# Broadcasting with an Energy Harvesting Rechargeable Transmitter

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***Abstract*—In this paper, we investigate the *transmission completion time* minimization problem in an additive white Gaussian noise (AWGN) broadcast channel, where the transmitter is able to harvest energy from the nature, using a rechargeable battery. The harvested energy is modeled to arrive at the transmitter during the course of transmissions. The transmitter has a fixed number of packets to be delivered to each receiver. The objective is to minimize the time by which all of the packets are delivered to their respective destinations. To this end, we optimize the transmit powers and transmission rates in a deterministic setting. We first analyze the structural properties of the optimal transmission policy in a two-user broadcast channel via the dual problem of maximizing the departure region by a fixed time $T$. We prove that the optimal *total* transmit power sequence has the same structure as the optimal single-user transmit power sequence in [1], [2]. In addition, the total power is split optimally based on a *cut-off* power level; if the total transmit power is lower than this cut-off level, all transmit power is allocated to the stronger user; otherwise, all transmit power above this level is allocated to the weaker user. We then extend our analysis to an $M$-user broadcast channel. We show that the optimal total power sequence has the same structure as the two-user case and optimally splitting the total power among $M$ users involves $M-1$ *cut-off* power levels. Using this structure, we propose an algorithm that finds the globally optimal policy. Our algorithm is based on reducing the broadcast channel problem to a single-user problem as much as possible. Finally, we illustrate the optimal policy and compare its performance with several suboptimal policies under different settings.**

***Index Terms*—Energy harvesting, rechargeable wireless networks, broadcast channels, transmission completion time minimization, throughput maximization.**

## I. Introduction

WE consider a wireless communication network where users are able to harvest energy from nature using rechargeable batteries. Such energy harvesting capabilities will make sustainable and environmentally friendly deployment of wireless communication networks possible. While energy-efficient scheduling policies have been well-investigated in traditional battery powered (un-rechargeable) systems [3]–[8], energy-efficient scheduling in energy harvesting networks with nodes that have rechargeable batteries has only recently been considered [1], [2]. References [1], [2] consider a single-user communication system with an energy harvesting transmitter, and develop a packet scheduling scheme that minimizes the time by which all of the packets are delivered to the receiver.

In this paper, we consider a multi-user extension of the work in [1], [2]. In particular, we consider a wireless broadcast channel with an energy harvesting transmitter. As shown in Fig. 1, we consider a broadcast channel with one transmitter and $M$ receivers, where the transmitter node has $M+1$ queues. The $M$ data queues store the data arrivals intended for the individual receivers, while the energy queue stores the energy harvested from the environment. The $M$ receivers have different channel gains, and the broadcast channel is *degraded* [9]. Our objective is to adaptively change the transmission rates that go to the users according to the instantaneous data and energy queue sizes, such that the *transmission completion time* is minimized.

In [1], [2], we prove that the optimal scheduling policy in a single-user energy harvesting communication system has a "majorization" structure. The transmit power is kept constant between energy harvests, the sequence of transmit powers increases monotonically, and only changes when the energy constraint is tight in the optimal schedule. The optimal transmit power sequence is the most majorized energy-feasible power sequence during the transmission duration. We develop, in [1], [2], an algorithm to obtain the optimal off-line scheduling policy based on these properties. Reference [10] extends [1], [2] to the case where rechargeable batteries have finite storage capacities. We extend [1], [2] in [11] to a fading channel, and develop optimal *off-line* and *on-line* scheduling policies under stochastic fading and energy arrival processes. References [12], [13] address single-user energy harvesting rechargeable systems as well, with a slotted time system model. Although the slotted time system model can model a more practical scenario, e.g., a scenario where block encoding is used, a continuous time system model is more general in the sense that the continuous time model can be used to analyze a slotted time system after proper rearrangements. Therefore, we consider a continuous time energy harvesting system model as in our previous work [1], [2], [11]. We focus on the *off-line* problem in this paper with the goal of determining the structural properties of the optimum *broadcast scheduler*, and developing an iterative optimal off-line scheduling algorithm. We also provide benchmark *on-line* scheduling algorithms for performance comparison, while we leave the solution of the optimal *on-line* policy for future work.

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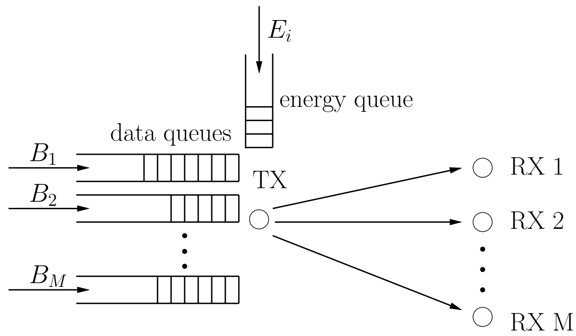


Fig. 1. An energy harvesting $M$-user broadcast channel.

Another line of research in wireless communications with energy harvesting nodes has been presented in [14]–[16] which considers the simultaneous transmission of information and energy. While this is a novel and interesting direction, in this paper, we do not consider the transmission of energy. We only consider the transmission of data. In particular, we address the issue of the replenishment of the transmission energy during the course of the communication session, and obtain the optimal off-line adaptation of the transmit power to the fluctuating levels of energy in a broadcasting scenario.

References [10], [11] investigate two related problems in single-user communication with an energy harvesting transmitter. The first problem is to maximize the throughput (number of bits transmitted) with a given deadline constraint, and the second problem is to minimize the transmission completion time with a given number of bits to transmit. These two problems are dual: given the energy arrival profile, if the maximum number of bits that can be sent by $T$ is $B^*$ in the first problem, then the minimum time to transmit $B^*$ bits in the second problem must be $T$, and the optimal transmission policies for these two problems must be identical. In this paper, we follow the same dual problem approach to solve the transmission completion time minimization problem in the broadcast channel. An alternative solution method for the second problem in the broadcast channel is provided in the independent and concurrent work [17]. This method divides the problem into local sub-problems that consider only two energy arrival epochs at a time. Iterations on the local problems are shown to converge to the global optimum.

We first analyze the structural properties of the optimal policy for the first problem in a two-user broadcast channel. Our goal is to determine the *maximum departure region* with a given deadline constraint $T$. The maximum departure region is defined as the set of all $(B_1, B_2)$ that can be transmitted to users reliably with a given deadline $T$. By using the convexity of the departure region and a Lagrangian approach, we show that the optimal *total* transmit power policy is independent of the operating point on the boundary, and has the same "majorization" structure as the single-user solution. As for the way of splitting the total transmit power between the two users, we prove that there exists a *cut-off* power level for the stronger user, i.e., only the power above this *cut-off* power level is allocated to the weaker user. We then investigate the maximum departure region in an $M$-user broadcast channel. We show that there exist $M-1$ *cut-off* power levels and the total power is split according to these *cut-off* power levels and the hierarchy among the channel gains of the users.

Next, we consider the second problem, where our goal is to minimize the time, $T$, by which a given number of bits are delivered to their intended receivers. Due to the duality between these problems, the optimal policy in the second problem has the same structural properties as those in the first problem. Using these optimal structural properties, we develop an iterative algorithm that finds the optimal schedule efficiently. In particular, we start with the two-user case and we obtain the optimal total power in the first epoch, $P_1$. Given the fact that there exists a *cut-off* power level, $P_c$, for the stronger user, the optimal policy depends on whether $P_1$ is smaller or larger than $P_c$, which, at this point, is unknown. Performing iterations on $P_c$ that alternates according to whether $P_c$ should be increased or decreased, we approach the optimal policy iteratively. At each iteration, single-user problems as in [1], [2] are solved. The algorithm naturally extends to the $M$-user broadcast channel. We perform alternating iterations on the cut-off level for the strongest user and determine whether it should be decreased or increased in a similar fashion. We also discuss the computational requirement of the proposed algorithm. Finally, we provide numerical illustrations and performance analysis for the optimal off-line policy. Specifically, we compare the performance of the optimal off-line policy with that of three practical semi-on-line sub-optimal policies which require no or partial off-line knowledge about the energy harvesting process.

## II. System Model and Problem Formulation

The system model is as shown in Figs. 1 and 2. The transmitter has an energy queue and $M$ data queues (Fig. 1). The physical layer is modeled as an AWGN broadcast channel, where the received signals at the $M$ receivers are

$$Y_m = X + Z_m, \qquad m = 1, \ldots, M \tag{1}$$

where $X$ is the transmit signal, and $Z_m$ is a Gaussian noise with zero-mean and variance $\sigma_m^2$, and without loss of generality $\sigma_1^2 \leq \sigma_2^2 \leq \ldots \leq \sigma_M^2$. Therefore, the first user is the strongest and the $M$th user is the weakest user in our broadcast channel. Next, for clarity of exposition we write the capacity region for this broadcast channel for $M = 2$, and subsequently generalize it to $M$ users. Assuming that the transmitter transmits with power $P$, the capacity region for the two-user AWGN broadcast channel is [9]

$$r_1 \leq \frac{1}{2}\log_2\left(1 + \frac{\alpha P}{\sigma_1^2}\right) \tag{2}$$

$$r_2 \leq \frac{1}{2}\log_2\left(1 + \frac{(1-\alpha)P}{\alpha P + \sigma_2^2}\right) \tag{3}$$

where $\alpha$ is the fraction of the total power spent for the message transmitted to the first user. Let us denote $f(p) \triangleq \frac{1}{2}\log_2(1+p)$ for future use. Then, the capacity region is $r_1 \leq f(\frac{\alpha P}{\sigma_1^2})$, $r_2 \leq f\left(\frac{(1-\alpha)P}{\alpha P + \sigma_2^2}\right)$. Working on the boundary of the capacity region, we have

$$P = \sigma_1^2 2^{2(r_1+r_2)} + (\sigma_2^2 - \sigma_1^2)2^{2r_2} - \sigma_2^2 \tag{4}$$

$$\triangleq g(r_1, r_2) \tag{5}$$

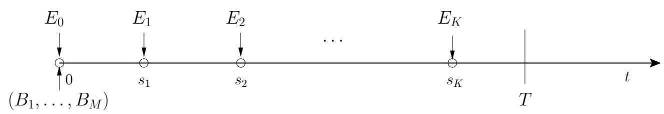


Fig. 2. System model. $(B_1, \ldots, B_M)$ bits to be transmitted to the users are available at the transmitter at the beginning. Energies arrive (are harvested) at points denoted by $\circ$. $T$ denotes the transmission completion time by which all of the bits are delivered to their respective destinations.

Therefore, $P$ is the smallest power necessary to transmit at rates $r_1$ and $r_2$ in this broadcast channel. We note that $g(r_1, r_2)$ is a strictly convex function of $(r_1, r_2)$.

The rate region for the $M$-user broadcast channel is obtained using an $M$-level superposition code [9]. The capacity region for the $M$-user AWGN broadcast channel is the set of rate vectors $(r_1, \ldots, r_M)$:

$$r_i = \frac{1}{2}\log_2\left(1 + \frac{\alpha_i P}{\sum_{j<i}\alpha_j P + \sigma_i^2}\right), \quad i = 1 \ldots, M \tag{6}$$

where $\alpha_i \geq 0$ and $\sum_i \alpha_i = 1$. Similar to the derivation in (3)-(5), we obtain the minimum power to achieve the rate vector $(r_1, \ldots, r_M)$ by a recursive formula as follows:

$$\begin{aligned} g_{(M)}(r_1, \ldots, r_M) = 2^{2r_M} g_{(M-1)}(r_1, \ldots, r_{M-1}) \\ + \sigma_M^2(2^{2r_M} - 1) \end{aligned} \tag{7}$$

where $g_{(M)}(r_1, \ldots, r_M)$ is the minimum power for the $M$-user AWGN broadcast channel. Note that (7) reduces to (5) for $M = 2$ where $g_{(1)}(r_1) = \sigma_1^2(2^{2r_1} - 1)$. Also note that $g_{(M)}(r_1, \ldots, r_M)$ is strictly convex in $(r_1, \ldots, r_M)$ by induction.

As shown in Fig. 1, the transmitter has $B_m$ bits destined to the $m$th receiver. Energy is harvested at times $s_k$ with amounts $E_k$, $k \geq 1$. $E_0$ denotes the initial energy available in the battery before the communication starts. Our goal is to select a transmission policy that minimizes the time, $T$, by which all of the bits are delivered to their intended receivers. The transmitter adapts its transmit power and the portions of the total transmit power used to transmit signals to the $M$ users according to the available energy level and the remaining number of bits. The energy consumed must satisfy the causality constraints, i.e., at any given time $t$, the total amount of energy consumed up to time $t$ must be less than or equal to the total amount of energy harvested up to time $t$.

Before we proceed to give a formal definition of the optimization problem and its solution, we start with the dual problem of finding the maximum departure region of the bits the transmitter can deliver to the users by any fixed time $T$. As we will observe in the next section, solving the dual problem enables us to identify the optimal structural properties for the original problem, and these properties help us reduce the original problem into simple scenarios, which can be solved efficiently. In the next section, we consider the two-user case, and generalize it to the most general $M$-user case in Section IV.

## III. CHARACTERIZING $\mathcal{D}(T)$: LARGEST $(B_1, B_2)$ REGION FOR A GIVEN $T$

In this section, our goal is to characterize the maximum departure region for a given deadline $T$. Let us denote the number of bits transmitted to the $i$th user by time $t$ via a rate function $r_i(\tau)$, in $0 \leq \tau \leq t$, as $B_i(r_i(t)) = \int_0^t r_i(\tau)d\tau$, $i = 1, 2$.

**Definition 1** *For any fixed transmission duration $T$, the maximum departure region, denoted as $\mathcal{D}(T)$, is the union of $(B_1, B_2)$ under any feasible rate allocation policy over the duration $[0, T)$, i.e., $\mathcal{D}(T) = \bigcup_{r_1(\tau), r_2(\tau)}(B_1(r_1(T)), B_2(r_2(T)))$, subject to the energy causality constraint $\int_0^t g(r_1, r_2)(\tau)d\tau \leq \sum_{i:s_i<t} E_i$, for $0 \leq t \leq T$.*

We call any policy which achieves the boundary of $\mathcal{D}(T)$ to be optimal. In the single-user scenario in [1], [2], we first examined the structural properties of the optimal policy. Based on these properties, we developed an algorithm to find the optimal scheduling policy. In this broadcast scenario also, we first analyze the structural properties of the optimal policy, and then obtain the optimal solution based on these structural properties. The following lemma which was proved for a single-user problem in [1], [2] was also proved for the broadcast problem in [17].

**Lemma 1** *Under the optimal policy, the transmission rate remains constant between energy harvests, i.e., the rate only potentially changes at an energy harvesting instant.*

**Proof:** We prove this using the strict convexity of $g(r_1, r_2)$. If the transmission rate for any user changes between two energy harvesting instants, then, we can always equalize the transmission rate over that duration without contradicting with the energy constraints. Based on the convexity of $g(r_1, r_2)$, after equalization of rates, the energy consumed over that duration decreases, and the saved energy can be allocated to both users to increase the departures. Therefore, changing rates between energy harvests is sub-optimal. ■

Therefore, in the following, we only consider policies where the rates are constant between any two consecutive energy arrivals. We denote the rates that go to both users as $(r_{1n}, r_{2n})$ over the duration $[s_{n-1}, s_n)$. An illustration of the maximum departure region is shown in Fig. 3.

**Lemma 2** *$\mathcal{D}(T)$ is a convex region.*

**Proof:** Proving the convexity of $\mathcal{D}(T)$ is equivalent to proving that, given any two achievable points $(B_1, B_2)$ and $(B_1', B_2')$ in $\mathcal{D}(T)$, any point on the line between these two points is also achievable, i.e., in $\mathcal{D}(T)$. Assume that $(B_1, B_2)$ and $(B_1', B_2')$ can be achieved with rate allocation policies $(\mathbf{r}_1, \mathbf{r}_2)$ and $(\mathbf{r}_1', \mathbf{r}_2')$, respectively. Consider the policy $(\lambda\mathbf{r}_1 + \bar{\lambda}\mathbf{r}_1', \lambda\mathbf{r}_2 + \bar{\lambda}\mathbf{r}_2')$, where $\bar{\lambda} = 1 - \lambda$. Then, the energy consumed up to $s_n$

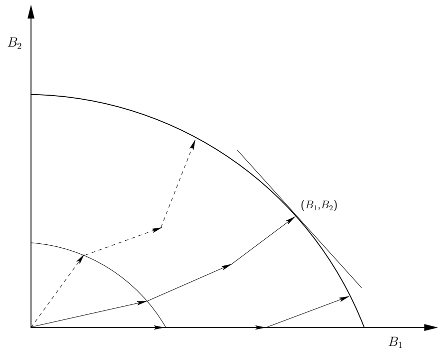


Fig. 3. The maximum departure region and trajectories to reach the boundary. Dotted trajectory is not possible.

is

$$\sum_{i=1}^{n} g(\lambda r_{1i} + \bar{\lambda} r'_{1i}, \lambda r_{2i} + \bar{\lambda} r'_{2i}) l_i$$
$$\leq \lambda \sum_{i=1}^{n} g(r_{1i}, r_{2i}) l_i + \bar{\lambda} \sum_{i=1}^{n} g(r'_{1i}, r'_{2i}) l_i \qquad (8)$$
$$\leq \lambda \sum_{i=0}^{n-1} E_i + \bar{\lambda} \sum_{i=0}^{n-1} E_i = \sum_{i=0}^{n-1} E_i \qquad (9)$$

Therefore, the energy causality constraint is satisfied for any $\lambda \in [0, 1]$, and the new policy is energy-feasible. Any point on the line between $(B_1, B_2)$ and $(B'_1, B'_2)$ can be achieved. When $\lambda \neq 0, 1$, the inequality in (8) is strict. Therefore, we save some amount of energy under the new policy, which can be used to increase the throughput for both users. This implies that $\mathcal{D}(T)$ is strictly convex. ■

In order to simplify the notation, for any given $T$, we assume that there are $N-1$ energy arrivals over $(0, T)$. We denote the last energy arrival time before $T$ as $s_{N-1}$, and $s_N = T$. We use $l_n$ to denote the length of the duration between two consecutive energy arrival instances $s_n$ and $s_{n-1}$, i.e., $l_n = s_n - s_{n-1}$, with $l_1 = s_1$ and $l_N = T - s_{N-1}$, as shown in Fig. 4.

Since $\mathcal{D}(T)$ is a strictly convex region, its boundary can be characterized by solving the following optimization problem for all $\mu_1, \mu_2 \geq 0$,

$$\max_{\mathbf{r}_1, \mathbf{r}_2} \quad \mu_1 \sum_{n=1}^{N} r_{1n} l_n + \mu_2 \sum_{n=1}^{N} r_{2n} l_n$$
$$\text{s.t.} \quad \sum_{n=1}^{j} g(r_{1n}, r_{2n}) l_n \leq \sum_{n=0}^{j-1} E_n, \quad 0 < j \leq N \qquad (10)$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ denote the rate sequences $r_{1n}$ and $r_{2n}$ for users 1 and 2, respectively. For $\mu_1 = 0$ or $\mu_2 = 0$, the problem in (10) reduces to the throughput maximization problem for the user which has the non-zero coefficient. The solution of this single-user problem is provided in [11]. We will refer to this problem as the *single-user problem* and its solution as the *single-user solution* in the rest of the paper. Due to the duality between solving the throughput maximization and transmission completion time minimization problems in the single-user scenario, we also refer to the solution in [1], [2] as the single-user solution.

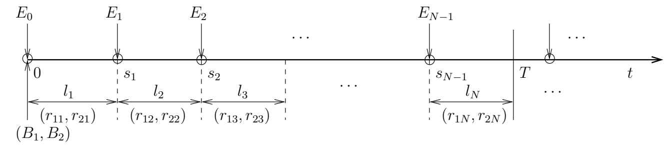


Fig. 4. Rates $(r_{1n}, r_{2n})$ and corresponding durations $l_n$ with a given deadline $T$.

The problem in (10) is a convex optimization problem with a convex cost function and a convex constraint set, therefore, the unique global solution should satisfy the extended KKT conditions. The Lagrangian is

$$\begin{aligned}\mathcal{L}(\mathbf{r}_1, \mathbf{r}_2, \boldsymbol{\lambda}, \boldsymbol{\gamma}) =& \mu_1 \sum_{n=1}^{N} r_{1n} l_n + \mu_2 \sum_{n=1}^{N} r_{2n} l_n \\ & - \sum_{j=1}^{N} \lambda_j \left( \sum_{n=1}^{j} g(r_{1n}, r_{2n}) l_n - \sum_{n=0}^{j-1} E_n \right) \\ & + \sum_{n=1}^{N} \gamma_{1n} r_{1n} + \sum_{n=1}^{N} \gamma_{2n} r_{2n} \end{aligned} \qquad (11)$$

Taking the derivatives with respect to $r_{1n}$ and $r_{2n}$, and setting them to zero, we have the optimality conditions

$$\mu_1 + \gamma_{1n} - \left( \sum_{j=n}^{N} \lambda_j \right) \sigma_1^2 2^{2(r_{1n} + r_{2n})} = 0 \qquad (12)$$

$$\mu_2 + \gamma_{2n} - \left( \sum_{j=n}^{N} \lambda_j \right) \left( \sigma_1^2 2^{2(r_{1n} + r_{2n})} + (\sigma_2^2 - \sigma_1^2) 2^{2 r_{2n}} \right) = 0 \qquad (13)$$

for $n = 1, \ldots, N$, with the complementary slackness conditions

$$\lambda_j \left( \sum_{n=1}^{j} g(r_{1n}, r_{2n}) l_n - \sum_{n=0}^{j-1} E_n \right) = 0, \quad j = 1, \ldots, N \qquad (14)$$

$$\gamma_{1n} r_{1n} = 0, \quad \gamma_{2n} r_{2n} = 0, \quad n = 1, \ldots, N \qquad (15)$$

Based on (12)-(15), we first prove an important property of the optimal policy.

**Lemma 3** *The optimal total transmit power of the transmitter is independent of the values of* $\mu_1, \mu_2$, *and it is the same as the single-user optimal transmit power. Specifically,*

$$i_n = \arg \min_{i_{n-1} < i \leq N} \left\{ \frac{\sum_{j=i_{n-1}}^{i-1} E_j}{s_i - s_{i_{n-1}}} \right\} \qquad (16)$$

$$P_n = \frac{\sum_{j=i_{n-1}}^{i_n - 1} E_j}{s_{i_n} - s_{i_{n-1}}} \qquad (17)$$

*i.e., at* $t = s_{i_n}$, $P_n$ *switches to* $P_{n+1}$.

The proof of Lemma 3 is provided in Appendix A.

Since the total transmit power can be obtained irrespective of the values of $\mu_1$, $\mu_2$, the optimization problem in (10) is separable over each duration $[s_{n-1}, s_n)$. Specifically, for $1 \leq$

$n \leq N$, the local optimization problem becomes

$$\max_{r_{1n}, r_{2n}} \quad \mu_1 r_{1n} + \mu_2 r_{2n}$$
$$\text{s.t.} \quad g(r_{1n}, r_{2n}) \leq P_n \tag{18}$$

We relax the power constraint to be an inequality to make the constraint set convex. Thus, this becomes a convex optimization problem. This does not affect the solution since the objective function is always maximized on the boundary of its constraint set, i.e., the capacity region defined by the transmit power $P_n$.

We first note that due to the degradedness of the second user, when $\frac{\mu_2}{\mu_1} \leq 1$, the total power $P_n$ is allocated to the first user only and no bits are transmitted to the second user. When $1 < \frac{\mu_2}{\mu_1}$, we define

$$P_c \triangleq \left( \frac{\mu_1 \sigma_2^2 - \mu_2 \sigma_1^2}{\mu_2 - \mu_1} \right)^+ \tag{19}$$

After a first order derivative analysis, we find the solution of (18) in terms of $P_c$ as follows

$$r_{1n} = \frac{1}{2} \log_2 \left(1 + \min\{P_c, P_n\}\right) \tag{20}$$
$$r_{2n} = \frac{1}{2} \log_2 \left(1 + \frac{(P_n - P_c)^+}{P_c + \sigma_2^2}\right) \tag{21}$$

In the optimal solution, power allocated to the first user can be at most $P_c$, and the remaining power is allocated to the second user. Hence, we call $P_c$ the *cut-off* power level.

**Lemma 4** *For fixed $\mu_1$, $\mu_2$, under the optimal power policy, there exists a constant cut-off power level, $P_c$, for the first user. If the total power level is below $P_c$, then, all the power is allocated to the first user; if the total power level is higher than $P_c$, then, all the power above $P_c$ is allocated to the second user.*

In the proof of Lemma 3 in Appendix A, we note that the optimal power $P_n$ monotonically increases in $n$. Combining Lemma 3 and Lemma 4, we illustrate the structure of the optimal policy in Fig. 5. Moreover, the optimal way of splitting the power in each epoch is such that both users' shares of the power monotonically increase in time. In particular, the second user's share is monotonically increasing in time. Hence, the path followed in the $(B_1, B_2)$ plane is such that it changes direction to get closer to the second user's departure axis as shown in Fig. 3. The dotted trajectory cannot be optimal, since the path does not get closer to the second user's departure axis in the middle (second) power epoch.

**Corollary 1** *Under the optimal policy, the transmission rate for the first user, $\{r_{1n}\}_{n=1}^N$, is either a constant sequence (zero or a positive constant), or an increasing sequence. Moreover, before $r_{1n}$ achieves the value at which it stays constant, we have $r_{2n} = 0$; and after $r_{1n}$ achieves the value at which it stays constant, $r_{2n}$ becomes a monotonically increasing sequence.*

Based on Lemma 3, we observe that for fixed $T$, $\mu_1$ and $\mu_2$, the optimal *total* power allocation is unique, i.e., does not depend on $\mu_1$ and $\mu_2$. However, the way the total power is split between the two users depends on $\mu_1$, $\mu_2$. In fact, the *cut-off* power level $P_c$ varies depending on the value of $\mu_2/\mu_1$. For different values of $\mu_2/\mu_1$, the optimal policy achieves different boundary points on the maximum departure region, and varying the value of $\mu_2/\mu_1$ traces the boundary of this region.

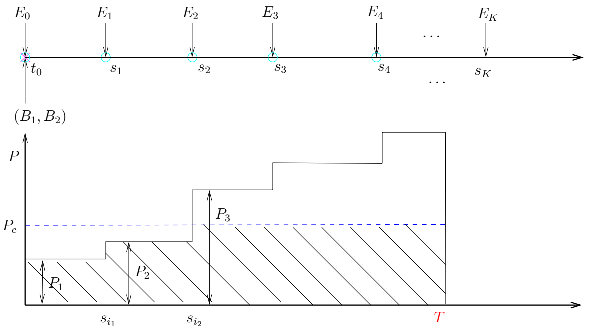


Fig. 5. Optimally splitting the total power between the signals that go to the two users.

## IV. $\mathcal{D}(T)$ FOR AN $M$-USER BROADCAST CHANNEL

The maximum departure region $\mathcal{D}(T)$ in the $M$-user broadcast channel is defined similar to the two-user case as the union of achievable $(B_1, \ldots, B_M)$ pairs where $B_i = \int_0^T r_i(\tau) d\tau$ and the instantaneous rates are subject to the energy causality constraint

$$\int_0^t g_{(M)}(r_1, \ldots, r_M)(\tau) d\tau \leq \sum_{i: s_i < t} E_i, \quad 0 \leq t \leq T \tag{22}$$

The structures of the optimal policy that achieves the boundary of $\mathcal{D}(T)$ established in Lemmas 3-4 and Corollary 1 naturally extend to the $M$-user case. First, note that Lemmas 1 and 2 immediately extend to the $M$-user case as $g_{(M)}(r_1, \ldots, r_M)$ is a strictly convex function. Hence, the boundary of $\mathcal{D}(T)$ is achieved by the unique solution of the following optimization problem for all $\mu_1, \ldots, \mu_M \geq 0$,

$$\max_{\mathbf{r}_1, \ldots, \mathbf{r}_M} \quad \mu_1 \sum_{n=1}^N r_{1n} l_n + \ldots + \mu_M \sum_{n=1}^N r_{Mn} l_n$$
$$\text{s.t.} \quad \sum_{n=1}^j g_{(M)}(r_{1n}, \ldots, r_{Mn}) l_n \leq \sum_{n=0}^{j-1} E_n, \ \forall j \tag{23}$$

where $\mathbf{r}_m$ denotes the rate sequence $\{r_{mn}\}_{n=1}^N$ for user $m$ and $N$ is the number of epochs in $[0, T]$. The corresponding Lagrangian is

$$\mathcal{L}(\mathbf{r}_1, \ldots, \mathbf{r}_M, \boldsymbol{\lambda}, \boldsymbol{\gamma}) = \mu_1 \sum_{n=1}^N r_{1n} l_n + \ldots + \mu_M \sum_{n=1}^N r_{Mn} l_n$$
$$- \sum_{j=1}^N \lambda_j \left( \sum_{n=1}^j g_{(M)}(r_{1n}, \ldots, r_{Mn}) l_n - \sum_{n=0}^{j-1} E_n \right)$$
$$+ \sum_{n=1}^N \gamma_{1n} r_{1n} + \ldots + \sum_{n=1}^N \gamma_{Mn} r_{Mn} \tag{24}$$

Taking the derivatives with respect to $r_{mn}$ for all $m, n$ and setting them to zero, we get the necessary KKT optimality conditions. By using the recursive formula in (7) and the

KKT optimality conditions, we can show that the optimal total power allocation is

$$g_{(M)}(r_{1n}^*, \ldots, r_{Mn}^*) = \max_m \left\{ \frac{\mu_m}{\sum_{i=n}^N \lambda_i} - \sigma_m^2 \right\} \quad (25)$$

As the complementary slackness conditions in (14)-(15) hold in the $M$-user case as well, we recover Lemma 3, i.e., the optimal total power sequence is exactly the same sequence as in (17) for the $M$-user case irrespective of the values of $\mu_i$, $i = 1, \ldots, M$.

Splitting the total power among $M$ users requires a cut-off power structure as in the two-user case that is stated in Lemma 4. Since the optimal total power is obtained irrespective of the values of $\mu_i$, the optimization problem in (23) is separable over each duration $[s_{n-1}, s_n)$. Specifically, for $1 \leq n \leq N$, the corresponding local optimization problem is

$$\begin{aligned} \max_{r_{1n},\ldots,r_{Mn}} \quad & \mu_1 r_{1n} + \ldots + \mu_M r_{Mn} \\ \text{s.t.} \quad & g_{(M)}(r_{1n}, \ldots, r_{Mn}) \leq P_n \end{aligned} \quad (26)$$

Whenever $\mu_j \leq \mu_i$ for any $1 \leq i < j \leq M$, i.e., whenever a degraded user has a smaller coefficient, the solution of (26) is such that $r_{jn}^* = 0$ for any value of $P_n$. Hence, we remove those users. The remaining $R \leq M$ users are such that $\sigma_1^2 \leq \sigma_2^2 \leq \ldots \leq \sigma_R^2$ with $\mu_1 < \mu_2 < \ldots < \mu_R$. Using a first order differential analysis, the optimal cut-off power levels for the remaining $R$ users must satisfy the following equations (see Appendix B): For $m = 1, \ldots, R-1$

$$P_{cm} = \max \left\{ \left( \frac{\mu_m \sigma_{\bar{m}}^2 - \mu_{\bar{m}} \sigma_m^2}{\mu_{\bar{m}} - \mu_m} \right)^+, P_{c(m-1)} \right\} \quad (27)$$

where by convention, we set $P_{c0} = 0$, $P_{cR} = \infty$ and $\bar{m}$ is the smallest user index with $P_{c\bar{m}} > P_{cm}$. We note that $P_{c0} \leq P_{c1} \leq \ldots \leq P_{c(R-1)} \leq P_{cR}$. For given $P_n$, the optimal solution is

$$r_{1n} = \frac{1}{2} \log \left( 1 + \frac{\min\{P_n, P_{c1}\}}{\sigma_1^2} \right) \quad (28)$$

$$r_{2n} = \frac{1}{2} \log \left( 1 + \frac{\min\{(P_n - P_{c1})^+, P_{c2} - P_{c1}\}}{P_{c1} + \sigma_2^2} \right) \quad (29)$$

$$\vdots$$

$$r_{Rn} = \frac{1}{2} \log \left( 1 + \frac{(P_n - P_{c(R-1)})^+}{P_{c(R-1)} + \sigma_R^2} \right) \quad (30)$$

We show the structure of optimally splitting the total power among the users for $R = M$ in Fig. 6. Note that the hierarchy among the channels of the users can be directly observed in Fig. 6. The top portion of the total power is allocated to the user with the worst channel and the power below it is interference for this user. The bottom portion of the total power is allocated to the user with the best channel and this user experiences no interference. We also note that the time sequence of the total power is the same as in the two-user case and the cut-off power levels are independent of the values of the varying total power levels. We also remark that the $R-1$ cut-off power levels are not necessarily distinct. When $P_{c(m+1)} = P_{cm}$ for some $1 \leq m \leq R-2$, we must have $r_{(m+1)n} = 0$ for all epochs $n$.

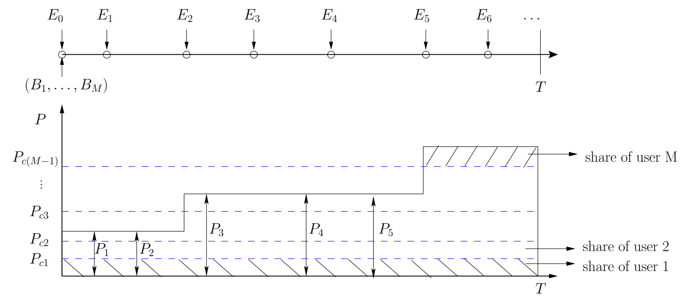


Fig. 6. Optimally splitting the total power for $M$ users.

## V. Minimizing the Transmission Completion Time $T$ for a Given $(B_1, \ldots, B_M)$

In this section, our goal is to minimize the transmission completion time of $(B_1, \ldots, B_M)$ bits. We start with the two-user case, and then generalize the algorithm to the $M$-user scenario.

### A. Two-User Scenario

We formulate the optimization problem as follows:

$$\begin{aligned} \min_{\mathbf{r}_1, \mathbf{r}_2} \quad & T \\ \text{s.t.} \quad & \sum_{n=1}^{j} g(r_{1n}, r_{2n}) l_n \leq \sum_{n=1}^{j-1} E_n, \quad 0 < j \leq N(T) \\ & \sum_{n=1}^{N(T)} r_{1n} l_n = B_1, \quad \sum_{n=1}^{N(T)} r_{2n} l_n = B_2 \end{aligned} \quad (31)$$

where $N(T) - 1$ is the number of energy arrivals (excluding $t = 0$) over $(0, T)$, and $l_{N(T)} = T - s_{N(T)-1}$. Since $N(T)$ depends on $T$, the optimization problem in (31) is not a convex optimization problem in general. Therefore, we cannot solve it using standard convex optimization tools.

We first note that this is exactly the dual problem of maximizing the departure region for fixed $T$. They are dual in the sense that, if the minimum transmission completion time for $(B_1, B_2)$ is $T$, then $(B_1, B_2)$ must lie on the boundary of $\mathcal{D}(T)$, and the transmission policy should be exactly the same for some $(\mu_1, \mu_2)$. This is based on the fact that $\mathcal{D}(T) \subset \mathcal{D}(T')$ for any $T < T'$. Assume $(B_1, B_2)$ does not lie on the boundary of $\mathcal{D}(T)$. Then, either $(B_1, B_2)$ cannot be achieved by $T$ or $(B_1, B_2)$ is strictly inside $\mathcal{D}(T)$ and hence $(B_1, B_2)$ can be achieved by $T' < T$. Therefore, if $(B_1, B_2)$ does not lie on the boundary of $\mathcal{D}(T)$, then $T$ cannot be the minimum transmission completion time. We have the following lemma.

**Lemma 5** *When $B_1, B_2 \neq 0$, under the optimal policy, the transmissions to both users must be finished at the same time.*

**Proof:** This lemma can be proved based on Corollary 1. If the transmission completion time for both users is not the same, then over the last duration, we transmit only to one of the users, while the transmission rate to the other user is zero. This contradicts with the monotonicity of the transmission rates for both users. Therefore, under the optimal policy, the transmitter must finish transmitting to both users at the same time. ■

Lemma 5 is proved in [17] also, by using a different approach. The authors prove it in [17] mainly based on the convexity of the capacity region of the broadcast channel.

According to Lemma 5, the problem of optimal selection of $P_c$ requires solving a *fixed point* equation. In particular, $P_c$ must be chosen such that the resulting transmission completion time for the first and second user are equal. Therefore, we propose the following algorithm to solve the transmission completion time minimization problem.

First, we aim to identify $P_1$, the first total transmit power starting from $t = 0$ in the system. This is exactly the same as identification of $P_1$ in the corresponding single-user problem. For this, as in [1], [2], we treat the energy arrivals as if they have arrived at time $t = 0$, and obtain a lower bound for the transmission completion time as in [1], [2]. We compute the minimum amount of energy required to finish $(B_1, B_2)$ by $s_1$. This requires to transmit at constant rates $(\frac{B_1}{s_1}, \frac{B_2}{s_1})$, and the amount of energy is equal to $g\left(\frac{B_1}{s_1}, \frac{B_2}{s_1}\right) s_1$, denoted as $A_1$. Then, we compare $A_1$ with $E_0$. If $E_0$ is greater than $A_1$, this implies that the transmitter can finish the transmission before $s_1$, and future energy arrivals are not needed. In this case, the minimum transmission completion time is the solution of the following equation

$$g\left(\frac{B_1}{T}, \frac{B_2}{T}\right) T = E_0 \tag{32}$$

If $A_1$ is greater than $E_0$, this implies that the final transmission completion time is greater than $s_1$, and some of the future energy arrivals must be utilized to complete the transmission. We calculate the amount of energy required to finish $(B_1, B_2)$ by $s_2, s_3, \ldots$, and denote them as $A_2, A_3, \ldots$, and compare these with $E_0 + E_1, \sum_{j=0}^{2} E_j, \sum_{j=0}^{3} E_j, \ldots$, until the first $A_i$ that becomes smaller than $\sum_{j=0}^{i-1} E_j$. We denote the corresponding time index as $\tilde{i}_1$. Then, we assume that we can use $\sum_{i=0}^{\tilde{i}_1-1} E_i$ to transmit $(B_1, B_2)$ at constant rates. And, the corresponding transmission completion time is the solution of the following equation

$$g\left(\frac{B_1}{T}, \frac{B_2}{T}\right) T = \sum_{i=0}^{\tilde{i}_1-1} E_i \tag{33}$$

We denote the solution of (33) as $\tilde{T}$, and the corresponding power as $\tilde{P}_1$. From our analysis, we know that the solution to this equation is a lower bound for the minimum transmission completion time. We check whether this constant power $\tilde{P}_1$ is feasible, when the actual energy arrival times are imposed. If it is feasible, it gives us the minimal transmission completion time; otherwise, we get $P_1$ by selecting the minimal slope according to (17). That is to say, we draw all of the lines from $t = 0$ to the corner points of the energy arrival instances before $\tilde{T}$, and choose the line with the smallest slope. We denote by $s_{i_1}$ the corresponding duration associated with $P_1$. Please see [2, Fig. 8] for a visualization of the algorithm.

Once $P_1$ is selected, it is the optimal total transmit power over the duration $[0, s_{i_1})$ in our broadcast channel problem. This is due to Lemma 3 and the fact that $(B_1, B_2)$ must lie on the boundary of the departure curve at the minimum transmission completion time. We defer the rigorous proof of optimality to Theorem 1 which immediately follows the algorithm. Next, we need to divide this total power between the signals transmitted to the two users. Based on Lemma 4 and Corollary 1, if the *cut-off* power level $P_c$ is higher than $P_1$, then, the transmitter spends all $P_1$ for the stronger user; otherwise, the first user finishes its transmission with a constant power $P_c$.

We will first determine whether $P_c$ lies in $[0, P_1]$ or it is higher than $P_1$. Assume $P_c = P_1$. The transmission completion time for the first (stronger) user is

$$T_1 = \frac{B_1}{f(P_1)} \tag{34}$$

Next, we calculate the maximum number of bits departed from the second user by $T_1$ given the set $P_c$, and denote it as $D_2(T_1, P_c)$:

$$D_2(T_1, P_c) = \sum_{i=1}^{N(T_1)} \frac{1}{2} \log\left(1 + \frac{[P_i^* - P_c]^+}{P_c + \sigma^2}\right)(s_i - s_{i-1}) \tag{35}$$

where $P_1^*, P_2^*, \ldots, P_{N(T_1)}^*$ is the optimal total power allocation by deadline $T_1$. $\{P_i^*\}_{i=1}^{N(T_1)}$ is found via single-user optimal power allocation (c.f. Lemma 3) by deadline $T_1$ as in [11]. Note that $P_1^* = P_1$.

If $D_2(T_1, P_1)$ is smaller than $B_2$, we need to decrease the rate for the first user. In this case, the transmission power for the first user is constant $P_c \in [0, P_1]$ throughout the entire duration. In particular, $P_c$ is the unique solution of

$$B_2 = D_2\left(\frac{B_1}{f(P_c)}, P_c\right) \tag{36}$$

Note that $D_2\left(\frac{B_1}{f(P_c)}, P_c\right)$ is a continuous, strictly monotonically decreasing function of $P_c$, hence the solution for $P_c$ in (36) exists and it is unique. We can use bisection method on $P_c$ to solve (36).

If $D_2(T_1, P_c)$ is larger than $B_2$, that implies $T_2 < T_1$, and we need to increase the power allocated for the first user, i.e., $P_c > P_1$. Therefore, according to Lemma 4, over the duration $[0, s_{i_1})$, the optimal policy is to allocate the entire $P_1$ to the first user only. We allocate $P_1$ to the first user, calculate the number of bits departed for the first user, and remove them from $B_1$. This simply reduces the problem to that of transmitting $(B_1', B_2)$ bits starting at time $t = s_{i_1}$, where $B_1' = B_1 - f(P_1)s_{i_1}$. The process is illustrated in Fig. 7. Then, the minimum transmission completion time is

$$T = s_{i_L} + \frac{B_1 - \sum_{k=1}^{L} f(P_k)(s_{i_k} - s_{i_{k-1}})}{f(P_c)} \tag{37}$$

where $L$ is the number of recursions needed to get $P_c$. In both scenarios, we reduce the problem into a simple form, and obtain the final optimal policy. We state our algorithm formally as Algorithm 1 below.

We have the following theorem which proves the optimality of the proposed algorithm. We provide the proof of this theorem in Appendix C.

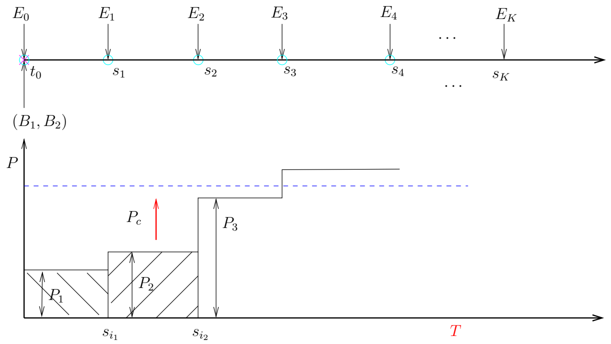

Fig. 7. Search for the cut-off power level $P_c$ iteratively.

**Algorithm 1** The algorithm to minimize the transmission completion time for a given $(B_1, B_2)$

Initialization: Set $n = 0$, $s_{i_0} = 0$, $P_0 = 0$.
**while** $B_1 > 0$ **do**
  $n = n + 1$;
  Determine $P_n$ and $s_{i_n}$ through the single-user method in [1], [2].
  Set $P_c = P_n$, $T_1 = \frac{B_1}{f(P_c)}$.
  Calculate $D_2(T_1, P_c)$, the maximum departures from the second user by $T_1 + s_{i_{n-1}}$ given $P_c$.
  **if** $B_2 \geq D_2(T_1, P_c)$ **then**
    Allocate $P_i$ to the first user over $[s_{i_{n-1}}, s_{i_n})$, update $B_1$.
  **else**
    Search for $P_c \in [P_{i-1}, P_i]$ s.t. $D_2(T_1, P_c) = B_2$ through bisection method.
  **end if**
**end while**

**Theorem 1** *The algorithm is feasible and optimal.*

### B. Generalizing the Algorithm for $M$ Users

In the $M$-user case, there are $M - 1$ *cut-off* power levels as shown in Fig. 6. Using this structure, we generalize the algorithm to find the minimum $T$. We first determine the total power level at the first epoch using the same approach with $g_{(M)}(r_1, \ldots, r_M)$ function. In particular, we assume that the energies are available at time $t = 0$ and calculate the energy necessary to send $(B_1, \ldots, B_M)$ by time $t = s_1$. Comparing this energy with $E_0$, we decide if minimum $T$ is smaller or larger than $s_1$ and proceed similarly to the two-user case to determine the initial total power level $P_1$.

Having determined $P_1$, we now decide whether $P_{c1} > P_1$ or otherwise. We set $P_{c1} = P_1$ and calculate

$$T_1 = \frac{B_1}{f(P_{c1})} \tag{38}$$

We need to determine whether the remaining bits $(B_2, \ldots, B_M)$ can be sent by $T_1$. Because of the cut-off structure of the optimal policy in the $M$-user scenario, we search for the cut-off power level for users 2, 3, …, $M$ in a sequential way. The cut-off power levels are selected to ensure that the maximum number of departures from the $m$th user equals $B_m$, $m \geq 2$. If such feasible cut-off power levels for all of the remaining $M - 1$ users can be obtained over $[0, T_1)$, it implies that the minimum transmission completion time $T \leq T_1$, and $P_{c1} \leq P_1$. Otherwise, once we find that the cut-off power level for a user is infeasible, it implies that not all of the users can be served by $T_1$, and $P_{c1} > P_1$.

First, we obtain the optimal total power allocation by deadline $T_1$, denoted as $P_2^*, \ldots, P_{N(T_1)}^*$. We set $P_{c2} = P_2^*$. If $f(\frac{P_{c2}-P_{c1}}{P_{c1}+\sigma_2^2})(T_1 - s_{i_1}) > B_2$, the optimal cut-off power level $P_{c2} \in [P_{c1}, P_2^*]$ and it satisfies $f(\frac{P_{c2}-P_{c1}}{P_{c1}+\sigma_2^2})(T_1 - s_{i_1}) = B_2$. Otherwise, we set $P_{c2} = P_3^*$ and repeat the same procedure until the optimal $P_{c2}$ is achieved. The remaining cut-off power levels can be determined in a similar manner. If any of these optimal power levels becomes infeasible, i.e., takes a value above $P_{N(T_1)}^*$, we get back to $P_{c1}$ and adjust it accordingly. The feasibility and the optimality of the algorithm can be proved through similar steps in Appendix C and is omitted for the brevity of the paper.

### C. Computation Requirement of the Proposed Algorithm

In a real life implementation of the algorithm, the iterations of the algorithm are stopped when sufficient accuracy is reached. For given $P_{c1}$, the main computational block of the algorithm is composed of two stages: In the first stage, we calculate the single-user optimal solution, i.e., the optimal power sequence $P_1^*, \ldots, P_{N(T_1)}^*$ given the deadline $T_1$. In the second stage, we determine whether the remaining bits $(B_2, \ldots, B_M)$ can be sent by $T_1$. The optimal power sequence $P_1^*, \ldots, P_{N(T_1)}^*$ can be found as in [1], [2] by a geometric framework which requires drawing lines to the energy arrival points and selecting the line with the minimum slope. Denoting the number of epochs as $N$, calculation of the minimum of $O(N)$ elements requires $O(N)$ comparisons and we repeat it $O(N)$ times. Hence, the first stage of the main computational block requires $O(N^2)$ operations. We can perform the single-user optimization also by the directional water-filling algorithm in [11] which also has $O(N^2)$ complexity. In the second stage, the algorithm goes through $P_i^*$, calculates the remaining cut-off power levels and determines whether the remaining bits $(B_2, \ldots, B_M)$ can be sent by $T_1$. Since the optimal cut-off power level increases as the user index increases, and we always start with the lowest possible power level, this stage requires $O(M + N)$ operations in the worst case. Hence, the main block of the algorithm requires $O(N^2 + M + N)$ computations.

The main block is run each time the first user's cut-off power level $P_{c1}$ is updated. In the algorithm that we presented, $P_{c1}$ is updated recursively as in (37). After deciding the range that $P_{c1}$ lies, we perform bisection method until the desired accuracy is reached. The number of recursions $L$ in (37) is $O(N)$ in the worst case and the bisection search is at most $O(\log(\frac{1}{\epsilon}))$ where $\epsilon$ is the desired accuracy. Therefore, the total number of iterations scale as $O(N)$ and the algorithm requires $O((N^2+M+N)(N-\log \epsilon))$ operations overall. Here, the total number of epochs $K$ upper bounds the number of epochs $N$. Therefore, the overall number of operations needed is upper bounded by $O((K^2 + M + K)(K - \log \epsilon))$.

## VI. Simulations

We consider a band-limited AWGN broadcast channel with $M = 3$ users. The bandwidth is $B_W = 1$ MHz and the noise power spectral density is $N_0 = 10^{-19}$ W/Hz. We assume that the path losses between the transmitter and the receivers are 100 dB, 105 dB and 110 dB.

$$r_1 = B_W \log_2\left(1 + \frac{\alpha_1 P h_1}{N_0 B_W}\right) = \log_2\left(1 + \frac{\alpha_1 P}{10^{-3}}\right) \text{ Mbps} \quad (39)$$

$$r_2 = B_W \log_2\left(1 + \frac{\alpha_2 P h_2}{\alpha_1 P h_2 + N_0 B_W}\right) = \log_2\left(1 + \frac{\alpha_2 P}{\alpha_1 P + 10^{-2.5}}\right) \text{ Mbps} \quad (40)$$

$$r_3 = B_W \log_2\left(1 + \frac{(1-\alpha_1-\alpha_2) P h_2}{(\alpha_1+\alpha_2) P h_2 + N_0 B_W}\right) = \log_2\left(1 + \frac{(1-\alpha_1-\alpha_2) P}{(\alpha_1+\alpha_2) P + 10^{-2}}\right) \text{ Mbps} \quad (41)$$

Therefore, we have

$$g(r_1, r_2, r_3) = 10^{-3} 2^{r_1+r_2+r_3} + (10^{-2.5} - 10^{-3}) 2^{r_2+r_3} + (10^{-2} - 10^{-2.5}) 2^{r_3} - 10^{-2} \quad \text{W} \quad (42)$$

### A. Deterministic Energy Arrivals

In this subsection, we illustrate the off-line optimal policy in a deterministic energy arrival sequence setting. In particular, we assume that at times $\mathbf{t} = [0, 5, 6, 8, 9, 11]$ s, energies with the following amounts are harvested: $\mathbf{E} = [20, 10, 3.5, 8, 10, 10]$ mJ.

We first study the two-user broadcast channel by removing the third user, i.e., setting $B_3 = 0$. We find the maximum departure region of the two-user broadcast channel $\mathcal{D}(T)$ for $T = 6, 8, 9, 10$ s, and plot them in Fig. 8. Note that the maximum departure regions are convex, and as $T$ increases, $\mathcal{D}(T)$ monotonically expands.

We next consider the same energy arrival sequence with $(B_1, B_2) = (21, 2)$ Mbits. We have the optimal transmission policy, as shown in Fig. 9. In this example, the cut-off power is less than $P_1$ and hence the power share (and the rate) of the first user is constant throughout the interval in which the bits are transmitted. The transmitter finishes its transmission by time $T = 9.28$ s, and the last energy harvest is not used. Note that $(21, 2)$ Mbits point (marked with *) in Fig. 8 is not included in $\mathcal{D}(T)$ at $T = 9$ s while it is strictly included in $\mathcal{D}(T)$ at $T = 10$ s.

Finally, we consider the same energy arrival sequence with $(B_1, B_2, B_3) = (12, 6, 3)$ Mbits and the optimal policy is shown in Fig. 10. We calculate the cut-off power levels as $P_{c1} = 0.963$ mW and $P_{c2} = 2.619$ mW. The bits of all three users are always transmitted throughout the communication. The last energy arrival is used in this case and the transmission is finished by $T = 12.33$ s.

### B. Stochastic Energy Arrivals

In this subsection, we consider stochastic energy arrivals in the two-user case, i.e., we set $B_3 = 0$. We compare the performance of the off-line optimal policy with those of three suboptimal policies. These policies are inspired by the optimal off-line policy while they require partial or no off-line knowledge of the energy arrivals.

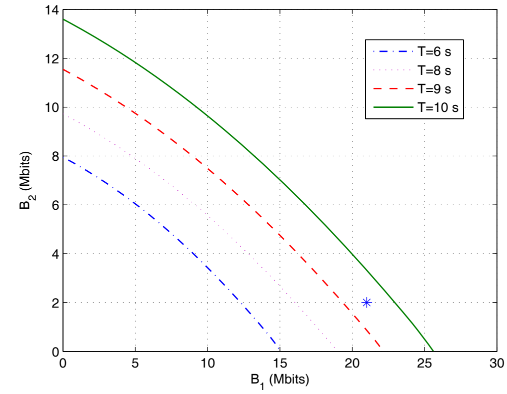


Fig. 8. The maximum departure region of the broadcast channel for various $T$.

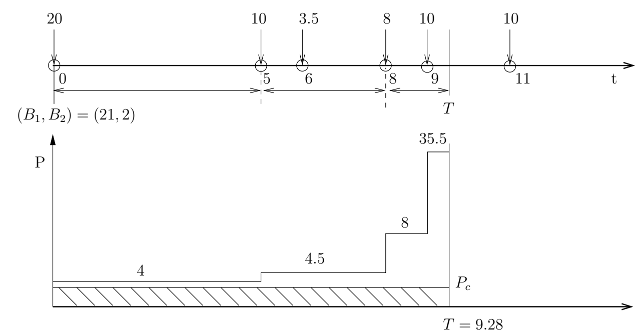


Fig. 9. Cut-off power $P_c = 3.798$ mW. Optimal transmit rates $\mathbf{r}_1 = [2.262, 2.262, 2.262, 2.262]$ Mbps and $\mathbf{r}_2 = [0.041, 0.1386, 0.6814, 2.4723]$ Mbps, with durations $\mathbf{l} = [5, 3, 1, 0.28]$ s.

*1) Constant Power Constant Share (CPCS) Policy:* This policy transmits with constant power equal to the average recharge rate, $P = \mathbb{E}[E]$, whenever the battery energy is non-zero and the transmitter is silent otherwise. In addition, the strong user's power share is constant whenever the transmitter is non-silent. In particular, the constant power share is set to $\alpha^*$ which is the solution of the following equation:

$$\frac{B_1}{B_2} = \frac{\log_2\left(1 + \frac{\alpha \mathbb{E}[E]}{\sigma_1^2}\right)}{\log_2\left(1 + \frac{(1-\alpha)\mathbb{E}[E]}{\alpha \mathbb{E}[E] + \sigma_2^2}\right)} \quad (43)$$

Note that CPCS does not require off-line or on-line knowledge of the energy arrivals. It requires the first order statistics, i.e., the mean, of the energy arrival process.

*2) Greedy Power Constant Share (GPCS) Policy:* This policy also keeps the total transmit power share constant equal to the solution of (43) throughout the transmission. However, different from the CPCS policy, the power is greedily updated. The policy assumes knowledge of the time instant at which the next energy arrival occurs and at the start of the $i$th epoch, the available energy is allocated to the next epoch only and hence the power is set to $P_i = \frac{E_{i-1}}{\ell_i}$. Note that GPCS requires partial

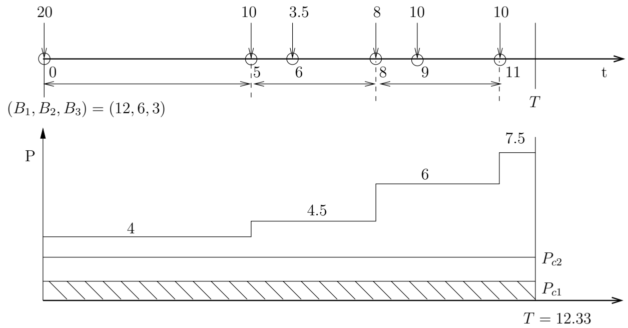


Fig. 10. Cut-off power levels $P_{c1} = 0.963$ mW and $P_{c2} = 2.619$ mW. Optimal transmit rates $\mathbf{r}_1 = [0.9731, 0.9731, 0.9731, 0.9731]$ Mbps, $\mathbf{r}_2 = [0.4869, 0.4869, 0.4869, 0.4869]$ Mbps and $\mathbf{r}_3 = [0.1498, 0.2005, 0.3425, 0.4718]$ Mbps with durations $\mathbf{l} = [5, 3, 3, 1.33]$ s.

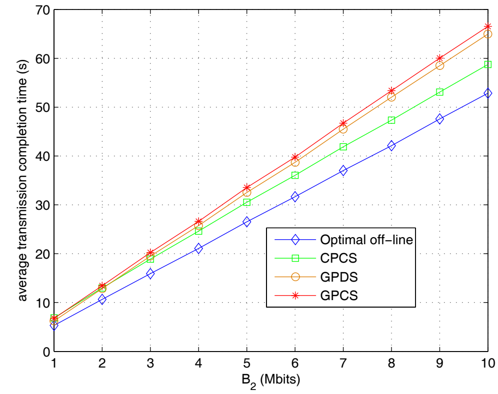


Fig. 11. Average transmission completion time versus $B_2$ when $d = 3$ and $P_{avg} = 1$ mJ/s.

off-line knowledge of the energy arrivals as well as the first order statistics, i.e., the mean, of the energy arrival process.

*3) Greedy Power Dynamic Share (GPDS) Policy:* This policy allocates power greedily in each epoch and the power shares are dynamically updated. In particular, the policy assumes knowledge of the time instant at which the next energy arrival occurs and at the start of the $i$th epoch, the arriving energy $E_{i-1}$ is spread over $\ell_i$. For $P_i = \frac{E_{i-1}}{\ell_i}$, $\alpha_i^*$ is calculated as the solution of the following equation:

$$\frac{B_{1i}}{B_{2i}} = \frac{\log_2(1 + \alpha P_i)}{\log_2\left(1 + \frac{(1-\alpha)P_i}{\alpha P_i + \sigma^2}\right)} \tag{44}$$

where $B_{1i}$ and $B_{2i}$ are the remaining bits of user 1 and user 2, respectively, at the beginning of epoch $i$. Note that this policy essentially performs the initialization in [17] and it requires partial off-line knowledge of the energy arrivals as well as on-line knowledge of the data backlog.

In the simulations, we consider a compound Poisson energy arrival process. The average inter-arrival time is 1 s and the arriving energy is a random variable which is distributed uniformly in $[0, 2P_{avg}]$ mJ, where $P_{avg}$ is the average recharge rate. The performance metric of the policies is the average transmission completion time over 1000 realizations of the stochastic energy arrival process. We first set the $d = \frac{B_1}{B_2}$ ratio constant, i.e., $B_1 = dB_2$. We plot the performances for $d = 3$ and $P_{avg} = 1$ mJ/s with varying $B_2$ in Fig. 11. We observe that the average transmission completion times of the policies increase with the number of bits. It is notable that CPCS policy performs better with respect to the greedy power policies even though greedy power policies use partial off-line information. Hence, transmitting with constant power proves to be useful as observed in the single-user case in [1], [2], [11]. We also observe that dynamically varying the power shares of the users yields better performance compared to the constant case. Next, we plot the variation of the average transmission completion time with respect to the average recharge rate in Fig. 12. CPCS policy performs better with respect to the greedy power policies for small average recharge rates; however, GPDS policy outperforms CPCS in the high average recharge rate regime. Therefore, adapting the power share according to user loads proves to be useful in the high average recharge rate regime.

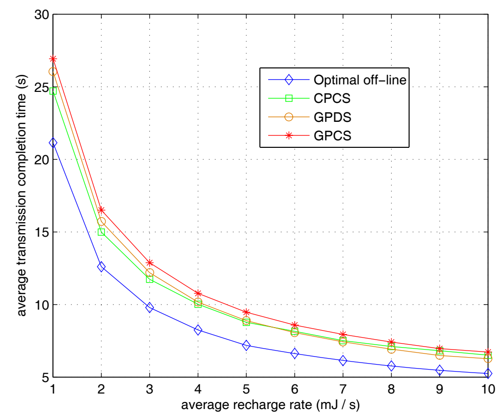


Fig. 12. Average transmission completion time versus average recharge rate when $B_1 = 12$ Mbits and $B_2 = 4$ Mbits.

## VII. Conclusions

We investigated the transmission completion time minimization problem in an energy harvesting broadcast channel. We first analyzed the structural properties of the optimal transmission policy, and proved that the optimal total transmit power has the same structure as in the single-user channel. We also proved that there exists a *cut-off* power for the stronger user. If the optimal total transmit power is lower than this cut-off level, all power is allocated to the stronger user, and otherwise, all power above this level is allocated to the weaker user. We then extended our results to the $M$-user broadcast channel for which total power sequence has the same structure as the two-user case and the optimal splitting of the total power requires $M - 1$ cut-off levels. Based on the structure of the optimal policy, we developed an iterative algorithm to obtain the globally optimal off-line transmission policy. Finally, we provided an extensive numerical analysis of the optimal policy and compared its performance with suboptimal policies under different settings.

## APPENDIX A
## PROOF OF LEMMA 3

According to the expression of $g(r_{1n}, r_{2n})$ in (5) and the KKT conditions in (12)-(13), we have

$$g(r_{1n}, r_{2n}) = \frac{\mu_2 + \gamma_{2n}}{\sum_{j=n}^N \lambda_j} - \sigma_2^2 \tag{45}$$
$$\geq \sigma_1^2 \left(2^{2(r_{1n}+r_{2n})} - 1\right) \tag{46}$$
$$= \frac{\mu_1 + \gamma_{1n}}{\sum_{j=n}^N \lambda_j} - \sigma_1^2 \tag{47}$$
$$\geq \frac{\mu_1}{\sum_{j=n}^N \lambda_j} - \sigma_1^2 \tag{48}$$

where (46) becomes an equality when $r_{2n} = 0$. Therefore, when $r_{2n} > 0$, (45)-(48) imply

$$g(r_{1n}, r_{2n}) = \frac{\mu_2}{\sum_{j=n}^N \lambda_j} - \sigma_2^2 > \frac{\mu_1}{\sum_{j=n}^N \lambda_j} - \sigma_1^2 \tag{49}$$

When $r_{2n} = 0$, we must have $r_{1n} > 0$. Otherwise, if $r_{1n} = 0$, we can always let the weaker user transmit with some power over this duration without contradicting with any energy constraints. Since there is no interference from the stronger user, the departure from the weaker user can be improved, thus it contradicts with the optimality of the policy. Therefore, when $r_{2n} = 0$, $\gamma_{1n} = 0$, and (45)-(48) imply

$$g(r_{1n}, r_{2n}) = \frac{\mu_1}{\sum_{j=n}^N \lambda_j} - \sigma_1^2 > \frac{\mu_2}{\sum_{j=n}^N \lambda_j} - \sigma_2^2 \tag{50}$$

Therefore, we can express $g(r_{1n}, r_{2n})$ in terms of the Lagrange multipliers as follows:

$$g(r_{1n}, r_{2n}) = \max\left\{\frac{\mu_1}{\sum_{j=n}^N \lambda_j} - \sigma_1^2, \frac{\mu_2}{\sum_{j=n}^N \lambda_j} - \sigma_2^2\right\} \tag{51}$$

If $\frac{\mu_2}{\sum_{j=n}^N \lambda_j} - \sigma_2^2 > \frac{\mu_1}{\sum_{j=n}^N \lambda_j} - \sigma_1^2$ for some $\bar{n}$, then, we have

$$\frac{\mu_2 - \mu_1}{\sum_{j=n}^N \lambda_j} \geq \frac{\mu_2 - \mu_1}{\sum_{j=\bar{n}}^N \lambda_j} > \sigma_2^2 - \sigma_1^2, \quad \forall n > \bar{n} \tag{52}$$

where the first inequality follows from $\lambda_j \geq 0$ for $j = 1, 2, \ldots N$. Therefore, we conclude that there exists an integer $\bar{n}$, $0 \leq \bar{n} \leq N$, such that, when $n \leq \bar{n}$, $r_{2n} = 0$; and when $n > \bar{n}$, $r_{2n} > 0$.

Furthermore, (49)-(50) imply that the energy constraint at $t = s_{\bar{n}}$ must be tight. Otherwise, by the complementary slackness conditions in (14), $\lambda_{\bar{n}} = 0$, and (50) implies

$$g(r_{1\bar{n}}, r_{2\bar{n}}) = \frac{\mu_1}{\sum_{j=\bar{n}+1}^N \lambda_j} - \sigma_1^2$$
$$> \frac{\mu_2}{\sum_{j=\bar{n}+1}^N \lambda_j} - \sigma_2^2 = g(r_{1,\bar{n}+1}, r_{2,\bar{n}+1}) \tag{53}$$

which contradicts with (49). Therefore, in the following, when we consider the energy constraints, we only need to consider two segments $[0, s_{\bar{n}})$ and $[s_{\bar{n}+1}, s_N)$ separately.

When $n < \bar{n}$, based on (49), if $\lambda_n = 0$, we have $g(r_{1n}, r_{2n}) = g(r_{1,n+1}, r_{2,n+1})$. Starting from $n = 1$, $g(r_{1n}, r_{2n})$ remains constant until an energy constraint becomes tight. Therefore, between any two consecutive epochs, when the energy constraints are tight, the power level remains constant. Similar arguments hold when $n \geq \bar{n}$. Thus, the corresponding power level is

$$P_n = \frac{\sum_{j=i_{n-1}}^{i_n-1} E_j}{s_{i_n} - s_{i_{n-1}}} \tag{54}$$

where $s_{i_{n-1}}$ and $s_{i_n}$ are two consecutive epochs with tight energy constraint.

Finally, we need to determine the epochs when the energy constraint becomes tight. We observe that $g(r_{1\bar{n}}, r_{2\bar{n}})$ must monotonically increase in $n$, as $\lambda_n \geq 0$. Hence, the individual terms in the $\max\{.,.\}$ function in (51) are monotonically increasing. In addition, both terms in the $\max\{.,.\}$ function strictly increases when energy constraint becomes tight. Therefore, we conclude that

$$i_n = \arg \min_{i_{n-1} < i \leq N} \left\{\frac{\sum_{j=i_{n-1}}^{i-1} E_j}{s_i - s_{i_{n-1}}}\right\} \tag{55}$$

This completes the proof.

## APPENDIX B
## THE CUT-OFF POWER LEVELS

We solve the $M$-variable local optimization problem in (26) by using a Lagrangian analysis. Specifically, the Lagrangian function is

$$H(r_{1n}, \ldots, r_{Mn}, \boldsymbol{\alpha}, \gamma) = \sum_{m=1}^M \mu_m r_{mn} + \sum_{m=1}^M \alpha_m r_{mn} - \gamma[g_{(M)}(r_{1n}, \ldots, r_{Mn}) - P_n]$$

where the Lagrange multipliers $\gamma$ and $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_M]$ satisfy $\gamma, \alpha_m \geq 0$ with the complimentary slackness conditions

$$\gamma[g_{(M)}(r_{1n}, \ldots, r_{Mn}) - P_n] = 0, \quad \alpha_m r_{mn} = 0 \tag{56}$$

Taking the derivative of the Lagrangian $H$ with respect to $r_{mn}$, and setting them to zero, we have

$$\mu_m + \alpha_m - \gamma' 2^{2\sum_{i=m}^M r_{in}}(g_{(m-1)} + \sigma_m^2) = 0, \tag{57}$$

for $m = 1, 2, \ldots, M$, where $\gamma' = (2\ln 2)\gamma$. Because of the nonnegativity of $\mu_m$ and $\alpha_m$, in order to have a solution satisfying the KKT conditions, we must have $\gamma' > 0$. Then, considering two consecutive equations, we have

$$\frac{\mu_m + \alpha_m}{\mu_{m-1} + \alpha_{m-1}} = \frac{g_{(m-1)} + \sigma_m^2}{g_{(m-2)} + \sigma_{m-1}^2} \cdot \frac{1}{2^{2r_{(m-1)n}}}$$
$$= \frac{g_{(m-1)} + \sigma_m^2}{g_{(m-2)} + \sigma_{m-1}^2} \cdot \frac{1}{1 + \frac{g_{(m-1)} - g_{(m-2)}}{g_{(m-2)} + \sigma_{m-1}^2}}$$
$$= \frac{g_{(m-1)} + \sigma_m^2}{g_{(m-1)} + \sigma_{m-1}^2} \tag{58}$$

If $\alpha_m = 0$, i.e., $g_{(m)} > g_{(m-1)}$, we have

$$g_{(m-1)} = \max\left\{\left(\frac{\mu_{m-1}\sigma_m^2 - \mu_m \sigma_{m-1}^2}{\mu_m - \mu_{m-1}}\right)^+, g_{(m-2)}\right\}$$

If $\alpha_m \neq 0$, i.e., $g_{(m)} = g_{(m-1)}$, and $\overline{m-1}$ is the smallest user index with $g_{(\overline{m-1})} > g_{(m-1)}$, we have

$$\frac{\mu_{\overline{m-1}}}{\mu_{m-1}+\alpha_{m-1}} = \frac{g_{(\overline{m-1}-1)}+\sigma^2_{\overline{m-1}}}{g_{(m-2)}+\sigma^2_{m-1}} \cdot \frac{1}{2^{2r_{(m-1)n}}} = \frac{g_{(m-1)}+\sigma^2_{\overline{m-1}}}{g_{(m-1)}+\sigma^2_{m-1}} \tag{59}$$

where (59) follows from the fact that $g_{(\overline{m-1}-1)} = g_{(m-1)}$. Thus,

$$g_{(m-1)} = \max\left\{\left(\frac{\mu_{m-1}\sigma^2_{\overline{m-1}} - \mu_{\overline{m-1}}\sigma^2_{m-1}}{\mu_{\overline{m-1}} - \mu_{m-1}}\right)^+, g_{(m-2)}\right\}$$

Therefore, the cut-off power levels are determined in the general form as in (27).

## Appendix C
## Proof of Theorem 1

Before we proceed to prove the optimality of the algorithm, we introduce the following lemma first, which is useful in the proof of the optimality of the algorithm.

**Lemma 6** *For any* $\alpha \in [0,1]$, $f\left(\frac{\alpha E/T}{(1-\alpha)E/T+\sigma^2}\right)T$ *monotonically increases in* $T$.

**Proof:** The monotonicity can be verified by taking derivatives. We have

$$\left(f\left(\frac{\alpha E/T}{(1-\alpha)E/T+\sigma^2}\right)T\right)' = \frac{1}{2}\log_2\left(\sigma^2+E/T\right) - \frac{1}{2}\log_2\left(\sigma^2+(1-\alpha)E/T\right) - \frac{E}{2\ln 2}\frac{E}{E+\sigma^2 T} + \frac{E}{2\ln 2}\frac{(1-\alpha)E}{(1-\alpha)E+\sigma^2 T} \tag{60}$$

and

$$\left(f\left(\frac{\alpha E/T}{(1-\alpha)E/T+\sigma^2}\right)T\right)'' = \frac{E^2}{2T\ln 2}\left(\frac{1}{(\sigma^2 T/(1-\alpha)+E)^2} - \frac{1}{(\sigma^2 T+E)^2}\right) < 0$$

where the last inequality follows as $E > 0$. Hence, $f\left(\frac{\alpha E/T}{(1-\alpha)E/T+\sigma^2}\right)T$ is a strictly concave function of $T$. Note that each term in the right hand side of (60) goes to 0 as $T \to \infty$. Thus,

$$\lim_{T\to\infty}\left(f\left(\frac{\alpha E/T}{(1-\alpha)E/T+\sigma^2}\right)T\right)' = 0.$$

Combining with the strict concavity, we conclude that the first derivative is positive when $T < \infty$, and the monotonicity follows. ■

We then prove the optimality of the algorithm. In order to prove that the algorithm is optimal, we need to prove that $P_1$ is optimal. Once we prove the optimality of $P_1$, the optimality of $P_2$, $P_3$, ... follows. Since the solution obtained using our algorithm always has the optimal structure described in Lemma 4, the optimality of the power allocation also implies the optimality of the rate selection, thus, the optimality of the algorithm follows. Therefore, in the following, we prove that $P_1$ is optimal.

First, we note that $P_1$ is the minimal slope up to $\widetilde{T}$. We need to prove that $P_1$ is also the minimal slope up to the final transmission completion time, $T$. Let us define $T'$ as follows

$$T' = \frac{\sum_{n=0}^{\tilde{i}_1} E_n}{P_1} \tag{61}$$

Assume that with $\tilde{P}_1$, we allocate $\alpha\tilde{P}_1$ to the first user, and finish $(B_1, B_2)$ using constant rates. Then, we allocate $\alpha P_1$ to the first user, and the rest to the second user. Based on Lemma 6, we have

$$f(\alpha P_1)T' \geq f(\alpha\tilde{P}_1)\tilde{T} = B_1 \tag{62}$$

$$f\left(\frac{\alpha P_1}{(1-\alpha)P_1+\sigma^2}\right)T' \geq f\left(\frac{\alpha\tilde{P}_1}{(1-\alpha)\tilde{P}_1+\sigma^2}\right)\tilde{T} = B_2 \tag{63}$$

Therefore, $T'$ is an upper bound for the optimal transmission completion time. Since $P_1$ is the minimal slope up to $T'$, we conclude that $P_1$ is optimal throughout the transmission. Following similar arguments, we can prove the optimality of the rest of the power allocations. This completes the proof of optimality.

In order to prove that the allocation is feasible, we need to show that the power allocation for the first user is always feasible in each step. Therefore, in the following, we first prove that $P_1$ is feasible when we assume that $P_c = P_1$. The feasibility of $P_1$ also implies the feasibility of the rest of the power allocation. With the assumption that $P_c = P_1$, the final transmission time for the first user is

$$T_1 = \frac{B_1}{f(P_1)} \leq \frac{B_1}{f(\alpha P_1)} \tag{64}$$

Based on (62) and (64), we know that $T_1 < T'$. Since $P_1$ is feasible up to $T'$, therefore, $P_1$ is feasible when we assume that $P_c = P_1$. The feasibility of the rest of the power allocations follows in a similar way. This completes the feasibility part of the proof.

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