),p(x_2)} \{(R_1,R_2) : R_1 \le I(X_1;Y_1|X_2) \qquad R_2 \le I(X_2;Y_2)\}\right] \tag{19}$$

where $\overline{\text{co}}$ denotes the closure of the convex hull operation. This achievable region corresponds to the achievability scheme that the "bad" receiver treats the signal for the "good" receiver as pure noise, and the "good" receiver decodes both messages as if it is the receiver in a multiple-access channel.

A time-sharing random variable may be used in place of the closure of the convex-hull operation [9, Sec. 14.3], and thus we may write the region in (19) as

$$\bigcup_{p(q),p(x_1|q),p(x_2|q)} \{(R_1,R_2) : R_1 \le I(X_1;Y_1|X_2,Q) \qquad R_2 \le I(X_2;Y_2|Q)\} \tag{20}$$

where the mutual informations are calculated according to the distribution

$$p(q)p(x_1|q)p(x_2|q)p(y_1|x_1,x_2)p'(y_2|y_1) \tag{21}$$

where Q is a time-sharing random variable. Random variable Q may take values from any arbitrary set, denoted as $\mathcal{Q}$. For calculation purposes, without of loss of generality, we may restrict $|\mathcal{Q}|$ to $|\mathcal{Q}| \le 2$ [13]. For DDICs that satisfy Condition 3, (20) reduces to

$$\bigcup_{p(q),p(x_1|q),p(x_2|q)} \{(R_1,R_2) : R_1 \le H(Y_1|X_2,Q) - \eta \qquad R_2 \le H(Y_2|Q) - H(Y_2|X_2,Q)\}. \tag{22}$$

We note that (22) remains an achievable region if we choose $p(x_2|q)$ to be the uniform distribution, for all q. Furthermore, for this choice of $p(x_2|q)$, we have

$$p(y_1|q) = \sum_{x_1,x_2} p(y_1|x_1,x_2)p(x_1|q)\frac{1}{|\mathcal{X}_2|} \tag{23}$$

$$= \frac{1}{|\mathcal{X}_2|}\sum_{x_1} p(x_1|q)\sum_{x_2} p(y_1|x_1,x_2) \tag{24}$$

$$= \frac{1}{|\mathcal{Y}_1|} \tag{25}$$

where (25) uses Condition 4. Thus, when $p(x_2|q)$ is chosen as the uniform distribution independent of q, $p(y_1|q)$ results in a uniform distribution as well, for all $q \in \mathcal{Q}$. With τ defined as in (16), using the fact that the DDIC under consideration satisfies Condition 1, i.e., it satisfies (8), we have that when $p(x_2|q)$ is uniform, independent of q, and consequently $p(y_1|q)$ is uniform, independent of q

$$H(Y_2|Q=q) = \tau \tag{26}$$

which means

$$H(Y_2|Q) = \sum_{q\in\mathcal{Q}} H(Y_2|Q=q)p(q) = \tau. \tag{27}$$

Hence, choosing $p(x_2|q)$ to be the uniform distribution, independent of q, in (22), yields the following as an achievable region:

$$\bigcup_{p(q),p(x_1|q)} \Bigg\{(R_1,R_2): R_1 \le \frac{1}{|\mathcal{X}_2|}\sum_{x_2,q} p(q)H(Y_1|X_2=x_2,Q=q)-\eta \quad R_2 \le \tau - \frac{1}{|\mathcal{X}_2|}\sum_{x_2,q} p(q)H(Y_2|X_2=x_2,Q=q)\Bigg\} \tag{28}$$

Due to Condition 2, for any q, any $p(x_1|q) = \boldsymbol{p}^q$, and any $x_2', x_2'' \in \mathcal{X}_2$, there exists a permutation matrix $G \in \mathcal{G}$ such that

$$H(Y_1|X_2=x_2',Q=q) = H\left(T_{x_2'}\boldsymbol{p}^q\right) \tag{29}$$
$$= H\left(GT_{x_2''}\boldsymbol{p}^q\right) \tag{30}$$
$$= H\left(T_{x_2''}\boldsymbol{p}^q\right) \tag{31}$$
$$= H\left(Y_1|X_2=x_2'',Q=q\right) \tag{32}$$

which means that for any q and $p(x_1|q)$, $H(Y_1|X_2=x_2,Q=q)$ does not depend on x_2. Furthermore, for any q, any $p(x_1|q)=\boldsymbol{p}^q$, and any $x_2', x_2'' \in \mathcal{X}_2$, there exist permutation matrices $G \in \mathcal{G}$ and Π, of order $|\mathcal{Y}_1|$ and $|\mathcal{Y}_2|$, respectively, such that

$$H(Y_2|X_2=x_2',Q=q) = H\left(T'T_{x_2'}\boldsymbol{p}^q\right) \tag{33}$$
$$= H\left(T'GT_{x_2''}\boldsymbol{p}^q\right) \tag{34}$$
$$= H\left(\Pi T'T_{x_2''}\boldsymbol{p}^q\right) \tag{35}$$
$$= H\left(T'T_{x_2''}\boldsymbol{p}^q\right) \tag{36}$$
$$= H\left(Y_2|X_2=x_2'',Q=q\right) \tag{37}$$

where (35) follows from the fact that $G \in \mathcal{G}$. Equation (37) means that for any q and $p(x_1|q)$, $H(Y_2|X_2=x_2,Q=q)$ does not depend on x_2 either. Hence, the achievable region in (28) can further be written as

$$\bigcup_{p(q),p(x_1|q)} \Bigg\{(R_1,R_2): R_1 \le \sum_{q} p(q)H(Y_1|X_2=x_2,Q=q)-\eta \quad R_2 \le \tau - \sum_{q} p(q)H(Y_2|X_2=x_2,Q=q)\Bigg\} \tag{38}$$

for any $x_2 \in \mathcal{X}_2$. Since we will use Condition 5 later, we choose to write the region in (38) as

$$\bigcup_{p_Q(q),p_{X_1|Q}(x_1|q)} \Bigg\{(R_1,R_2): R_1 \le \sum_{q} p_Q(q)H(Y_1|X_2=\tilde{x}_2,Q=q)-\eta \quad R_2 \le \tau - \sum_{q} p_Q(q)H(Y_2|X_2=\tilde{x}_2,Q=q)\Bigg\} \tag{39}$$

where $\tilde{x}_2$ is given in Condition 5.

V. The Capacity Region

In this section, we show that the achievable region in (39) contains the outer bound in (18), and thus, (18) and (39) are both, in fact, single-letter characterizations of the capacity region of DDICs satisfying Conditions 1–5.

To show (39) contains (18), it is sufficient to show that for any $p_U(u)$ and $p(x_1,x_2|u)$, the region

$$\Bigg\{(R_1,R_2): R_1 \le \sum_{u\in\mathcal{U}} p_U(u)H(Y_1|U=u)-\eta \quad R_2 \le \tau - \sum_{u\in\mathcal{U}} p_U(u)H(Y_2|U=u)\Bigg\} \tag{40}$$

is the same as the region

$$\Bigg\{(R_1,R_2): R_1 \le \sum_{q\in\mathcal{Q}} p_Q(q)H(Y_1|X_2=\tilde{x}_2,Q=q)-\eta \quad R_2 \le \tau - \sum_{q\in\mathcal{Q}} p_Q(q)H(Y_2|X_2=\tilde{x}_2,Q=q)\Bigg\} \tag{41}$$

for some $p_Q(q)$ and $p_{X_1|Q}(x_1|q)$. More specifically, it suffices to show that for any distribution $p_U(u)$ and $p(x_1,x_2|u)$, and picking $\mathcal{Q}=\mathcal{U}$ and $p_Q(u)=p_U(u)$ for all $u \in \mathcal{U}$, there exists some $p_{X_1|Q}(x_1|u)$, which we will call $p_0(x_1|u)$, such that

$$H(Y_1|U=u) = H(Y_1|X_2=\tilde{x}_2,Q=q) \tag{42}$$
$$H(Y_2|U=u) = H(Y_2|X_2=\tilde{x}_2,Q=q) \tag{43}$$

for all $u \in \mathcal{U}$, where the entropies on the left-hand side of (42) and (43) are the quantities in region (40), evaluated with the marginals of the joint distribution

$$p_U(u)p(x_1,x_2|u)p(y_1|x_1,x_2)p'(y_2|y_1) \tag{44}$$

and the entropies on the right-hand side of (42) and (43) are the quantities in region (41), evaluated with the marginals of the joint distribution

$$p_U(u)p_0(x_1|u)p(y_1|x_1,X_2=\tilde{x}_2)p'(y_2|y_1). \tag{45}$$

Using Condition 5, for each $u \in \mathcal{U}$, there exists a $p^u(x_1)=\boldsymbol{p}^u$ and a permutation matrix $G^u \in \mathcal{G}$, such that

$$\sum_{x_1,x_2} p(x_1,x_2|U=u)\boldsymbol{p}_{x_1,x_2} = G^uT_{\tilde{x}_2}\boldsymbol{p}^u. \tag{46}$$

By choosing $p_0(x_1|u)=\boldsymbol{p}^u$, we have

$$H(Y_1|U=u) = H\left(G^uT_{\tilde{x}_2}\boldsymbol{p}^u\right) = H\left(T_{\tilde{x}_2}\boldsymbol{p}^u\right) = H(Y_1|X_2=\tilde{x}_2,Q=u) \tag{47}$$

and we also have

$$H(Y_2|U=u) = H\left(T'G^uT_{\tilde{x}_2}\boldsymbol{p}^u\right) \tag{48}$$
$$= H\left(\Pi^uT'T_{\tilde{x}_2}\boldsymbol{p}^u\right) \tag{49}$$
$$= H\left(T'T_{\tilde{x}_2}\boldsymbol{p}^u\right) \tag{50}$$
$$= H(Y_2|X_2=\tilde{x}_2,Q=u) \tag{51}$$

for all $u \in \mathcal{U}$, where (47) and (48) follow from (46), and (49) is due to the fact that $G \in \mathcal{G}$. Hence, based on (47) and (51), we conclude that for any $p_U(u)$ and $p(x_1,x_2|u)$, there exists a $\boldsymbol{p}^u$ for each $u \in \mathcal{U}$, such that by choosing $\mathcal{Q}=\mathcal{U}$, $p_Q(u)=p_U(u)$ and $p_{X_1|Q}(x_1|u)=\boldsymbol{p}^u$, the regions (40) and (41) are the same. Thus, we have proved that region (39) contains region (18).

Therefore, we conclude that the single-letter characterization of the capacity region of DDICs satisfying Conditions 1–5 is (39) and (18). Furthermore, we note that for these DDICs, encoder cooperation cannot enlarge the capacity region.

VI. EXAMPLES

In this section, we will provide three examples of DDICs for which Conditions 1–5 are satisfied. The first example is the channel model adopted in [1], for which the capacity region is already known. In the second and third examples, the capacity regions were previously unknown, and using the results of this correspondence, we are able to determine the capacity regions.

A. Example 1

A DADIC is defined as [1]

$$Y_1 = X_1 \oplus X_2 \oplus V_1 \tag{52}$$
$$Y_2 = X_1 \oplus X_2 \oplus V_1 \oplus V_2 \tag{53}$$

where

$$\mathcal{X}_1 = \mathcal{X}_2 = \mathcal{Y}_1 = \mathcal{Y}_2 = \mathcal{S} = \{0, 1, \ldots, s-1\} \tag{54}$$

and $\oplus$ denotes modulo-s sum, and V_1 and V_2 are independent noise random variables defined over $\mathcal{S}$ with distributions

$$\boldsymbol{p}_i = (p_i(0), p_i(1), \ldots, p_i(s-1)), \quad i = 1, 2. \tag{55}$$

Since $Y_2 = Y_1 \oplus V_2$, matrix T' is circulant, and thus input symmetric [10, Sec. II-D]. Hence, Condition 1 is satisfied. It is straightforward to check that Conditions 2–5 are also satisfied. For example, when $s = 3$, we have

$$T' = \begin{bmatrix} p_2(0) & p_2(2) & p_2(1) \\ p_2(1) & p_2(0) & p_2(2) \\ p_2(2) & p_2(1) & p_2(0) \end{bmatrix} \tag{56}$$

and the input symmetry group for T' is

$$\mathcal{G} = \left\{ G_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, G_1 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, G_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \right\} \tag{57}$$

which is transitive, i.e., $1 \xrightarrow{G_2} 2, 1 \xrightarrow{G_1} 3, 2 \xrightarrow{G_1} 1, 2 \xrightarrow{G_2} 3, 3 \xrightarrow{G_2} 1, 3 \xrightarrow{G_1} 2$. From (52)

$$T_0 = \begin{bmatrix} p_1(0) & p_1(2) & p_1(1) \\ p_1(1) & p_1(0) & p_1(2) \\ p_1(2) & p_1(1) & p_1(0) \end{bmatrix}$$
$$T_1 = \begin{bmatrix} p_1(2) & p_1(1) & p_1(0) \\ p_1(0) & p_1(2) & p_1(1) \\ p_1(1) & p_1(0) & p_1(2) \end{bmatrix}$$
$$T_2 = \begin{bmatrix} p_1(1) & p_1(0) & p_1(2) \\ p_1(2) & p_1(1) & p_1(0) \\ p_1(0) & p_1(2) & p_1(1) \end{bmatrix}. \tag{58}$$

Conditions 2–4 are satisfied because

$$T_1 = G_1 T_0, \quad T_2 = G_2 T_0 \tag{59}$$
$$\eta = H(V_1) \tag{60}$$
$$\sum_{x_2} p(y_1|x_1, x_2) = p_1(0) + p_1(1) + p_1(2) = 1. \tag{61}$$

Next, we check Condition 5.

$$\left\{ \sum_{x_1,x_2} a_{x_1,x_2} \boldsymbol{p}_{x_1,x_2} : \sum_{x_1,x_2} a_{x_1,x_2} = 1, a_{x_1,x_2} \geq 0 \right\} \tag{62}$$
$$= \left\{ a \begin{pmatrix} p_1(0) \\ p_1(1) \\ p_1(2) \end{pmatrix} + b \begin{pmatrix} p_1(2) \\ p_1(0) \\ p_1(1) \end{pmatrix} + c \begin{pmatrix} p_1(1) \\ p_1(2) \\ p_1(0) \end{pmatrix} : a + b + c = 1, a, b, c \geq 0 \right\} \tag{63}$$

because even though (62) is a convex combination of nine vectors, due to vectors repeating themselves in the columns of T_0, T_1, and T_2, the set, in fact, consists of convex combinations of only three vectors. On the other hand, for $\tilde{x}_2 = 0$

$$\left\{ G \left(\sum_{x_1} b_{x_1} \boldsymbol{p}_{x_1,\tilde{x}_2} \right) : \sum_{x_1} b_{x_1} = 1, b_{x_1} \geq 0, G = G_0 \right\} \tag{64}$$
$$= \left\{ a \begin{pmatrix} p_1(0) \\ p_1(1) \\ p_1(2) \end{pmatrix} + b \begin{pmatrix} p_1(2) \\ p_1(0) \\ p_1(1) \end{pmatrix} + c \begin{pmatrix} p_1(1) \\ p_1(2) \\ p_1(0) \end{pmatrix} : a + b + c = 1, a, b, c \geq 0 \right\} \tag{65}$$

because (64) is the convex combinations of the columns of T_0, with the unitary permutation. Thus

$$\left\{ \sum_{x_1,x_2} a_{x_1,x_2} \boldsymbol{p}_{x_1,x_2} : \sum_{x_1,x_2} a_{x_1,x_2} = 1, a_{x_1,x_2} \geq 0 \right\}$$
$$= \left\{ G \left(\sum_{x_1} b_{x_1} \boldsymbol{p}_{x_1,\tilde{x}_2} \right) : \sum_{x_1} b_{x_1} = 1, b_{x_1} \geq 0, G = G_0 \right\} \tag{66}$$
$$\subseteq \left\{ G \left(\sum_{x_1} b_{x_1} \boldsymbol{p}_{x_1,\tilde{x}_2} \right) : \sum_{x_1} b_{x_1} = 1, b_{x_1} \geq 0, G \in \mathcal{G} \right\} \tag{67}$$

and Condition 5 is satisfied.

B. Example 2

Next, we consider the following DDIC. We have $|\mathcal{X}_1| = |\mathcal{X}_2| = |\mathcal{Y}_1| = 2, |\mathcal{Y}_2| = 3$, and $p(y_1|x_1, x_2)$ is characterized by

$$Y_1 = X_1 \oplus X_2 \oplus V_1 \tag{68}$$

where V_1 is Bernoulli with p. $p'(y_2|y_1)$ is an erasure channel with parameter $0 \leq \alpha \leq 1$, i.e., the transition probability matrix is

$$T' = \begin{bmatrix} 1-\alpha & 0 \\ \alpha & \alpha \\ 0 & 1-\alpha \end{bmatrix}. \tag{69}$$

Thus, the channel is such that the "bad" receiver cannot receive all the bits that the "good" receiver receives. More specifically, α proportion of the time, whether the bit is a 0 or 1 is unrecognizable, and thus denoted as an erasure e.

It is easy to see that T' is input symmetric because the input symmetry group

$$\mathcal{G} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\} \tag{70}$$

is transitive. Conditions 2–5 are satisfied because $p(y_1|x_1, x_2)$ is the same as in Example 1 in Section VI-A.

C. Example 3

Let a, b, c, d, e, f be nonnegative numbers such that $a + b + c = 1$ and $d + e + f = 1/2$. We have $|\mathcal{X}_1| = 4, |\mathcal{X}_2| = |\mathcal{Y}_1| = 3$, and $|\mathcal{Y}_2| = 6$. The DDIC is described as

$$T' = \begin{bmatrix} d & e & f \\ e & f & d \\ d & f & e \\ f & e & d \\ e & d & f \\ f & d & e \end{bmatrix}, \quad T_0 = \begin{bmatrix} a & b & c & c \\ b & c & a & b \\ c & a & b & a \end{bmatrix},$$
$$T_1 = \begin{bmatrix} c & a & b & a \\ a & b & c & c \\ b & c & a & b \end{bmatrix}, \quad T_2 = \begin{bmatrix} b & c & a & b \\ c & a & b & a \\ a & b & c & c \end{bmatrix}. \tag{71}$$

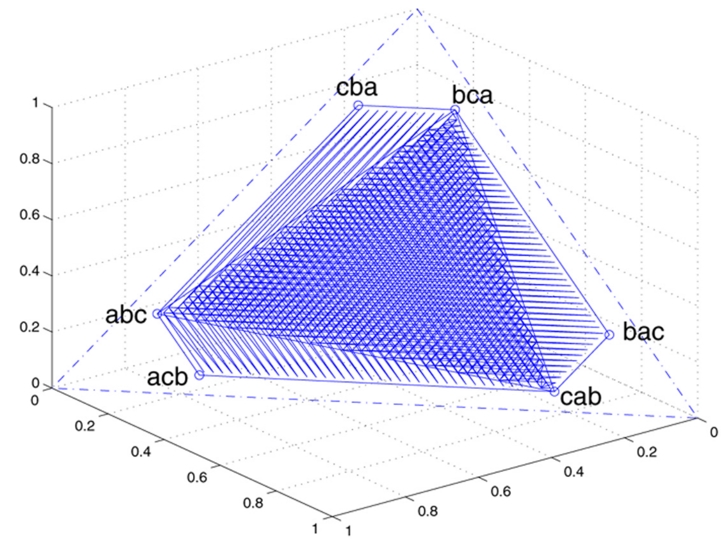

Fig. 1. Explanation of Condition 5 in Example 3.

It is straightforward to see that T' is input symmetric because the input symmetry group

$$\mathcal{G} = \left\{ G_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad G_1 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \right.$$
$$G_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad G_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix},$$
$$\left. G_4 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad G_5 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \right\} \tag{72}$$

is transitive. Conditions 2–4 are satisfied because

$$T_1 = G_1 T_0, \quad T_2 = G_2 T_0 \tag{73}$$
$$\eta = -a \log a - b \log b - c \log c \tag{74}$$
$$\sum_{x_2} p(y_1|x_1, x_2) = a + b + c = 1. \tag{75}$$

To show Condition 5, we use Fig. 1. The set on the first line of (7) in Condition 5 is the convex combination of the following six points:

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix}, \begin{bmatrix} a \\ c \\ b \end{bmatrix}, \begin{bmatrix} c \\ a \\ b \end{bmatrix}, \begin{bmatrix} b \\ a \\ c \end{bmatrix}, \begin{bmatrix} b \\ c \\ a \end{bmatrix}, \begin{bmatrix} c \\ b \\ a \end{bmatrix} \tag{76}$$

resulting in all the points within the hexagon in Fig. 1. The three sets

$$\left\{ G\left(\sum_{x_1} b_{x_1} \boldsymbol{p}_{x_1,\tilde{x}_2}\right) : \sum_{x_1} b_{x_1} = 1, b_{x_1} \geq 0, G = G_0 \right\}$$
$$= \left\{ \mu_1 \begin{bmatrix} a \\ b \\ c \end{bmatrix} + \mu_2 \begin{bmatrix} b \\ c \\ a \end{bmatrix} + \mu_3 \begin{bmatrix} c \\ a \\ b \end{bmatrix} \right.$$
$$\left. + \mu_4 \begin{bmatrix} c \\ b \\ a \end{bmatrix} : \sum_{i=1}^{4} \mu_i = 1, \mu_i \geq 0 \right\} \tag{77}$$

and

$$\left\{ G\left(\sum_{x_1} b_{x_1} \boldsymbol{p}_{x_1,\tilde{x}_2}\right) : \sum_{x_1} b_{x_1} = 1, b_{x_1} \geq 0, G = G_1 \right\}$$
$$= \left\{ \mu_1 \begin{bmatrix} c \\ a \\ b \end{bmatrix} + \mu_2 \begin{bmatrix} a \\ b \\ c \end{bmatrix} + \mu_3 \begin{bmatrix} b \\ c \\ a \end{bmatrix} \right.$$
$$\left. + \mu_4 \begin{bmatrix} a \\ c \\ b \end{bmatrix} : \sum_{i=1}^{4} \mu_i = 1, \mu_i \geq 0 \right\} \tag{78}$$

and

$$\left\{ G\left(\sum_{x_1} b_{x_1} \boldsymbol{p}_{x_1,\tilde{x}_2}\right) : \sum_{x_1} b_{x_1} = 1, b_{x_1} \geq 0, G = G_2 \right\}$$
$$= \left\{ \mu_1 \begin{bmatrix} b \\ c \\ a \end{bmatrix} + \mu_2 \begin{bmatrix} c \\ a \\ b \end{bmatrix} + \mu_3 \begin{bmatrix} a \\ b \\ c \end{bmatrix} \right.$$
$$\left. + \mu_4 \begin{bmatrix} b \\ a \\ c \end{bmatrix} : \sum_{i=1}^{4} \mu_i = 1, \mu_i \geq 0 \right\} \tag{79}$$

correspond to the points in the three shaded areas,

$$[abc, cba, bca, cab], [acb, abc, bca, cab], \text{ and } [bac, cab, abc, bca]$$

respectively. Since the three shaded areas cover the entire hexagon, and $\{G_0, G_1, G_2\} \subset \mathcal{G}$, Condition 5 is satisfied.

VII. Conclusion

We provided a single-letter characterization for the capacity region of a class of DDICs, which is more general than the class of DADICs studied by Benzel [1]. We showed that for the class of DDICs studied, encoder cooperation does not enlarge the capacity region, and the best way to manage the interference is through random codebook design and treating the signal intended for the "good" receiver as pure noise at the "bad" receiver.

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An Algorithm to Compute the Nearest Point in the Lattice A_n^*

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***Abstract*—The lattice A_n^* is an important lattice because of its covering properties in low dimensions. Clarkson described an algorithm to compute the nearest lattice point in A_n^* that requires $O(n \log n)$ arithmetic operations. In this correspondence, we describe a new algorithm. While the complexity is still $O(n \log n)$, it is significantly simpler to describe and verify. In practice, we find that the new algorithm also runs faster.**

***Index Terms*—Channel coding, direction-of-arrival estimation, frequency estimation, lattice theory, nearest point algorithm, quantization, synchronization.**

I. INTRODUCTION

The study of point lattices is of great importance in several areas of number theory, particularly the studies of quadratic forms, the geometry of numbers, and simultaneous Diophantine approximation, and also to the practical engineering problems of quantization and channel coding. They are also important in studying the sphere packing problem and the kissing number problem [1], [2].

A *lattice*, L, is a set of points in $\mathbb{R}^n$ such that

$$L = \{\boldsymbol{x} \in \mathbb{R}^n | \boldsymbol{x} = \boldsymbol{B}\boldsymbol{w}, \boldsymbol{w} \in \mathbb{Z}^n\}$$

where $\boldsymbol{B}$ is termed the *generator matrix*.

The lattice A_n^* is an interesting lattice due to its covering properties in low dimensions. It gives the thinnest covering in all dimensions up to 8 [2]. A_n^* has also found application in a number of estimation problems including period estimation from sparse timing data [3], frequency estimation [4], and direction of arrival estimation [5].

The nearest lattice point problem is as follows. Given $\boldsymbol{y} \in \mathbb{R}^n$ and some lattice L whose lattice points lie in $\mathbb{R}^n$, find the lattice point $\boldsymbol{x} \in L$ such that the Euclidean distance between $\boldsymbol{y}$ and $\boldsymbol{x}$ is minimized. If the lattice is used for vector quantization then the nearest lattice point corresponds to the minimum distortion point. If the lattice is used as a code for a Gaussian channel, then the nearest lattice point corresponds to maximum-likelihood decoding [6].

Conway and Sloane [6] appear to have been the first to study the problem of computing the nearest lattice point in A_n^*. By decomposing A_n^* into a union of translations of its dual lattice A_n, they discovered an algorithm for computing the nearest lattice point to a given point in $O(n^2 \log n)$ arithmetic operations. Later [7], they were able to improve the execution time of the algorithm to $O(n^2)$ operations.

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