Information-Theoretic Analysis of an Energy Harvesting Communication System

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Abstract*—In energy harvesting communication systems, an exogenous recharge process supplies energy for the data transmission and arriving energy can be buffered in a battery before consumption. Transmission is interrupted if there is not sufficient energy. We address communication with such random energy arrivals in an information-theoretic setting. Based on the classical additive white Gaussian noise (AWGN) channel model, we study the coding problem with random energy arrivals at the transmitter. We show that the capacity of the AWGN channel with stochastic energy arrivals is equal to the capacity with an average power constraint equal to the average recharge rate. We provide two different capacity achieving schemes:** *save-and-transmit* **and** *best-effort-transmit. Next, we consider the case where energy arrivals have time-varying average in a larger time scale. We derive the optimal offline power allocation for maximum average throughput and provide an algorithm that finds the optimal power allocation.**

I. Introduction

In this paper, we analyze point-to-point communication of energy harvesting nodes from an information-theoretic perspective. We focus on wireless networking applications where nodes (e.g., sensors nodes) can harvest energy from nature through various different sources, such as solar cells, vibration absorption devices, water mills, thermoelectric generators, microbial fuel cells, etc. In such systems, energy that becomes available for data transmission can be modeled as an exogenous recharge process. Therefore, unlike traditional battery-powered systems, in these systems, energy is not a deterministic quantity, but is a random process which varies stochastically in time at a scale on the order of symbol duration. The transmission can be interrupted due to lack of energy in the battery. On the other hand, excess energy can be buffered in the battery before consumption for transmission. This model requires a major shift in terms of the power constraint imposed on the channel input compared to those in the existing literature.

To illustrate, in information-theoretic approaches, there are two widely used input constraints on the channel inputs of continuous-alphabet channels: average power constraint and amplitude constraint. If the input is average power constrained, then any codeword should be such that while each symbol can take any real value, the average power of the entire codeword should be no more than the power constraint. On the other hand, if the input is amplitude constrained, then every code symbol should be less than the constraint in amplitude. It is clear that in an energy harvesting model, the constraint imposed on the channel input is different than these constraints, in that, while code symbols are instantaneously amplitude constrained, energy can be saved in the battery for later use. This amounts to an unprecedented input constraint on the channel input. In this context, the main goal of this paper is to investigate the effect of stochastic energy arrivals on the communication in an information-theoretic framework. In particular, we augment an energy buffer to the classical AWGN system and study information-theoretically achievable rates.

First, we consider the setting where energy arrives at the transmitter as a stochastic process, which varies on the order of symbol duration. The problem is posed as design of a codebook that complies with all of the input constraints. The input constraint imposed by stochastic energy arrivals is a random one and in this sense it generalizes classical deterministic average or amplitude power constraints. The recharge process together with past code symbols determine the allowable range of inputs in each channel use. We start with showing that the capacity of the AWGN channel with an average power constraint equal to the average recharge rate is an upper bound for the capacity in the energy harvesting system. Then, we develop a scheme called *save-and-transmit* scheme that achieves this upper bound and hence the capacity. The *save-and-transmit* scheme relies on sending zero code symbols in a portion of the total block length, which becomes negligible as the block length gets larger. Next, based on feasibility to send the code symbol in a channel use, we provide an alternative scheme called the *best-effort-transmit* scheme for achieving the capacity. Whenever the available energy is sufficient to send the code symbol, it is put to the channel, while a zero is put to the channel if there is not enough energy in the battery. We show that this scheme achieves rates arbitrarily close to the capacity.

Secondly, we address a typical behavior in certain energy harvesting sensors, such as solar-powered sensors, where the recharge process is not ergodic or stationary. In this case, we assume that the average recharging rate is not a constant in time, but rather fosters time variation in a scale much larger than the time scale that communication takes place. Accordingly, we extend the formulation to the case in which recharge

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process has a mean value that is varying in sufficiently long time and we call each such sufficiently long time a slot. We derive the optimal power control for maximum average throughput in this case, and provide a geometric interpretation for the resulting power allocation. We illustrate the advantage of the optimal power allocation in a numerical study.

A. Relevant Literature

To the best of our knowledge, this work is the first attempt to analyze an energy harvesting communication system from an information-theoretic perspective. There are many motivating works in the networking literature. In [1], Lei *et. al.* address replenishment in one hop transmission. Formulating transmission strategy as a Markov decision process, [1] uses dynamic programming techniques for optimization of transmission policy under replenishment. In [2], Gatzianas *et. al.* extend classical wireless network scheduling results to a network with users having rechargeable batteries. Each battery is considered as an energy queue, and data and energy queues are updated simultaneously in an interaction determined by rate versus power relationship. Stability of data queues is studied using Lyapunov techniques. In [3], [4], in a similar energy harvesting setting, a dynamic power management policy is proposed and is shown to stabilize data queues. In each slot, energy spent is equal to the average recharge rate. Moreover, under a linear approximation, some delay-optimal schemes are proposed.

The capacity of scalar AWGN channel has been extensively studied in the literature under different constraints on the input signal. Average power (or the second moment) constraint on the input yields the well-known result that the capacity achieving input distribution is Gaussian with variance equal to the power constraint. Smith [5], [6] considers amplitude constraints in addition to average power constraints and concludes that the capacity achieving input distribution function is a step function with finite number of increase points. Moreover, Shamai and Bar-David [7] extend Smith's result to amplitude constrained quadrature Gaussian channel for which the optimal input distribution is concentrated to finite number of uniform phase circles within the amplitude constraint.

II. AWGN Channel with Random Energy Arrivals

We consider scalar AWGN channel characterized by the input X, output Y, additive noise N with unit normal distribution $\mathcal{N}(0,1)$ and a battery (see Fig. 1). Input and output alphabets are taken as real numbers.

Energy enters the system from a power source that supplies E_i units of energy in the ith channel use where $E_i \geq 0$. $\{E_1, E_2, \ldots, E_n\}$ is the time sequence of supplied energy in n channel uses. E_i is an i.i.d. sequence with average value P, i.e., $E[E_i] = P$, for all i.

We assume that the energy stored and depleted from the battery are for only communication purposes (for example, the energy required for processing is out of the model). Existing energy in the battery can be retrieved without any loss and the battery capacity is large enough so that every quanta of incoming energy can be stored in the battery. This assumption is especially valid for the current state of the technology in which batteries have very large capacities compared to the rate of harvested energy flow [3]: $E_{max} \gg P$.

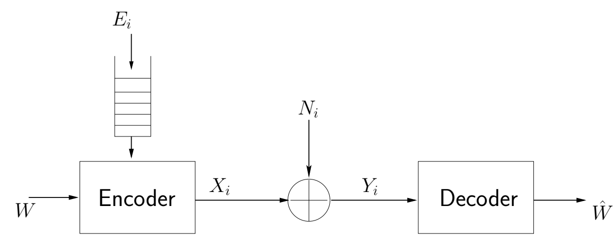

Fig. 1. AWGN channel with random energy arrivals.

The battery is initially empty and energy needed for communication of a message is obtained from the arriving energy during transmission of the corresponding codeword subject to causality. In particular, E_i units of energy is added to the battery and X_i^2 units of energy is depleted from the battery in the ith channel use. This brings us to the following cumulative power constraints on the input based on causality:

$$\sum_{i=1}^{k} X_i^2 \leq \sum_{i=1}^{k} E_i, \qquad k = 1, \ldots, n \tag{1}$$

This is illustrated in Fig. 2, where in each channel use, E_i amount of energy arrives and X_i^2 amount of energy is used. Note that the constraints in (1) are upon the support set of the random variables X_i. The first constraint restricts the support set of X_1 to $[-\sqrt{E_1}, \sqrt{E_1}]$. The second constraint is $X_1^2 + X_2^2 \leq E_1 + E_2$. In general, letting S_i denote $[\sum_{j=1}^{i-1}(E_j - X_j^2)]^+$, in channel use i, the symbol X_i is subject to the constraint $X_i^2 \leq E_i + S_i$.

The input constraints in (1) are challenging because E_i are random and these constraints introduce memory (in time) in the channel inputs. Randomness in E_i makes the problem similar to fading channels in that state of the recharge process (i.e., low or high E_i) affects instantaneous quality of communication. Moreover, this time variation in recharge process allows opportunistic control of energy as in fading channels. However, recharge process can be collected in battery unlike a fading state. In fact, we will see that, this nature of energy arrivals makes it more advantageous to save energy in the battery for future use when a peak occurs in the recharge process, as opposed to opportunistically *ride* the peaks.

III. The Capacity

We will invoke the general capacity formula of Verdu and Han [8]. For fixed n, let f^n be the joint density function of random variables $\{X_i\}_{i=1}^n$ and let $\mathcal{F}^n$ be the set of n variable joint density functions that satisfy the constraints in (1). Since AWGN is an information-stable channel [8], the capacity of the channel in Fig. 1 with constraints in (1) is:

$$C = \lim_{n\to\infty} \frac{1}{n} \max_{f^n \in \mathcal{F}^n} I(X^n; Y^n) \tag{2}$$

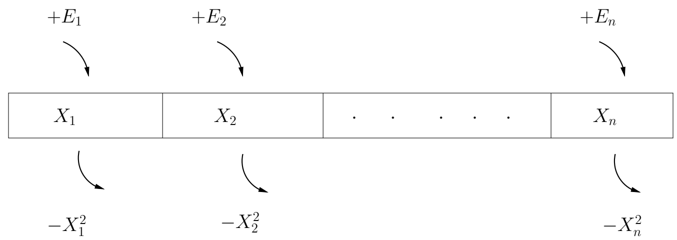

Fig. 2. For time i, E_i denotes the energy arriving, and X_i denotes the channel input. Therefore, X_i^2 is the energy used at time i.

In general, capacity achieving input distribution is in the form of product of marginal distributions (independent distribution) [8]. However, note that the power constraints create dependence among the random variables. The constraint on X_{i+1} is dependent on given value of X_j, $j \leq i$. Though independent processes achieve higher mutual information than the ones with the same marginal distribution but with correlation [8], the capacity that we seek in this problem does not let the process be independent. This problem falls in the family of problems of finding capacity under dependence constraints on code symbols which is by itself interesting and less studied.

An upper bound for C is the corresponding AWGN capacity with average power P, as $\frac{1}{n}\sum_{i=1}^{n} X_i^2 \leq \frac{1}{n}\sum_{i=1}^{n} E_i$ and by the i.i.d. nature of E_i, invoking strong law of large numbers [9], $\frac{1}{n}\sum_{i=1}^{n} E_i \rightarrow P$ with probability 1. Therefore, each codeword satisfying constraints in (1) automatically satisfies $\lim_{n\rightarrow\infty}\frac{1}{n}\sum_{i=1}^{n} X_i^2 \leq P$ with probability 1. However, if a codeword satisfies average power constraint, it does not necessarily satisfy the constraints in (1). Hence, we get the following bound:

$$C \leq \frac{1}{2}\log\left(1+P\right) \tag{3}$$

Next, we state and prove that the above upper bound can be achieved.

Theorem 1 *The capacity of the AWGN channel under i.i.d. random energy arrivals E_i, where $E[E_i] = P$, is independent of the realizations of E_i and equal to the channel capacity under average power constraint P:*

$$C = \frac{1}{2}\log\left(1+P\right) \tag{4}$$

To prove the theorem, we need an achievable scheme. Achieving the capacity requires design of an $(n, 2^{nR_n}, \epsilon_n)$ codebook with encoding function $\{f_k^n(.)\}_{k=1}^n$ and decoding function $g_n(.)$ where n is the code length, 2^{nR_n} is the code size and ϵ_n is the probability of error such that $\epsilon_n \rightarrow 0$ and $R_n \rightarrow C$.

There are two separate causes of error. The first one is that any codeword does not satisfy the input constraints. In this case, the encoder does not send that codeword but sends an all zero codeword[1]. Hence, all zero codeword is assumed to be in the codebook and it is decoded to message 0 at the receiver. If the decoder maps received signal to message 0, it is accepted as an error. Second cause of error is the actual decoding error at the receiver. If the received signal is decoded to a message that is different from the message sent, then an error occurs. Accordingly, the error event is defined as union of these two events.

[1]Note that zero is costless (requires no energy) and therefore all zero codeword satisfies the input constraints for all realizations of the energy arrival process.

While designing the codebook and the encoding/decoding rule, a first approach can be to optimize the codebook design subject to the input constraints. Therefore, the occurrence of the first type of error (i.e., insufficient energy) is eliminated from the beginning. However, we will show that, it is not necessary to solve this difficult optimization problem. Instead, the asymptotic behavior of the constraints allows us to design codes that do not obey the input constraints for a particular n with a small probability but the requirement is that small probability goes to zero asymptotically. For the error probability to go to zero, we need to average out the error due to randomness introduced by the channel and due to the randomness in the energy arrivals. To do this, we propose a scheme that implements a *save-and-transmit* principle in that error due to randomness in energy is averaged out first and then the channel coding is performed.

In addition, it is possible to maintain error-free communication even if some code symbols cannot be put to the channel correctly. Therefore, we also investigate achievable rates using a *best-effort-transmit* scheme, which can interestingly approach ϵ neighborhood of the capacity for any $\epsilon > 0$.

IV. The Save-and-Transmit Scheme

We now present an achievable scheme. Let $h(n) \in o(n)$ such that $\lim_{n\rightarrow\infty} h(n) = \infty$. Here, $o(n)$ denotes the class of functions $y(n)$ such that $\lim_{n\rightarrow\infty}\frac{y(n)}{n} = 0$. Consider the sequence of codes with code length n and rate R_n such that the first $h(n)$ symbols of each codeword is zero and the remaining $n - h(n)$ codewords are chosen as independent random variables from the (capacity achieving) Gaussian distribution with variance P where P is the average recharge rate. That is, for $k = 1, 2, ..., h(n)$, we have, the encoding function, $f_k^n(m) = 0$ for all $m \in \{1, 2, ..., 2^{nR}\}$. For $k = h(n)+1, ..., n$, $f_k^n(m)$ comes from realizations of a Gaussian distributed random variable of variance P for all $m \in \{1, 2, ..., 2^{nR}\}$.

As n grows large, by strong law of large numbers, at time index $h(n)$, about $h(n)P$ amount of energy is collected in the battery with very high probability. Since no energy is consumed up to this time, the codebook is feasible until time $h(n)$. After time $h(n)$, the probability that a code symbol requires more energy than the energy available in the battery goes to zero and energy consumed is accounted for the energy arrived. In the next $h(n)$ channel uses, $h(n)P$ units of energy is collected in the battery and another $h(n)P$ is consumed for data transmission. We repeat this procedure for a total of $n/h(n)$ times, where in each interval of $h(n)$ symbols, we collect about $h(n)P$ units of energy and use about $h(n)P$ units of energy collected in the previous block of $h(n)$ symbols. The first collected amount guarantees that the codewords are

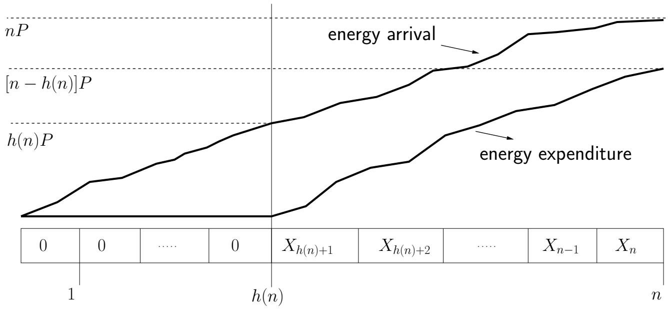

Fig. 3. The codewords in the save-and-transmit scheme. First $h(n)$ code symbols are identically zero for all codewords. Remaining $n - h(n)$ code symbols are selected as i.i.d. Gaussian distributed. Evolution of a sample energy arrival and energy expenditure for a particular codeword is illustrated. Collected energy in the first $h(n)$ channel uses makes it asymptotically impossible to deem any codeword infeasible.

always feasible (see Fig. 3). More specifically, as the length of the codewords gets large, by strong law of large numbers, we have $\sum_{i=h(n)}^{u} X_i^2 \leq \sum_{i=1}^{u} E_i$ almost surely for all $u > h(n)$. For a precise proof of these arguments, we need to make use of Marcinkiewicz-Zygmund type strong laws for sums of i.i.d. random variables. Please see [10] for a complete proof.

The achievable rate for this scheme is

$$\frac{1}{n} I(X^n; Y^n) = \frac{1}{n} \sum_{j=h(n)}^{n} I(X_j; Y_j) = \frac{n - h(n)}{2n} \log(1 + P) \rightarrow \frac{1}{2} \log(1 + P) \quad (5)$$

In other words, communication can start with $h(n)$ amount of delay and the capacity of a corresponding average power constrained AWGN channel can be achieved. Since, this scheme is asymptotically feasible, this proves the achievability part and completes the proof of Theorem 1.

It should be emphasized that the above achievable scheme is obtained without using causal or noncausal information of energy arrivals. For any realization of E_i, we introduce a reasonable delay $h(n)$ to save energy and afterwards transmit with average power P. This scheme obeys the input constraint with probability 1 and achieves the upper bound in (3). The ability to collect energy in the battery allows designer to apply a save-and-transmit strategy so that the uncertainty in the energy arrivals is eliminated first and then the uncertainty in the channel is dealt with by means of appropriate channel coding.

V. The Best-Effort-Transmit Scheme

Let $X^n = (X_1, X_2, \ldots, X_n)$ be a codeword of length n where X_i is the code symbol to be sent in channel use i and the codebook be $\mathcal{C}^n$. The codebook that two parties agree upon is determined by generating independent Gaussian distributed random variables with mean zero and variance $P - \epsilon$ for each code symbol. Let $S(i)$ be the battery energy just before the ith channel use starts. In the best-effort-transmit scheme, the code symbol X_i can be put to the channel if $S(i) \geq X_i^2$. Otherwise, transmitter puts a code symbol 0 to the channel as battery does not have sufficient energy to send symbol X_i. Hence, the input to the channel is $X_i \mathbf{1}(S(i) \geq X_i^2)$ and therefore there is a possible mismatch. Yet, we will show in the following that the mismatch does not affect the correct decoding of the message at the receiver.

The battery energy is updated as follows

$$S(i+1) = S(i) + E_i - X_i^2 \mathbf{1}(S(i) \geq X_i^2) \quad (6)$$

The energy updates in (6) are analogous to the queue updates in classical slotted systems [2] where the notion of a slot is replaced with channel use. Therefore, battery acts like an energy queue. Unlike in the classical systems with packet queues, our aim is to keep the energy queue unstable so that the availability of the energy to send a code symbol is asymptotically guaranteed.

The key to the updates in (6) is to put 0 symbol on the channel as it guarantees that infeasible code symbols are observed only finitely many times, which is stated in the following theorem, and detailed proof is again skipped for brevity and can be found in [10].

Theorem 2 *Let X^n be a codeword such that $\{X_i\}$ is an i.i.d. real random sequence with mean zero and variance $P - \epsilon$ for some $\epsilon > 0$ sufficiently small. Let E_i be the energy arrivals and $S(i)$ be the battery energy which is updated as in (6). Then, only finitely many code symbols are infeasible.*

It is well known that we can pack about $2^{\frac{n}{2}\log(1+(P-\epsilon))}$ codewords with block length n, each having average power less than or equal to $P-\epsilon$, in the $\mathbf{R}^n$ space such that, whenever one of them is sent by the transmitter, a joint typicality decoder [9] which checks whether a codeword is jointly typical with the received signal vector, can reconstruct the codeword at the receiver with probability of error approaching zero. In the *best-effort-transmit* scheme, the received signal vector will still be jointly typical with the sent codeword, since the number of infeasible code symbols is only finitely many due to Theorem 2. Hence, the same decoder can recover the codeword if the channel input is $X_i \mathbf{1}(S(i) \geq X_i^2)$ in each channel use. Therefore, the code rate $R = \frac{1}{2} \log(1 + (P - \epsilon))$ is achievable, and hence by continuity of $\log(.)$, $R < \frac{1}{2} \log(1 + P)$ can be achieved.

VI. Optimal Power Control in a Large Time Scale

We have seen that classical AWGN capacity with average power constraint can be achieved with $o(n)$ delay for fueling the battery with energy if the recharge process is i.i.d. However, the recharge process can deviate from its i.i.d. characteristic in a large time scale. In particular, the mean value of the recharge process may vary after a long time. In the classical example of sensor nodes fueled with solar power, mean recharge rate changes depending on the time of the day. As an example, the mean recharge rate may vary in one-hour slots and the sensor may be on for twelve hours a day, in which case, a careful management of energy expenditure in each slot will be required to optimize the average performance during the day.

We generalize the system model for L large time slots (see Fig. 4). We assume that the duration of each slot is T_s (large enough). For each slot $i = 1, 2, ..., L$, average recharge rate is $P_{in}(i)$ and $P_{tr}(i)$ units of power is allocated for data transmission. In slot i, $P_{in}(i)T_s$ units of energy enters the battery and $P_{tr}(i)T_s$ units of energy is spent for communication. If $P_{in}(i) > P_{tr}(i)$, $(P_{in}(i) - P_{tr}(i))T_s$ units of energy is saved, or otherwise $(P_{tr}(i) - P_{in}(i))T_s$ units of energy is depleted from the battery. Assuming zero initial energy in the battery and large enough battery capacity, every unit of incoming energy is saved in the battery. The causality of energy arrivals requires

$$\sum_{i=1}^{\ell} P_{tr}(i) \leq \sum_{i=1}^{\ell} P_{in}(i), \quad \ell = 1, \ldots, L \tag{7}$$

Around $\frac{1}{2}\log(1 + P_{tr}(i))\, T_s$ bits of data are sent in slot i. Before the communication starts, suppose the designer knows the mean recharge rates $P_{in}(i)$ for all i, calculates $P_{tr}(i)$ and adjusts the average power of codewords in slot i to $P_{tr}(i)$ during transmission[2]. We allocate transmit power to each slot subject to causality constraint so that average throughput in L slots is optimized:

$$\begin{aligned} \max \quad & \frac{1}{L}\sum_{i=1}^{L} \frac{1}{2}\log(1 + P_{tr}(i)) \\ \text{s.t.} \quad & \sum_{i=1}^{\ell} P_{tr}(i) \leq \sum_{i=1}^{\ell} P_{in}(i), \quad \ell = 1, \ldots, L \end{aligned} \tag{8}$$

We will denote the solution of the above optimization problem as $\mathbf{P}_{tr}^* = [P_{tr}^*(1), P_{tr}^*(2), ..., P_{tr}^*(L)]$. Note that the slot duration T_s does not appear in the optimization problem. The rest of this section is devoted to characterizing $\mathbf{P}_{tr}^*$. After developing necessary concepts, we present an algorithm to find $\mathbf{P}_{tr}^*$ and illustrate it for several cases.

A. Solution of the Optimization Problem

In order to understand how the optimal solution may look like, we will first pose the simplest version of the problem. Suppose there is cLT_S amount of energy available in the battery where $c > 0$ is a constant and the recharge process is zero. Then, the optimal power allocation strategy $\mathbf{P}_{tr}^*$ is the one that maximizes $\frac{1}{L}\sum_{i=1}^{L}\frac{1}{2}\log(1 + P_{tr}(i))$ subject to $\sum_{i=1}^{L} P_{tr}(i) \leq cL$. By Jensen's inequality [9], optimal strategy is $P_{tr}^*(i) = c$. That is, if $P_{tr}(i) \neq P_{tr}(j)$ for $i \neq j$, then we can always achieve higher average throughput by setting $P_{tr}'(i) = P_{tr}'(j) = \frac{P_{tr}(i)+P_{tr}(j)}{2}$. Returning to the original problem, above observation translates to a general optimality criterion for the problem, stated in the following theorem. Please see [10] for a proof of this theorem.

[2]Changing the average power of codewords requires using different codebooks in each slot. However, scaling a common codebook by slot power $P_{tr}(i)$ works as well. This can also be interpreted as a codebook with dynamic power allocation [11] in slow time variation.

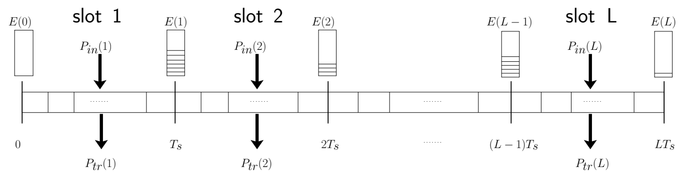

Fig. 4. L large time slots. In each slot, sufficiently large time passes to achieve AWGN capacity with average power constrained to allocated power in that slot.

Theorem 3 *Let $\mathbf{P}_{tr} = [P_{tr}(1), ..., P_{tr}(L)]$ be any power allocation such that $\sum_{i=1}^{\ell} P_{tr}(i) \leq \sum_{i=1}^{\ell} P_{in}(i)$, for $\ell = 1, \ldots, L$. Assume $\mathbf{P}_{tr}' \neq \mathbf{P}_{tr}$ be another power allocation such that $\sum_{i=1}^{\ell} P_{tr}'(i) \leq \sum_{i=1}^{\ell} P_{in}(i)$, $\ell = 1, \ldots, L$ and for some $e, s \in \{1, 2, .., L\}$ with $e < s$*

$$P_{tr}'(i) = \begin{cases} c, & i \in \{e, e+1, ..., s\} \\ P_{tr}(i), & i \notin \{e, e+1, ..., s\}. \end{cases}$$

Then, $\sum_{i=1}^{L}\frac{1}{2}\log(1 + P_{tr}'(i)) > \sum_{i=1}^{L}\frac{1}{2}\log(1 + P_{tr}(i))$.

This theorem proposes a method to optimize the power allocation. It should be performed such that variation in power from slot to slot is avoided as much as possible and the energy should be consumed in as smooth a way as possible. We will now make this more precise by describing the method of finding the solution $\mathbf{P}_{tr}^*$.

It is inferred that $\mathbf{P}_{tr}^*$ must take a constant value (not necessarily the same value) in each slot. In addition, we can partition the whole interval into disjoint intervals over which $P_{tr}^*(i)$ is constant as follows:

$$P_{tr}^*(i) = \frac{\text{energy recharged in } [n_k T_s, n_{k+1} T_s]}{(n_{k+1} - n_k)T_s} \tag{9}$$

for all i in $\{n_k + 1, ..., n_{k+1}\}$ for some subset $\{n_k\}$ of $\{1, 2, .., L\}$ with $n_{k+1} > n_k$. Thus, the problem can also be cast as finding the right subset $\{n_k^*\}$ that optimizes the average throughput. Note that 0 and L are always in this subset. Hence, $n_1^* = 0$ and the element with last order is L. Define the cumulative energy arrival rate as $e(i) = \sum_{j=1}^{i} P_{in}(j)$ for all $i \in \{1 : L\}$ and reset $e(0) = 0$. In each point, we have to use the knowledge of the amount of energy that is going to arrive in the remaining time interval and determine feasible constant power strategies and find the optimal one among them. We have $n_1^* = 0$ and next n_i^* are determined for optimal strategy as follows [10]:

$$n_i^* = \arg \min_{k \in \{n_{i-1}^*+1:L\}} \frac{e(k) - e(n_{i-1}^*)}{k - n_{i-1}^*} \tag{10}$$

Then, the optimal power allocation is as follows:

$$P_{tr}^*(i) = \frac{e(n_i^*) - e(n_{i-1}^*)}{n_i^* - n_{i-1}^*}, \qquad \text{for all } i \in \{n_{i-1}^* : n_i^*\} \tag{11}$$

An illustration of the operation of the algorithm is presented in Fig. 5. The operation of the algorithm has the following geometric structure. At each point i, lines are drawn from

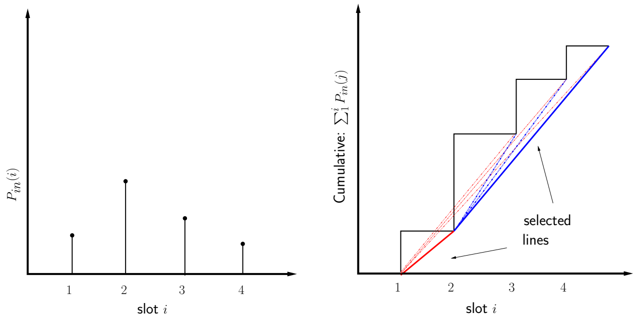

Fig. 5. Operation of the optimal power allocation algorithm. On the left figure, the power impulses are shown and on the right figure, cumulative energy arrival rate is shown and construction of optimal power allocation as minimum feasible piecewise linear curve that is under the cumulative arrivals is illustrated. $P^*_{tr}(i)$ is the slope of the selected line in slot i.

cumulative energy point $(i, e(i))$ to future points $(k, e(k))$ for all $k > i$ and the one with minimum slope is chosen, say k^*. That slope is the allocated power for all slots between i and k^*.

B. A Numerical Study of the Algorithm

The optimal power management algorithm takes the arrival power vector $[P_{in}(1), ..., P_{in}(L)]$ and outputs the transmit power vector $[P^*_{tr}(1), ..., P^*_{tr}(L)]$. We let the arrival rates of energy in all slots, $P_{in}(i)$, follow an i.i.d. exponential distribution.

A benchmark algorithm is simply no power management algorithm, i.e., $P_{tr}(i) = P_{in}(i)$. In this simple scheme, the energy arrival rate in each slot is taken as the communication power in that slot. This scheme yields an average throughput

$$T^{lb} = \frac{1}{L}\sum_{i=1}^{L}\frac{1}{2}\log\left(1 + P_{in}(i)\right) \tag{12}$$

which is a lower bound. However, if the designer has the information of arrival rates in future slots, then optimal power management algorithm can improve the average throughput. It is clear that an upper bound for the average throughput is

$$T^{ub} = \frac{1}{2}\log\left(1 + \frac{1}{L}\sum_{i=1}^{L} P_{in}(i)\right) \tag{13}$$

The comparison of performances of optimal power management with the upper bound T^{ub} and the lower bound T^{lb} (no power management) is given in Fig. 6 for a $L = 20$ slot system. We observe that as the variance of the arrival rates increases, the advantage of optimal power management becomes more apparent with respect to no power management. Because, the power is more peaky as the variance is increased yet for better throughput, transmitter should not ride the peaks but rather save the energy for future use. Another observation is that the difference between upper bound and average throughput with optimal power management also increases as the standard deviation of the arrival rate is increased. Hence, the causality constraint becomes more restrictive as the variation in the arrival rate is increased.

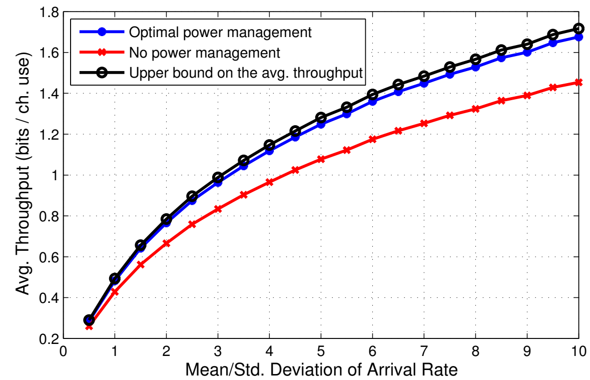

Fig. 6. Average throughput in $L = 20$ slots is plotted for different values of mean/standard deviation of the arrival rate. As the variation in the arrival rate increases, optimal power management algorithm yields higher advantage over no power management.

VII. Conclusion

We studied communication in an AWGN channel under random energy arrivals using an information-theoretic framework. We showed that the capacity of AWGN under energy harvesting is the same as the capacity of the AWGN channel, where the average power is constrained to the average recharge rate. This upper bound can be achieved by save-and-transmit and best-effort-transmit schemes. Next, we addressed time varying recharge rates in large time scales. We obtained an algorithm to find the optimal power management for maximum average throughput. We illustrated its operation and provided a numerical study to visualize the advantage of the algorithm.

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