Age of Information for Updates with Distortion

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***Abstract*—We consider an information update system where an information receiver requests updates from an information provider in order to minimize its age of information. The updates are generated at the transmitter as a result of completing a set of tasks such as collecting data and performing computations. We refer to this as the update generation process. We model the *quality* (i.e., *distortion*) of an update as an increasing (resp. decreasing) function of the processing time spent while generating the update at the transmitter. While processing longer at the transmitter results in a better quality (lower distortion) update, it causes the update to age. We determine the age-optimal policies for the update request times at the receiver and update processing times at the transmitter subject to a minimum required quality (maximum allowed distortion) constraint on the updates.**

I. Introduction

We consider an information update system where an information receiver requests updates from an information provider in order to minimize the age of information at the receiver. To generate an update, the information provider completes a set of tasks as in [1]. We study the *quality* of updates as introduced in [2], [3] via their *distortion*. We model the *quality* (i.e., the *distortion*) of an update as a monotonically increasing (resp., monotonically decreasing) function of the processing time spent to generate the update at the transmitter.

Examples of such systems can be found in sensor networking and distributed computation applications. For instance, in a sensor networking application where multiple sensors observe the realization of a common underlying random variable (e.g., temperature in an area), if the information provider generates an update using the observation of a single sensor, the update will be generated faster, but will have large distortion; and conversely, if the information provider generates an update using the observations of all sensors, the update will be generated with a delay, but will have small distortion (Fig. 1). Here, we assume that retrieving the observation from a sensor takes a unit of time. Similarly, in a distributed computation system with stragglers, the master can generate an update using faster servers with lower quality, or utilize all servers to generate a better quality update with a delay [4].

References [1]–[3], [5] consider this trade-off between service performance and information freshness: [1] emphasizes the difference between service time and the age. [2] considers the joint optimization of information freshness, quality of information, and total energy consumption. [2] models the age and distortion as a convex function and develops an algorithm

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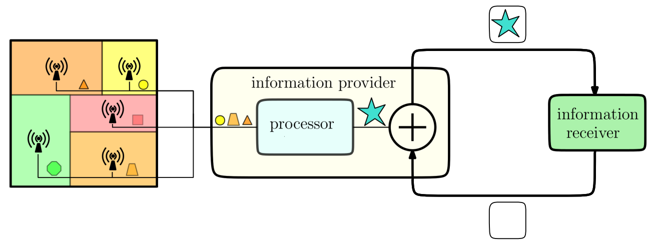


Fig. 1. An Information updating system which consists of an information provider which collects/processes data and an information receiver.

which is 2-competitive. In our paper, there is no energy constraint. We minimize the average age of information, which is inherently non-convex, subject to a distortion constraint for each update which is convex.

In [3], service performance is measured by how *quickly* the provider responds to the queries of the receiver. In [3], the performance of the system is considered to be the highest when the service provider responds immediately upon a request. However, by responding quickly, the service provider may be using available, but perhaps outdated, information resulting in larger age; on the other hand, if the provider waits for processing new data and responds to the queries a bit later, information of the update may be fresher. Thus, in the model of [3], processing data degrades quality of service as it worsens response time, but improves the age. In contrast, in our model, processing data improves service performance (the quality of updates), but worsens the age, as the age at the receiver grows while the transmitter processes the data. Finally, [5] minimizes the age subject to minimum throughput requirements.

We model distortion as a monotonically decreasing function of processing time, c_i, motivated by the diminishing returns property [2], [6], [7]. We consider exponentially and inverse linearly decaying distortion functions in numerical results. In particular, inverse linearly decaying distortion function arises in sensor networking applications, where all sensors observe an underlying random variable distorted by independent Gaussian noise, and the information provider combines sensor observations linearly to minimize the mean squared error.

We consider the information update system shown in Fig. 1. The information provider connects to multiple units (sensors, servers, etc.) to generate an update. When there is no update, the information at the receiver gets stale over time, i.e., the age increases linearly. The information receiver requests an update from the information provider similar to [1], [3], [8]. After receiving the update request, the information provider

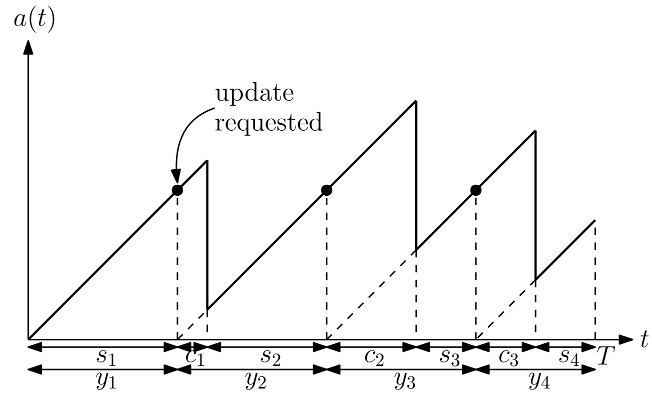


Fig. 2. Age evolution at the receiver.

allocates c_i amount of time as shown in Fig. 2 for processing the information analogous to [1], [9]. During this processing time, the information used to generate the update ages by c_i. When the information provider sends the update to the receiver, the age at the receiver reduces down to the age of the update which is c_i by noting that the communication time between the transmitter and receiver is zero as in [10]–[21].

In this paper, we determine age-optimum updating schemes for a system with a distortion constraint on each update. We are given a total time duration over which the average age is calculated T, the total number of updates N, the threshold for the distortion β, and the distortion function. We solve for the optimum request times of the updates at the receiver and the optimum processing times of the updates at the transmitter in order to minimize the overall age. If there is no constraint on distortion, i.e., the distortion constraint is infinity, then we show that the optimal policy is to request updates with equal inter-update times. If there is an active constraint on distortion, we show that the optimal policy is to allocate the minimum processing time for each update that satisfies the distortion constraint. When the distortion constraint is relatively large, i.e., the required processing time is relatively small compared to the total time period, then the optimal policy is to request updates regularly following a waiting (request) time after receiving each update, with a longer request time for the first update than others. When the distortion constraint is relatively small, i.e., the required processing time is relatively large compared to the total time period, the optimal policy is to request an update immediately once the previous update is received, i.e., back-to-back, except for a potentially non-zero requesting time for the first update.

II. System Model and Problem Formulation

Let $a(t)$ be the instantaneous age at time t, with $a(0) = 0$. When there is no update, the age increases linearly over time. When an update is received, the age at the receiver reduces down to the age of the latest received update. There is no delay in the channel between the information provider and the receiver. However, in order to generate an update, the provider needs to allocate a processing time. For update i, the provider allocates c_i amount of processing time.

We model the distortion function as a monotonically decreasing function of processing time due to the diminishing returns property. For instance, in numerical results, we consider an exponentially decaying distortion function, D_e,

$$D_e(c_i) = a\left(e^{-bc_i} - d\right) \tag{1}$$

where c_i are chosen such that $d \leq e^{-bc_i}$ to make sure that distortion function is always nonnegative. In addition, we consider inverse linearly decaying distortion function, D_ℓ,

$$D_\ell(c_i) = \frac{a}{bc_i + d} \tag{2}$$

which arises in sensor networking applications. In particular, consider a system with M sensors placed in an area, measuring a common random variable X with mean μ_X and variance σ_X^2. The measurement at each sensor, Y_j, is perturbed by an independent identically distributed zero-mean Gaussian noise with variance σ^2. Information provider uses a linear estimator, $\hat{X} = \sum_{j=1}^{M} w_j Y_j$ to minimize the distortion (mean squared error) defined as $D_\ell = \mathbb{E}[(\hat{X} - X)^2]$. In this model, we assume that the information provider connects to one sensor at a time and spends one unit of time to retrieve the measurement from that sensor. Thus, if the information provider connects to c_i sensors, it spends c_i units of time for processing (i.e., retrieving data) and achieves a distortion of $D_\ell(c_i) = \sigma^2/(c_i + \frac{\sigma^2}{\mu_X^2+\sigma_X^2})$ for the ith update, which has the inverse linearly decaying form in (2).

Let s_i be the time interval between the requesting time for the ith update and the receiving time of the $(i-1)$th update at the receiver, and let c_i be the processing time of the ith update at the transmitter; see Fig 2. We take $s_0 = c_0 = c_{N+1} = 0$. Our objective is to minimize the average age of information at the information receiver subject to having a desired level of distortion for each update over a total time period T, given that there are N updates. We formulate the problem as,

$$\begin{aligned} \min_{\{s_i, c_i\}} \quad & \frac{1}{T}\int_0^T a(t)dt \\ \text{s.t.} \quad & \sum_{i=0}^{N+1} s_i + c_i = T \\ & D(c_i) \leq \beta, \quad i = 1, \dots, N \\ & s_i \geq 0, \quad c_i \geq 0 \end{aligned} \tag{3}$$

where $a(t)$ is the instantaneous age, and $D(c_i)$ is the distortion function which is monotonically decreasing in c_i; the distortion function $D(c_i)$ may be $D_e(c_i)$ or $D_\ell(c_i)$ defined above, or any other appropriate distortion function depending on the application. Let $A_T \triangleq \int_0^T a(t)dt$ be the total age. Note that minimizing $\frac{A_T}{T}$ is equivalent to minimizing A_T since T is a known constant.

III. The Optimal Solution

The age function evolves as in Fig. 2. Total age, A_T, is

$$A_T = \frac{1}{2}\sum_{i=1}^{N+1}(s_i + c_{i-1})^2 + \sum_{i=1}^{N} c_i(s_i + c_{i-1}) \tag{4}$$

Since $D(c_i)$ is a monotonically decreasing function of c_i, $D(c_i) \leq \beta$ is equivalent to $c_i \geq c$ where c is a constant. Thus, we replace the distortion constraint given in (3) with $c_i \geq c$. In addition, we define $y_i = s_i + c_{i-1}$ for $i = 1, \dots, N+1$. Then, we rewrite the optimization problem in (3) as,

$$\begin{aligned} \min_{\{y_i, c_i\}} \quad & \frac{1}{2} \sum_{i=1}^{N+1} y_i^2 + \sum_{i=1}^{N} c_i y_i \\ \text{s.t.} \quad & \sum_{i=1}^{N+1} y_i = T \\ & y_1 \geq 0, \quad y_i \geq c_{i-1}, \quad i = 2, \dots, N+1 \\ & c_i \geq c, \quad i = 1, \dots, N \end{aligned} \tag{5}$$

Note that the optimization problem in (5) is not convex due to the multiplicative terms of c_i and y_i. We choose $c_i = c$ for $i = 1, \dots, N$ as an optimization step to minimize the second term of the objective function and at the same time to obtain the largest feasible set for the problem in (5). Thus, the optimization problem in (5) becomes as follows,

$$\begin{aligned} \min_{\{y_i\}} \quad & \frac{1}{2} \sum_{i=1}^{N+1} y_i^2 + \sum_{i=1}^{N} c y_i \\ \text{s.t.} \quad & \sum_{i=1}^{N+1} y_i = T \\ & y_1 \geq 0, \quad y_i \geq c, \quad i = 2, \dots, N+1 \end{aligned} \tag{6}$$

When $c = 0$, i.e., there is no active distortion constraint, the optimal solution is to choose $y_i = \frac{T}{N+1}$ for all i. Thus, for the rest of this section, we consider the case when $c > 0$.

We write the Lagrangian for the problem in (6) as,

$$\begin{aligned} \mathcal{L} =& \frac{1}{2} \sum_{i=1}^{N+1} y_i^2 + \sum_{i=1}^{N} c y_i - \lambda \left(\sum_{i=1}^{N+1} y_i - T \right) \\ & - \sum_{i=2}^{N+1} \theta_i (y_i - c) - \theta_1 y_1 \end{aligned} \tag{7}$$

where $\theta_i \geq 0$ and λ can be anything. The problem in (6) is convex. Thus, KKT conditions are necessary and sufficient for the optimal solution. The KKT conditions are,

$$\frac{\partial \mathcal{L}}{\partial y_i} = y_i + c - \lambda - \theta_i = 0, \quad i = 1, \dots, N \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial y_{N+1}} = y_{N+1} - \lambda - \theta_{N+1} \tag{9}$$

The complementary slackness conditions are,

$$\lambda \left(\sum_{i=1}^{N+1} y_i - T \right) = 0 \tag{10}$$

$$\theta_1 y_1 = 0 \tag{11}$$

$$\theta_i (y_i - c) = 0, \quad i = 2, \dots, N+1 \tag{12}$$

When $y_i > c$ for all i, we have $\theta_i = 0$ due to (11) and (12). Then, from (8) and (9), we obtain $y_i = \lambda - c$ for $i = 1, \dots, N$, and $y_{N+1} = \lambda$. Since $\sum_{i=1}^{N+1} y_i = T$, we find $\lambda = \frac{T+Nc}{N+1}$. Thus, the optimal solution becomes,

$$y_i = \frac{T-c}{N+1}, \quad i = 1, \dots, N \tag{13}$$

$$y_{N+1} = \frac{T+Nc}{N+1} \tag{14}$$

In order to have $y_i > c$, we need $T > (N+2)c$. Viewing this condition from the perspective of c, this is the case when c is small in comparison to T. Therefore, we note that, in this case, when minimum processing time, c, is relatively small, the optimal policy is to choose y_i as equal as possible except for y_{N+1}. When c becomes larger compared to T, $y_i - c$ decreases. Specifically, when $T = (N+2)c$, $y_i = c$ for $i = 1, \dots, N$.

In the remaining case, i.e., when $T < (N+2)c$, $y_1 < c$ and $y_{N+1} > c$, we have $\theta_1 = 0$ and $\theta_{N+1} = 0$ by (11) and (12). Then, by solving $y_i = \lambda - c$, $y_{N+1} = \lambda$, and $\sum_{i=1}^{N+1} y_i = T$, we obtain,

$$y_1 = \frac{T - Nc}{2} \tag{15}$$

$$y_i = c, \quad i = 2, \dots, N \tag{16}$$

$$y_{N+1} = \frac{T - (N-2)c}{2} \tag{17}$$

Since $y_1 > 0$, we need $Nc < T$. Thus, this solution applies when $Nc < T \leq (N+2)c$. Finally, when $T = Nc$, the optimal solution becomes,

$$y_1 = 0 \tag{18}$$

$$y_i = c, \quad i = 2, \dots, N+1 \tag{19}$$

In summary, when $c = 0$, i.e., we do not have any distortion constraints, then the optimal solution is to update in every $\frac{T}{N+1}$ units of time, i.e., $y_i = \frac{T}{N+1}$ for all i. When $c > 0$ but, relatively small compared to T, i.e., $(N+2)c < T$, the optimal solution is to wait for $\frac{T-c}{N+1}$ to request the first update. For the remaining updates, the receiver waits for $\frac{T-(N+2)c}{N+1}$ time to request another update after the previous update is received. After requesting N updates, the optimal policy is to let the age grow for $\frac{T-c}{N+1}$ units of time. When c becomes large compared to T, i.e., $Nc < T \leq (N+2)c$, the optimal policy is to wait for $\frac{T-Nc}{2}$ to request the first update and request the remaining updates when the previous update is received. After updating N times, we let the age grow for the remaining $\frac{T-Nc}{2}$ time. Finally, when $T = Nc$, the optimal policy is to request the first update at $t = 0$ and request the remaining updates as soon as the previous update is received. We note that when $Nc > T$, there is no feasible policy. The possible optimal policies are shown as numerical results in Fig. 3.

IV. Numerical Results

In this section, we provide five numerical results for an exponentially decaying distortion function, D_e, defined in (1) with $a = \frac{1}{1-e^{-1}}$, $b = \frac{1}{4}$ and $d = e^{-1}$. For the first four simulations, we cover each optimal policy given in Section III. In these simulations, we take $T = 10$ and $N = 3$.

In the first example, we take $c = 0$. In other words, we optimize the age without considering the distortion on the

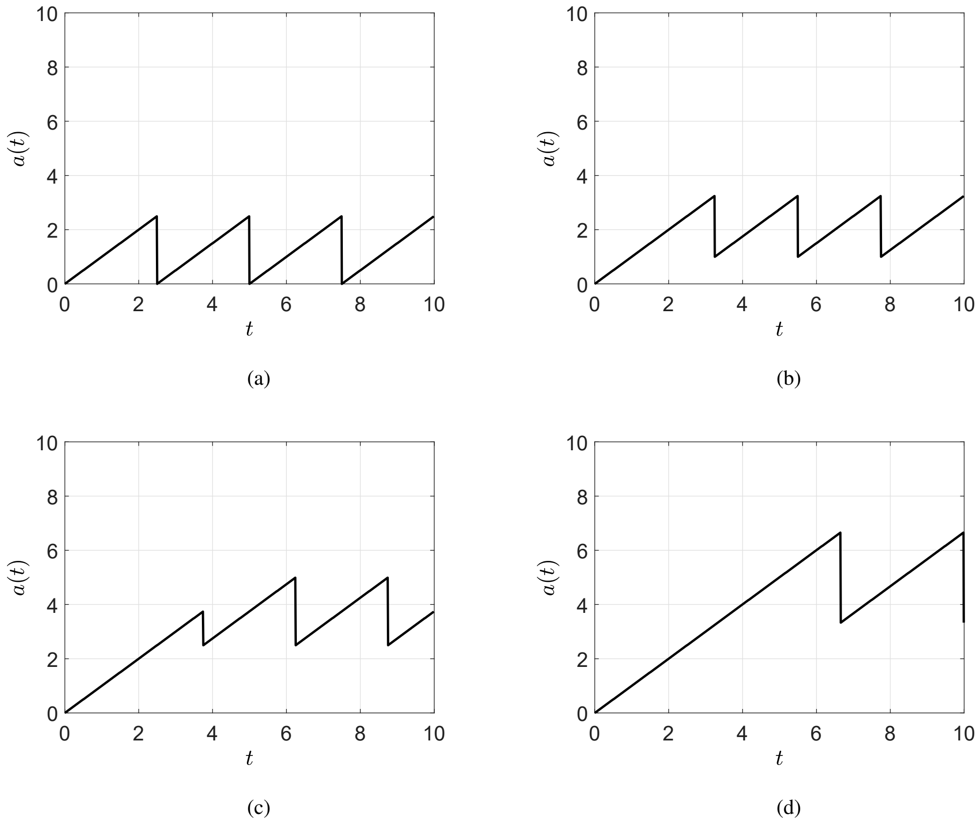


Fig. 3. Evolution of $a(t)$ with optimal update policies when $T = 10$, $N = 3$, (a) $c = 0$, (b) $c = 1$, (c) $c = 2.5$, (d) $c = \frac{10}{3}$.

updates. The optimal solution is to choose $y_i = 2.5$ and $c_i = 0$ for all i. The optimal age evolves as in Fig. 3(a). We note that the optimal policy is to request an update in equal time periods and the provider sends the updates immediately.

In the second example, we take $c = 1$. This is the case where c is small compared to the total time duration T, i.e., $(N + 2)c < T$. The optimal policy is to choose $y_i = 2.25$ for $i = 1, 2, 3$, $y_4 = 3.25$ and $c_i = 1$ for all i. The optimal age evolution is given in Fig. 3(b). We note that the optimal policy is to request the first update after $s_1 = 2.25$ time. For the remaining updates, after the previous update is received, the receiver waits for $s_2 = s_3 = 1.25$ time to request another update. After receiving a request, the provider generates the updates after processing $c = 1$ time.

For the third example, we take $c = 2.5$. In this case, we wish to receive the updates with lower distortion compared to previous cases. The optimal policy is to choose $y_1 = 1.25$, $y_2 = y_3 = 2.5$, $y_4 = 3.75$, and $c_i = 2.5$ for all i. The optimal age evolution is shown in Fig. 3(c). We note that the optimal policy is to request the first update after waiting $s_1 = 1.25$. The receiver requests the remaining updates as soon as the previous update is received (back-to-back) since the provider uses relatively large amount of time to generate updates.

For the fourth example, we take $c = \frac{10}{3}$. In this case, there is only one feasible solution, which is to choose $y_1 = 0$, $y_i = \frac{10}{3}$ for $i = 2, 3, 4$ and $c_i = \frac{10}{3}$ for all i. The optimal age evolves as in Fig. 3(d). We note that the optimal policy is to request the first update at $t = 0$ and the remaining updates as soon as the previous update is received (back-to-back).

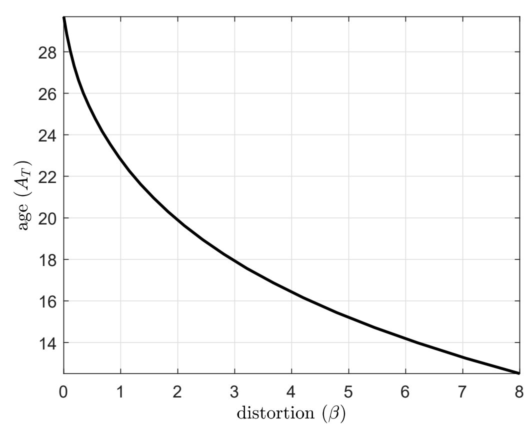
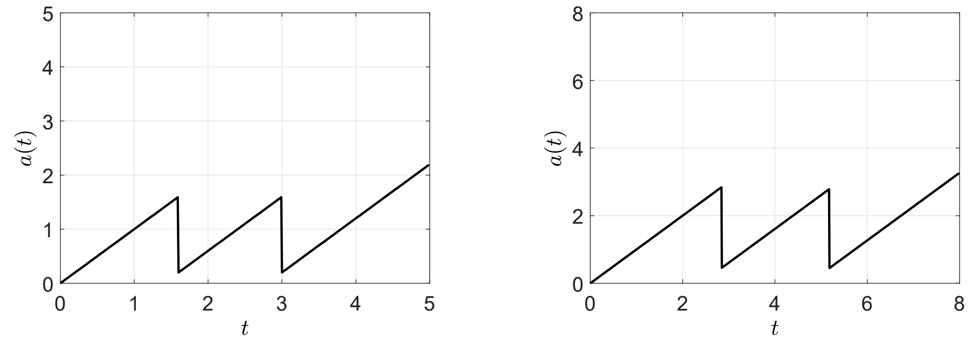


Fig. 4. Age versus distortion of the updates when $a = \frac{8}{1-e^{-3}}$, $b = 1.2$, and $d = e^{-3}$. We vary β and find minimum age for each β.

Finally, we note that there is a trade-off between age and distortion. If we increase the distortion constraint β (hence decrease the processing time constraint c), then the achievable age decreases. On the other hand, if we decrease the distortion constraint β (hence increase the processing time constraint c), then the achievable age increases. We show this trade-off between age and distortion in our fifth simulation in Fig. 4.



(a)

(b)

Fig. 5. Evolution of $a(t)$ with optimal update policies when $N = 2$, $a = \frac{a(t)}{1-e^{-1}}$, $b = 1$, $d = e^{-1}$, (a) $T = 5$, and (b) $T = 8$.

V. Conclusion and Future Directions

In this paper, we considered the concept of status updating with update packets subject to distortion. In this model, updates are generated at the transmitter following an update generation process that involves collecting data and performing computations. The distortion in each update is inversely proportional with the processing time during update generation at the transmitter; while processing longer generates a better-precision update, the long processing time increases the age of information. This implies that there is a trade-off between precision (quality) of information and freshness of information. The system may be designed to strike a desired balance between quality and freshness of information. In this paper, we determined this design, by solving for the optimum update scheme subject to a desired distortion level.

For future work, the distortion function can be modeled with considering the current age. Even though the update generation process is independent of the age of the system, the receiver might desire for updates with different level of distortion depending on its current age. For example, if the receiver does not get any updates for a long time, its current age becomes large, and the receiver might wish to receive an update with low distortion compared to other updates that it receives when the age is small. Thus, distortion function for an update that is requested at time t and generated after processing c_i amount of time at the provider can be modeled by replacing a in (1) by the instantaneous age, $a(t)$. We provide simulation results for the optimization problem defined in (3) with the distortion function in (1) where $a = \frac{a(t)}{1-e^{-1}}$, $b = 1$, $d = e^{-1}$. We consider $N = 2$ and $\beta = 1$ and take $T = 5$ and $T = 8$ for the first and second simulations, respectively. The simulation results are shown in Fig. 5. We see that even though we applied the same threshold for the distortion, the processing times for the second simulation where $c_i = 0.46$ is higher than the processing times for the first simulation where $c_i = 0.2$.

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