Energy and Data Cooperation in Energy Harvesting Multiple Access Channel

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***Abstract*—We consider the energy harvesting two user Gaussian multiple access channel (MAC), where both users harvest energy from nature. The users cooperate at the physical layer (data cooperation) by establishing common messages through overheard signals and then cooperatively sending them. In addition, the users cooperate at the battery level (energy cooperation) by wirelessly transferring energy to each other. We find the jointly optimal offline transmit power and rate allocation policy together with the energy transfer policy that maximizes the departure region. We provide necessary conditions for energy transfer, and prove some properties of the optimal transmit policy, thereby shedding some light on the interplay between energy and data cooperation.**

I. Introduction

Power control for energy harvesting communications has received considerable attention in recent literature, such as in [1]–[19]. In [1], the transmission completion time minimization problem is solved for an unlimited-sized battery. In [2], the throughput maximization problem is solved and its equivalence to the transmission completion time minimization problem is shown for an arbitrarily-sized battery. In [3], [5]–[10] the problem is extended to fading, broadcast, multiple access and interference channels. Two-hop communication is considered with energy harvesting nodes for half- or full-duplex relay settings in [11]–[15].

Optimal scheduling problems on a multiple access channel (MAC) are investigated in [20], [8], [9], [16], [17]. Reference [20] considers a minimum energy scheduling problem over a MAC where data packets arrive over time. In [8], an energy harvesting MAC is considered. In [9], an energy harvesting MAC with additional maximum power constraints on each user is considered. Recently, in [16], a MAC with both energy and data arrivals and in [17] a cooperative MAC with only energy arrivals is considered.

The concept of energy cooperation and energy sharing is introduced in [18] and communication systems with energy exchange are investigated in [18], [19], [21]–[25]. In this paper, we consider a cooperative MAC with both energy and data cooperation as shown in Fig. 1. We use this system model to investigate interactions of data and energy cooperation, and study their joint optimization.

We first show that in this scenario, the cooperative powers in all slots must be non-zero for both users. Then, we derive a one-to-one relation between the optimal transmission rates and optimal transmission powers. Next, we show that, data cooperation always precedes energy cooperation. In other words, excess energy must first be used to increase cooperative powers and then to further assist the other user by means of direct energy transfer. We determine necessary conditions for energy transfer to take place. We then propose an algorithm which solves the offline energy transfer and power allocation problem iteratively based on these conditions.

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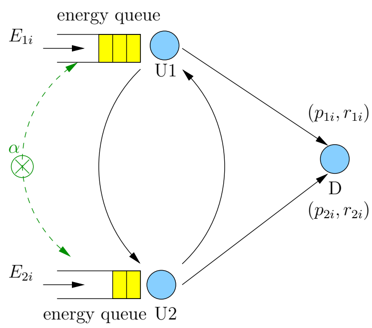

Fig. 1. Cooperative MAC with joint energy and data cooperation

II. System Model and Problem Formulation

We consider the energy harvesting cooperative MAC with bidirectional energy cooperation as shown in Fig. 1. The physical layer is a cooperative Gaussian MAC with unit-variance Gaussian noises at the users and σ^2 variance Gaussian noise at the receiver. With the encoding and decoding policy in [17, Section II], the achievable rate region with transmitter sub-powers $p_{12i}, p_{21i}, p_{U1i}, p_{U2i}$ in each slot i is [17], [26]:

$$\mathcal{C}(p_{12i}, p_{21i}, p_{U1i}, p_{U2i}) = \begin{cases} r_{1i} \leq f(1 + p_{12i}), & (1) \\ r_{2i} \leq f(1 + p_{21i}), & (2) \\ r_{1i} + r_{2i} \leq f(1 + s_i/\sigma^2) & (3) \end{cases}$$

where $f(x) = \frac{1}{2}\log(x), p_{1i} = p_{12i} + p_{U1i}, p_{2i} = p_{21i} + p_{U2i}$,

$$s_i = p_{1i} + p_{2i} + 2\sqrt{p_{U1i}p_{U2i}} \quad (4)$$

The operational meaning of the sub-powers will be important to us: p_{12i} and p_{21i} denote the powers used in slot i to build up common information at the cooperative partner, while p_{U1i}

and p_{U2i} are cooperative powers used for jointly conveying the common information to the receiver.

There are N equal length slots. In slot i, there are energy arrivals to both users with amounts E_{1i}, E_{2i}. Energy transfers from user 1 (2) to user 2 (1) are denoted by δ_{1i} (δ_{2i}). Energy transfer efficiency is $0 \leq \alpha < 1$: when user 1 (2) transfers δ_{1i} (δ_{2i}) Joules of energy to user 2 (1), $\alpha\delta_{1i}$ ($\alpha\delta_{2i}$) Joules of energy enters the energy queue of user 2 (1). We denote the transmission powers, energy transfers and data rates of users 1 and 2 as $p_{12i}, p_{U1i}, \delta_{1i}, r_{1i}$ and $p_{21i}, p_{U2i}, \delta_{2i}, r_{2i}$, respectively. We use boldface letters to denote vectors of these variables. When there is wireless energy transfer, this is done by two separate orthogonal energy transfer units whose coupling frequencies are set differently [27]. Finally, data transmission and energy transfer channels are orthogonal, i.e., energy transfer does not create interference to data communication.

Energy arrivals, as well as energy transfers occur at the beginning of each slot. Hence, the net energy available for user $\ell \in \{1, 2\}$ in each slot $k \in \{1, \dots, N\}$ is given by $\sum_{i=1}^{k}(E_{\ell i} - \delta_{\ell i} + \alpha\delta_{mi})$ where m is the other user. The energy that has not arrived yet cannot be used for data transmission or energy transfer, leading to the following *energy causality constraints*:

$$\sum_{i=1}^{k} p_{1i} \leq \sum_{i=1}^{k}(E_{1i} - \delta_{1i} + \alpha\delta_{2i}), \quad 1 \leq k \leq N \tag{5}$$

$$\sum_{i=1}^{k} p_{2i} \leq \sum_{i=1}^{k}(E_{2i} - \delta_{2i} + \alpha\delta_{1i}), \quad 1 \leq k \leq N \tag{6}$$

The rate allocations (r_{1i}, r_{2i}) must be achievable in each slot:

$$(r_{1i}, r_{2i}) \in \mathcal{C}(p_{12i}, p_{21i}, p_{U1i}, p_{U2i}), \quad 1 \leq i \leq N \tag{7}$$

We aim to maximize the departure region, which can be performed by maximizing the weighted sum rate for given priorities $0 \leq \mu_1, \mu_2 \leq 1$:

$$\max_{\mathbf{p}_{12}, \mathbf{p}_{21}, \mathbf{p}_{U1}, \mathbf{p}_{U2}, \boldsymbol{\delta}_1, \boldsymbol{\delta}_2, \mathbf{r}_1, \mathbf{r}_2 \geq \mathbf{0}} \quad \mu_1 \sum_{i=1}^{N} r_{1i} + \mu_2 \sum_{i=1}^{N} r_{2i} \quad \text{s.t.} \quad (5)\text{-}(7) \tag{8}$$

III. Necessary Conditions and Optimal Profile

In this section, we state the necessary conditions for the optimal profile. These conditions lead to interesting interpretations regarding the nature of energy exchange, including its direction, timing and physical relation to data cooperation. We relax the equality in (4) to reformulate (8) as follows.

$$\max \quad \mu_1 \sum_{i=1}^{N} r_{1i} + \mu_2 \sum_{i=1}^{N} r_{2i}$$

$$\text{s.t.} \quad \sum_{i=1}^{k} p_{12i} + p_{U1i} \leq \sum_{i=1}^{k} E_{1i} - \delta_{1i} + \alpha\delta_{2i}, \quad \forall k \tag{9}$$

$$\sum_{i=1}^{k} p_{21i} + p_{U2i} \leq \sum_{i=1}^{k} E_{2i} - \delta_{2i} + \alpha\delta_{1i}, \quad \forall k \tag{10}$$

$$r_{1i} \leq f(1 + p_{12i}), \quad \forall i \tag{11}$$

$$r_{2i} \leq f(1 + p_{21i}), \quad \forall i \tag{12}$$

$$r_{1i} + r_{2i} \leq f(1 + s_i/\sigma^2), \quad \forall i \tag{13}$$

$$s_i \leq p_{12i} + p_{U1i} + p_{21i} + p_{U2i} + 2\sqrt{p_{U1i}p_{U2i}}, \quad \forall i \tag{14}$$

$$\mathbf{p}_{12}, \mathbf{p}_{21}, \mathbf{p}_{U1}, \mathbf{p}_{U2}, \boldsymbol{\delta}_1, \boldsymbol{\delta}_2, \mathbf{r}_1, \mathbf{r}_2, \mathbf{s} \geq \mathbf{0} \tag{15}$$

The problem in (15) is a convex optimization problem, however it is non-differentiable due to the term $\sqrt{p_{U1i}p_{U2i}}$ when $p_{U1i} = 0$ or $p_{U2i} = 0$. Now, we show that in the optimal solution, the cooperative powers p_{U1i}, p_{U2i} are non-zero at all slots. The proof is given in Appendix A.

Lemma 1 *The cooperative powers are strictly positive at all slots, i.e.,* $p_{U1i} > 0, p_{U2i} > 0, \forall i$.

Utilizing Lemma 1, the functions $\sqrt{p_{U1i}p_{U2i}}$ are now differentiable. Then, the KKT optimality conditions are found as:

$$-\mu_1 + \theta_{1i} + \theta_{3i} - \gamma_{5i} = 0, \forall i \tag{16}$$

$$-\mu_2 + \theta_{2i} + \theta_{3i} - \gamma_{6i} = 0, \forall i \tag{17}$$

$$\sum_{k=i}^{N} \lambda_{1k} - \frac{\theta_{1i}}{(1 + p_{12i})} - \beta_i - \gamma_{1i} = 0, \forall i \tag{18}$$

$$\sum_{k=i}^{N} \lambda_{2k} - \frac{\theta_{2i}}{(1 + p_{21i})} - \beta_i - \gamma_{2i} = 0, \forall i \tag{19}$$

$$\sum_{k=i}^{N} \lambda_{1k} - \beta_i \left(1 + \frac{\sqrt{p_{U2i}}}{\sqrt{p_{U1i}}}\right) - \gamma_{3i} = 0, \forall i \tag{20}$$

$$\sum_{k=i}^{N} \lambda_{2k} - \beta_i \left(1 + \frac{\sqrt{p_{U1i}}}{\sqrt{p_{U2i}}}\right) - \gamma_{4i} = 0, \forall i \tag{21}$$

$$\sum_{k=i}^{N} \lambda_{1k} - \alpha \sum_{k=i}^{N} \lambda_{2k} - \gamma_{7i} = 0, \forall i \tag{22}$$

$$\sum_{k=i}^{N} \lambda_{2k} - \alpha \sum_{k=i}^{N} \lambda_{1k} - \gamma_{8i} = 0, \forall i \tag{23}$$

$$-\frac{\theta_{3i}}{\sigma^2 + s_i} + \beta_i - \gamma_{9i} = 0, \forall i \tag{24}$$

with complementary slackness conditions:

$$\lambda_{1k} \left(\sum_{i=1}^{k} p_{12i} + p_{U1i} - E_{1i} + \delta_{1i} - \alpha\delta_{2i}\right) = 0, \forall k \tag{25}$$

$$\lambda_{2k} \left(\sum_{i=1}^{k} p_{21i} + p_{U2i} - E_{2i} + \delta_{2i} - \alpha\delta_{1i}\right) = 0, \forall k \tag{26}$$

$$\theta_{1i} \left(r_{1i} - f(1 + p_{12i})\right) = 0, \forall i \tag{27}$$

$$\theta_{2i} \left(r_{2i} - f(1 + p_{21i})\right) = 0, \forall i \tag{28}$$

$$\theta_{3i} \left(r_{1i} + r_{2i} - f(1 + s_i/\sigma^2)\right) = 0, \forall i \tag{29}$$

$$\beta_i(s_i - p_{12i} - p_{U1i} - p_{21i} - p_{U2i} - 2\sqrt{p_{U1i}p_{U2i}}) = 0 \tag{30}$$

$$\gamma_{1i}p_{12i} = \gamma_{2i}p_{21i} = \gamma_{3i}p_{U1i} = \gamma_{4i}p_{U2i} = 0 \tag{31}$$

$$\gamma_{5i}r_{1i} = \gamma_{6i}r_{2i} = \gamma_{7i}\delta_{1i} = \gamma_{8i}\delta_{2i} = \gamma_{9i}s_i = 0 \tag{32}$$

From Lemma 1, $\gamma_{3i} = \gamma_{4i} = 0, \forall i$. Now, we investigate the optimal Lagrange multipliers in the following two lemmas.

Lemma 2 *We have* $\beta_i > 0, \forall i$.

Proof: Assume $\beta_i = 0$. From (24), $\theta_{3i} = 0$, from (16), $\theta_{1i} = \mu_1 + \gamma_{5i} > 0$ and from (17), $\theta_{2i} = \mu_2 + \gamma_{6i} > 0$, which imply from (32) $r_{1i} = r_{2i} = 0$, which cannot be optimal. ■

We note that Lemma 2 further means, from (20) and (21), that $\sum_{k=i}^{N} \lambda_{1k} > 0, \sum_{k=i}^{N} \lambda_{2k} > 0, \forall i$.

Lemma 3 *We have* $\gamma_{9i} = 0, \forall i$.

Proof: Assume $\gamma_{9i} > 0$ for some i. This implies $s_i = 0$ and from (13), $r_{1i} = r_{2i} = 0$, which cannot be optimal. ■

Using the structure of the optimal Lagrange multipliers, the following lemma shows the properties of the optimal solution.

Lemma 4 *The optimal profile must satisfy:*

1) $s_i = p_{12i} + p_{U1i} + p_{21i} + p_{U2i} + 2\sqrt{p_{U1i}p_{U2i}}, \forall i$.
2) $r_{1i} + r_{2i} = f(1 + s_i/\sigma^2), \forall i$
3) $r_{1i} = f(1 + p_{12i}), r_{2i} = f(1 + p_{12i}), \forall i$

Proof: We prove the lemma as follows:
1) Follows from Lemma 2 and (30).
2) From Lemma 3 and (24), we have $\theta_{3i} = \beta_i(\sigma^2 + s_i)$. Since $\beta_i > 0$ from Lemma 2, $\theta_{3i} > 0$ which implies $r_{1i} + r_{2i} = f(1 + s_i/\sigma^2)$ from (29).
3) If $p_{12i} = 0$, then we must have $r_{1i} = 0$ and $r_{1i} = f(1 + p_{12i})$ is satisfied. If $p_{12i} > 0$, then $\gamma_{1i} = 0$ from (31). From (18) and (20), $\theta_{1i} = \beta_i\sqrt{p_{U2i}/p_{U1i}}(1+p_{12i}) > 0$. From (27), $r_{1i} = f(1 + p_{12i})$. Similarly, if $p_{21i} = 0$, then we must have $r_{2i} = 0$ and $r_{2i} = f(1+p_{21i})$. If $p_{21i} > 0$, then $\gamma_{2i} = 0$ from (31). From (19) and (21), $\theta_{2i} = \beta_i\sqrt{p_{U1i}/p_{U2i}}(1+p_{21i}) > 0$. From (28), $r_{2i} = f(1 + p_{21i})$. ■

Lemma 4 shows that there is a one-to-one correspondence between the transmission rates and transmission powers. Furthermore, the transmission powers should satisfy

$$f(1 + p_{12i}) + f(1 + p_{21i}) = f(1 + s_i/\sigma^2), \ \forall i. \tag{33}$$

Now, we show that, data cooperation always precedes energy cooperation. In other words, a user with excess energy to be invested in cooperation in a given slot, must first invest more energy for data cooperation than its partner; only then can it invest energy for direct energy cooperation.

Lemma 5 *The optimal profile satisfies the following:*

1) If $\delta_{2i} > 0$ *then* $p_{U2i} > p_{U1i}$.
2) If $\delta_{1i} > 0$ *then* $p_{U1i} > p_{U2i}$.

Proof: We start with the first item. If $\delta_{2i} > 0$, then from (32), we have $\gamma_{8i} = 0$. From (23), we have $\sum_{k=i}^{N} \lambda_{2k} = \alpha \sum_{k=i}^{N} \lambda_{1k}$. This implies from (20) and (21),

$$\beta_i \left(1 + \frac{\sqrt{p_{U1i}}}{\sqrt{p_{U2i}}}\right) = \alpha\beta_i \left(1 + \frac{\sqrt{p_{U2i}}}{\sqrt{p_{U1i}}}\right) \tag{34}$$

Since $\beta_i > 0$ and $\alpha < 1$, (34) implies:

$$\left(1 + \frac{\sqrt{p_{U1i}}}{\sqrt{p_{U2i}}}\right) < \left(1 + \frac{\sqrt{p_{U2i}}}{\sqrt{p_{U1i}}}\right) \tag{35}$$

which implies $p_{U2i} > p_{U1i}$. The second item is proved similarly. ■

Now, we show that if, in a given slot, a user with high priority transfers energy to a user with lower priority, the user with higher priority must already be transmitting at a higher data rate in that slot than the user with lower priority.

Lemma 6 *The optimal profile satisfies the following:*

1) For $\mu_2 \geq \mu_1$*, if* $\delta_{2i} > 0$*, then* $r_{2i} \geq r_{1i}$.
2) For $\mu_1 \geq \mu_2$*, if* $\delta_{1i} > 0$*, then* $r_{1i} \geq r_{2i}$.

Proof: We start with the first item. Assume $\mu_1 \geq \mu_2$ and $\delta_{2i} > 0$. If $p_{12i} = 0$, then $r_{1i} = 0$ and the statement holds trivially. We will assume $p_{12i} > 0$. From (32), $\gamma_{8i} = 0$. From (23), $\sum_{k=i}^{N} \lambda_{2k} < \sum_{k=i}^{N} \lambda_{1k}$. From (18) and (19), this implies

$$\frac{\theta_{2i}}{(1 + p_{21i})} + \beta_i + \gamma_{2i} < \frac{\theta_{1i}}{(1 + p_{12i})} + \beta_i + \gamma_{1i} \tag{36}$$

$$= \frac{\theta_{1i}}{(1 + p_{12i})} + \beta_i \tag{37}$$

where the equality follows since $p_{12i} > 0$ implies $\gamma_{1i} = 0$. Then we have,

$$\frac{\theta_{2i}}{(1 + p_{21i})} < \frac{\theta_{1i}}{(1 + p_{12i})} \tag{38}$$

From (16) and (17) we have,

$$\theta_{1i} = \mu_1 + \gamma_{5i} - \theta_{3i} = \mu_1 - \theta_{3i} \tag{39}$$

$$\theta_{2i} = \mu_2 + \gamma_{6i} - \theta_{3i} \geq \mu_2 - \theta_{3i} \tag{40}$$

where (39) follows from $r_{1i} > 0 = f(1+p_{12i}) > 0$, therefore $\gamma_{5i} = 0$. Since $\mu_2 \geq \mu_1$, we have $\theta_{2i} \geq \theta_{1i}$. Together with (38), this implies we have $p_{21i} > p_{12i}$ and therefore $r_{2i} > r_{1i}$. The second item is proved similarly. ■

IV. Procrastinating Policies

In this section, we show the existence of *procrastinating policies* that solve this problem. Procrastinating policies are introduced in [19] and they have the property that any energy transferred at slot i, must be immediately consumed by the receiving party at slot i. We formalize this definition below.

Definition 1 *A policy is called procrastinating if it satisfies the following property:*

$$p_{12i} + p_{U1i} \geq \alpha\delta_{2i}, \quad p_{21i} + p_{U2i} \geq \alpha\delta_{1i}, \forall i \tag{41}$$

Lemma 7 *There exists a procrastinating policy that solves the problem in (8).*

The proof of Lemma 7 follows from similar arguments as in [19, Lemma 1].

We split the energy transfers δ_{1i}, δ_{2i} into two components $\pi_{1i} \geq 0, \pi_{2i} \geq 0$ and $\nu_{1i} \geq 0, \nu_{2i} \geq 0$ as follows:

$$\delta_{1i} = \pi_{1i} + \nu_{1i}, \quad \delta_{2i} = \pi_{2i} + \nu_{2i}, \quad \forall i \tag{42}$$

In this decomposition π_{1i}, π_{2i} represent the portion of energy transfer that is consumed in the *direct* transmission, i.e., to increase p_{12i}, p_{21i}. Similarly, ν_{1i}, ν_{2i} represent the portion of energy transfer that is consumed in the *cooperative* transmission, i.e., to increase p_{U1i}, p_{U2i}. Any procrastinating policy can now be written as

$$p_{12i} \geq \alpha\pi_{2i}, \quad p_{U1i} \geq \alpha\nu_{2i}, \quad \forall i \tag{43}$$

$$p_{21i} \geq \alpha\pi_{1i}, \quad p_{U2i} \geq \alpha\nu_{1i}, \quad \forall i \tag{44}$$

Lemma 8 *The optimal profile satisfies the following properties,*

1) *For $\mu_2 \geq \mu_1$, if $\pi_{2i} > 0$ then $p_{21i} > 0$.*
2) *For $\mu_1 \geq \mu_2$, if $\pi_{1i} > 0$, then $p_{12i} > 0$.*

Proof: We start with the first item. If $\pi_{2i} > 0$ then $\delta_{2i} > 0$ and from Lemma 6 we have $r_{2i} \geq r_{1i}$ which implies $p_{21i} \geq p_{12i}$. From procrastinating policies, we have that this transferred energy must be used immediately in direct power, therefore $p_{12i} > 0$, which implies $p_{21i} > 0$. The second item is proved similarly. ■

Lemma 8 shows that if any direct energy is transferred from a user with high priority to a user with low priority, then the sending party must be consuming at least some amount in direct transmission.

V. Algorithmic Solution

While we have shown several important properties of the optimal solution, we still need to solve the problem to obtain the transmit scheduling and energy transfer policy. We do this using an algorithmic approach based on the KKT conditions given earlier. We determine the conditions under which energy transfer occurs. Then, we develop an algorithm to compute the optimal energy transfer and power allocation policy. Now, we show that energy transfers are never bidirectional, i.e., in any slot energy transfer happens only in a single direction.

Lemma 9 *In the optimal profile if $\delta_{1i} > 0$ then $\delta_{2i} = 0$ and if $\delta_{2i} > 0$ then $\delta_{1i} = 0$, i.e. $\delta_{1i}\delta_{2i} = 0$, $\forall i$.*

Proof: Assume for some slot i, $\delta_{1i} > 0, \delta_{2i} > 0$. Then, from (32), $\gamma_{7i} = \gamma_{8i} = 0$, and from (22) and (23), $\sum_{k=i}^{N} \lambda_{1k} = \alpha \sum_{k=i}^{N} \lambda_{2k} = \alpha(\alpha \sum_{k=i}^{N} \lambda_{1k})$ which cannot happen unless $\alpha = 1$. ■

Lemma 10 *If $\alpha < \frac{\sum_{k=i}^{N} \lambda_{1k}}{\sum_{k=i}^{N} \lambda_{2k}} < \frac{1}{\alpha}$, there is no energy transfer in either direction at slot i, i.e., $\delta_{1i} = \delta_{2i} = 0$.*

Proof: Let $\alpha < \frac{\sum_{k=i}^{N} \lambda_{1k}}{\sum_{k=i}^{N} \lambda_{2k}} < \frac{1}{\alpha}$, or equivalently $\sum_{k=i}^{N} \lambda_{1k} > \alpha \sum_{k=i}^{N} \lambda_{2k}$ and $\sum_{k=i}^{N} \lambda_{2k} > \alpha \sum_{k=i}^{N} \lambda_{1k}$. From (22) and (23), $\gamma_{7i} > 0, \gamma_{8i} > 0$ and from (32) $\delta_{1i} = \delta_{2i} = 0$. ■

Lemma 11 *A power allocation policy which yields $\frac{\sum_{k=i}^{N} \lambda_{1k}}{\sum_{k=i}^{N} \lambda_{2k}} < \alpha$ or $\frac{\sum_{k=i}^{N} \lambda_{1k}}{\sum_{k=i}^{N} \lambda_{2k}} > \frac{1}{\alpha}$ is strictly suboptimal.*

Proof: Follows from (22), (23) and $\gamma_{7i} \geq 0, \gamma_{8i} \geq 0$. ■

Lemma 12 *In a given slot $i \in 1, \ldots, N$, user $\ell \in \{1, 2\}$ transfers energy to user $m \in \{1, 2\}, m \neq \ell$, i.e., $\delta_{li} > 0$, if $\frac{\sum_{k=i}^{N} \lambda_{\ell k}}{\sum_{k=i}^{N} \lambda_{mk}} = \alpha$.*

Proof: If $\delta_{1i} > 0$ then from (32) we have $\gamma_{7i} = 0$. If $\delta_{2i} > 0$ then from (32) we have $\gamma_{8i} = 0$. The result then follows from (22) and (23). ■

Lemmas 10, 11 and 12 have the following physical interpretation: the ratio of the generalized water levels $v_{\ell i} \triangleq \left(\sum_{k=i}^{N} \lambda_{\ell k}\right)^{-1}$, $\ell \in \{1, 2\}$, determines whether or not there should be energy cooperation in each given slot. In particular, for slot i in which the generalized water level ratio $v_{\ell i}/v_{mi}$ without energy transfer is below the energy transfer efficiency α, energy should be transferred from user m to user ℓ, until the ratio is exactly equal to α. If there is not much discrepancy between the water levels, i.e., the ratio is between α and $1/\alpha$, then there should be no energy transfer.

Note that, the KKT conditions pertaining to the energy transfer policy do not explicitly depend on the powers, and the KKT conditions pertaining to the optimal power distribution policy do not explicitly depend on the energy transfer variables. Since these two sets of conditions are coupled only through the generalized water levels, it is possible to develop an iterative algorithm that iterates over power distribution and energy transfer steps, updating the generalized water levels in each energy transfer step based on Lemmas 10, 11 and 12. Such an algorithm, that provably converges, is given in Algorithm 1.

VI. Numerical Results

In this section, we demonstrate that energy and data cooperation improve the achievable departure region of a MAC. In Fig. 2 we plot the achievable departure region of the proposed cooperative MAC model with energy and data cooperation. For comparison, we also plot the departure region of a cooperative MAC channel with only data cooperation which was studied in [17]. For direct comparison, we use capacity and achievable rate formulas for bandlimited Gaussian channels, that yield the capacity and achievable rates in bits per second. We select the bandwidth and equivalent noise variance as in [17]. The Gaussian noise variances on the direct links are 10^{-2} W and the transmission bandwidth is 1 MHz. For the cooperative MAC only, the inter-user channels are assumed to be AWGN channels with variance 5×10^{-3} W, which translates to inter-user links having a 3-dB SNR advantage over the direct links.

The energy arrivals are $\mathbf{E}_1 = [5, 0, 5, 0, 0, 0, 0, 10, 0, 0]$ mJ, $\mathbf{E}_2 = [5, 0, 0, 0, 0, 10, 0, 0, 5, 0]$ mJ, with energy transfer efficiency of $\alpha = 0.6$ and the transmission deadline is chosen as 10 seconds. Energy cooperation together with data

Algorithm 1 Optimal energy and data cooperation algorithm

Initialize

for $i = 1 : N$ **do**
    $p_{1i} \leftarrow E_{1i}, p_{2i} \leftarrow E_{2i}$
    Determine subpowers $p_{12i}, p_{U1i}, p_{21i}, p_{U2i}$
    Determine water levels $\sum_{k=i}^{N} \lambda_{1k}, \sum_{k=i}^{N} \lambda_{2k}$ from (18), (19)
end for

Main Algorithm

repeat
    for $i = 1 : N$ **do**
If $\sum_{k=i}^{N} \lambda_{1k} < \alpha \sum_{k=i}^{N} \lambda_{2k}$, transfer energy from user 1 to user 2
If $\sum_{k=i}^{N} \lambda_{2k} < \alpha \sum_{k=i}^{N} \lambda_{1k}$, transfer energy from user 2 to user 1
        Determine new subpowers $p_{12i}, p_{U1i}, p_{21i}, p_{U2i}$
        Determine new water levels
    end for
until $\sum_{k=i}^{N} \lambda_{1k} = \alpha \sum_{k=i}^{N} \lambda_{2k}$ or $\sum_{k=i}^{N} \lambda_{2k} = \alpha \sum_{k=i}^{N} \lambda_{1k}$

cooperation has enhanced the departure region of the MAC. It is interesting that this effect is more pronounced in single user optimal points rather than sum rate optimal point. At the sum rate optimal point, $\sum_{i=1}^{N} r_{1i} + r_{2i}$ is optimized and the discrepancies in the energy arrival patterns are negated due to the powers appearing as a summation term.

Now, we investigate the case when user priorities are fixed at $\mu_1 = 0.6, \mu_2 = 1$. In Figs. 3 and 4 we plot the energy usage curve, where we plot the cumulative energy consumption for each user. We separately plot the energy used for direct power components, p_{12}, p_{21} and the cooperative power components, p_{U1}, p_{U2}. We also compare the effect of energy cooperation. From Fig. 3 we see that with energy cooperation, user 1 has transferred considerable amount of energy to user 2 and set its direct powers to zero. This means that user 1 no longer transmits any independent data, but has become a dedicated relay for user 2. From Fig. 4 we see that with energy cooperation, the direct power of user 2 has exceeded the available energy at slot 5. The cooperative powers p_{U2} did not change with energy cooperation and therefore all the transferred energy from user 1 has been consumed in direct transmission.

VII. Conclusions

We considered the energy harvesting two user Gaussian multiple access channel with joint energy and data cooperation. We found the optimal offline transmit power and rate allocation policy that maximizes the departure region. We first showed that, the cooperative powers in each slot must be non-zero for both users. Next, we showed that, data cooperation always precedes energy cooperation. In other words, excess energy must first be used to increase cooperative powers and then to assist the other user. Then, we showed that if a high priority user transfers energy to a low priority user, the higher priority user must already be transmitting at a higher data rate than the other user. Finally, we showed the existence of procrastinating policies, which have the property that energy transferred in a slot must be consumed in that slot immediately. Using the KKT conditions, we devised an iterative solution.

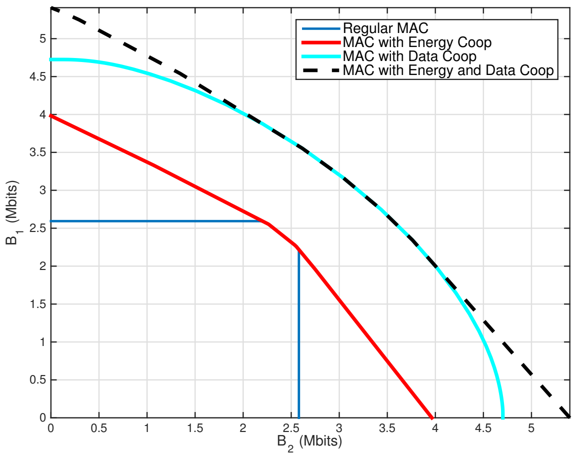

Fig. 2. Departure regions of regular MAC, MAC with energy cooperation, MAC with data cooperation and MAC with energy and data cooperation

Appendix A
Proof of Lemma 1

We discuss three cases to reach a contradiction in each case. Case 1: Let $\exists k$ such that $p_{U1k} = 0, p_{U2k} > 0$. Then, $s_k = p_{12k} + p_{21k} + p_{U2k}$. We define a new power allocation vector as $\tilde{p}_{U2k} = p_{U2k} - \epsilon_1 - \epsilon_2, \tilde{p}_{21k} = p_{21k} + \epsilon_1, \tilde{p}_{U1k} = \alpha\epsilon_2, \tilde{p}_{12k} = p_{12k}$, for some $\epsilon_1 > 0, \epsilon_2 > 0$. Here, we have transferred ϵ_2 amount of energy from user 2 to user 1 and consumed it in the cooperative power of user 1. Additionally, we decreased p_{U2k} by ϵ_1 and increased p_{21k} by ϵ_1. The energy causality constraints are satisfied for the new power allocation. Rate region constraints (11) and (12) become:

$$r_{1k} \leq f(1 + \tilde{p}_{12k}) = f(1 + p_{12k}) \quad (45)$$

$$r_{2k} < f(1 + \tilde{p}_{21k}) = f(1 + p_{21k} + \epsilon_1) \quad (46)$$

For constraint (13), we have

$$\tilde{s}_k = \tilde{p}_{12k} + \tilde{p}_{U1k} + \tilde{p}_{21k} + \tilde{p}_{U2k} + 2\sqrt{\tilde{p}_{U1k}\tilde{p}_{U2k}} \quad (47)$$

$$= p_{12k} + \alpha\epsilon_2 + p_{21k} + \epsilon_1 + p_{U2k} - \epsilon_1 - \epsilon_2 + 2\sqrt{\alpha\epsilon_2(p_{U2k} - \epsilon_1 - \epsilon_2)} \quad (48)$$

$$= s_k + (\alpha - 1)\epsilon_2 + 2\sqrt{\alpha\epsilon_2(p_{U2k} - \epsilon_1 - \epsilon_2)} > s_k \quad (49)$$

where last inequality holds since $2\sqrt{\alpha\epsilon_2(p_{U2k} - \epsilon_1 - \epsilon_2)} > (1 - \alpha)\epsilon_2$ for small ϵ_1, ϵ_2. Therefore,

$$r_{1k} + r_{2k} < f(1 + \tilde{s}_k/\sigma^2) \quad (50)$$

The constraints (46), (50) are loose and we can increase r_{2k} to get a larger optimal value which contradicts the optimality of the original profile. Therefore, case 1 cannot happen.

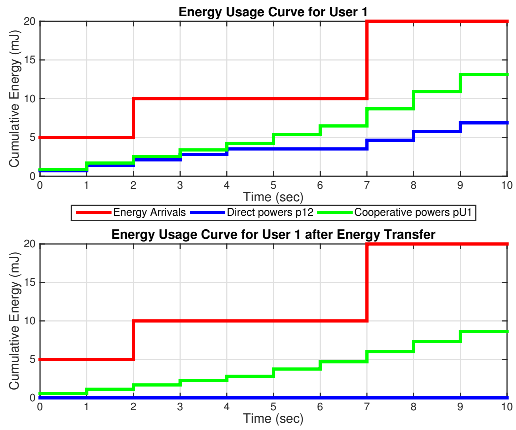

Fig. 3. Energy usage curve for user 1 with and without energy cooperation for $\mu_1 = 0.6$ and $\mu_2 = 1$

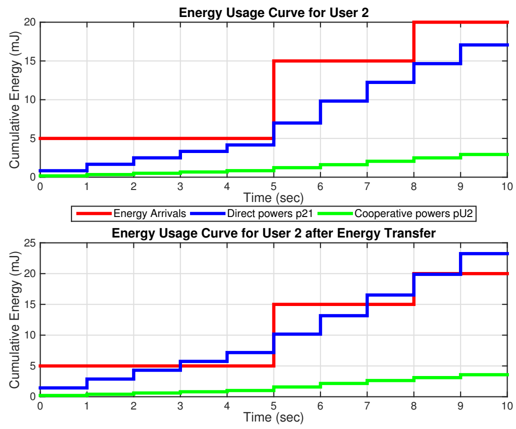

Fig. 4. Energy usage curve for user 2 with and without energy cooperation for $\mu_1 = 0.6$ and $\mu_2 = 1$

Case 2: Similar to case 1, we will reach a contradiction. Case 3: Let $\exists k$ such that $p_{U1k} = 0, p_{U2k} = 0$. Then, $s_k = p_{12k} + p_{21k}$. We cannot have $r_{1k} = f(1+p_{12k}), r_{2k} = f(1+p_{21k})$ because $f(1+p_{12k}) + f(1+p_{21k}) > f(1+s_k/\sigma^2)$ so this is not feasible. Without loss of generality, assume $r_{1k} < f(1+p_{12k})$. We define a new power allocation vector as $\tilde{p}_{12k} = p_{12k} - \epsilon_1 - \epsilon_2, \tilde{p}_{U1k} = \epsilon_1, \tilde{p}_{21k} = p_{21k}, \tilde{p}_{U2k} = \alpha\epsilon_2$. Here, we have transferred ϵ_2 amount of energy from user 1 to user 2 and consumed it in the cooperative power of user 2. Additionally, we decreased p_{12k} by ϵ_1 and increased p_{U1k} by ϵ_1.

For small ϵ_1, ϵ_2 we still have $r_{1k} < f(1+\tilde{p}_{12k})$ which implies (11) is satisfied. Since p_{21k} has not been changed, (12) is satisfied. For constraint (13) we have,

$$\tilde{s}_k = \tilde{p}_{12k} + \tilde{p}_{U1k} + \tilde{p}_{21k} + \tilde{p}_{U2k} + 2\sqrt{\tilde{p}_{U1k}\tilde{p}_{U2k}} \quad (51)$$
$$= p_{12k} - \epsilon_1 - \epsilon_2 + \epsilon_1 + \alpha\epsilon_2 + p_{21k} + 2\sqrt{\epsilon_1\alpha\epsilon_2} \quad (52)$$
$$= s_k + (\alpha - 1)\epsilon_2 + 2\sqrt{\epsilon_1\alpha\epsilon_2} > s_k \quad (53)$$

where last inequality holds for $\epsilon_1 > \epsilon_2(1-\alpha)^2/(4\alpha)$ which we enforce. Then, $r_{1k}+r_{2k} < f(1+\tilde{s}_k/\sigma^2)$. Now, we increase r_{1k} which is a contradiction. Therefore, case 3 cannot happen.

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