Scaling Laws for Age of Information in Wireless Networks

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Abstract

We study age of information in a multiple source-multiple destination setting with a focus on its scaling in large wireless networks. There are n nodes uniformly and independently distributed on a fixed area that are randomly paired with each other to form n source-destination (S-D) pairs. Each source node wants to keep its destination node as up-to-date as possible. To accommodate successful communication between all n S-D pairs, we first propose a three-phase transmission scheme which utilizes local cooperation between the nodes along with what we call *mega update packets* to serve multiple S-D pairs at once. We show that under the proposed scheme average age of an S-D pair scales as $O(n^{\frac{1}{4}}\log n)$ as the number of users, n, in the network grows. Next, we observe that communications that take place in Phases I and III of the proposed scheme are scaled-down versions of network-level communications. With this along with scale-invariance of the system, we introduce hierarchy to improve this scaling result and show that when *hierarchical cooperation* between users is utilized, an average age scaling of $O(n^{\alpha(h)}\log n)$ per-user is achievable where h denotes the number of hierarchy levels and $\alpha(h)=\frac{1}{3\cdot 2^h+1}$ which tends to 0 as h increases such that asymptotically average age scaling of the proposed hierarchical scheme is $O(\log n)$. To the best of our knowledge, this is the best average age scaling result in a status update system with multiple S-D pairs.

I. Introduction

From smart homes to stock market and unmanned aerial vehicle (UAV) systems many recent applications involve real-time monitoring of a phenomenon of interest. In these applications,

This work was supported by NSF Grants CNS 15-26608, CCF 17-13977 and ECCS 18-07348. This paper was presented in part at IEEE ICC, Shanghai, China, May 2019.

resulting measurements obtained by the sources are sent to the interested recipient nodes in the form of status update packets. Common to all these applications is the need of timely delivery of these packets since more recent measurements better capture the source process. In other words, in these systems information loses its value as it becomes stale. In order to measure the freshness of the received information, age of information (AoI) metric has been proposed. Age tracks the time elapsed since the most recent update packet at the destination node was generated at the source node. In other words, at time t, age $\Delta(t)$ of a packet which has a timestamp $u(t)$ is $\Delta(t) = t - u(t)$. Age of information has been extensively studied in the literature in queueing, optimization and scheduling settings in references [1]–[11] and in an energy harvesting setting in references [12]–[21].

With increasing connectivity in communication networks and rapidly growing number of information sources (both people and sensors), the issue of scalability of age of information has emerged. In early 00's, followed by the pioneering work of Gupta and Kumar [22] a similar issue had come up for scaling laws of *throughput* in large networks. This work uses a multi-hop scheme that achieves a total throughput of $O(\sqrt{n})$ for the network, and hence, $O(\frac{1}{\sqrt{n}})$ throughput per-user. References [23]–[26] studied throughput scaling in dense and extended networks considering static and mobile nodes. This line of research has culminated in the seminal papers of Ozgur et al. [27], [28] which achieved $O(1)$ throughput per-user by utilizing hierarchical cooperation between nodes. In this paper, we study scaling of *age of information* in large wireless networks.

What makes age analysis in large networks challenging is the fact that good age performance corresponds to neither high throughput nor low delay. As argued in references [29] and [30], for the optimized age performance we need regular packet delivery with low delay. The way to achieve the maximum throughput is to send as many updates as possible from the source. However, this may cause congestion in the system resulting in stale packet deliveries at the destination. Likewise, packet delay in the network can be reduced by decreasing the update frequency which in turn yields outdated information at the destination since the update delivery rate is low. In this paper, we balance these two opposing objectives, and develop an achievable scheme that strikes a balance between the two in large networks.

References that are most closely related to our work study the scaling of age of information in the broadcast setting [31]–[35]. These works study a single source node which sends status updates to multiple receiver nodes. Reference [31] studies a mobile social network with a single service provider and n communicating users, and shows that under Poisson *contact processes* among users and uniform rate allocation from the service provider, the average age of the content at the users is $O(\log n)$. Without the contact process between users, however, age grows linearly in n since the service provider serves only one user at a time. In [32]–[35], single and multihop multicast networks are considered and $O(1)$ average age is obtained at the end nodes by using special transmission schemes such as the earliest k transmission scheme in which the source node waits for delivery to the earliest k out of the total n receiver nodes. Reference [36], on the other hand, studies age scaling in the multiaccess setting with massive number of source nodes.

In this work, we focus on a multiple source-multiple destination setting and study a fixed area network of n randomly located source-destination (S-D) pairs that want to send time-sensitive update packets to each other. Each node is both a source and a destination. We aim to find a transmission scheme which allows all n S-D pairs to successfully communicate and achieves the smallest average age scaling per-user.

As studied in [36], a straightforward way to achieve successful communication between all S-D pairs is to use a round-robin policy such that at each turn only one source transmits to its destination and stays silent while all other sources transmit successively during their respective turns. This direct method achieves an age scaling of $O(n)$ meaning that age increases linearly in n since under this policy average inter-update times at a destination node increases linearly as n grows making the updates less frequent and causing age to increase.

As in the setting of [22], a multihop scheme that involves successive transmissions between the source and destination nodes can be utilized. In that work, the network is divided into cells and hops take place in between these cells such that $O(\sqrt{n})$ messages are carried by each cell. Each of these cells can be considered a queue with multiple sources. As studied in [37], age of a single update packet that is served by a queue with $O(\sqrt{n})$ different packet streams is also $O(\sqrt{n})$ under LCFS with preemption policy. Therefore, in the multihop scheme, after one hop, age of an update becomes $O(\sqrt{n})$ since the queue is shared by $O(\sqrt{n})$ other packets. Considering the fact that the number of hops needed is $O(\sqrt{n})$, using a multihop scheme, the average age scales as $O(n)$ as in [36].

In this paper, considering all these previous results, we first propose a three-phase transmission scheme to serve all n S-D pairs such that time average age of each node is small. Our scheme utilizes local cooperation between the users as in [27]. We divide the network into cells of M users each. In the first phase, nodes from the same cell communicate to create a *mega update packet*, which contains the updates of all nodes from that cell. In the second phase, inter-cell communication takes place and each cell sends its mega packet to the corresponding destination cells. The main idea behind the mega update packets is to serve many nodes at once to decrease inter update time. In the third and final phase, individual packets are extracted from the received mega update packets and relayed to the intended recipient nodes in the cells. During all these phases, we make use of the spatial separation of the nodes to allow multiple simultaneous transmissions provided that there is no destructive interference caused by others. Using this scheme, we achieve an average age scaling of $O(n^{\frac{1}{4}} \log n)$ per-user.

Next, we observe that the first and third phases of the proposed transmission scheme essentially require successful communication between pairs but among M nodes rather than n. With this observation and the fact that the system is scale-invariant, we introduce hierarchy in Phases I and III to improve the age scaling result. In other words, we can further divide cells into smaller subcells and apply the proposed three-phase transmission scheme again in Phases I and III. Although hierarchical cooperation was shown to result in poor delay performance in [28], by utilizing mega update packets better age scaling can be achieved here. In fact, using this scheme, we show that an average age scaling of $O\left(n^{\alpha(h)} \log n\right)$ per-user is achievable where $\alpha(h) = \frac{1}{3 \cdot 2^h + 1}$ and h denotes the number of hierarchy levels. We note that when this hierarchical cooperation is not utilized, i.e., $h = 0$, we retrieve the performance of the initial scheme which achieves an age scaling of $O(n^{\frac{1}{4}} \log n)$ per-user. In the asymptotic case when $h \to \infty$, the proposed scheme with hierarchical cooperation achieves an average age scaling of $O(\log n)$. To the best of our knowledge, this is the best average age scaling result in a status update system with multiple S-D pairs.

II. System Model and Age Metric

We consider n nodes that are uniformly and independently distributed on a square of fixed area S. Every node is both a source and a destination. These sources and destinations are paired randomly irrespective of their locations to form n S-D pairs. Sources create time-sensitive status update packets and transmit them to their respective destinations using the common wireless channel. Each source wants to keep its destination as up-to-date as possible. Thus, destination nodes need to be updated regularly with low transmission delays. We use the age of information metric to measure the freshness of the status update packets. Age is measured for each destination node and for node i at time t age is the random process $\Delta_i(t) = t - u_i(t)$ where $u_i(t)$ is the timestamp of the most recent update at that node. The metric we use, time averaged age, is given by,

$$\Delta_i = \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau \Delta_i(t) dt \tag{1}$$

for node i. We use a graphical average age analysis to derive the average age for a single S-D pair assuming ergodicity similar to [1] and [6].

Inspired by [27], we first propose a scheme based on clustering nodes and making use of what we call *mega update packets* to increase the spatial reuse of the common wireless channel. This entails dividing n users into $\frac{n}{M}$ cells with M users in each cell with high probability. The users communicate locally within cells to form the mega update packets. We model the delay in these intra-cell communications as i.i.d. exponential with parameter λ. Then, mega packets are transmitted between the cells. We model the delay in these inter-cell communications as i.i.d. exponential with parameter $\tilde{\lambda}$. Finally, the individual updates are extracted from mega updates and distributed to the intended destinations within cells again via intra-cell communications. While intra-cell communications occur simultaneously in parallel across the cells (see Section VI for details), inter-cell updates occur sequentially one-at-a-time.

Second, we observe that the bottleneck in average age scaling in this three-phase scheme is M since during intra-cell transmissions M transmissions are needed, one for each node in a cell. Noting that each cell is a scaled-down version of the whole system, we then propose introducing hierarchy by forming subcells from the cells and applying the three-phase transmission scheme on a cell level to overcome this bottleneck. Thus, when $h = 1$ hierarchy level is utilized, this hierarchical scheme includes inter-cell, inter-subcell (within cells) and intra-subcell transmissions. We first analyze the case with $h = 1$ level of hierarchy and then generalize the result to h hierarchy levels. We again model the delay in communications as i.i.d. exponential random variables with varying parameters depending on the type, e.g., intra-subcell, inter-subcell within cells or inter-cell, of the communication (see Section V for details).

Due to i.i.d. nature of service times in all types of communications with or without hierarchy, all destination nodes experience statistically identical age processes and will have the same average age. Thus, we will drop user index i in the average age expression and use Δ instead of Δ_i in the following analysis.

Finally, we denote the kth order statistic of random variables $X_1, \ldots, X_n$ as $X_{k:n}$. Here, $X_{k:n}$ is the kth smallest random variable, e.g., $X_{1:n} = \min\{X_i\}$ and $X_{n:n} = \max\{X_i\}$. For i.i.d. exponential random variables X_i with parameter λ,

$$\mathbb{E}[X_{k:n}] = \frac{1}{\lambda}(H_n - H_{n-k}) \tag{2}$$

$$\mathrm{Var}[X_{k:n}] = \frac{1}{\lambda^2}(G_n - G_{n-k}) \tag{3}$$

where $H_n = \sum_{j=1}^{n} \frac{1}{j}$ and $G_n = \sum_{j=1}^{n} \frac{1}{j^2}$. Using these,

$$\mathbb{E}[X_{k:n}^2] = \frac{1}{\lambda^2}\left((H_n - H_{n-k})^2 + G_n - G_{n-k}\right). \tag{4}$$

Note that, for large n, $H_n \to \log n$ and $G_n \to \frac{\pi^2}{6}$.

III. Age Analysis of a Single S-D Pair

The network operates in sessions such that during each session all n sources successfully send their update packets to their corresponding destinations. Each session lasts for Y units of time. Here, we derive the average age of a single S-D pair (s, d) since each pair experiences statistically identical age as explained in Section II.

Session j starts at time T_{j-1} and all sources including s generate their respective jth update packets. This session lasts until time $T_j = T_{j-1} + Y_j$, at which, all n packets are received by their designated recipient nodes including node d. In other words, a session ends when the last

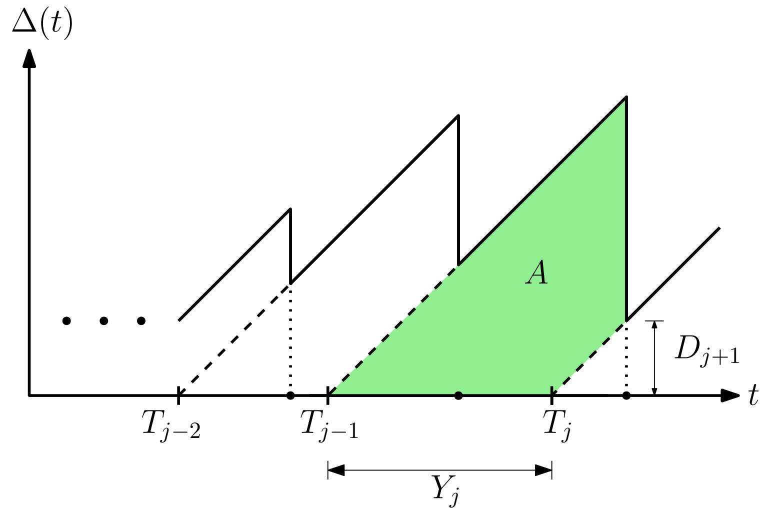

Fig. 1. Sample age $\Delta(t)$ evolution for a single S-D pair. Update deliveries are shown with symbol $\bullet$. Session j starts at time T_{j-1} and lasts until $T_j = Y_j + T_{j-1}$.

S-D pair finishes the packet transmission. Thus, in the proposed scheme every destination node but one receive their packets before the session ends. Fig. 1 shows the evolution of the age at a destination node over time. It is in the usual sawtooth shape with the age increasing linearly over time and dropping to a smaller value as the updates are received at the destination. The process repeats itself at time T_j when all sources including s generate the next update packet, namely update $j+1$.

Using Fig. 1, the average age for an S-D pair is given by

$$\Delta = \frac{\mathbb{E}[A]}{\mathbb{E}[L]} \tag{5}$$

where A denotes the shaded area and L is its length. From the figure, we observe that

$$\mathbb{E}[A] = \frac{\mathbb{E}[Y^2]}{2} + \mathbb{E}[D]\,\mathbb{E}[Y] \tag{6}$$

$$\mathbb{E}[L] = \mathbb{E}[Y] \tag{7}$$

since the system is ergodic and Y_j and D_{j+1} are independent. Here, D denotes the time interval between the generation of an update and its arrival at the destination node. Using these in (5) as in [38], the average age for an S-D pair is given by

$$\Delta = \mathbb{E}[D] + \frac{\mathbb{E}[Y^2]}{2\,\mathbb{E}[Y]} \tag{8}$$

Note that in some systems D may be directly equal to the link delay. However, as in our model here, D may capture some additional delays that may occur during the service time of an update. This will be further clarified in the next section.

IV. Three-Phase Transmission Scheme

Proposed scheme involves clustering nodes and making use of *mega update packets* to serve many S-D pairs at once to reduce the session time. In this section, we describe the proposed three-phase transmission scheme and define mega update packets. As in [27], we divide the square network area into $\frac{n}{M}$ cells of equal area such that each cell includes M nodes with high probability which tends to 1 as n increases.[1] The transmission delays between the nodes belonging to the same cell are denoted by X_i whereas the transmission delays between the nodes from different cells are denoted by $\tilde{X}_i$. Note that X_i and $\tilde{X}_i$ are independent; X_i are i.i.d. exponential with parameter λ and $\tilde{X}_i$ are i.i.d. exponential with parameter $\tilde{\lambda}$.

Phase I. Creating mega update packets: In a cell, each one of the M nodes distributes its current update packet to remaining $M-1$ nodes through $M-1$ links with independent random delays. This operation resembles the wait-for-all scheme studied in [32] since each node keeps transmitting until all $M-1$ nodes receive its packet. Thus, the time needed for each node to distribute its update packet to other nodes in the cell is $U = X_{M-1:M-1}$. Considering M successive transmissions for each node in the cell, this phase is completed in $V = \sum_{i=1}^{M} U_i$ units of time. By the end of this phase in a cell, each one of the M nodes has M different update packets one from each other node in that cell. Each node combines all these M packets to create what we call a *mega update packet* (see Fig. 2). In order to reduce the session time, cells work in parallel during Phase I (see Section VI for a detailed description of this operation). This phase ends when the slowest cell among simultaneously operating cells finishes creating its mega update packet. Phase I takes $Y_I = V_{\frac{n}{M}:\frac{n}{M}}$ units of time, where Y_I denotes the duration of Phase I.

Phase II. MIMO-like transmissions: In this phase, each cell successively performs MIMO-like transmissions using the mega update packets created in Phase I. In each cell, all M

[1] We note that it is sufficient to have $O(M)$ nodes in each cell for the proposed scheme to work. However, from hereafter, we assume that each cell has exactly M nodes for ease of exposition.

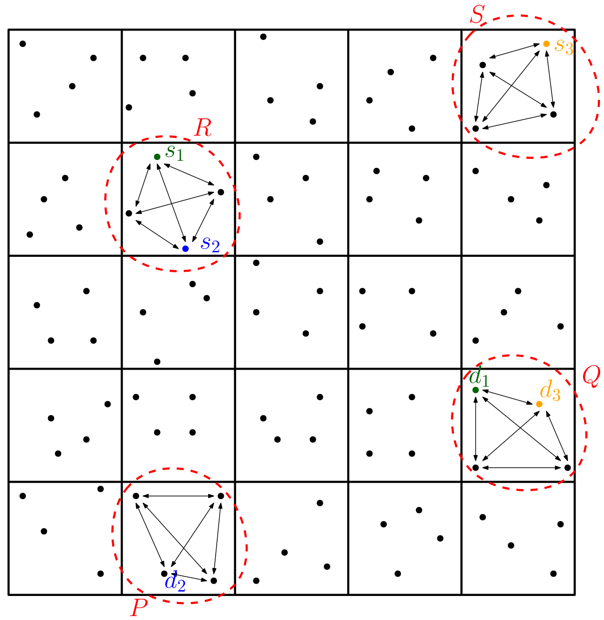

Fig. 2. Cell formation for $M = 4$ and $n = 100$. Simultaneous intra-cell transmissions are depicted for three S-D pairs from cells P, Q, R and S.

source nodes send the mega update packet through the channel simultaneously to the respective destination cells in which the destination nodes are located. Since every node sends the same mega packet which includes all M packets to be transmitted from that cell, this does not create interference. Thus, this is equivalent to sending update packets of all M sources with M copies each all at once (see Fig. 3). Hence, this significantly reduces the time needed to transmit updates of all M sources from that cell to their respective destinations. Note that this stage does not require the destination nodes to be in the same cell. In fact, considering that we have M nodes in a cell, each cell can at most have M different destination cells. Since we send M copies of each update to a destination cell in which there are M receiver nodes, only the earliest successful transmission is important. In other words, among the M^2 active links only the one with the smallest delay is critical in delivering the mega update packet to a destination cell. Thus, it takes $\tilde{U} = \tilde{X}_{1:M^2}$ units of time for a source node s from cell j to send its update to the destination cell where the destination node d lies in. This MIMO-like transmissions of cell j continues until all M destination cells receive the mega packet. Hence, for each cell, this phase lasts for $\tilde{V} = \tilde{U}_{M:M}$. We repeat this for each cell, making the session time of this phase $Y_{II} = \sum_{i=1}^{\frac{n}{M}} \tilde{V}_i$.

Phase III. In-cell relaying to the destination nodes: By the end of Phase II, each cell receives a mega packet for each one of its nodes. These packets may be received directly by

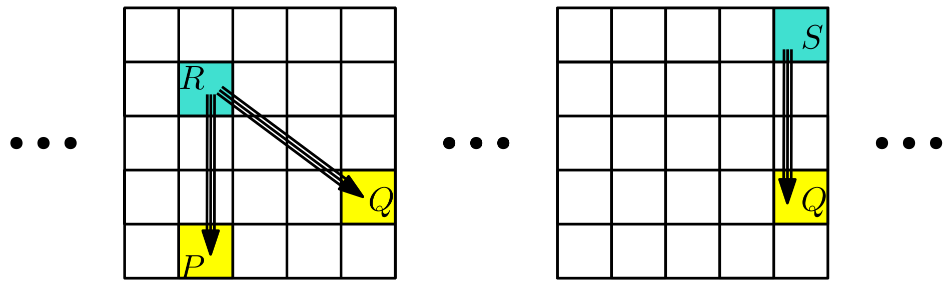

Fig. 3. In Phase II cells take turns to perform inter-cell transmissions. These inter-cell transmissions are shown for the same three S-D pairs depicted in Fig. 2.

their intended destination nodes. However, considering the worst case where they are received by any other node, we need to relay them to their actual designated recipient nodes. Thus, in this phase, M actual messages are extracted from the corresponding M mega update packets received during Phase II and sent to their recipients one at a time. Since this phase has intra-cell transmissions, it is performed in parallel across cells. For a single node this takes X units of time, consequently we need $\hat{V} = \sum_{i=1}^{M} X_i$ to finish this process in a cell. As in Phase I, we need to wait for the slowest cell to finish this operation. Then, this phase lasts for $Y_{III} = \hat{V}_{\frac{n}{M}:\frac{n}{M}}$ units of time.

The total session time of the proposed scheme is,

$$Y = Y_I + Y_{II} + Y_{III} = V_{\frac{n}{M}:\frac{n}{M}} + \sum_{i=1}^{\frac{n}{M}} \tilde{V}_i + \hat{V}_{\frac{n}{M}:\frac{n}{M}} \tag{9}$$

where V, $\tilde{V}$, and $\hat{V}$ are defined above. Note that in our proposed scheme we have,

$$D = Y_I + Y_{II} + Z \tag{10}$$

where, as noted earlier, D denotes the time between generation of an update at certain source node till its arrival at the corresponding destination node. Assuming no S-D pair is in the same cell, in our scheme, arrivals to destination nodes occur in Phase III. Note that when an S-D pair is in the same cell, corresponding D is smaller which consequently leads to a smaller age. Therefore, by assuming no S-D pair is in the same cell, we essentially consider the worst case. Thus, any successful packet delivery will be no earlier than the duration of the first two phases $Y_I + Y_{II}$. On top of that, Phase III involves M successive in-cell transmissions for each node of a particular cell. Hence, depending on the cell that the source node lies in, as well as the realization of the transmission delay X, the corresponding destination node may receive the packet some time after Phase III starts. For example, if a packet is the $j+1$th to be transmitted in Phase III, then delivery will be at $Y_I + Y_{II} + \sum_{i=1}^{j} X_i + X$. Then, the random variable Z is of the form $Z = \sum_{i=1}^{j} X_i + X$.

Substituting (9)-(10) in (8) we obtain,

$$\Delta = \mathbb{E}[Y_I] + \mathbb{E}[Y_{II}] + \mathbb{E}[Z] + \frac{\mathbb{E}[Y^2]}{2\,\mathbb{E}[Y]} \tag{11}$$

which is the average age of an S-D pair under the proposed transmission scheme.

Before we perform the explicit age calculation using (11), we make some observations to simplify our analysis. First, we note that, when the transmission delays $\tilde{X}$ are i.i.d. exponential with rate $\tilde{\lambda}$, then $\tilde{U} = \tilde{X}_{1:M^2}$ is also exponential with rate $M^2\tilde{\lambda}$ [39]. Second, we have the following upper bound for the duration of Phase I.

Lemma 1 *Y_I satisfies the following inequality,*

$$Y_I \leq \bar{V} \tag{12}$$

where $\bar{V} = \sum_{i=1}^{M} \bar{U}_i$ and $\bar{U} = X_{n:n}$.

Proof: Recall that $Y_I = V_{\frac{n}{M}:\frac{n}{M}}$, where $V = \sum_{i=1}^{M} U_i$ and $U = X_{M-1:M-1}$. To show the inequality we make the following observation: In Phase I, $\frac{n}{M}$ cells operate simultaneously. First nodes of each of these cells start transmitting their packets to all other $M-1$ nodes of their cell at the same time. Since intra-cell transmission delays are all i.i.d. across cells and packets, what we essentially have in this case is simultaneous transmission to $\frac{n}{M}(M-1) \approx n$ nodes, and therefore all first nodes will be done in $X_{n:n}$ units of time.

We repeat this for the second nodes of each cell and so on to get $\bar{V} = \sum_{i=1}^{M} (X_{n:n})_i = \sum_{i=1}^{M} \bar{U}_i$. In this way of operation, a cell waits for all other cells to finish distributing the update packet of the first node and then continues with the second node and so on. In a way, for each of its nodes it waits for the slowest cell to finish. However, in our constructed scheme during Phase I, inside a cell, nodes distribute their packets to other nodes of that cell without considering other cells and phase ends when all cells finish this process for all their M nodes. Thus, $\bar{V}$ is an upper bound on Y_I. ■

Although our proposed Phase I lasts shorter than the scheme described in Lemma 1, for tractability and ease of calculation we worsen our scheme in terms of session time, and take the upper bound in Lemma 1 as our Phase I duration such that from now on $Y_I = \bar{V}$. Third, we have the following upper bound for the duration of Phase III.

Lemma 2 *Y_{III} satisfies the following inequality,*

$$Y_{III} \leq \bar{\bar{V}} \tag{13}$$

where $\bar{\bar{V}} = \sum_{i=1}^{M} \bar{\bar{U}}_i$ and $\bar{\bar{U}} = X_{\frac{n}{M}:\frac{n}{M}}$.

We omit the proof of Lemma 2 since it follows similar to the proof of Lemma 1. Due to the same tractability issues, we worsen Phase III as well in terms of duration and take $Y_{III} = \bar{\bar{V}}$ from now on.

As a result of Lemmas 1 and 2, (9) becomes

$$Y = \bar{V} + \sum_{i=1}^{\frac{n}{M}} \tilde{V}_i + \bar{\bar{V}}. \tag{14}$$

Now, we are ready to derive an age expression using Lemmas 1 and 2 in (11). This is stated in the following theorem.

Theorem 1 *Under the constructed transmission scheme, the average age of an S-D pair is given by,*

$$\begin{aligned}\Delta =& \frac{M}{\lambda} H_n + \frac{n}{M^3\tilde{\lambda}} H_M + \frac{M-1}{2\lambda} H_{\frac{n}{M}} + \frac{1}{\lambda} \\ &+ \frac{\frac{M^2}{\lambda^2} H_n^2 + \frac{n^2}{M^6\tilde{\lambda}^2} H_M^2 + \frac{M^2}{\lambda^2} H_{\frac{n}{M}}^2 + \frac{M}{\lambda^2} G_n + \frac{n}{M^5\tilde{\lambda}^2} G_M + + \frac{M}{\lambda^2} G_{\frac{n}{M}}}{2\left(\frac{M}{\lambda} H_n + \frac{n}{M^3\tilde{\lambda}} H_M + \frac{M}{\lambda} H_{\frac{n}{M}}\right)} \\ &+ \frac{\frac{n}{M^2\lambda\tilde{\lambda}} H_n H_M + \frac{M^2}{\lambda^2} H_n H_{\frac{n}{M}} + \frac{n}{M^2\lambda\tilde{\lambda}} H_M H_{\frac{n}{M}}}{\frac{M}{\lambda} H_n + \frac{n}{M^3\tilde{\lambda}} H_M + \frac{M}{\lambda} H_{\frac{n}{M}}}.\end{aligned} \tag{15}$$

Proof: The proof follows upon substituting (14) back in (11) and taking expectations of order statistics of exponential random variables as in Section II. Doing these, we obtain

$$\mathbb{E}[Y_I] = \frac{M}{\lambda} H_n \tag{16}$$

$$\mathbb{E}[Y_{II}] = \frac{n}{M^3 \tilde{\lambda}} H_M \tag{17}$$

$$\mathbb{E}[Y_{III}] = \frac{M}{\lambda} H_{\frac{n}{M}} \tag{18}$$

$$\mathbb{E}[Y_I^2] = \frac{M^2}{\lambda^2} H_n^2 + \frac{M}{\lambda^2} G_n \tag{19}$$

$$\mathbb{E}[Y_{II}^2] = \frac{n^2}{M^6 \tilde{\lambda}^2} H_M^2 + \frac{n}{M^5 \tilde{\lambda}^2} G_M \tag{20}$$

$$\mathbb{E}[Y_{III}^2] = \frac{M^2}{\lambda^2} H_{\frac{n}{M}}^2 + \frac{M}{\lambda^2} G_{\frac{n}{M}}. \tag{21}$$

Lastly, we need to calculate $\mathbb{E}[Z]$ where the random variable Z is the additional amount of time after Phase II ends until the destination node receives the update. Let us take an S-D pair (s, d) where source node s is from cell $j+1$. In Phase III, d has to wait for all other j mega packets from the first j cells to be distributed among the nodes. When its turn comes, d just needs X amount of time to get its packet. Then, d has $Z = \sum_{i=1}^{j} \bar{\bar{U}}_i + X$. Here, we have $\bar{\bar{U}}$ inside the summation as opposed to X as in the discussion preceding (11) because of Lemma 2. Taking expectation on $\bar{\bar{U}}$, j and X by noting their mutual independence we get

$$\mathbb{E}[Z] = \left(\frac{1}{M}\sum_{j=0}^{M-1} j\right)\mathbb{E}[\bar{\bar{U}}] + \mathbb{E}[X] = \frac{M-1}{2\lambda} H_{\frac{n}{M}} + \frac{1}{\lambda}. \tag{22}$$

Using (16)-(22) in (11) yields the expression. ■

Having derived the expression for the average age Δ of an S-D pair, we are now ready to work with large n.

Theorem 2 *For large n and with $M = n^b$, where $0 < b \leq 1$, average age Δ in Theorem 1 approximately becomes,*

$$\Delta \approx \frac{n^b}{\lambda} \log n + \frac{n}{n^{3b} \tilde{\lambda}} b \log n + \frac{n^b - 1}{2\lambda}(1-b)\log n + \frac{1}{\lambda} \\ + \frac{(1 + (1-b)^2)\frac{n^{2b}}{\lambda^2}(\log n)^2 + \frac{n^2}{n^{6b}\tilde{\lambda}^2} b^2 (\log n)^2 + \left(\frac{2n^b}{\lambda^2} + \frac{n}{n^{5b}\tilde{\lambda}^2}\right)\frac{\pi^2}{6}}{2\left((2-b)\frac{n^b}{\lambda}\log n + \frac{n}{n^{3b}\tilde{\lambda}} b \log n\right)}$$

$$+\frac{b(2-b)\frac{n}{n^{2b}\lambda\tilde{\lambda}}(\log n)^2+\frac{n^{2b}}{\lambda^2}(1-b)(\log n)^2}{(2-b)\frac{n^b}{\lambda}\log n+\frac{n}{n^{3b}\tilde{\lambda}}b\log n}. \tag{23}$$

Proof: The expression follows upon substituting $M = n^b$ in (15) and noting that for large n, we have $H_n \approx \log n$. Further, G_n is monotonically increasing and converges to $\frac{\pi^2}{6}$. Since we have $M = n^b$, as n grows large M does too, resulting in $H_M \approx b \log n$ and G_M converging to $\frac{\pi^2}{6}$. ■

Theorem 3 *For large n, and for $\frac{1}{4} \leq b \leq 1$, the average age of an S-D pair Δ given in (23) reduces to,*

$$\Delta \approx cn^b \log n \tag{24}$$

with a constant c. That is, age is $O(n^b \log n)$, for $\frac{1}{4} \leq b \leq 1$.

Proof: By analyzing the result of Theorem 2 we note that the first and third terms are $O(n^b \log n)$, and the second term is $O(n^{1-3b} \log n)$, and fourth term is a constant independent of n. The fifth term can be written as

$$\frac{n^{2b}(\log n)^2\left(\frac{1+(1-b)^2}{\lambda^2}+\frac{b^2}{n^{2(4b-1)}\tilde{\lambda}^2}+\frac{\pi^2}{6}\left(\frac{2}{n^b(\log n)^2\lambda^2}+\frac{1}{n^{7b-1}(\log n)^2\tilde{\lambda}^2}\right)\right)}{n^b\log n\left(\frac{2(2-b)}{\lambda}+\frac{2b}{\tilde{\lambda}n^{4b-1}}\right)} \tag{25}$$

which is $O(n^b \log n)$ when $b \geq 1 - 3b$. Continuing similarly for the remaining term shows that it is also $O(n^b \log n)$ which gives the overall scaling result for $\frac{1}{4} \leq b \leq 1$. ■

Thus, the proposed transmission scheme, which involves intra-cell cooperation and inter-cell MIMO-like transmissions of mega update packets, allows the successful communication of n S-D pairs, and achieves an average age scaling of $O(n^{\frac{1}{4}} \log n)$ per-user.

Next corollary analyzes the performance of the proposed three-phase scheme without the utilization of mega update packets.

Corollary 1 *When mega update packets are not utilized, the proposed three-phase transmission policy achieves an average age scaling of $O(n^{\frac{1}{3}} \log n)$ per-user.*

Proof: In the proposed scheme, mega update packets are formed during Phase I and are transmitted to destination cells in Phase II. By transmitting a mega update packet, all M nodes of a particular cell send their messages to the corresponding destination cells at once. When the mega update packets are not utilized, however, each node of a cell still stores all other messages received in Phase I in its buffer but instead of combining these messages to create the mega update packet, these nodes can send the individual packets to corresponding M destination cells sequentially. In other words, rather than sending all M packets as a mega packet simultaneously, nodes in a cell collectively can send the packets one by one. This requires M successive transmissions for a cell and a total of n transmissions for $\frac{n}{M}$ cells. If we call this operation Phase II′, corresponding session length is $Y_{II'} = \sum_{i=1}^{n}(\tilde{X}_{1:M^2})_i$. Note that when $M = n^b$ where $0 < b \leq 1$ we have

$$\mathbb{E}[Y_{II'}] = \frac{n}{M^2\tilde{\lambda}} \tag{26}$$

$$\mathbb{E}[Y_{II'}^2] = \frac{n}{M^4\tilde{\lambda}^2} + \frac{n^2}{M^4\tilde{\lambda}^2}. \tag{27}$$

Repeating a similar analysis as above with Phase II′ instead of Phase II for large n yields the result. Note that even if the mega update packets are not utilized, nodes in a cell still need enough buffer space to store all M messages received in Phase I so that these messages can be sent out one by one in Phase II′. ■

In the next section, we propose introducing hierarchy to the proposed three-phase transmission scheme to improve the average age scaling.

V. Three-Phase Transmission Scheme with Hierarchy

A. *Motivation and Outline of the Scheme*

The three-phase transmission scheme proposed in Section IV allows successful communication of n S-D pairs. Following the analysis to obtain the average age expression by substituting the first and second order moments of the phase durations given in (16)-(22) into the average age expression given in (8), we observe that the resulting per-user average age scaling, when n is large, with $M = n^b$ where $0 < b \leq 1$ and exponential link delays, is characterized by the expected scaling of the phases. As derived in (16)-(18) expected durations of the phases are $O(n^b \log n)$, $O(n^{1-3b} \log n)$ and $O(n^b \log n)$ which in turn result in an average age scaling of $O(n^{\frac{1}{4}} \log n)$ upon selecting $b = 1 - 3b$. Thus, to obtain a better average age scaling we need to

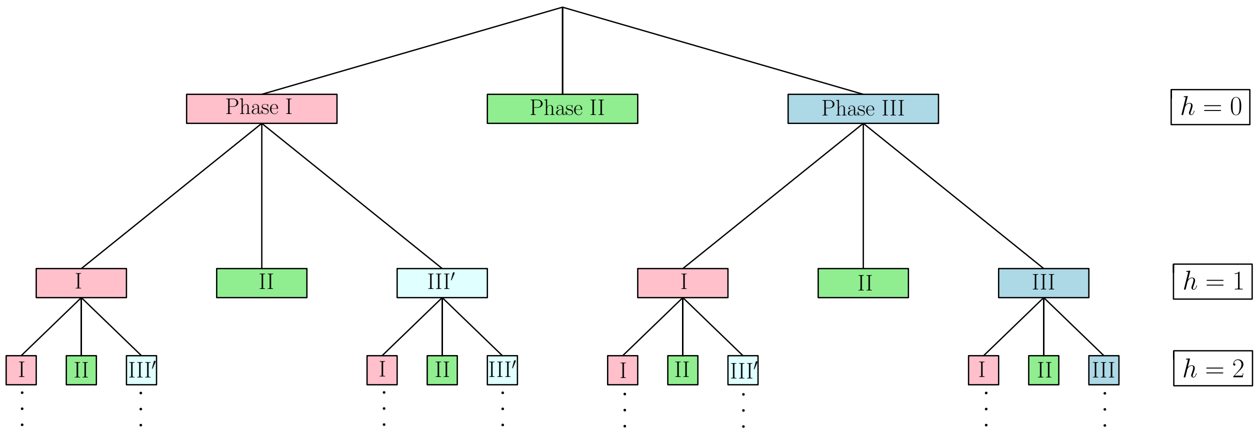

Fig. 4. Proposed three-phase hierarchical transmission scheme.

improve the expected length of each phase. This motivates the hierarchical cooperation in the proposed scheme.

In the Phase I of Section IV, the communication takes place in between $M = n^b$ nodes rather than n nodes and a simple TDMA operation is performed among these nodes which results in an average scaling of $O(n^b \log n)$. Instead, we introduce the first level of hierarchy by dividing each of these cells into n^{b-a} further subcells with n^a users each where $0 < a < b$. Then, we apply the same three-phase scheme with one difference to this cell to accommodate Phase I transmissions of Section IV. In particular, to create the mega packet of the cell, first local communication among the nodes is performed within subcells and MIMO-like transmissions are carried out in between subcells within a cell. Then, instead of relaying the received packet to a single node as in Phase III of Section IV, received packets are relayed to every other node in the subcell to create the mega update packet. With this operation, Phase I of Section IV is completed in three phases, Phase I, Phase II, Phase III′, each of which are scaled down versions of the overall scheme with the corresponding difference in the third phase which is denoted as Phase III′ to highlight this difference. The expected length of the first phase with $h = 1$ level of hierarchy is then $O(n^a \log n) + O(n^{b-3a} \log n) + O(n^{b-a} \log n)$ (see Section V-B for a detailed derivation) all of which are smaller than $O(n^b \log n)$ achieved in Section IV.

Similarly, Phase III of Section IV can also be completed in three phases under $h = 1$ level of hierarchy. However, this time in the third step we need Phase III rather than Phase III′ since we need to relay the received update packet within subcell to its intended recipient node to conclude the delivery.

Fig. 4 shows the proposed hierarchy structure in which Phases I and III of level h can be performed by applying the three-phase scheme on a smaller scale at level $h+1$ accordingly. The advantage of the hierarchical transmission is summarized in Table I.

	$h=0$	$h=1$
Phase I	$O(n^b \log n)$	$O(n^a \log n)$ + $O(n^{b-3a} \log n)$ + $O(n^{b-a} \log n)$
Phase II	$O(n^{1-3b} \log n)$	$O(n^{1-3b} \log n)$
Phase III	$O(n^b \log n)$	$O(n^a \log n)$ + $O(n^{b-3a} \log n)$ + $O(n^a \log n)$

TABLE I
COMPARISON OF THE EXPECTED DURATIONS OF THE PHASES WITH $h=0$ AS IN SECTION IV AND $h=1$ HIERARCHY LEVEL WITH $0<a<b\leq 1$.

B. Detailed Description of the Scheme for $h=1$

In this section, we describe the proposed hierarchical transmission scheme with $h=1$ level of hierarchy in detail. Later, we generalize the average age scaling result for $h>1$ levels of hierarchy using the fact that the system is scale-invariant. We note that the scheme in Section IV does not utilize hierarchical cooperation, i.e., $h=0$. As in Section IV, we start with a square network that is divided into $\frac{n}{M}$ cells of equal area with M nodes in each cell with high probability that tends to 1 as n increases. Selecting $M=n^b$ where $0<b\leq 1$ results in n^{1-b} equal area cells with n^b users each cell. Introducing the first level of hierarchy, we further divide each cell into n^{b-a} equal area subcells to get a total of n^{1-a} subcells with n^a nodes each where $0<a<b$.

Remember that when there is no hierarchical cooperation we denote the transmission delays within cells as X_i and in between cells as $\tilde{X}_i$. In this section, in order to accommodate hierarchical structure for $h=1$ level, we change our notation so that transmission delays between the nodes from different cells are now denoted by $X_i^{(0)}$, between the nodes from different subcells within the same cell are denoted by $X_i^{(1)}$, and between the nodes belonging to the same subcell are denoted by $X_i^{(2)}$. Note that $X_i^{(j)}$ are independent; and $X_i^{(j)}$ are i.i.d. exponential with parameter λ_j for $j=0,1,2$. Note that we have $\lambda_2 \geq \lambda_1 \geq \lambda_0$ so that link delays are proportional to the distances between nodes on average. Also note that in what follows we use Y_I, Y_{II} and Y_{III} to denote the phase durations under the three-phase scheme with hierarchy.

Phase I. Creating mega update packets: In this phase, each cell generates its mega update packet which includes all $M=n^b$ messages to be sent from that cell. Unlike the scheme in

Section IV, we create mega update packets in three successive phases by applying the three-phase transmission scheme to each cell.

First, each node in a subcell distributes its update packet to remaining $n^a - 1$ nodes in its subcell which takes $U^I = X^{(2)}_{n^a-1:n^a-1}$ units of time. Considering n^a successive transmissions for each node of the subcell, this step is completed in a subcell in $V^I = \sum_{i=1}^{n^a} U_i^I$ units of time. This operation is analogous to the Phase I in Section IV but performed among n^a nodes in a subcell rather than among n^b nodes within a cell. Upon completion of this step in a subcell, each node of that subcell has n^a different update packets one from each node. Each node combines all these update packets to form a *preliminary* mega update packet which includes all n^a messages to be sent out from this subcell. This operation is performed in parallel among all subcells in the network (see Section VI for a detailed description of this operation) and ends when the slowest simultaneously operating subcell finishes creating its *preliminary* mega update packet. Hence, it takes $Y_I^I = V^I_{n^{1-a}:n^{1-a}}$ units of time, where Y_I^I denotes the duration of the first phase at $h = 1$.

When all *preliminary* mega update packets are formed, all n^{b-a} subcells of a cell perform MIMO-like transmissions among each other to distribute their *preliminary* mega update packets to remaining subcells within the cell. Since this requires cell-level transmissions in between subcells, this step is performed in parallel among cells and thus, subcells take turns. As in the Phase II of Section IV, all n^a nodes of a subcell start transmitting the *preliminary* mega update packet to remaining $n^{b-a}-1$ subcells. Since every node sends the same *preliminary* mega update packet this does not create interference. This transmission continues until the earliest node in each remaining subcell receives the *preliminary* mega update packet. In other words, among the n^{2a} active links between the source and destination subcells, only the one with the smallest delay is critical. Thus, for a single subcell it takes $U^{II} = (X^{(1)}_{1:n^{2a}})_{n^{b-a}-1:n^{b-a}-1}$ units of time. Since subcells take turns, in a cell this step is completed in $V^{II} = \sum_{i=1}^{n^{b-a}} U_i^{II}$ units of time. Finally, on the network-level these MIMO-like transmissions continue until the slowest of the simultaneously operating cells finishes which corresponds to $Y_I^{II} = V^{II}_{n^{1-b}:n^{1-b}}$.

By the end of the MIMO-like transmissions among subcells, each subcell receives *preliminary* mega update packets of remaining $n^{b-a}-1$ subcells that lie in its cell. In this step, these packets are distributed within the subcell in parallel among the subcells of the network. This is identical to the operation of Phase III of Section IV on subcell-level except that each *preliminary* mega update packet received is transmitted to all nodes of that subcell to successfully form the mega update packet of the corresponding cell. To highlight this difference we denote this step as Phase III′ in Fig. 4 at $h = 1$ level. Distributing one *preliminary* mega update packet takes $U^{III'} = X^{(2)}_{n^a-1:n^a-1}$ units of time. By repeating this for all *preliminary* mega update packets received this step is completed in a subcell in $V^{III'} = \sum_{i=1}^{n^{b-a}-1} U_i^{III'}$ units of time. We wait for the slowest subcell and thus on the network-level this step is completed in $Y_I^{III'} = V^{III'}_{n^{1-a}:n^{1-a}}$ units of time.

With this, each node in a subcell receives remaining $n^{b-a} - 1$ *preliminary* mega update packets of n^a message each. Combining these with their own *preliminary* mega update packet, every node in a subcell forms the mega update packet which includes all n^b messages to be sent out from that cell. Thus, the first phase lasts for $Y_I = Y_I^I + Y_I^{II} + Y_I^{III'}$ units of time.

Phase II. MIMO-like transmissions: Identical to Phase II of Section IV, in this phase each cell successively performs MIMO-like transmissions using the mega update packets created in Phase I. This phase requires network-level transmissions between cells. Thus, only one cell operates at a time. As in Section IV, a source node s from cell j needs $\tilde{U} = X^{(0)}_{1:n^{2b}}$ units of time to send its update to the destination cell where the destination node d lies in. Transmissions of cell j continue until all n^b destination cells receive the mega update packet. Hence, for each cell, this phase lasts for $\tilde{V} = \tilde{U}_{n^b:n^b}$. This operation is repeated for each cell and hence the session time of this phase $Y_{II} = \sum_{i=1}^{n^{1-b}} \tilde{V}_i$. At the end of this phase, each cell delivers its mega update packet to one node in each of the corresponding destination cells.

Phase III. In-cell relaying to the destination nodes: By the end of Phase II, each cell receives a total of n^b mega update packets, one for each node. In Section IV, relevant packets which have a destination node in that cell are extracted from these mega update packets and relayed to their respective designated recipient nodes by a simple TDMA operation which scales as $O(n^b \log n)$. However, as in Phase I we can introduce hierarchy to this phase and apply the three-phase scheme again. Thus, extracted relevant packets are first distributed within subcells of the nodes which received them in Phase II. Then, these packets are delivered to their corresponding destination subcells in which the destination nodes are located through MIMO-like transmissions and finally, they are relayed to the corresponding recipient nodes within subcells.

Noting that each subcell receives on average n^a mega update packets, with one relevant packet each, distribution of these n^a packets within subcell takes $\hat{V}^I = \sum_{i=1}^{n^a} \hat{U}_i^I$ with $\hat{U}^I = X^{(2)}_{n^a-1:n^a-1}$ and on the network-level is completed in $Y_{III}^I = \hat{V}^I_{n^{1-a}:n^{1-a}}$ units of time. With this operation, the *secondary* mega update packet of that subcell is formed which includes all n^a update packets with destinations in that cell. Then, these *secondary* mega update packets are transmitted to the respective destination subcells in parallel among cells (subcells take turns) through MIMO-like transmissions until all n^a destination subcells receive them. In a cell, this is completed in $\hat{V}^{II} = \sum_{i=1}^{n^{b-a}} \hat{U}_i^{II}$ units of time where $\hat{U}^{II} = (X^{(1)}_{1:n^{2a}})_{n^a:n^a}$ and therefore, on the network-level is completed in $Y_{III}^{II} = \hat{V}^{II}_{n^{1-b}:n^{1-b}}$ when all cells finish. Thus, each subcell receives a total of n^a *secondary* mega update packets each of which includes one update destined to a node in that subcell. Finally, these packets are relayed to their actual recipient nodes within subcell. For a subcell it takes $\hat{V}^{III} = \sum_{i=1}^{n^a} \hat{U}_i^{III}$ units of time where $\hat{U}^{III} = X^{(2)}$ and hence on the network-level it is completed in $Y_{III}^{III} = \hat{V}^{III}_{n^{1-a}:n^{1-a}}$. Note that since in the last step we relay the packets to their destination node rather than all nodes in the subcell, this step is the subcell-level equivalent of Phase III of Section IV. As a result, the third phase lasts for $Y_{III} = Y_{III}^I + Y_{III}^{II} + Y_{III}^{III}$ and finishes when every S-D pair of the network is served.

Total session time of the proposed scheme is, therefore, $Y = Y_I + Y_{II} + Y_{III}$. Before we perform the explicit age calculation, we again make some observations to simplify our analysis.

Lemma 3 *Y_I satisfies the following inequality,*

$$Y_I \leq \bar{V}^I + \bar{V}^{II} + \bar{V}^{III'} \tag{28}$$

where

$$\bar{V}^I = \sum_{i=1}^{n^a} \bar{U}_i^I, \qquad \bar{U}^I = X^{(2)}_{n:n} \tag{29}$$

$$\bar{V}^{II} = \sum_{i=1}^{n^{b-a}} \bar{U}_i^{II}, \qquad \bar{U}^{II} = (X^{(1)}_{1:n^{2a}})_{n^{1-a}:n^{1-a}} \tag{30}$$

$$\bar{V}^{III'} = \sum_{i=1}^{n^{b-a}} \bar{U}_i^{III'}, \qquad \bar{U}^{III'} = X^{(2)}_{n:n} \tag{31}$$

The proof of this lemma follows similarly from that of Lemma 1. We show that $Y_I^I \leq \bar{V}^I$, $Y_I^{II} \leq \bar{V}^{II}$ and $Y_I^{III'} \leq \bar{V}^{III'}$ which yields (28).

We worsen our scheme in terms of session time and hereafter take the upper bound in Lemma 3 as our Phase I duration for tractability and ease of calculation. Thus, from now on $Y_I = \bar{V}^I + \bar{V}^{II} + \bar{V}^{III'}$. Next, we have the following upper bound for the duration of Phase III.

Lemma 4 *Y_{III} satisfies the following inequality,*

$$Y_{III} \leq \bar{\bar{V}}^I + \bar{\bar{V}}^{II} + \bar{\bar{V}}^{III} \tag{32}$$

where

$$\bar{\bar{V}}^I = \sum_{i=1}^{n^a} \bar{\bar{U}}_i^I, \qquad \bar{\bar{U}}^I = X_{n:n}^{(2)} \tag{33}$$

$$\bar{\bar{V}}^{II} = \sum_{i=1}^{n^{b-a}} \bar{\bar{U}}_i^{II}, \qquad \bar{\bar{U}}^{II} = (X_{1:n^{2a}}^{(1)})_{n^{1-b+a}:n^{1-b+a}} \tag{34}$$

$$\bar{\bar{V}}^{III} = \sum_{i=1}^{n^a} \bar{\bar{U}}_i^{III}, \qquad \bar{\bar{U}}^{III} = X_{n^{1-a}:n^{1-a}}^{(2)} \tag{35}$$

We omit the proof of Lemma 4 since it follows similar to the proof of Lemma 1. We worsen Phase III as well in terms of duration and take $Y_{III} = \bar{\bar{V}}^I + \bar{\bar{V}}^{II} + \bar{\bar{V}}^{III}$ from now on because of similar tractability issues.

As a result of Lemmas 3 and 4, total session time becomes

$$Y = \bar{V}^I + \bar{V}^{II} + \bar{V}^{III'} + Y_{II} + \bar{\bar{V}}^I + \bar{\bar{V}}^{II} + \bar{\bar{V}}^{III} \tag{36}$$

Taking expectations of order statistics of exponential random variables as in (2)-(4) and using the fact that for large n, we have $H_n \approx \log n$ and G_n is monotonically increasing and converges to $\frac{\pi^2}{6}$, first two moments of the subphase and phase durations approximately become

$$\mathbb{E}\left[\sum_{i \in \mathcal{I}'} \bar{V}^{(i)}\right] = \left(\frac{n^a + n^{b-a}}{\lambda_2} + \frac{(1-a)n^{b-3a}}{\lambda_1}\right) \log n \tag{37}$$

$$\mathbb{E}\left[\sum_{i \in \mathcal{I}} \bar{\bar{V}}^{(i)}\right] = \left(\frac{(2-a)n^a}{\lambda_2} + \frac{(1-b+a)n^{b-3a}}{\lambda_1}\right) \log n \tag{38}$$

$$\mathbb{E}[Y_{II}] = \frac{bn^{1-3b}}{\lambda_0} \log n \tag{39}$$

$$\mathbb{E}\left[Y_{II}^2\right] = \frac{n^{1-5b}}{\lambda_0^2}\frac{\pi^2}{6} + \frac{b^2 n^{2(1-3b)}}{\lambda_0^2}\log^2 n \tag{40}$$

$$\mathbb{E}\left[\left(\bar{V}^I\right)^2\right] = \frac{n^a}{\lambda_2^2}\frac{\pi^2}{6} + \frac{n^{2a}}{\lambda_2^2}\log^2 n \tag{41}$$

$$\mathbb{E}\left[\left(\bar{V}^{II}\right)^2\right] = \frac{n^{b-5a}}{\lambda_1^2}\frac{\pi^2}{6} + \frac{(1-a)^2 n^{2(b-3a)}}{\lambda_1^2}\log^2 n \tag{42}$$

$$\mathbb{E}\left[\left(\bar{V}^{III'}\right)^2\right] = \frac{n^{b-a}}{\lambda_2^2}\frac{\pi^2}{6} + \frac{n^{2(b-a)}}{\lambda_2^2}\log^2 n \tag{43}$$

$$\mathbb{E}\left[\left(\bar{\bar{V}}^I\right)^2\right] = \frac{n^a}{\lambda_2^2}\frac{\pi^2}{6} + \frac{n^{2a}}{\lambda_2^2}\log^2 n \tag{44}$$

$$\mathbb{E}\left[\left(\bar{\bar{V}}^{II}\right)^2\right] = \frac{n^{b-5a}}{\lambda_1^2}\frac{\pi^2}{6} + \frac{(1-b+a)^2 n^{2(b-3a)}}{\lambda_1^2}\log^2 n \tag{45}$$

$$\mathbb{E}\left[\left(\bar{\bar{V}}^{III}\right)^2\right] = \frac{n^a}{\lambda_2^2}\frac{\pi^2}{6} + \frac{(1-a)^2 n^{2a}}{\lambda_2^2}\log^2 n \tag{46}$$

where in (37), $i \in \mathcal{I}' = \{I, II, III'\}$ and in (38), $i \in \mathcal{I} = \{I, II, III\}$.

Now, we are ready to derive an average age expression using (8). For ease of exposition, we assume that every node updates its age at the end of each session when the hierarchy is implemented and take $D_{j+1} = Y_{j+1}$. Then, (8) becomes

$$\Delta = E[Y] + \frac{E[Y^2]}{2E[Y]} \tag{47}$$

Note that this assumption can only result in a higher average age as all nodes but one receive their update packets before the session ends, i.e., $D \leq Y$ for all updates and nodes.

Theorem 4 *Under the constructed transmission scheme with $h = 1$ level of hierarchy, for large n, the average age of an S-D pair is given by,*

$$\begin{aligned}\Delta =& \mathbb{E}\left[\sum_{i\in\mathcal{I}'}\bar{V}^{(i)}\right] + \mathbb{E}[Y_{II}] + \mathbb{E}\left[\sum_{i\in\mathcal{I}}\bar{\bar{V}}^{(i)}\right] \\ &+ \frac{\mathbb{E}\left[\left(\sum_{i\in\mathcal{I}'}\bar{V}^{(i)} + Y_{II} + \sum_{i\in\mathcal{I}}\bar{\bar{V}}^{(i)}\right)^2\right]}{2\left(\mathbb{E}\left[\sum_{i\in\mathcal{I}'}\bar{V}^{(i)}\right] + \mathbb{E}[Y_{II}] + \mathbb{E}\left[\sum_{i\in\mathcal{I}}\bar{\bar{V}}^{(i)}\right]\right)}\end{aligned} \tag{48}$$

The proof of Theorem 4 follows upon substituting (36) back in (47). Moments follow from (37)-(46).

Theorem 5 *For large n, with $a = \frac{b}{2}$ and $\frac{1}{7} \leq a \leq \frac{1}{2}$, the average age of an S-D pair when $h = 1$ hierarchy level is implemented, Δ, given in (48) reduces to,*

$$\Delta \approx \tilde{c} n^a \log n \tag{49}$$

with a constant $\tilde{c}$. That is, age is $O(n^a \log n)$, for $\frac{1}{7} \leq a \leq \frac{1}{2}$.

Proof: Using (37)-(46) in (48), we observe that in the average age expression we have terms with $O(n^a \log n)$, $O(n^{b-a} \log n)$, $O(n^{b-3a} \log n)$, and $O(n^{1-3b} \log n)$. Among first three types, noting that $b - 3a < b - a$, dominating terms are $O(n^a)$ and $O(n^{b-a})$. Thus, by choosing $a = b - a$ we can minimize the resulting scaling. With this selection, the first and third terms in (48) are $O(n^a \log n)$ whereas the second one is $O(n^{1-6a} \log n)$. Looking at the fourth term we observe that it has the following form when $b = 2a$,

$$\frac{c_1 n^{2a} \log^2 n + c_2 n^{2(1-6a)} \log^2 n + c_3 n^{1-5a} \log^2 n}{c_4 n^a \log n + c_5 n^{1-6a} \log n}, \tag{50}$$

where $c_1, \dots, c_5$ are constants. We observe that for $\frac{1}{7} \leq a \leq \frac{1}{2}$, first three terms and the fourth term given in (50) are $O(n^a \log n)$ which yields the result. ∎

Thus, the proposed hierarchical scheme with $h = 1$ hierarchy levels achieves an average age scaling of $O(n^{\frac{1}{7}} \log n)$ per-user when $a = \frac{1}{7}$ and $b = \frac{2}{7}$. This implies that if the cells have M nodes each, each subcell has $\sqrt{M}$ nodes when $h = 1$. Note that in Section IV it is shown that $\frac{1}{4} \leq b \leq 1$. Here, resulting b not only satisfies this but also gives a better scaling in the end because of the hierarchy we utilized. In Theorem 6 below, we generalize this scaling result to h levels of hierarchy.

Theorem 6 *For large n, when the proposed scheme is implemented with h hierarchy levels, the average age scaling of $O\left(n^{\alpha(h)} \log n\right)$ per-user is achievable where $\alpha(h) = \frac{1}{3 \cdot 2^h + 1}$.*

Proof: We observe that when $h = 1$ hierarchy level is utilized, the scaling result comes from $a = 1 - 6a$. Since $b = 2a$, another way to express this is $\frac{b}{2^h} = 1 - 3b$. As h increases with $b = 2a$ structure in each hierarchy level, we see that subcells at level h have $n^{\frac{b}{2^h}}$ nodes. Thus, when h levels of hierarchy is utilized, subcell transmissions take place among $n^{\frac{b}{2^h}}$ nodes and inter-subcell transmissions have $n^{\frac{b}{2^h}}$ turns. However, the second phase is still $O(n^{1-3b})$ as each cell at the top of the hierarchy has n^b nodes. Thus, $\frac{b}{2^h} = 1 - 3b$ yields $\alpha(h) = \frac{1}{3 \cdot 2^h + 1}$. ■

Thus, when hierarchy is utilized, the proposed transmission scheme, which involves local cooperation and MIMO-like inter-cell transmissions, allows the successful communication of n S-D pairs, and achieves an average age scaling of $O\left(n^{\alpha(h)} \log n\right)$ per-user where $h = 0, 1, \ldots$ is the number of hierarchy levels. Note that in the asymptotic case when h tends to ∞, this scheme gives an average age scaling of $O(\log n)$ per-user. This is the case because as h increases, number of turns in each phase, $n^{\frac{b}{2^h}}$, decreases such that eventually $\log n$ term which comes from the fact that packets are distributed locally to all other nodes in the same subcell in Phases I and III$'$ dominates. We also observe that when the hierarchy is not utilized, i.e. $h = 0$, Theorem 6 yields the result in Theorem 3 in Section IV.

VI. Note on Phases I and III

We use the protocol model introduced in [22] to model the interference such that two nodes can be active if they are sufficiently spatially separated from each other. In other words, we allow simultaneous transmissions provided there is no destructive interference caused by other active nodes. Suppose that node i transmits its update to node j. Then, node j can successfully receive this update if the following is satisfied for any other node k that is simultaneously transmitting,

$$d(j,k) \geq (1+\gamma) d(j,i) \tag{51}$$

where function $d(x,y)$ denotes the distance between nodes x and y and γ is a positive constant determining the guard zone.

The proposed three-phase scheme with h levels of hierarchy utilizes parallelized transmissions in Phases I and III where $h \geq 0$. When the hierarchy is not utilized, i.e., $h = 0$, parallel intra-cell transmissions take place during these phases. In order to implement these parallel communications in Phases I and III, we follow a 9-TDMA scheme as in [27]. Specifically, $\frac{n}{9M}$ of the total $\frac{n}{M}$ cells work simultaneously so that Phases I and III are completed in 9 successive subphases. Using the protocol model, cells that are at least $(1+\gamma) r\sqrt{2}$ away from a cell can operate simultaneously during these phases, where $r = \sqrt{SM/n}$ is the length of each square cell and S is the network area. Noting that there are at least two inactive cells in between two active cells under a 9-TDMA operation, this scheme satisfies (51) if the guard zone parameter $\gamma \leq \sqrt{2}-1$.

On the other hand, when $h=1$ level of hierarchy is utilized, the proposed scheme includes within subcell transmissions that are parallelized across subcells and within cell transmissions that are parallelized among cells (subcells take turns) in Phases I and III (or III$'$). Similar 9-TDMA scheme again is used to accommodate these simultaneous transmissions. When $\gamma \leq \sqrt{2}-1$, parallel 9-TDMA operation among subcells is still allowed since from cell-level to subcell-level both distance terms in (51) decrease proportionally. Extending this, we see that for h level of hierarchy, by selecting an appropriate guard zone parameter γ, parallelized Phase I and III (or III$'$) operation under 9-TDMA scheme is allowed. Noting that 9 here is a constant and valid for any n, it does not change the scaling results.

VII. Conclusions and Discussions

Given a large wireless network of fixed area consisting of n randomly located source-destination pairs that want to send time-sensitive status update packets to each other, we have studied the scalability of age of information. To accommodate the communication between the S-D pairs, we have proposed a three-phase transmission scheme which uses local cooperation between nodes and mega update packets to achieve an average age scaling of $O(n^{\frac{1}{4}}\log n)$. Our scheme divides the network into $\frac{n}{M}$ cells of M nodes each. The first and third phases include intra-cell transmissions and take place simultaneously across all cells. The second phase includes inter-cell transmissions and therefore during this phase cells operate one at a time.

We observe that the bottleneck in the resulting age scaling result is caused by $O(M)$ transmissions in Phases I and III. Furthermore, we note that each cell is a scaled-down version of the whole network. With these, we introduce hierarchy to the system and apply the three-phase scheme on a cell-level in these phases. In other words, Phases I and III of the hth level of the hierarchy are completed in three successive steps in the next level of hierarchy. We have shown that this scheme with hierarchical cooperation improves the scaling result and achieves an average age scaling of $O\left(n^{\alpha(h)}\log n\right)$ where $\alpha(h)=\frac{1}{3\cdot 2^h+1}$ and h is the number of hierarchy levels. In the asymptotic case when h tends to ∞ resulting average age per-user scales as $O(\log n)$.

We conclude this section with discussions on the model and the results. We emphasize that the focus of this paper is the scaling of age of information in large wireless networks. Thus, presented scaling results hold with high probability when the number of nodes in the network grows beyond a certain threshold. The random network model used in this work is rather idealized. Although it is of theoretical interest on its own right, a more realistic model may adopt a physical interference model that is based on signal-to-interference ratio requirements and possibly mobile nodes rather than static nodes. Moreover, we make use of the mega update packets in the proposed transmission scheme to serve multiple S-D pairs at once without considering the growing mega update packet size as the network population increases. Likewise, link delays are modeled as i.i.d. exponentials with constant parameters that are not affected by the transmission distance or packet size. It would be an interesting direction to analyze the effects of packet size and the distance between S-D pairs on the average age scaling. Further, the schemes discussed in this paper are not private since a packet intended for a certain destination node is observed by other nodes in the network. To ensure privacy, updates can be encrypted in a way that the routing of a packet to the correct destination node is still maintained but only the intended destination node can fully decrypt the message.

References

[1] S. K. Kaul, R. D. Yates, and M. Gruteser. Status updates through queues. In *CISS*, March 2012.

[2] M. Costa, M. Codrenau, and A. Ephremides. Age of information with packet management. In *IEEE ISIT*, June 2014.

[3] L. Huang and E. Modiano. Optimizing age-of-information in a multi-class queueing system. In *IEEE ISIT*, June 2015.

[4] V. Tripathi, R. Talak, and E. Modiano. Age of information for discrete time queues. January 2019. Available on arXiv: arXiv:1901.10463.

[5] A. M. Bedewy, Y. Sun, and N. B. Shroff. Optimizing data freshness, throughput, and delay in multi-server information-update systems. In *IEEE ISIT*, July 2016.

[6] E. Najm, R. D. Yates, and E. Soljanin. Status updates through M/G/1/1 queues with HARQ. In *IEEE ISIT*, June 2017.

[7] A. Soysal and S. Ulukus. Age of information in G/G/1/1 systems. In *Asilomar Conference*, November 2019.

[8] Y. Sun, E. Uysal-Biyikoglu, R. D. Yates, C. E. Koksal, and N. B. Shroff. Update or wait: How to keep your data fresh. In *IEEE Infocom*, April 2016.

[9] M. Bastopcu and S. Ulukus. Age of information with soft updates. In *Allerton Conference*, October 2018.

[10] M. Bastopcu and S. Ulukus. Age of information for updates with distortion. In *IEEE ITW*, August 2019.

[11] P. Zou, O. Ozel, and S. Subramaniam. On the benefits of waiting in status update systems. In *IEEE Infocom*, April 2019.

[12] R. D. Yates. Lazy is timely: Status updates by an energy harvesting source. In *IEEE ISIT*, June 2015.

[13] A. Arafa and S. Ulukus. Age minimization in energy harvesting communications: Energy-controlled delays. In *Asilomar Conference*, October 2017.

[14] A. Arafa and S. Ulukus. Age-minimal transmission in energy harvesting two-hop networks. In *IEEE Globecom*, December 2017.

[15] B. T. Bacinoglu, E. T. Ceran, and E. Uysal-Biyikoglu. Age of information under energy replenishment constraints. In *UCSD ITA*, February 2015.

[16] X. Wu, J. Yang, and J. Wu. Optimal status update for age of information minimization with an energy harvesting source. *IEEE Transactions on Green Communications and Networking*, 2(1):193–204, March 2018.

[17] A. Arafa, J. Yang, S. Ulukus, and H. V. Poor. Age-minimal online policies for energy harvesting sensors with incremental battery recharges. In *UCSD ITA*, February 2018.

[18] A. Arafa, J. Yang, and S. Ulukus. Age-minimal online policies for energy harvesting sensors with random battery recharges. In *IEEE ICC*, May 2018.

[19] A. Arafa, J. Yang, S. Ulukus, and H. V. Poor. Online timely status updates with erasures for energy harvesting sensors. In *Allerton Conference*, October 2018.

[20] A. Baknina and S. Ulukus. Coded status updates in an energy harvesting erasure channel. In *CISS*, March 2018.

[21] A. Baknina, O. Ozel, J. Yang, S. Ulukus, and A. Yener. Sending information through status updates. In *IEEE ISIT*, June 2018.

[22] P. Gupta and P. R. Kumar. Capacity of wireless networks. *IEEE Transactions on Information Theory*, 46(2):388–404, March 2000.

[23] A. El Gamal, J. Mammen, B. Prabhakar, and D. Shah. Optimal throughput-delay scaling in wireless networks-part I: The fluid model. *IEEE Transactions on Information Theory*, 52(06):2568–2592, June 2006.

[24] M. Grossglauser and D. N. C. Tse. Mobility increases the capacity of ad hoc wireless networks. *IEEE Transactions on Networking*, 10(04):477–486, August 2002.

[25] M. J. Neely and E. Modiano. Capacity and delay trade-offs for ad hoc mobile networks. *IEEE Transactions on Information Theory*, 51(06):1917–1937, June 2005.

[26] G. Sharma, R. Mazumdar, and N. B. Shroff. Delay and capacity trade-offs in mobile ad hoc networks: A global perspective. In *IEEE Infocom*, April 2006.

[27] A. Ozgur, O. Leveque, and D. N. C. Tse. Hierarchical cooperation achieves optimal capacity scaling in ad hoc networks. *IEEE Transactions on Information Theory*, 53(10):3549–3572, September 2007.

[28] A. Ozgur and O. Leveque. Throughput-delay trade-off for hierarchical cooperation in ad hoc wireless networks. *IEEE Transactions on Information Theory*, 56(03):1369–1377, March 2010.

[29] I. Kadota, E. Uysal-Biyikoglu, R. Singh, and E. Modiano. Minimizing the age of information in broadcast wireless networks. In *Allerton Conference*, September 2016.

[30] N. Pappas, J. Gunnarsson, L. Kratz, M. Kountouris, and V. Angelakis. Age of information of multiple sources with queue management. In *IEEE ICC*, June 2015.

[31] S. Ioannidis, A. Chaintreau, and L. Massoulie. Optimal and scalable distribution of content updates over a mobile social network. In *IEEE Infocom*, April 2009.

[32] J. Zhong, E. Soljanin, and R. D. Yates. Status updates through multicast networks. In *Allerton Conference*, October 2017.

[33] B. Buyukates, A. Soysal, and S. Ulukus. Age of information in two-hop multicast networks. In *Asilomar Conference*, October 2018.

[34] B. Buyukates, A. Soysal, and S. Ulukus. Age of information in multihop multicast networks. *Journal of Communications and Networks, special issue on Age of Information*, 21(3):256–267, July 2019.

[35] B. Buyukates, A. Soysal, and S. Ulukus. Age of information in multicast networks with multiple update streams. In *Asilomar Conference*, November 2019.

[36] Z. Jiang, B. Krishnamachari, X. Zheng, S. Zhou, and Z. Niu. Timely status update in massive IoT systems: Decentralized scheduling for wireless uplinks. In *IEEE ISIT*, June 2018.

[37] R. D. Yates and S. K. Kaul. Real-time status updating: Multiple sources. In *IEEE ISIT*, July 2012.

[38] B. Buyukates, A. Soysal, and S. Ulukus. Age of information scaling in large networks. In *IEEE ICC*, May 2019.

[39] R. D. Yates and D. J. Goodman. *Probability and Stochastic Processes*. Wiley, 2007.