Capacity Bounds for the Gaussian Interference Channel with Transmitter Cooperation

Ravi Tandon Sennur Ulukus
Department of Electrical and Computer Engineering
University of Maryland, College Park, MD 20742
ravit@umd.edu *ulukus@umd.edu*

***Abstract*—We obtain a new outer bound on the capacity region of the two-user interference channel with generalized feedback (IC-GF). This outer bound is based on the idea of dependence balance which was proposed by Hekstra and Willems [1]. We explicitly evaluate our outer bound for the Gaussian IC with user-cooperation (IC-UC), where each transmitter receives an additive white Gaussian noise corrupted version of the channel input of the other transmitter [2]. We show that for all non-zero values of cooperation noise variances, our outer bound strictly improves upon the cut-set outer bound.**

I. Introduction

It is well known that noiseless feedback can increase the capacity region of the discrete memoryless multiple access channel as was shown by Gaarder and Wolf in [3]. The multiple access channel with generalized feedback (MAC-GF) was first introduced by Carleial [4]. The model therein allows for different feedback signals at the two transmitters. For this channel model, Carleial [4] obtained an achievable rate region using block Markov superposition encoding and windowed decoding. An improvement over this achievable rate region was obtained by Willems et. al. in [5] by using block Markov superposition encoding combined with backwards decoding.

Inspired from the uplink MAC-GF channel model, the IC-GF was studied in [6], [7], (also see the references therein) where achievable rate regions were obtained. It was shown in [6] and [7] that for the Gaussian IC-UC, the overheard information at the transmitters has a dual effect of enabling cooperation and mitigating interference, thereby providing improved achievable rates compared to the best known evaluation of the Han-Kobayashi achievable rate region [8], [9]. As far as the converse is concerned for IC-GF, a well known outer bound is the cut-set outer bound. The cut-set bound allows all input distributions, thereby permitting arbitrary correlation between the channel inputs and hence is seemingly loose. In this paper, we use the idea of dependence balance [1] to obtain a new outer bound for the capacity region of the IC-GF.

We focus on the Gaussian IC-UC and explicitly evaluate our outer bound for this channel model. For all non-zero and finite values of cooperation noise variances, our outer bound strictly improves upon the cut-set outer bound. We should remark here that the approach of dependence balance was also used in [10] to obtain an improved sum-rate upper bound for the Gaussian IC with common, noisy feedback from the receivers.

This work was supported by NSF Grants CCF 04-47613, CCF 05-14846, CNS 07-16311 and CCF 07-29127.

Evaluation of our outer bound for the Gaussian IC-UC is not straightforward since our outer bound is expressed in terms of a union of probability densities of three random variables, one of which is an auxiliary random variable. Moreover, this union is over all such densities which satisfy a non-trivial dependence balance constraint. We overcome this difficulty by proving that it is sufficient to consider jointly Gaussian input distributions, satisfying the dependence balance constraint, when evaluating our outer bound. The proof of this part closely follows the proof of a recent result by Bross, Lapidoth and Wigger [11], [12] for the Gaussian MAC with conferencing encoders.

II. System Model

A discrete memoryless IC-GF (see Figure 1) is defined by: two input alphabets $\mathcal{X}_1$ and $\mathcal{X}_2$, two output alphabets $\mathcal{Y}_1$ and $\mathcal{Y}_2$ at receivers 1 and 2, respectively, two feedback output alphabets $\mathcal{Y}_{F_1}$ and $\mathcal{Y}_{F_2}$ at transmitters 1 and 2, respectively, and a probability transition function $p(y_1, y_2, y_{F_1}, y_{F_2}|x_1, x_2)$, defined for all quadruples $(y_1, y_2, y_{F_1}, y_{F_2}) \in \mathcal{Y}_1\times\mathcal{Y}_2\times\mathcal{Y}_{F_1}\times\mathcal{Y}_{F_2}$, for every pair $(x_1, x_2) \in \mathcal{X}_1 \times \mathcal{X}_2$.

A $(n, M_1, M_2, P_e^{(1)}, P_e^{(2)})$ code for IC-GF consists of two sets of encoding functions $f_{1i} : \mathcal{M}_1 \times \mathcal{Y}_{F_1}^{i-1} \rightarrow \mathcal{X}_1$, $f_{2i} : \mathcal{M}_2 \times \mathcal{Y}_{F_2}^{i-1} \rightarrow \mathcal{X}_2$ for $i = 1, \ldots, n$ and two decoding functions $g_1 : \mathcal{Y}_1^n \rightarrow \mathcal{M}_1$ and $g_2 : \mathcal{Y}_2^n \rightarrow \mathcal{M}_2$. The two transmitters produce independent and uniformly distributed messages $W_1 \in \{1, \ldots, M_1\}$ and $W_2 \in \{1, \ldots, M_2\}$, respectively, and transmit them through n channel uses. The average error probability at receivers 1 and 2 are defined as, $P_e^{(k)} = \Pr[\hat{W}_k \neq W_k]$ for $k = 1, 2$. A rate pair (R_1, R_2) is said to be achievable for IC-GF if for any pair $\epsilon_1 \geq 0, \epsilon_2 \geq 0$, there exists a pair of n encoding functions $\{f_{1i}\}_{i=1}^n$, $\{f_{2i}\}_{i=1}^n$, and a pair of decoding functions (g_1, g_2) such that $R_k \leq \log(M_k)/n$ and $P_e^{(k)} \leq \epsilon_k$ for sufficiently large n, for $k = 1, 2$. The capacity region of IC-GF is the closure of the set of all achievable rate pairs (R_1, R_2).

III. Cut-set Outer Bound

A general outer bound on the capacity region of a multi-terminal network is the cut-set outer bound [13]. The cut-set outer bound for IC-GF is given by

$$\mathcal{CS} = \big\{(R_1, R_2) : \; R_1 \leq I(X_1, X_2; Y_1) \tag{1}$$

$$R_2 \leq I(X_1, X_2; Y_2) \tag{2}$$

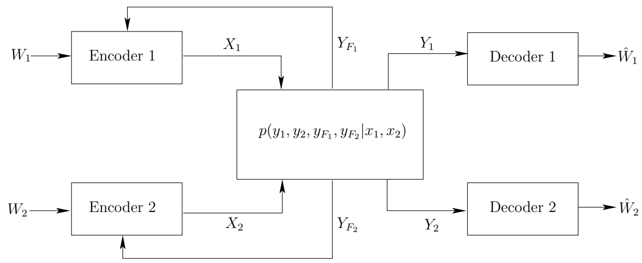

Fig. 1. The interference channel with generalized feedback (IC-GF).

$$R_1 \leq I(X_1; Y_1, Y_2, Y_{F_2}|X_2) \quad (3)$$
$$R_2 \leq I(X_2; Y_1, Y_2, Y_{F_1}|X_1) \quad (4)$$
$$R_1 + R_2 \leq I(X_1, X_2; Y_1, Y_2)\Big\} \quad (5)$$

where the random variables X_1, X_2 and $(Y_1, Y_2, Y_{F_1}, Y_{F_2})$ have the joint distribution

$$p(x_1, x_2, y_1, y_2, y_{F_1}, y_{F_2}) = p(x_1, x_2)p(y_1, y_2, y_{F_1}, y_{F_2}|x_1, x_2) \quad (6)$$

The cut-set bound is seemingly loose since it allows arbitrary correlation among channel inputs by permitting arbitrary input distributions $p(x_1, x_2)$. Using the approach of dependence balance, we will obtain an outer bound for IC-GF which will restrict the set of input distributions. In particular, our outer bound only permits those input distributions which satisfy the non-trivial dependence balance constraint.

IV. A New Outer Bound for IC-GF

Theorem 1: The capacity region of IC-GF is contained in the region

$$\mathcal{DB} = \Big\{(R_1, R_2): \; R_1 \leq I(X_1, X_2; Y_1) \quad (7)$$
$$R_2 \leq I(X_1, X_2; Y_2) \quad (8)$$
$$R_1 \leq I(X_1; Y_1, Y_2, Y_{F_2}|X_2, T_2) \quad (9)$$
$$R_2 \leq I(X_2; Y_1, Y_2, Y_{F_1}|X_1, T_1) \quad (10)$$
$$R_1 + R_2 \leq I(X_1, X_2; Y_1, Y_2, Y_{F_1}, Y_{F_2}|T_1, T_2) \quad (11)$$
$$R_1 + R_2 \leq I(X_1, X_2; Y_1, Y_2)\Big\} \quad (12)$$

where the random variables $(T_1, T_2, X_1, X_2, Y_1, Y_2, Y_{F_1}, Y_{F_2})$ have the joint distribution

$$p(t_1, t_2, x_1, x_2, y_1, y_2, y_{F_1}, y_{F_2}) = p(t_1, t_2, x_1, x_2). \; p(y_1, y_2, y_{F_1}, y_{F_2}|x_1, x_2) \quad (13)$$

and also satisfy the following dependence balance bound

$$I(X_1; X_2|T_1, T_2) \leq I(X_1; X_2|Y_{F_1}, Y_{F_2}, T_1, T_2) \quad (14)$$

The proof of Theorem 1 is omitted due to space limitations and can be found in [14].

V. Gaussian IC with User Cooperation

In this section, we will evaluate our outer bound for a user cooperation setting [6], [7], where the transmitters receive noisy versions of the other transmitter's channel input. The IC-UC (see Figure 2) is a special instance of IC-GF, where the channel outputs are described as,

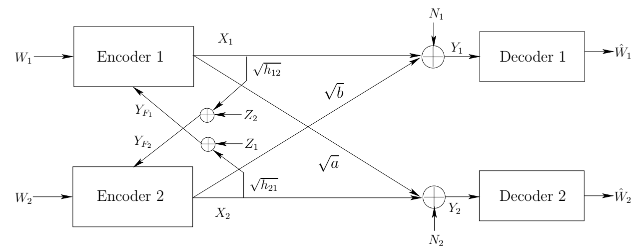

Fig. 2. The Gaussian IC with user cooperation (IC-UC).

$$Y_1 = X_1 + \sqrt{b}X_2 + N_1 \quad (15)$$
$$Y_2 = \sqrt{a}X_1 + X_2 + N_2 \quad (16)$$
$$Y_{F_1} = \sqrt{h_{21}}X_2 + Z_1 \quad (17)$$
$$Y_{F_2} = \sqrt{h_{12}}X_1 + Z_2 \quad (18)$$

where N_1, N_2, Z_1 and Z_2 are independent, zero-mean, Gaussian random variables with variances $\sigma^2_{N_1}, \sigma^2_{N_2}$, $\sigma^2_{Z_1}$ and $\sigma^2_{Z_2}$, respectively. The channel gains a, b, h_{12} and h_{21} are assumed to be fixed and known at all terminals. Moreover, the channel inputs are subject to average power constraints, $E[X_1^2] \leq P_1$ and $E[X_2^2] \leq P_2$. Note that the channel model described above has a special probability structure, namely,

$$p(y_1, y_2, y_{F_1}, y_{F_2}|x_1, x_2) = p(y_1, y_2|x_1, x_2). \; p(y_{F_1}|x_2)p(y_{F_2}|x_1) \quad (19)$$

For any IC-GF with a transition probability in the form of (19), we have a strengthened version of Theorem 1.

Theorem 2: The capacity region of any IC-GF with a transition probability in the form of (19), is contained in the region

$$\mathcal{DB}_{UC} = \Big\{(R_1, R_2): \; R_1 \leq I(X_1, X_2; Y_1) \quad (20)$$
$$R_2 \leq I(X_1, X_2; Y_2) \quad (21)$$
$$R_1 \leq I(X_1; Y_1, Y_2, Y_{F_2}|X_2, T) \quad (22)$$
$$R_2 \leq I(X_2; Y_1, Y_2, Y_{F_1}|X_1, T) \quad (23)$$
$$R_1 + R_2 \leq I(X_1, X_2; Y_1, Y_2, Y_{F_1}, Y_{F_2}|T) \quad (24)$$
$$R_1 + R_2 \leq I(X_1, X_2; Y_1, Y_2)\Big\} \quad (25)$$

where the random variables $(T, X_1, X_2, Y_1, Y_2, Y_{F_1}, Y_{F_2})$ have the joint distribution

$$p(t, x_1, x_2, y_1, y_2, y_{F_1}, y_{F_2}) = p(t, x_1, x_2)p(y_1, y_2|x_1, x_2). \; p(y_{F_1}|x_2)p(y_{F_2}|x_1) \quad (26)$$

and also satisfy the following dependence balance bound

$$I(X_1; X_2|T) \leq I(X_1; X_2|Y_{F_1}, Y_{F_2}, T) \quad (27)$$

where the random variable T is subject to a cardinality constraint $|\mathcal{T}| \leq |\mathcal{X}_1||\mathcal{X}_2| + 3$.

The proof of Theorem 2 can be found in [14]. The main idea behind the proof of Theorem 2 is to use (19) to obtain an outer bound in terms of only one auxiliary random variable. In the next section, we will show that it suffices to consider jointly Gaussian (T, X_1, X_2) satisfying (27) when evaluating Theorem 2 for the Gaussian IC-UC described in (15)-(18).

VI. Outline for Evaluating $\mathcal{DB}_{UC}$

The main difficulty in evaluating our outer bound, $\mathcal{DB}_{UC}$ for the Gaussian IC-UC is to identify the optimal selection of joint densities of (T, X_1, X_2). Our aim will be to prove that it is sufficient to consider jointly Gaussian (T, X_1, X_2) satisfying (27) while evaluating the outer bound.

We begin by considering the set of all distributions of three random variables (T, X_1, X_2) which satisfy the power constraints, $E\big[X_1^2\big] \leq P_1$ and $E\big[X_2^2\big] \leq P_2$. Let us formally define this set of input distributions as

$$\mathcal{P} = \{p(t, x_1, x_2) : E\big[X_1^2\big] \leq P_1, E\big[X_2^2\big] \leq P_2\}$$

For simplicity, we abbreviate jointly Gaussian distributions as $\mathcal{JG}$ and distributions which are not jointly Gaussian as $\mathcal{NG}$. We first partition $\mathcal{P}$ into two disjoint subsets,

$$\mathcal{P}_G = \{p(t, x_1, x_2) \in \mathcal{P} : (T, X_1, X_2) \text{ are } \mathcal{JG}\}$$
$$\mathcal{P}_{NG} = \{p(t, x_1, x_2) \in \mathcal{P} : (T, X_1, X_2) \text{ are } \mathcal{NG}\}$$

We further individually partition the sets $\mathcal{P}_G$ and $\mathcal{P}_{NG}$, respectively, as

$$\mathcal{P}_G^{DB} = \{p(t, x_1, x_2) \in \mathcal{P}_G : (T, X_1, X_2) \text{ satisfy (27)}\}$$
$$\mathcal{P}_G^{\overline{DB}} = \{p(t, x_1, x_2) \in \mathcal{P}_G : (T, X_1, X_2) \text{ do not satisfy (27)}\}$$
$$\mathcal{P}_{NG}^{DB} = \{p(t, x_1, x_2) \in \mathcal{P}_{NG} : (T, X_1, X_2) \text{ satisfy (27)}\}$$
$$\mathcal{P}_{NG}^{\overline{DB}} = \{p(t, x_1, x_2) \in \mathcal{P}_{NG} : (T, X_1, X_2) \text{ do not satisfy (27)}\}$$

Finally, we partition the set $\mathcal{P}_{NG}^{DB}$ into two disjoint sets $\mathcal{P}_{NG}^{DB(a)}$ and $\mathcal{P}_{NG}^{DB(b)}$ with $\mathcal{P}_{NG}^{DB} = \mathcal{P}_{NG}^{DB(a)} \bigcup \mathcal{P}_{NG}^{DB(b)}$, as

$$\begin{aligned}\mathcal{P}_{NG}^{DB(a)} = \{&p(t, x_1, x_2) \in \mathcal{P}_{NG}^{DB} : \text{ covariance matrix of} \\ &p(t, x_1, x_2) \text{ is } Q \text{ and there exists a } \mathcal{JG} \\ &(T_G, X_{1G}, X_{2G}) \text{ with covariance matrix } Q \\ &\text{satisfying (27)}\}\end{aligned}$$

$$\begin{aligned}\mathcal{P}_{NG}^{DB(b)} = \{&p(t, x_1, x_2) \in \mathcal{P}_{NG}^{DB} : \text{ covariance matrix of} \\ &p(t, x_1, x_2) \text{ is } Q \text{ and there does not exist} \\ &\text{a } \mathcal{JG} \ (T_G, X_{1G}, X_{2G}) \text{ with covariance} \\ &\text{matrix } Q \text{ satisfying (27)}\}\end{aligned}$$

So far, we have partitioned the set of input distributions into five disjoint sets: $\mathcal{P}_G^{DB}$, $\mathcal{P}_G^{\overline{DB}}$, $\mathcal{P}_{NG}^{DB(a)}$, $\mathcal{P}_{NG}^{DB(b)}$ and $\mathcal{P}_{NG}^{\overline{DB}}$ (see Figure 3). It is clear that the input distributions which fall into the sets $\mathcal{P}_G^{\overline{DB}}$ and $\mathcal{P}_{NG}^{\overline{DB}}$ need not be considered since they do not satisfy the constraint (27). Therefore, we only need to restrict our attention on the three remaining sets $\mathcal{P}_G^{DB}$, $\mathcal{P}_{NG}^{DB(a)}$, and $\mathcal{P}_{NG}^{DB(b)}$ i.e., those input distributions which satisfy the dependence balance bound.

We explicitly evaluate our outer bound in the following three steps:

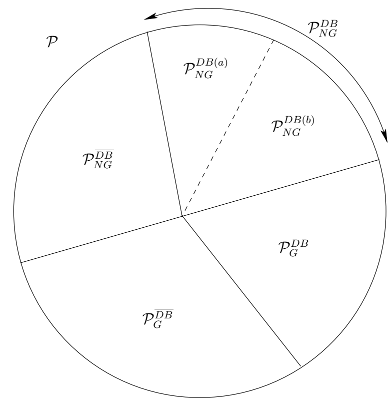

Fig. 3. A partition of the set of input distributions $\mathcal{P}$.

1. We first explicitly characterize the region of rate pairs provided by our outer bound for the probability distributions in the set $\mathcal{P}_G^{DB}$.

2. In the second step, we will show that for any input distribution belonging to the set $\mathcal{P}_{NG}^{DB(a)}$, there exists an input distribution in the set $\mathcal{P}_G^{DB}$ which yields a set of larger rate pairs. Therefore, we do not need to consider the input distributions in the set $\mathcal{P}_{NG}^{DB(a)}$ in evaluating our outer bound.

3. We next focus on the set $\mathcal{P}_{NG}^{DB(b)}$ and show that for any non-Gaussian input distribution $p(t, x_1, x_2) \in \mathcal{P}_{NG}^{DB(b)}$, we can construct a jointly Gaussian input distribution satisfying (27), i.e., we can find a corresponding input distribution in $\mathcal{P}_G^{DB}$, which yields a set of rates which includes the set of rates of the fixed non-Gaussian input distribution $p(t, x_1, x_2)$. Therefore, we do not need to consider the input distributions in the set $\mathcal{P}_{NG}^{DB(b)}$ either in evaluating our outer bound.

To set the stage for our evaluation in steps $1-3$, let us define $\mathcal{Q}$ as the set of all valid 3×3 covariance matrices of three random variables (T, X_1, X_2). A typical element Q in the set $\mathcal{Q}$ takes the following form,

$$\begin{aligned}Q &= E\big[(X_1 \ X_2 \ T)(X_1 \ X_2 \ T)^T\big] \\ &= \begin{pmatrix} P_1 & \rho_{12}\sqrt{P_1P_2} & \rho_{1T}\sqrt{P_1P_T} \\ \rho_{12}\sqrt{P_1P_2} & P_2 & \rho_{2T}\sqrt{P_2P_T} \\ \rho_{1T}\sqrt{P_1P_T} & \rho_{2T}\sqrt{P_2P_T} & P_T \end{pmatrix} \end{aligned} \tag{28}$$

A necessary condition for Q to be a valid covariance matrix is that it is positive semi-definite, i.e., det$(Q) \geq 0$. This is equivalent to saying that,

$$\text{det}(Q) = P_1P_2P_T\Delta \geq 0 \tag{29}$$

where we have defined for simplicity,

$$\Delta = 1 - \rho_{12}^2 - \rho_{1T}^2 - \rho_{2T}^2 + 2\rho_{1T}\rho_{2T}\rho_{12} \tag{30}$$

VII. Evaluation of $\mathcal{DB}_{UC}$

We start with step 1 and first characterize the set of jointly Gaussian triples (T_G, X_{1G}, X_{2G}) in $\mathcal{P}_G^{DB}$. For this purpose, we rewrite (27) as follows,

$$\begin{aligned}&h(Y_{F_1}, Y_{F_2}|T) + h(Y_{F_1}, Y_{F_2}|X_1, X_2, T) \\ &\leq h(Y_{F_1}, Y_{F_2}|X_1, T) + h(Y_{F_1}, Y_{F_2}|X_2, T)\end{aligned} \tag{31}$$

Making use of the following equalities,

$$h(Y_{F_1}, Y_{F_2}|X_1, X_2, T) = \frac{1}{2}\log\left((2\pi e)^2\sigma_{Z_1}^2\sigma_{Z_2}^2\right) \quad (32)$$

$$h(Y_{F_1}, Y_{F_2}|X_1, T) = \frac{1}{2}\log\left((2\pi e)\sigma_{Z_2}^2\right) + h(Y_{F_1}|X_1, T) \quad (33)$$

$$h(Y_{F_1}, Y_{F_2}|X_2, T) = \frac{1}{2}\log\left((2\pi e)\sigma_{Z_1}^2\right) + h(Y_{F_2}|X_2, T) \quad (34)$$

we obtain a simplified expression for (31) as,

$$h(Y_{F_1}, Y_{F_2}|T) \leq h(Y_{F_1}|X_1, T) + h(Y_{F_2}|X_2, T) \quad (35)$$

which can be further simplified by using (15)-(18) to the following two equalities,

$$I(Y_{F_1}; X_1|T) = 0 \quad (36)$$

$$I(Y_{F_2}; X_2|Y_{F_1}, T) = 0 \quad (37)$$

Using (15)-(18), it is straightforward to note that a jointly Gaussian triple (T, X_1, X_2) with a covariance matrix Q satisfies (36)-(37) iff $X_1 \rightarrow T \rightarrow X_2$, which is in turn equivalent to the statement that the covariance matrix Q satisfies $\rho_{12} = \rho_{1T}\rho_{2T}$.

We can now write the set of rate pairs provided by our outer bound for a jointly Gaussian triple (T_G, X_{1G}, X_{2G}) in the set $\mathcal{P}_G^{DB}$ as

$$R_1 \leq I(X_{1G}, X_{2G}; Y_1) \quad (38)$$

$$R_2 \leq I(X_{1G}, X_{2G}; Y_2) \quad (39)$$

$$R_1 \leq I(X_{1G}; Y_1, Y_2, Y_{F_2}|X_{2G}, T) \quad (40)$$

$$R_2 \leq I(X_{2G}; Y_1, Y_2, Y_{F_1}|X_{1G}, T) \quad (41)$$

$$R_1 + R_2 \leq I(X_{1G}, X_{2G}; Y_1, Y_2, Y_{F_1}, Y_{F_2}|T) \quad (42)$$

$$R_1 + R_2 \leq I(X_{1G}, X_{2G}; Y_1, Y_2)\} \quad (43)$$

where (T_G, X_{1G}, X_{2G}) satisfy $X_{1G} \rightarrow T_G \rightarrow X_{2G}$.

In step 2, we consider any non-Gaussian input distribution $p(t, x_1, x_2)$ in $\mathcal{P}_{NG}^{DB(a)}$ with a covariance matrix Q. For such an input distribution, we know by the maximum entropy theorem [13], that the rates provided by a jointly Gaussian triple with the same covariance matrix Q are always at least as large as the rates provided by the chosen non-Gaussian distribution. Therefore, for any input distribution in $\mathcal{P}_{NG}^{DB(a)}$, there always exists an input distribution in $\mathcal{P}_G^{DB}$, satisfying (27), which yields larger rates. This means that we can ignore the set $\mathcal{P}_{NG}^{DB(a)}$ altogether while evaluating our outer bounds.

We now arrive at step 3 of the evaluation of our outer bound. Consider any triple (T, X_1, X_2) with a non-Gaussian distribution $p(t, x_1, x_2) \in \mathcal{P}_{NG}^{DB(b)}$, with a valid covariance matrix Q. We first construct another triple (T', X_1, X_2) with a covariance matrix S by selecting

$$T' = E[X_1|T] \quad (44)$$

Following this step, we next make use of the Markov chain

$$T' \rightarrow T \rightarrow (X_1, X_2) \rightarrow (Y_1, Y_2, Y_{F_1}, Y_{F_2}) \quad (45)$$

to show the existence of a jointly Gaussian (T_G', X_{1G}, X_{2G}) with a covariance matrix S and which satisfies (27). The proof of this claim can be found in [14].

This particular selection of T' is closely related to a recent work in [11] where it was shown that jointly Gaussian distributions are sufficient to characterize the capacity region of Gaussian MAC with conferencing encoders. Although, when evaluating our outer bound for IC-UC, we do not have a conditionally independent structure among (T, X_1, X_2) to start with. This structure arises from the dependence balance constraint (27), permitting us to use this approach.

We now arrive at the final step of the evaluation. In particular, we will show that the rates of this jointly Gaussian triple (T_G', X_{1G}, X_{2G}) will include the rates of the given non-Gaussian triple (T, X_1, X_2). For the triple (T_G', X_{1G}, X_{2G}), we have the following set of inequalities,

$$\begin{aligned}
&I(X_{1G}; Y_1, Y_2, Y_{F_2}|X_{2G}, T_G') \\
&= h(Y_1, Y_2, Y_{F_2}|X_{2G}, T_G') - h(Y_1, Y_2, Y_{F_2}|X_{1G}, X_{2G}, T_G') \\
&= h(X_{1G} + N_1, \sqrt{a}X_{1G} + N_2, \sqrt{h_{12}}X_{1G} + Z_2|X_{2G}, T_G') \\
&\quad - h(Y_1, Y_2, Y_{F_2}|X_{1G}, X_{2G}, T_G') && (46) \\
&\geq h((X_1 + N_1, \sqrt{a}X_1 + N_2, \sqrt{h_{12}}X_1 + Z_2|X_2, T') \\
&\quad - h(Y_1, Y_2, Y_{F_2}|X_{1G}, X_{2G}, T_G') && (47) \\
&\geq h((X_1 + N_1, \sqrt{a}X_1 + N_2, \sqrt{h_{12}}X_1 + Z_2|X_2, T', T) \\
&\quad - h(Y_1, Y_2, Y_{F_2}|X_{1G}, X_{2G}, T_G') && (48) \\
&= h((X_1 + N_1, \sqrt{a}X_1 + N_2, \sqrt{h_{12}}X_1 + Z_2|X_2, T) \\
&\quad - h(Y_1, Y_2, Y_{F_2}|X_1, X_2, T) && (49) \\
&= I(X_1; Y_1, Y_2, Y_{F_2}|X_2, T) && (50)
\end{aligned}$$

where (47) follows from the fact that (T', X_1, X_2) and (T_G', X_{1G}, X_{2G}) have the same covariance matrix S and using the maximum entropy theorem. Next, (48) follows from the fact that conditioning reduces differential entropy and finally (49) follows from the fact that T' is a deterministic function of T and invoking the Markov chain in (45). Similarly, we also have

$$\begin{aligned}
&I(X_{2G}; Y_1, Y_2, Y_{F_1}|X_{1G}, T_G') \\
&\qquad \geq I(X_2; Y_1, Y_2, Y_{F_1}|X_1, T) && (51) \\
&I(X_{1G}, X_{2G}; Y_1, Y_2, Y_{F_1}, Y_{F_2}|T_G') \\
&\qquad \geq I(X_1, X_2; Y_1, Y_2, Y_{F_1}, Y_{F_2}|T) && (52)
\end{aligned}$$

Finally, we have

$$\begin{aligned}
I(X_{1G}, X_{2G}; Y_1) &= h(Y_1) - h(Y_1|X_{1G}, X_{2G}) && (53) \\
&= h(X_{1G} + \sqrt{b}X_{2G} + N_1) - h(N_1) && (54) \\
&\geq h(X_1 + \sqrt{b}X_2 + N_1) - h(N_1) && (55) \\
&= I(X_1, X_2; Y_1) && (56)
\end{aligned}$$

and similarly, we also have,

$$I(X_{1G}, X_{2G}; Y_2) \geq I(X_1, X_2; Y_2) \quad (57)$$

$$I(X_{1G}, X_{2G}; Y_1, Y_2) \geq I(X_1, X_2; Y_1, Y_2) \quad (58)$$

Therefore, we conclude that for any non-Gaussian distribution $p(t, x_1, x_2) \in \mathcal{P}_{NG}^{DB(b)}$, there exists a jointly Gaussian distribution $p(t, x_1, x_2) \in \mathcal{P}_G^{DB}$ which satisfies the dependence balance bound (27) and yields a set of rates which includes the set of rates given by the fixed non-Gaussian distribution. Hence, it suffices to consider jointly Gaussian distributions in $\mathcal{P}_G^{DB}$ to evaluate our outer bound.

The explicit expressions for our outer bound and the cut-set bound for the Gaussian IC-UC are rather tedious and we do not present them here. They can be found in [14]. Here, we plot these bounds for certain parameters. Figure 4 illustrates our outer bound, cut-set bound, an achievable rate region with cooperation [7], capacity region without cooperation [15] for the case when $P_1 = P_2 = \sigma_{N_1}^2 = \sigma_{N_1}^2 = 1$ and $\sigma_{Z_1}^2 = \sigma_{Z_2}^2 = 1$ and $a = b = 1$ and $h_{12} = h_{21} = 2$. Figure 5 illustrates our sum rate upper bound and the cut-set bound as function of h, where $h = h_{12} = h_{21}$ and $P_1 = P_2 = \sigma_{N_1}^2 = \sigma_{N_1}^2 = 1$ and $\sigma_{Z_1}^2 = \sigma_{Z_2}^2 = 1$, $a = b = 0.5$.

VIII. Conclusions

We obtained a new outer bound for the capacity region of the two-user IC with generalized feedback. We explicitly evaluated this outer bound for the Gaussian IC with user cooperation. Our outer bound strictly improves upon the cut-set bound for all non-zero values of cooperation noise variances.

To evaluate our outer bound for the Gaussian IC-UC, we have to consider all input distributions satisfying the dependence balance constraint. The main difficulty in evaluating our outer bound arises from the fact that there might exist some non-Gaussian input distribution $p(t, x_1, x_2)$ with a covariance matrix Q, such that $p(t, x_1, x_2)$ satisfies the dependence balance constraint but there does not exist a jointly Gaussian triple with the covariance matrix Q satisfying the dependence balance constraint. Therefore, the regular methodology of evaluating outer bounds, i.e., the approach of applying maximum entropy theorem [13] fails beyond this particular point. Through our explicit evaluation, we were able to show the existence of a jointly Gaussian triple with a covariance matrix S which satisfies the dependence balance constraint and yields larger rates than the fixed non-Gaussian distribution.

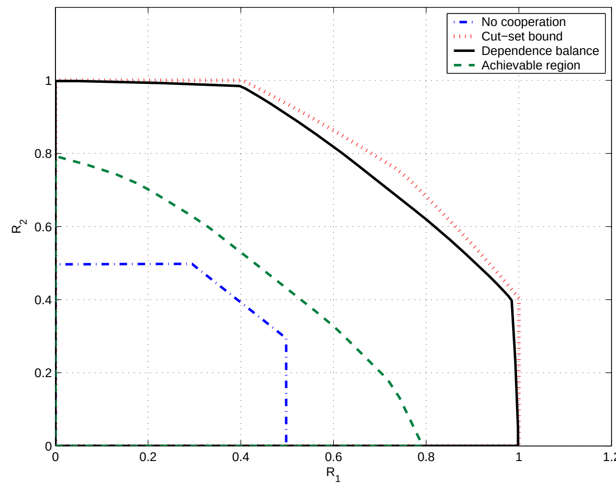

Fig. 4. Illustration of bounds for $P_1 = P_2 = \sigma_{N_1}^2 = \sigma_{N_2}^2 = 1$, $\sigma_{Z_1}^2 = \sigma_{Z_2}^2 = 1$ and $a = b = 1$ and $h_{12} = h_{21} = 2$.

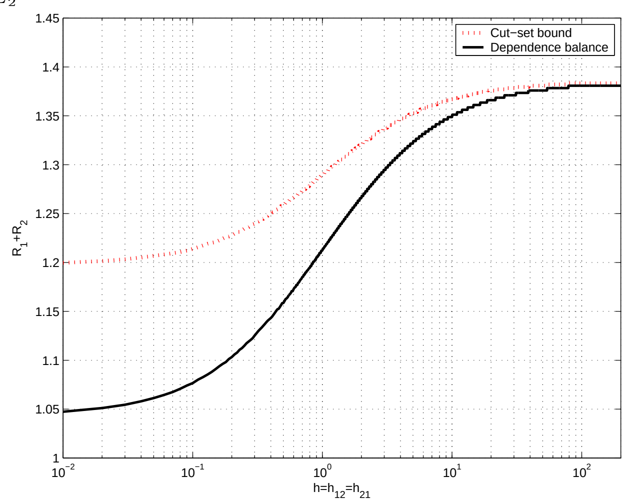

Fig. 5. Illustration of sum-rate upper bound and the cut-set bound as a function of h, where $h = h_{12} = h_{21}$.

References

[1] A. P. Hekstra and F. M. J. Willems. Dependence balance bounds for single output two-way channels. *IEEE Trans. on Information Theory*, 35(1):44–53, January 1989.

[2] A. Sendonaris, E. Erkip, and B. Aazhang. User cooperation diversity–Part I: System description. *IEEE Trans. on Communications*, 51(11):1927–1938, November 2003.

[3] N. Gaarder and J. Wolf. The capacity region of a multiple-access discrete memoryless channel can increase with feedback. *IEEE Trans. on Information Theory*, 21(1):100–102, Jan 1975.

[4] A. B. Carleial. Multiple-access channels with different genaralized feedback signals. *IEEE Trans. on Information Theory*, 28(6):841–850, November 1982.

[5] F. M. J. Willems, E. C. van der Meulen, and J. P. M. Schalkwijk. Achievable rate region for the multiple access channel with generalized feedback. In *Proc. Annual Allerton Conference on Communication, Control and Computing*, pages 284–292, 1983.

[6] D. Tuninetti. On interference channels with generalized feedback. In *Proc. IEEE ISIT*, June 2007.

[7] A. Host-Madsen. Capacity bounds for cooperative diversity. *IEEE Trans. on Information Theory*, 52(4):1522–1544, April 2006.

[8] T. Han and K. Kobayashi. A new achievable rate region for the interference channel. *IEEE Trans. on Information Theory*, 27(1):49–60, January 1981.

[9] I. Sason. On achievable rate regions for the Gaussian interference channel. *IEEE Trans. on Information Theory*, 50(6):1345–1356, June 2004.

[10] M. Gastpar and G. Kramer. On noisy feedback for interference channels. In *Proc. Asilomar Conf. on Signals, Systems, and Computers, Pacific Grove, CA, USA*, Oct. 29-Nov. 1 2006.

[11] S. I. Bross, A. Lapidoth, and M. Wigger. The Gaussian MAC with conferencing encoders. In *Proc. IEEE ISIT*, July 2008.

[12] V. Venkatesan. Optimality of Gaussian inputs for a multi-access achievable rate region. *Semester Thesis, ETH Zurich, Switzerland*, June 2007.

[13] T. M. Cover and J. A. Thomas. *Elements of Information Theory*. New York:Wiley, 1991.

[14] R. Tandon and S. Ulukus. Dependence balance based outer bounds for Gaussian networks with cooperation and feedback. [Arxiv:0812.1857]. *Submitted to IEEE Trans. on Information Theory*, Dec. 2008.

[15] H. Sato. The capacity of the Gaussian interference channel under strong interference. *IEEE Trans. on Information Theory*, 27(6):786–788, November 1981.