

Energy Harvesting Multiple Access Channel with Peak Temperature Constraints

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Abstract— We consider a two-user energy harvesting multiple access channel where the temperatures of the nodes are affected by the electromagnetic waves caused by the data transmission. To protect the nodes from excess heat, power allocation policies should take into consideration the temperature of all the nodes. In this paper, we study the optimal power allocations when the temperatures of the nodes are subject to peak temperature constraints. We first study the general case where each node has a different peak temperature requirement and the nodes have different temperature parameters. For this case, we show that the capacity region of the single energy arrival case is a single pentagon. We also show that the optimal policy for the multiple energy arrivals can be obtained by using generalized water-filling. We then study the temperature limited case where the transmitters have abundant energy and the only binding constraints are the temperature constraints. We study the optimal power allocation in this case and we derive sufficient conditions under which the rate region collapses to a single pentagon.

I. INTRODUCTION

We study the optimal power allocation for a two-user energy harvesting multiple access channel where the temperatures at the transmitters and the receiver are constrained by a peak value. The temperature constraint ensures that the nodes do not overheat as a result of data transmission.

Power scheduling for energy harvesting communication systems has gained attention recently due to the challenges which arise with having a finite amount of energy arriving spread in time. The single-user channel is studied in [1]–[4], the broadcast channel in [5], [6], the multiple access channel in [7], [8], the multi-hop channels in [9]–[12], and the two-way channel in [13], [14]. The effect of temperature on power allocation is studied in [15]–[18].

The effect of temperature on power allocation was first studied in [15]. In [15], a peak temperature constraint is considered and the optimal power allocation is studied for the continuous time setting. The cases of imperfect transmitter and receiver circuitry are studied in [16]. The discrete time version of [15] is introduced in [17], [18] for implicit and explicit temperature constraints. In this paper, we extend this explicit temperature constrained model for a multi-user setting.

In this paper, we study the optimal offline power allocation problem for the two-user multiple-access channel model in Fig. 1. We first show that the capacity region for the energy

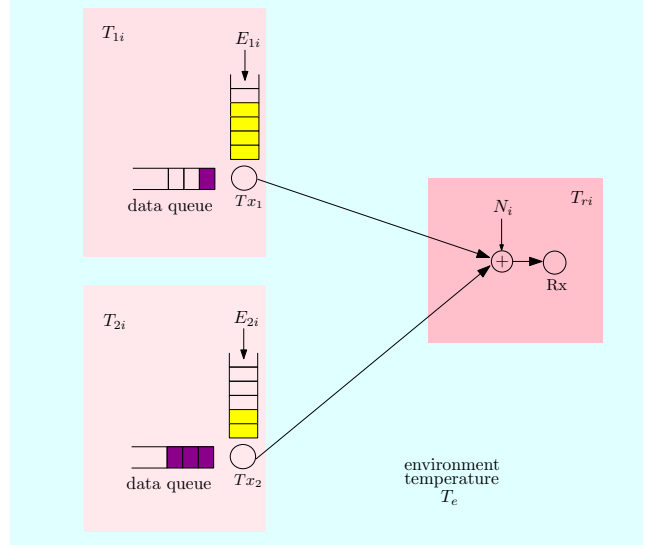


Fig. 1. An energy harvesting multiple access channel with peak temperature constraints. In general, the critical temperatures at the nodes are different.

harvesting multiple access channel with peak temperature constraints is a convex region, and hence, the region can be fully characterized by studying its tangent lines. For the case of single energy arrival, we show that the optimal achievable rate region is a single pentagon which is constructed by the intersection of two pentagons which result from the energy and temperature constraints. We show that for the case of multiple energy arrivals that the optimal power allocations can be obtained by generalized water-filling. We then study the special case when the energy is abundant and the only binding constraints are the peak temperature constraints at the nodes. In this case, we provide an explicit structure for the optimal power allocations; we show that at any point in the optimal rate region, at least one of the powers is non-increasing. We also show that and the sum of the powers is non-decreasing in most of the optimal rate region. Then, we derive sufficient conditions under which the optimal rate region of the multiple access channel reduces to a single pentagon.

II. SYSTEM MODEL

We consider the temperature model considered at [15]–[18]. In the two-user multiple access channel shown in Fig. 1, we assume that each node is placed in a different physical

environment with different heat characteristics in the presence of electromagnetic radiation. The temperature at node $j \in \{1, 2, r\}$, $T_j(t)$, is given by the following differential equation

$$\frac{dT_j(t)}{dt} = a_j p_j(t) - b_j(T_j(t) - T_e) \quad (1)$$

where T_e is the environment temperature, $p_j(t)$ is the power at user j , and a_j, b_j are non-negative constant parameters in the device temperature evolution model. These parameters determine the speed of heating up and cooling down in the presence of applied electromagnetic radiation. For example, if a_j is small, the device temperature will not change much by the electromagnetic radiations while if b_i is small, the device will cool down quickly. Incident power at the receiver is $p_r(t) = h_1 p_1(t) + h_2 p_2(t)$. For simplicity, we let $h_1 = h_2 = 1$.

With initial temperature $T_j(0) = T_e$, the solution of (1) is:

$$T_j(t) = e^{-b_j t} \int_0^t e^{b_j \tau} a_j p_j(\tau) d\tau + T_e \quad (2)$$

Similar to [17], [18], we consider temperature levels at discrete time instants $i\Delta$ where i is the slot index and Δ is the slot length. We define $T_{ji} \triangleq T_j(i\Delta)$ as the temperature level at the end of the i th slot, and $P_{ji} \triangleq p_j(i\Delta)$ as the power level used in the i th slot. Using (2), T_{ji} is expressed as:

$$T_{ji} = e^{-b_j i\Delta} \int_0^{i\Delta} e^{b_j \tau} a_j p_j(\tau) d\tau + T_e \quad (3)$$

$$= \alpha_j T_{j(i-1)} + \beta_j P_{ji} + \gamma_j \quad (4)$$

where $\alpha_j = e^{-b_j \Delta}$, $\beta_j = \frac{a_j}{b_j} [1 - \alpha_j]$ and $\gamma_j = T_e [1 - \alpha_j]$.

The goal of this paper is to determine the optimal power allocation policy which achieves the largest departure region for the Gaussian multiple access channel under temperature and energy constraints. Assuming a unit variance for the Gaussian noise, the achievable rates for the multiple access channel are given by:

$$\mathcal{C}(P_{1i}, P_{2i}) = \left\{ (r_{1i}, r_{2i}) : r_{1i} \leq \frac{1}{2} \log(1 + P_{1i}) \right. \\ \left. r_{2i} \leq \frac{1}{2} \log(1 + P_{2i}) \right. \\ \left. r_{1i} + r_{2i} \leq \frac{1}{2} \log(1 + P_{1i} + P_{2i}) \right\} \quad (5)$$

The aim is to maximize the cumulative achievable rate region, departure region, i.e., subject to a deadline D .

The maximum allowable peak temperature at node $j \in \{1, 2, r\}$ is T_{cj} . The peak temperature constraint can then be written as follows:

$$\mathcal{T}_k(\{a_j, b_j, T_{cj}\}, j = 1, 2, r) = \left\{ (T_{1i}, T_{2i}, T_{ri})_{i=1}^k : \right. \\ T_{1k} \leq T_{c1} \\ T_{2k} \leq T_{c2} \\ \left. T_{rk} \leq T_{cr} \right\} \quad (6)$$

Using (4), these temperature constraints can be written in

terms of only the transmission powers as follows:

$$\mathcal{T}_k(\{a_j, b_j, T_{cj}\}, j = 1, 2, r) = \left\{ (P_{1i}, P_{2i})_{i=1}^k : \right. \\ \sum_{i=1}^k \alpha_1^{k-i} P_{1i} \leq \frac{T_{c1} - T_e}{\beta_1} \\ \sum_{i=1}^k \alpha_2^{k-i} P_{2i} \leq \frac{T_{c2} - T_e}{\beta_2} \\ \left. \sum_{i=1}^k \alpha_r^{k-i} (P_{1i} + P_{2i}) \leq \frac{T_{cr} - T_e}{\beta_r} \right\} \quad (7)$$

Moreover, power allocations P_{1i} and P_{2i} must satisfy the following energy causality constraints:

$$\mathcal{E}_k(\mathbf{E}_1, \mathbf{E}_2) = \left\{ (P_{1i}, P_{2i})_{i=1}^k : \sum_{i=1}^k P_{1i} \leq \sum_{i=1}^k E_{1i} \right. \\ \left. \sum_{i=1}^k P_{2i} \leq \sum_{i=1}^k E_{2i} \right\} \quad (8)$$

In this paper, we characterize the maximum achievable multiple access rate region under the constraints (5), (7) and (8). We first establish the convexity of the region resulting from these constraints in the following lemma:

Lemma 1 *The optimal rate region formed by (5), (7) and (8) is a convex region.*

Proof: Let us consider two feasible power policies $(\bar{\mathbf{P}}_1, \bar{\mathbf{P}}_2)$ and $(\tilde{\mathbf{P}}_1, \tilde{\mathbf{P}}_2)$. Let us also consider a new policy which is a convex combination of the previous two feasible policies $(\mathbf{P}_1, \mathbf{P}_2) = \eta(\bar{\mathbf{P}}_1, \bar{\mathbf{P}}_2) + (1 - \eta)(\tilde{\mathbf{P}}_1, \tilde{\mathbf{P}}_2)$, where $\eta \in [0, 1]$. Since the constraints (7) and (8) are linear, this new policy is also feasible in the constraints.

Now assume that policies $(\mathbf{P}_1, \mathbf{P}_2)$, $(\bar{\mathbf{P}}_1, \bar{\mathbf{P}}_2)$ and $(\tilde{\mathbf{P}}_1, \tilde{\mathbf{P}}_2)$ achieve pentagons \mathcal{C} , $\bar{\mathcal{C}}$ and $\tilde{\mathcal{C}}$, respectively. Now, choose two points \bar{q} and \tilde{q} such that $\bar{q} \in \bar{\mathcal{C}}$ and $\tilde{q} \in \tilde{\mathcal{C}}$. Then for any $\eta \in [0, 1]$ we define $q = \eta\bar{q} + (1 - \eta)\tilde{q}$. We need to show that q is in \mathcal{C} . We show this as follows:

$$q_1 = \eta\bar{q}_1 + (1 - \eta)\tilde{q}_1 \quad (9)$$

$$= \eta \sum_{i=1}^D \frac{1}{2} \log(1 + \bar{P}_{1i}) + (1 - \eta) \sum_{i=1}^D \frac{1}{2} \log(1 + \tilde{P}_{1i}) \quad (10)$$

$$\leq \sum_{i=1}^D \frac{1}{2} \log(1 + \eta\bar{P}_{1i} + (1 - \eta)\tilde{P}_{1i}) \quad (11)$$

$$= \sum_{i=1}^D \frac{1}{2} \log(1 + P_{1i}) \quad (12)$$

which is feasible in \mathcal{C} . Similarly, we can show this for q_2 and $q_1 + q_2$. This concludes the proof. ■

Since the region is a convex region, we can characterize it by considering its tangent lines. The tangent lines can be expressed as $\mu_1 r_{1i} + \mu_2 r_{2i}$. Changing the ratio between μ_1 and μ_2 will change the slope of the tangent line. Hence, we

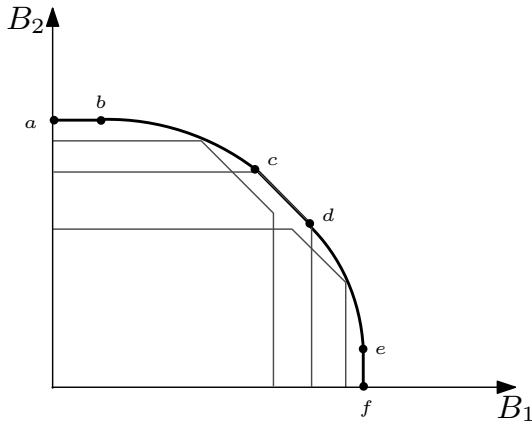


Fig. 2. The rate region for the multiple access channel.

formulate the problem as follows:

$$\begin{aligned}
 & \max_{\{r_i, P_i\}} \sum_{i=1}^D \mu_1 r_{1i} + \mu_2 r_{2i} \\
 & \text{s.t. } (P_{1i}, P_{2i})_{i=1}^k \in \mathcal{T}_k(\{a_j, b_j, T_{c_j}\}, j = 1, 2, r) \\
 & (r_{1k}, r_{2k}) \in \mathcal{C}(P_{1k}, P_{2k}) \\
 & (P_{1i}, P_{2i})_{i=1}^k \in \mathcal{E}_k(\mathbf{E}_1, \mathbf{E}_2), \forall k \in \{1, \dots, D\} \quad (13)
 \end{aligned}$$

for $\mu_1, \mu_2 \in [0, 1]$. For each different value of $\frac{\mu_1}{\mu_2}$ we get a point on the boundary of the optimal achievable rate region, the region is shown in Fig. 2, where $B_j = \sum_{i=1}^D r_{ji}$.

In what follows, we first study the general case. We show that for the single slot scenario, i.e., when $D = 1$, that the rate region is a single pentagon generated by the intersection of regions generated by the temperature and energy constraints. We then study the multiple slot setting and show that the optimal power allocation can be obtained by using generalized water-filling algorithms. We then study the temperature limited case and derive sufficient conditions under which the capacity region collapses to a single pentagon.

III. GENERAL CASE

In this section, we first study the single energy arrival. Then we study the multiple energy arrival case.

A. Single Slot Analysis

In this subsection, we study the case when there is only one energy arrival and one slot to use this incoming energy. The problem in this case is,

$$\begin{aligned}
 & \max_{r_1, r_2, P_1, P_2} \mu_1 r_1 + \mu_2 r_2 \\
 & \text{s.t. } (P_1, P_2) \in \mathcal{T}(\{a_j, b_j, T_{c_j}\}, j = 1, 2, r) \\
 & (P_1, P_2) \in \mathcal{E}(E_1, E_2), (r_1, r_2) \in \mathcal{C}(P_1, P_2) \quad (14)
 \end{aligned}$$

For this case, the optimal rate region is a single pentagon which we characterize in the next lemma which is also illustrated in Fig. 3.

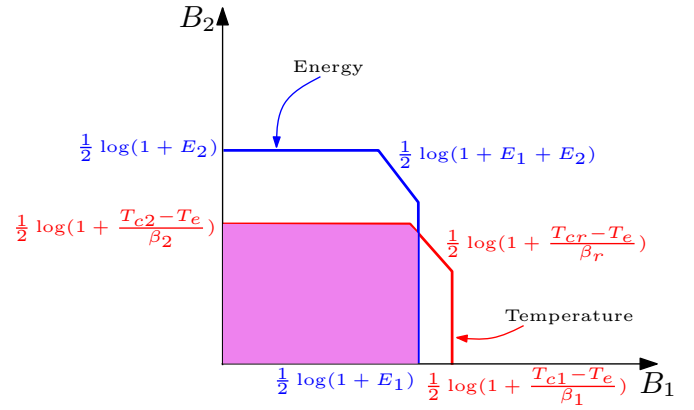


Fig. 3. The rate region for the multiple access channel with a single energy arrival.

Lemma 2 *The optimal rate region for problem (14) is a single pentagon and is given by:*

$$\begin{aligned}
 \mathcal{C}_s(E_1, E_2) = & \{(r_1, r_2): \\
 & r_1 \leq \frac{1}{2} \log \left(1 + \min \left\{ E_1, \frac{T_{c1} - T_e}{\beta_1} \right\} \right) \\
 & r_2 \leq \frac{1}{2} \log \left(1 + \min \left\{ E_2, \frac{T_{c2} - T_e}{\beta_2} \right\} \right) \\
 & r_1 + r_2 \leq \frac{1}{2} \log \left(1 + \min \left\{ E_1 + E_2, \frac{T_{cr} - T_e}{\beta_r} \right\} \right) \} \quad (15)
 \end{aligned}$$

The proof of Lemma 2 follows from the intersection of (7) and (8) for a single slot setting, i.e., when $D = 1$. The rate region is a single pentagon as there is only one transmission policy, which is to use the maximum allowable power in a way to make sure that both energy and temperature constraints are satisfied.

B. Multiple Slot Analysis

Now, we study the multiple energy arrival case and characterize the optimal solution for different parts of the rate region in Fig. 2.

1) *Point a:* Achieving points *a* (and similarly *f*) is similar to the single-user case in [18], however, here there is an extra constraint due to the difference between the transmitter and receiver's material. The problem can be written as follows:

$$\begin{aligned}
 & \max_{\{P_{2i}\}} \sum_{i=1}^D \frac{1}{2} \log(1 + P_{2i}) \\
 & \text{s.t. } \sum_{i=1}^k \alpha_2^{k-i} P_{2i} \leq \frac{T_{c2} - T_e}{\beta_2} \\
 & \sum_{i=1}^k \alpha_r^{k-i} P_{2i} \leq \frac{T_{cr} - T_e}{\beta_r} \\
 & \sum_{i=1}^k P_{2i} \leq \sum_{i=1}^k E_{2i}, \forall k \in \{1, \dots, D\} \quad (16)
 \end{aligned}$$

This problem is a convex optimization problem and the KKTs are necessary and sufficient. The Lagrangian of this problem can be written as follows:

$$\begin{aligned} \mathcal{L} = & - \sum_{i=1}^D \frac{1}{2} \log(1 + P_{2i}) + \sum_{k=1}^D \lambda_{2k} \left(\sum_{i=1}^k \alpha_2^{k-i} P_{2i} - \frac{T_{c2} - T_e}{\beta_2} \right) \\ & + \sum_{k=1}^D \lambda_{rk} \left(\sum_{i=1}^k \alpha_r^{k-i} P_{2i} - \frac{T_{cr} - T_e}{\beta_r} \right) \\ & + \sum_{k=1}^D \nu_{2k} \left(\sum_{i=1}^k P_{2i} - \sum_{i=1}^k E_{2i} \right) \end{aligned} \quad (17)$$

Differentiating with respect to P_{2i} gives the optimal power as:

$$P_{2i} = \left(\frac{1}{\sum_{k=i}^D \lambda_{2k} \alpha_2^{k-i} + \lambda_{rk} \alpha_r^{k-i} + \sum_{k=i}^D \nu_{2k}} - 1 \right)^+ \quad (18)$$

which can be solved by directional generalized water-filling. We next specify a special case in the following lemma.

Lemma 3 *When either $\alpha_r < \alpha_2$ and $\frac{T_{c2} - T_e}{\beta_2} \leq \frac{T_{cr} - T_e}{\beta_r}$ or $\alpha_r \leq \alpha_2$ and $\frac{T_{c2} - T_e}{\beta_2} < \frac{T_{cr} - T_e}{\beta_r}$ is satisfied, we have $\lambda_{rk} = 0$ and the problem reduces to the single-user problem in [18].*

Proof: Let us consider the case when $\alpha_r < \alpha_2$ and $\frac{T_{c2} - T_e}{\beta_2} \leq \frac{T_{cr} - T_e}{\beta_r}$ are satisfied. This implies the following:

$$\sum_{i=1}^k \alpha_r^{k-i} P_{2i} < \sum_{i=1}^k \alpha_2^{k-i} P_{2i} \leq \frac{T_{c2} - T_e}{\beta_2} \leq \frac{T_{cr} - T_e}{\beta_r} \quad (19)$$

From complementary slackness, this implies that $\lambda_{rk} = 0$. The proof follows similarly for the other case. ■

The problem reduces to a single-user problem when either $\alpha_2 < \alpha_r$ and $\frac{T_{cr} - T_e}{\beta_r} \leq \frac{T_{c2} - T_e}{\beta_2}$ or $\alpha_2 \leq \alpha_r$ and $\frac{T_{cr} - T_e}{\beta_r} < \frac{T_{c2} - T_e}{\beta_2}$.

2) *Point b:* We then study point b (or similarly point e). Point b represents the maximum rate the first user can achieve while user 2 is achieving its single-user rate. Hence, to achieve point b , we need to fix the second user's power allocation to be the optimal single-user power allocation P_{2i}^s and then solve for the maximum rate for user 1. This can be done by solving the following optimization problem:

$$\begin{aligned} \max_{\{P_{1i}\}} & \sum_{i=1}^D \frac{1}{2} \log \left(1 + \frac{P_{1i}}{1 + P_{2i}^s} \right) \\ \text{s.t.} & \sum_{i=1}^k \alpha_1^{k-i} P_{1i} \leq \frac{T_{c1} - T_e}{\beta_1} \\ & \sum_{i=1}^k \alpha_r^{k-i} (P_{1i} + P_{2i}^s) \leq \frac{T_{cr} - T_e}{\beta_r} \\ & \sum_{i=1}^k P_{1i} \leq \sum_{i=1}^k E_{1i}, \quad \forall k \end{aligned} \quad (20)$$

Here, the power suffers from a different fading of $\frac{1}{1 + P_{2i}^s}$ in each slot. Moreover, there is a time-varying peak temperature

constraint. Using a Lagrange analysis, the optimal power allocation is given by:

$$P_{1i} = \left(\frac{1}{\sum_{k=i}^D \lambda_{1k} \alpha_1^{k-i} + \lambda_{rk} \alpha_r^{k-i} + \sum_{k=i}^D \mu_k} - 1 - P_{2i}^s \right)^+ \quad (21)$$

The optimal solution can be found using generalized water-filling. Similar to Lemma 3, we conclude that if either $\alpha_1 < \alpha_r$ and $\frac{T_{cr} - T_e}{\beta_r} \leq \frac{T_{c1} - T_e}{\beta_1}$ or $\alpha_1 \leq \alpha_r$ and $\frac{T_{cr} - T_e}{\beta_r} < \frac{T_{c1} - T_e}{\beta_1}$ we have $\lambda_{1k} = 0$.

Also note that if for any slot m we have $\sum_{i=1}^m \alpha_r^{m-i} P_{2i}^s = \frac{T_{cr} - T_e}{\beta_r}$, this implies that in the optimal solution of (20) we have $P_{1i} = 0, \forall i \in \{1, \dots, m\}$.

3) *The points between b and c:* The points between b and c can be achieved by setting $\mu_2 > \mu_1 > 0$. In this case, we are aiming to characterize the left corner point of the resulting pentagon. The problem in this case is written as:

$$\begin{aligned} \max_{\{P_{1i}\}, \{P_{2i}\}} & \sum_{i=1}^D (\mu_2 - \mu_1) \frac{1}{2} \log(1 + P_{2i}) + \mu_1 \frac{1}{2} \log(1 + P_{1i} + P_{2i}) \\ \text{s.t.} & (P_{1i}, P_{2i})_{i=1}^k \in \mathcal{T}_k(\{a_j, b_j, T_{cj}\}, j = 1, 2, r) \\ & (P_{1i}, P_{2i})_{i=1}^k \in \mathcal{E}_k(\mathbf{E}_1, \mathbf{E}_2), \quad \forall k \in \{1, \dots, D\} \end{aligned} \quad (22)$$

Similar to the previous cases, the optimal power allocation can be obtained using generalized water-filling.

4) *Sum-rate (the line between c and d):* For the sum-rate, we have $\mu_1 = \mu_2 > 0$ and the problem can be written as:

$$\begin{aligned} \max_{\{P_i\}} & \sum_{i=1}^D \frac{1}{2} \log(1 + P_{1i} + P_{2i}) \\ \text{s.t.} & (P_{1i}, P_{2i})_{i=1}^k \in \mathcal{T}_k(\{a_j, b_j, T_{cj}\}, j = 1, 2, r) \\ & (P_{1i}, P_{2i})_{i=1}^k \in \mathcal{E}_k(\mathbf{E}_1, \mathbf{E}_2), \quad \forall k \in \{1, \dots, D\} \end{aligned} \quad (23)$$

The solution in general can be found using generalized water-filling. The problem reduces to a single-user problem in terms of the sum of the power, i.e., $P_{1i} + P_{2i}$, when either $\max\{\alpha_1, \alpha_2\} < \alpha_r$ and $\frac{T_{cr} - T_e}{\beta_r} \leq \min\{\frac{T_{c1} - T_e}{\beta_1}, \frac{T_{c2} - T_e}{\beta_2}\}$ or $\max\{\alpha_1, \alpha_2\} \leq \alpha_r$ and $\frac{T_{cr} - T_e}{\beta_r} < \min\{\frac{T_{c1} - T_e}{\beta_1}, \frac{T_{c2} - T_e}{\beta_2}\}$ is satisfied. The proof follows similar to Lemma 3.

IV. TEMPERATURE LIMITED CASE

In this section, we study the case when the system is temperature limited, i.e., when the energy constraint is never binding. This occurs when the following is satisfied:

$$\frac{T_{jc} - T_e}{\beta} < \frac{\sum_{i=1}^k E_{ji}}{k}, \quad \forall k \in \{1, \dots, D\}, \quad j = 1, 2 \quad (24)$$

The problem in this case can be written as:

$$\begin{aligned} \max_{\{r_i, P_i\}} & \sum_{i=1}^D \mu_1 r_{1i} + \mu_2 r_{2i} \\ \text{s.t.} & (P_{1i}, P_{2i})_{i=1}^k \in \mathcal{T}_k(\{a_j, b_j, T_{cj}\}, j = 1, 2, r) \\ & (r_{1k}, r_{2k}) \in \mathcal{C}(P_{1k}, P_{2k}), \quad \forall k \end{aligned} \quad (25)$$

We first study the case when a_i, b_i, T_{ic} are not equal.

A. General Case

In this part, we first study a general characteristic of the rate region. We first have the following lemma.

Lemma 4 *At any point of the boundary of the optimal achievable rate region, for any two slots, either P_{1i} or P_{2i} is non-increasing or both are non-increasing. Furthermore, for the region between points b and e , $P_{1i} + P_{2i}$ is non-increasing throughout.*

Proof: To prove this, we need to show it in each part of the rate. We begin with the sum-rate region. In this case, the problem can be written as follows:

$$\begin{aligned} \max_{\{P_i\}} \quad & \sum_{i=1}^D \frac{1}{2} \log(1 + P_{1i} + P_{2i}) \\ \text{s.t.} \quad & (P_{1i}, P_{2i})_{i=1}^k \in \mathcal{T}_k(\{a_j, b_j, T_{c_j}\}, j = 1, 2, r), \quad \forall k \end{aligned} \quad (26)$$

Now, assume for the sake of contradiction that the optimal powers for slots $j < k$ satisfy $P_{1j} + P_{2j} < P_{1k} + P_{2k}$. This means that at least one of the powers is also increasing. Assume without loss of generality that it is at user 2, i.e., $P_{2j} < P_{2k}$. We can then decrease P_{2k} by a small enough δ and increase P_{2j} with the same amount. This remains feasible in all the constraints as this makes room for more temperature budget. Moreover, due to concavity of the objective function this strictly increases the objective function. Hence, $P_t \triangleq P_{1i} + P_{2i}$ has to be non-increasing. This also proves that at least one of the powers has to be non-increasing, as otherwise P_t will be equal to the sum of two increasing sequences which has to be increasing also.

We now consider the case when $\mu_1 > \mu_2$. In this case, the problem can be written as:

$$\begin{aligned} \max_{\{r_i, P_i\}} \quad & \sum_{i=1}^D (\mu_1 - \mu_2) \frac{1}{2} \log(1 + P_{1i}) + \mu_2 \frac{1}{2} \log(1 + P_{1i} + P_{2i}) \\ \text{s.t.} \quad & (P_{1i}, P_{2i})_{i=1}^k \in \mathcal{T}_k(\{a_j, b_j, T_{c_j}\}, j = 1, 2, r), \quad \forall k \end{aligned} \quad (27)$$

The proof again follows by contradiction following the same steps as the previous case. This covers the statement for the curve between points b and e

It remains to consider the optimal individual power allocation for the curve between points a and b (or equivalently point e) in Fig. 2. At this point, the second user is transmitting with its single-user power allocation, and it follows from [18] that it is non-increasing. ■

B. Identical Temperature Parameters

In this part, we consider the case when all nodes have identical temperature dynamics parameters, i.e., $a_i = a$, $b_i = b$. We still allow different critical temperature constraints in each node.

In what follows we study sufficient conditions for which the multiple access rate region reduces to a single pentagon.

Lemma 5 *The optimal rate region is a single pentagon when the following condition is satisfied:*

$$\max\{T_{c1}, T_{c2}\} - T_e \leq (T_{cr} - T_e)(1 - \alpha) \leq (1 - \alpha) \min\{T_{c1}, T_{c2}\} \quad (28)$$

Proof: To prove this, we need to show that point b is the same as point c . To show this we need to first obtain the optimal sum-power allocation achieving the region between c and d . Then, we need to show that the optimal single user power allocation is feasible in the optimal sum-power allocation. This will imply that point c is at least the same height as point b , which implies that points b and c are the same point.

We first characterize the optimal power allocation for the sum-rate case. When the following condition is satisfied:

$$T_{cr} \leq \min\{T_{c1}, T_{c2}\} \quad (29)$$

the sum-rate problem reduces to only one constraint which is at the receiver. This follows because we have the following:

$$\begin{aligned} \sum_{i=1}^k \alpha^{k-i} \max\{P_{1i}, P_{2i}\} & \leq \sum_{i=1}^k \alpha^{k-i} (P_{1i} + P_{2i}) \\ & \leq \frac{T_{cr} - T_e}{\beta} \\ & \leq \frac{\min\{T_{c1}, T_{c2}\} - T_e}{\beta} \end{aligned} \quad (30)$$

Hence, whenever the temperature constraint is satisfied at the receiver, it will be also satisfied at both transmitters. Hence, this is reduced to the single-user problem studied in [18] with the optimization variable as $P_{1i} + P_{2i}$. From the properties of the single-user power allocations in [18], we have that the optimal powers satisfy:

$$(P_{1i} + P_{2i})^* \geq \frac{T_{cr} - T_e}{\beta} (1 - \alpha) \quad (31)$$

Now for the single-user optimal power allocation of user 2, from the feasibility we have:

$$P_{2i}^s \leq \frac{T_{c2} - T_e}{\beta} \quad (32)$$

Therefore, we have:

$$(P_{1i} + P_{2i})^* - P_{2i}^s \geq \frac{T_{cr} - T_e}{\beta} (1 - \alpha) - \frac{T_{c2} - T_e}{\beta} \quad (33)$$

$$\geq 0 \quad (34)$$

where the last inequality follows from our assumption. Similarly, this follows for the points d and e . ■

Note that only one side of the optimal rate region could indeed collapse, i.e., points b and c may coincide while points d and e are always different. We state this fact in the next corollary.

Corollary 1 *In the optimal rate region, point b and point c are the same point when the following is satisfied:*

$$T_{c2} - T_e \leq (T_{cr} - T_e)(1 - \alpha) \leq (1 - \alpha) \min\{T_{c1}, T_{c2}\} \quad (35)$$

C. Temperature Constraint Only at the Receiver

In this section, we study the case when there is a temperature constraint only at the receiver. In this case, the problem can be written as follows:

$$\begin{aligned} \max_{\{r_i, P_i\}} \quad & \sum_{i=1}^D \mu_1 r_{1i} + \mu_2 r_{2i} \\ \text{s.t.} \quad & \sum_{i=1}^k \alpha^{k-i} (P_{1i} + P_{2i}) \leq \frac{T_c - T_e}{\beta} \\ & (r_{1k}, r_{2k}) \in \mathcal{C}(P_{1k}, P_{2k}), \quad \forall k \end{aligned} \quad (36)$$

We now present the following lemma.

Lemma 6 *When $\mu_1 > \mu_2$, the optimal sum-power allocation $P_{1i} + P_{2i}$ is non-increasing. Moreover, user 1 power allocation, P_{1i} , is also non-increasing. Similarly, when $\mu_2 > \mu_1$ we have that the second single-user power allocation, P_{2i} , is non-increasing.*

Proof: When $\mu_1 > \mu_2$, the objective function reduced to:

$$\begin{aligned} \max_{\{r_i, P_i\}} \quad & \sum_{i=1}^D (\mu_1 - \mu_2) \frac{1}{2} \log(1 + P_{1i}) + \mu_2 \frac{1}{2} \log(1 + P_{1i} + P_{2i}) \\ \text{s.t.} \quad & \sum_{i=1}^k \alpha^{k-i} (P_{1i} + P_{2i}) \leq \frac{T_c - T_e}{\beta}, \quad \forall k \end{aligned} \quad (37)$$

The proof for having $P_{1i} + P_{2i}$ is non-increasing follows similar to Lemma 4.

Now, assume that for some $j < k$, we have that $P_{1j} < P_{1k}$. Since we have $P_{1j} + P_{2j} > P_{1k} + P_{2k}$ this implies that $P_{2j} > P_{2k}$. Now, we can decrease P_{2j} by a small enough δ while increasing P_{1j} with the same amount. This leaves the sum powers to be equal, and hence, does not change the constraint feasibility while strictly increasing P_{1j} and hence strictly increasing the objective function. ■

V. NUMERICAL RESULTS

In this section, we show a numerical result for the general setting in Fig. 4. The optimal power allocation does not possess any monotonicity in general. Moreover, even though the temperature is monotonically increasing at both the transmitters, the temperature is not monotonically increasing at the receiver. We observe a main reason for this as the differences among the system parameters defining the temperature dynamics for each node.

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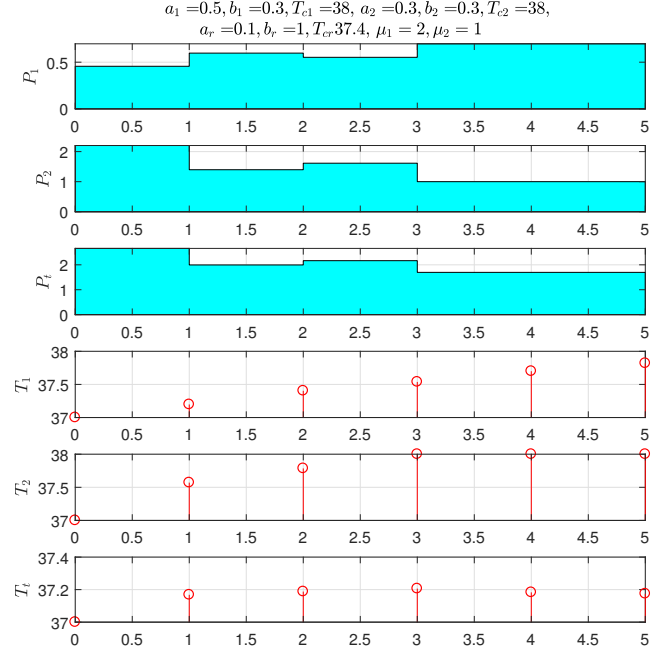


Fig. 4. Simulation result for the general setting with $\mu_1 > \mu_2$.

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