Energy Harvesting Communications Under Temperature Constraints

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***Abstract*— We consider throughput maximization in a single-user energy harvesting communication system under energy and temperature constraints. We model three main temperature related physical defects in wireless sensors mathematically, and investigate their impact on throughput maximization. Specifically, we consider temperature dependent energy leakage, effects of processing circuit power on temperature, and temperature increases due to the energy harvesting process itself. In each case, we determine the optimum power schedule.**

I. Introduction

Wireless sensors are prone to temperature increase caused by sensor operation. Temperature dynamics in such systems are typically determined by the temperature of the surrounding environment, transmit power for data transmission, and circuit power associated with processing. In energy harvesting wireless sensors, an additional cause of temperature increase is the energy harvesting process itself. The increase in temperature can have several undesired outcomes. In particular, it can cause damage to the device [1] or to its surrounding environment [2]. In this case, a hard peak temperature constraint could be more suitable. Another effect of temperature increase is the energy leakage or the energy lost without utilization. Typically, the energy leakage in sensors is temperature dependent [3]. In this paper, we specialize in energy harvesting communications and address the thermal effects related to energy leakage, processing costs and energy harvesting process.

Our work is motivated by and builds upon the recent work [4], which considers the throughput maximization problem for energy harvesting transmitters under temperature constraints. Data transmission with energy harvesting transmitters has been considered in recent work [5]–[13]. In particular, throughput maximization under offline and online knowledge of the energy arrivals is considered for single-user and multi-user energy harvesting communications. In [14]–[18], this problem is considered under imperfections such as energy leakage, charge/discharge inefficiency, and processing costs.

In order to address temperature sensitivity of such systems, we consider problem formulations that capture temperature related physical defects in energy harvesting communications. In particular, we investigate the impact of these physical defects on the optimal throughput for energy harvesting communications. We first address the temperature dependent energy leakage. Energy leakage is inevitable in wireless sensors due to power dissipated in the electronics circuitry of the system. It is well-known that the energy leakage increases with temperature. We adopt a linear leakage model as in [3], [19], [20] and investigate the optimal transmit power policy.

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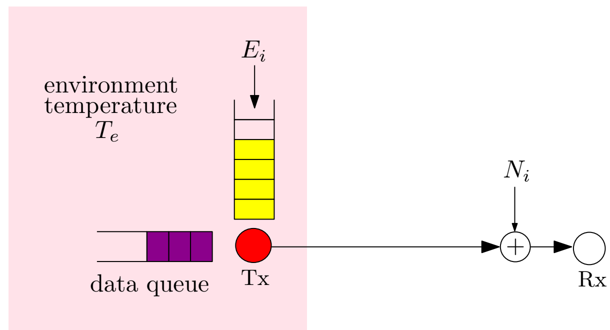

Fig. 1. System model representing an energy harvesting transmitter in an environment with temperature T_e.

Next, we consider the problem of processing cost. Processing cost has been studied earlier in energy harvesting communications [16], [17]. In view of [16], [21], it is well-known that the processing cost forces the optimal transmission to be bursty. In the absence of temperature constraints, the silent and active periods affect the throughput only through their lengths. However, with temperature constraints, their sequence has to be properly designed in addition to their lengths. We address this problem by allowing the transmitter to divide the transmission duration into two consecutive transmission and silence periods, and identify the optimal policy in this case.

Finally, we study the effects of the heat caused by the energy harvesting process itself. While harvesting more energy improves the throughput, it also increases the temperature. In this case, the transmitter has to determine the transmit power level as well as the amount of harvested energy (i.e., energy intake). Under a linear relation between temperature increase and harvested energy, we investigate the jointly optimal energy harvesting and transmit power policy.

II. Model and Problem Formulation

We consider a single-user energy harvesting channel, Fig. 1, subject to temperature constraints. The physical layer is an additive Gaussian noise channel where the noise variance is unity for convenience. We use a continuous time model: In an infinitesimal time duration dt in $[t, t + dt]$, the transmitter decides a feasible transmit power level $p(t)$, and $\frac{1}{2}\log\left(1+p(t)\right)dt$ units of data is sent to the receiver.

The battery at the transmitter has unlimited size and the initial energy available in the battery at time zero is E_0. Energy arrivals occur at times $\{s_1, s_2, \ldots\}$ in amounts $\{E_1, E_2, \ldots\}$. We call the time interval between two consecutive energy arrivals an *epoch*. D is the deadline. E_i and s_i are known offline. Let $h(t) = \max\{k : s_k < t\}$. Power policy $p(t)$ is subject to energy causality constraints as [5]:

$$\int_0^t p(\tau)d\tau \leq \sum_{i=0}^{h(t)} E_i, \qquad \forall t \in [0, D] \tag{1}$$

We adopt the following first order thermal model:

$$\frac{dT(t)}{dt} = ap(t) - b(T(t) - T_e) + c(t) \tag{2}$$

where T_e is the environment temperature, $T(t)$ is the temperature at time t, $c(t)$ represents additional heat sources, and a, b are non-negative constants. With the initial temperature $T(0) = T_e$, the solution of (2) is [4]:

$$T(t) = e^{-bt}\int_0^t e^{b\tau}\left(ap(\tau) + c(\tau)\right)d\tau + T_e \tag{3}$$

First, we consider the throughput maximization problem with temperature dependent energy leakage:

$$\begin{aligned} \max_{p(t)} \quad & \frac{1}{2}\int_0^D \log(1+p(\tau))d\tau \\ \text{s.t.} \quad & \int_0^t p(\tau)d\tau + \int_0^t \epsilon_l(T(t) - T_e)d\tau \leq \sum_{i=0}^{h(t)} E_i \\ & p(t) \geq 0, \ \forall t \in [0, D] \end{aligned} \tag{4}$$

where ϵ_l is the energy leakage coefficient. Energy leakage happens even when the transmitter is not transmitting. In particular, $\epsilon_l T_e D$ is the nominal energy leakage, which is the amount of energy that leaks when the transmitter is not transmitting and the temperature is T_e. We assume that $\tilde{E}_i \geq \epsilon_l T_e(s_{i+1} - s_i)$ is the actual harvested energy at $t = s_i$ and that E_i in the formulation in (4) is $E_i = \tilde{E}_i - \epsilon_l T_e(s_{i+1} - s_i)$.

Next, we consider the throughput maximization problem under temperature constraints in the presence of processing costs due to power spent for the transmitter to be on:

$$\begin{aligned} \max_{p(t), \{\theta_i\}} \quad & \int_{\theta_1}^{\theta_2} \frac{1}{2}\log(1+p(\tau))d\tau + \int_{\theta_3}^{\theta_4} \frac{1}{2}\log(1+p(\tau))d\tau \\ \text{s.t.} \quad & \int_{\theta_1}^{\theta_2} (\epsilon_p + p(\tau))d\tau + \int_{\theta_3}^{\theta_4} (\epsilon_p + p(\tau))d\tau \leq E \\ & T(t) \leq T_c \\ & 0 \leq \theta_1 \leq \theta_2 \leq \theta_3 \leq \theta_4 \leq D \\ & p(t) \geq 0, \ \forall t \in [0, D] \end{aligned} \tag{5}$$

where ϵ_p is the processing cost, E is the available energy at the transmitter. This problem is motivated by the approach in [21]. It is well-known that, when there is processing cost, the transmission becomes *bursty*. Under temperature constraints, in general, there may be many intervals of being on and off for the transmitter. In the formulation in (5), we allow the transmitter to divide the transmission session into two parts only and to *cool-down* in between transmissions. Here, in the single epoch formulation in (5), the transmitter is active in the intervals $[\theta_1, \theta_2]$ and $[\theta_3, \theta_4]$ and silent in the rest.

Finally, we consider the temperature increase due to energy harvesting. In this case, $c(t) = \sum_{i=1}^N \epsilon_h \alpha_i E_i \delta(t - s_i)$ where E_i is the available energy and $\alpha_i E_i$ is the amount of *harvested* energy (i.e., energy intake) with $\alpha_i \in [0, 1]$. Note that α_i is controlled by the transmitter. Here, ϵ_h is the coefficient that determines the temperature increase due to energy harvesting. In particular, at time $t = s_i$, temperature increases with amount $\epsilon_h \alpha_i E_i$. We impose a hard temperature constraint $T(t) \leq T_c$ for all $t \in [0, D]$. This constraint is equivalent to:

$$\int_0^t ae^{b\tau}p(\tau)d\tau + \sum_{i=1}^{h(t)} \epsilon_h \alpha_i E_i e^{bs_i} u(t - s_i) \leq T_\delta e^{bt}, \ \forall t \tag{6}$$

where $u(\cdot)$ is the unit step function. We consider the following problem:

$$\begin{aligned} \max_{p(t), \{\alpha_i\}} \quad & \frac{1}{2}\int_0^D \log(1+p(\tau))d\tau \\ \text{s.t.} \quad & \int_0^t p(\tau)d\tau \leq \sum_{i=0}^{h(t)} \alpha_i E_i \\ & \int_0^t ae^{b\tau}p(\tau)d\tau + \sum_{i=1}^{h(t)} \epsilon_h \alpha_i E_i e^{bs_i} u(t - s_i) \leq T_\delta e^{bt} \\ & p(t) \geq 0, \ \forall t \in [0, D] \end{aligned} \tag{7}$$

In the following sections, we specialize in the problems stated in (4), (5), and (7).

III. Temperature Dependent Energy Leakage: Problem in (4)

In this section, we focus on the throughput maximization problem in (4) with temperature dependent energy leakage. The problem is convex and the Lagrangian is:

$$\begin{aligned} \mathcal{L} = & -\int_0^D \log(1+p(\tau))d\tau + \int_0^D \lambda(t)\left(\int_0^t p(\tau)d\tau\right)dt \\ & + \int_0^D \lambda(t)\left(\int_0^t \epsilon_l(T(\tau) - T_e)d\tau - \sum_{i=0}^{h(t)} E_i\right)dt \end{aligned} \tag{8}$$

The KKT optimality conditions are:

$$\begin{aligned} -\frac{1}{1+p(t)} + & \int_t^D \lambda(\tau)d\tau \\ & + \epsilon_l a e^{bt}\int_t^D \int_t^x \lambda(x)e^{-b\tau}dx d\tau = 0 \end{aligned} \tag{9}$$

In the following sub-sections, we first investigate the solution for a single energy arrival, then for multiple energy arrivals, and then provide the general form of the solution.

A. Single Energy Arrival

In this case, (9) reduces to the following:

$$p(t) = \left(\frac{1}{\lambda \left(1 + \frac{\epsilon_l a}{b}(1 - e^{b(t-D)})\right)} - 1 \right)^+ \tag{10}$$

Lemma 1 *The optimal power, $p(t)$, is monotone increasing and convex.*

Proof: Since $1 + \frac{\epsilon_l a}{b}(1 - e^{b(t-D)})$ is a monotone decreasing, concave and positive function, its reciprocal is a monotone increasing and convex function. Hence, from (10), the optimum power is monotone increasing and convex. ■

Lemma 1 suggests that in the optimal policy, energy utilization is deferred to the future to the extent possible. This enables a controlled increase in the temperature and the energy loss due to leakage. Note that the linear dependence of the energy leakage on the temperature forms a positive feedback loop in that more energy is lost as the temperature increases.

Lemma 2 *The temperature, $T(t)$, resulting from the optimal power policy is monotone increasing.*

Proof: First, we need to calculate the temperature based on the optimal power policy (10). It suffices to consider only the term with the integration in the temperature expression in (3) since T_e is constant and will not affect the analysis. Since the power is increasing, there exists $t = t_0$ such that $p(t) > 0$ for all $t \in [t_0, D]$. Hence, using (10) we have,

$$\begin{aligned} T(t) - T_e =& \frac{e^{-b(t-D)}}{\lambda \epsilon_l} \log \left(\frac{\lambda + \frac{\epsilon_l \lambda a}{b}\left(1 - e^{-b(D-t_0)}\right)}{\lambda + \frac{\epsilon_l \lambda a}{b}\left(1 - e^{-b(D-t)}\right)} \right) \\ & - \frac{a}{b}\left(1 - e^{b(t_0 - t)}\right) \end{aligned} \tag{11}$$

Next, we need to show that this $T(t)$ is increasing. To check this, we evaluate the derivative of $T(t)$ as follows,

$$\begin{aligned} \frac{dT}{dt} =& - \frac{be^{-b(t-D)}}{\lambda \epsilon_l} \log \left(\frac{1 + \frac{\epsilon_l a}{b}\left(1 - e^{-b(D-t_0)}\right)}{1 + \frac{\epsilon_l a}{b}\left(1 - e^{-b(D-t)}\right)} \right) \\ & + \frac{a}{\lambda + \frac{\epsilon_l \lambda a}{b}\left(1 - \lambda e^{-b(D-t)}\right)} - ae^{b(t_0 - t)} \qquad (12) \\ =& - \frac{be^{-b(t-D)}}{\lambda \epsilon_l} \log \left(\frac{\frac{\epsilon_l a}{b}\left(e^{-b(D-t)} - e^{-b(D-t_0)}\right)}{1 + \frac{\epsilon_l a}{b}\left(1 - e^{-b(D-t)}\right)} + 1 \right) \\ & + \frac{a}{\lambda + \frac{\epsilon_l \lambda a}{b}\left(1 - \lambda e^{-b(D-t)}\right)} - ae^{b(t_0 - t)} \qquad (13) \\ \geq& - \frac{be^{-b(t-D)}}{\lambda \epsilon_l} \left(\frac{\frac{\epsilon_l a}{b}\left(e^{-b(D-t)} - e^{-b(D-t_0)}\right)}{1 + \frac{\epsilon_l a}{b}\left(1 - e^{-b(D-t)}\right)} \right) \\ & + \frac{a}{\lambda + \frac{\epsilon_l \lambda a}{b}\left(1 - e^{-b(D-t)}\right)} - ae^{b(t_0 - t)} \qquad (14) \\ =& ae^{b(t_0 - t)} p^*(t) > 0 \qquad (15) \end{aligned}$$

where the inequality follows since $\log(1 + x) \leq x$. Hence, the temperature is strictly increasing whenever the optimum power is non-zero. ■

Lemma 3 *Optimum Lagrange multiplier satisfies $\lambda \in (0, 1)$.*

Proof: From (10), we know that the optimum power is increasing, and if it is non-zero, then, it is non-zero specifically at $t = D$. Hence, we have,

$$\lambda \left(1 + \frac{\epsilon_l a}{b}(1 - e^{b(t-D)})\right) \Big|_{t=D} < 1 \tag{16}$$

which in turn implies that $\lambda < 1$. In addition, the Lagrange multiplier cannot be zero, as this would imply the power to be infinity. Combining this with the non-negativity of the Lagrange multiplier gives the desired result. ■

Note that the only variable in the expression in (10) is λ. The optimal power can be obtained by one-dimensional search on $\lambda \in (0, 1)$. The next lemmas will be useful in providing the optimal algorithm for the multiple energy arrival case. They state the monotonicity of the power with the harvested energy.

Lemma 4 *In the optimal power policy, the energy constraint is satisfied with equality.*

Proof: The proof follows by contradiction. If the optimal power does not satisfy the energy constraint with equality, then we can increase the power which strictly increases the objective function, and this violates the optimality. ■

Lemma 5 *$p(t)$ is monotonically increasing with E.*

Proof: Assume we have energy arrival E, and corresponding power $p(t)$. We know that the constraint will be satisfied with equality. Then, if we increase the energy to $E + \epsilon$ for any $\epsilon > 0$, the constraint is not satisfied. Hence, there exists $\delta > 0$ such that when λ is replaced with $\lambda - \delta$, equality is achieved. Decreasing λ increases $p(t)$ for all $t \in [0, D]$. ■

B. Multiple Energy Arrivals

Lemma 6 *$p(t)$ is monotonically increasing throughout the transmission duration.*

Proof: Using (9) we have

$$p^*(t) = \left(\frac{1}{\int_t^D \lambda(\tau) d\tau + \epsilon_l a e^{bt} \int_t^D \int_t^x \lambda(x) e^{-b\tau} d\tau dx} - 1 \right)^+ \tag{17}$$

which when the inner integral is evaluated becomes,

$$p^*(t) = \left(\frac{1}{\int_t^D \lambda(\tau) d\tau + \frac{\epsilon_l a}{b} \int_t^D \lambda(x)[1 - e^{-b(x-t)}] dx} - 1 \right)^+ \tag{18}$$

Since the denominator is a decreasing function of t, $p(t)$ is increasing in t. ■

Lemma 7 *The battery can be empty only at the energy arrival instants. It is certainly empty at the end.*

Proof: This follows since the optimal power is monotonically increasing throughout the transmission duration, hence, there does not exist a duration of non-zero measure with zero power. Hence, the battery can never be empty for a non-zero measure interval. Therefore, the battery cannot be empty except a duration of measure zero, which can only happen at energy arrival instants or at the end of the deadline. If the battery is not empty at the end, then we can always increase the power without violating the constraint, which violates optimality. ■

Lemma 8 *The transmission power may have possible positive jumps only at the energy arrival instants.*

Proof: From Lemma 7, we have that the energy can be consumed fully only at the energy arrivals or the deadline, hence, the energy constraint can be tight only at these instants. Therefore, from the complementary slackness, we have,

$$\lambda(t) = \sum_{i=1}^{N} \lambda_i \delta(t - s_i) \tag{19}$$

Then, substituting this in (18), we have,

$$p^*(t) = \left(\frac{1}{\sum_{i=1}^{N} \lambda_i u(s_i - t) \left(1 + \frac{\epsilon_l a}{b}[1 - e^{-b(s_i - t)}]\right)} - 1 \right)^+ \tag{20}$$

which may have positive jumps at the instants s_i due to the presence of the unit step function. ■

Lemma 9 *The temperature is monotone increasing throughout the communication session.*

Proof: We show this for a two epoch system, however, the proof for N epochs follows identical steps. It is clear that for the second slot temperature is increasing since the power in the second epoch has the form,

$$p^*(t) = \frac{1}{\lambda_2 \left(1 + \frac{\epsilon_l a}{b}[1 - e^{-b(s_2 - t)}]\right)} - 1 \tag{21}$$

which is the same form as in the case of a single epoch, hence we can proceed as in the proof of Lemma 2. Thus, it remains to show the same for the first epoch. The optimal power in the first epoch can be written as,

$$p^*(t) = \frac{1}{\sum_{i=1}^{2} \lambda_i \left(1 + \frac{\epsilon_l a}{b}[1 - e^{-b(s_i - t)}]\right)} - 1 \tag{22}$$

$$= \frac{1}{\tilde{\lambda} + \tilde{\lambda}\frac{\epsilon_l a}{b} - \frac{\epsilon_l a}{b}\left(e^{-bs_1}\lambda_1 + e^{-bs_2}\lambda_2\right) e^{bt}} - 1 \tag{23}$$

$$= \frac{1}{\tilde{\lambda}\left(1 + \frac{\epsilon_l a}{b}[1 - e^{-b(\tilde{D} - t)}]\right)} - 1 \tag{24}$$

where $\tilde{\lambda} = \lambda_1 + \lambda_2$ and $\tilde{D}$ is some number between s_1, s_2. The existence of $\tilde{D}$ is guaranteed since we have,

$$\tilde{\lambda} \min\{e^{-bs_1}, e^{-bs_2}\} \leq \lambda_1 e^{-bs_1} + \lambda_2 e^{-bs_2} \tag{25}$$

$$\leq \tilde{\lambda} \max\{e^{-bs_1}, e^{-bs_2}\} \tag{26}$$

and the exponential function is continuous. We can now apply again the proof in Lemma 2. Hence, the temperature is strictly increasing within each slot, and therefore, throughout the communication session. It also follows that the temperature can be constant only when the power is zero, which can happen only at the beginning of the transmission duration. ■

C. Optimal Policy

We provide the optimal policy first for the case of a single arrival and then for the case of multiple arrivals.

1) Single Energy Arrival: For the single energy arrival case, we showed that the optimal solution depends only on λ. We also showed in Lemma 3 that λ lies in the interval $(0, 1)$. Also, using the fact that the energy constraint is always satisfied with equality, λ satisfies the following equation:

$$\int_{p(t) \geq 0} (p(\tau) + \epsilon_l (T(\tau) - T_e)) d\tau = E \tag{27}$$

where $p(t)$ is given in (10). Hence, the optimal λ is found by a one-dimensional search on $(0, 1)$. Note also that since $p(t)$ is monotone in λ and (27) is linear in $p(t)$, (27) is monotone in λ. Hence, we can search for λ using the bisection method in the range $(0, 1)$ until this equation is satisfied with equality.

2) Multiple Energy Arrivals: We know from (19) that the optimal multiple epoch problem reduces to finding $\{\lambda_i\}_{i=1}^{N}$. The algorithm begins by solving each slot individually using the single epoch solution. However, the epochs are not completely independent of each other, due to the temperature accumulation throughout the transmission duration.

We begin by assuming an initial energy allocation, where we use only the energy that arrived in each epoch with no energy transfer between the epochs. We first solve for epoch 1 power allocation. In the first epoch, there is no temperature accumulation which should be taken into account. Next, we solve for epoch 2 power allocation, but with setting the temperature as follows,

$$T(t) = e^{-bt} \int_0^t e^{b\tau} a p(\tau) d\tau + T(s_1) \tag{28}$$

where $T(s_1)$ is the temperature of the system at the end of the epoch 1, i.e., the system starts at the second slot from temperature $T(s_1)$ instead of T_e. Note that this can be calculated using the optimal λ, t_0 from (11). In effect, this is equivalent to subtracting from the available energy in the second epoch an amount equal to $(s_2 - s_1)\epsilon_l(T(s_1) - T_e)$, where $s_2 - s_1$ is the duration of the second epoch. Similarly, we proceed to solve for power allocation in all epochs with setting the temperature at epoch i as,

$$T(t) = e^{-bt} \int_0^t e^{b\tau} a p(\tau) d\tau + T(s_{i-1}) \tag{29}$$

Until this point, the obtained solution may not be optimal. We need to check if the solution satisfies the optimality conditions. If the power allocations between the slots is increasing, then indeed this is the optimal solution, according to Lemma 6, since we can find the corresponding Lagrange multipliers.

Next, if the solution for the power is not increasing, then this is not the optimal solution, and it needs to be modified. If slots i and $i-1$ do not satisfy this, then we transfer energy from slot $i-1$ to slot i, and re-solve the problem until we equate the powers at time s_i. According to Lemma 5, transferring energy from one slot to another decreases the power in slot $i-1$ while increases the power in slot i, which guarantees the existence of an increasing solution by transferring energy. In this case, we have $\lambda_i = 0$. We repeat this procedure between every two consecutive slots which have non-increasing power until the power is increasing throughout the transmission duration.

IV. Non-zero Processing Power: Problem in (5)

In this section, we focus on the throughput maximization problem in (5) with processing cost. First, we discuss the interpretation of the possible solutions of problem (5). If $\theta_1^* = 0$, $\theta_2^* = \theta_3^*$ and $\theta_4^* = D$, then this corresponds to the case that the transmitter is on for the whole duration. This usually happens when the transmitter has sufficient energy. If $\theta_1^* = 0$, $\theta_1^* < \theta_2^* < \theta_3^*$ and $\theta_3^* = \theta_4^* = D$, then the problem reduces to a problem similar to the one proposed at [21]. If $\theta_1^* < \theta_2^* < \theta_3^* < \theta_4^*$, then there is a cooling down phase for the duration $[\theta_2, \theta_3]$.

We now argue that without loss of generality we can set $\theta_1^* = 0$ and $\theta_4^* = D$. The intuition behind this is that we want to separate the two transmissions as much as we can to give the system the longest time to cool down, and hence may achieve a better rate. The problem in this case is:

$$\begin{aligned} \max_{p(t),\{\theta_i\}} \quad & \int_0^{\theta_1} \frac{1}{2}\log(1+p(\tau))d\tau + \int_{\theta_2}^{D} \frac{1}{2}\log(1+p(\tau))d\tau \\ \text{s.t.} \quad & \int_0^{\theta_1} (\epsilon_p + p(\tau))d\tau + \int_{\theta_2}^{D} (\epsilon_p + p(\tau))d\tau \le E \\ & T(t) \le T_c \\ & 0 \le \theta_1 \le \theta_2 \le D \\ & p(t) \ge 0, \ \forall t \in [0, D] \end{aligned} \tag{30}$$

Lemma 10 *Problems (5) and (30) are equivalent.*

Proof: To prove this, we need to show that the optimal solution of each problem is feasible in the other problem with optimal value no less than the other.

It is clear that the optimal solution for (30) is always feasible in (5) with the same optimal value. Now, for (5), assume that the optimal solution is $\theta_1^* > 0$ and $\theta_4^* < D$. We now need to check the feasibility of it in (30). The feasibility is easy to check for the energy constraint, since it only depends on the duration and not the position. The temperature constraint is also feasible, since the heat generated for (5) in the duration $[\theta_1^*, \theta_2^*]$ is the same as when translated to $[0, \theta_2^* - \theta_1^*]$. Thus, we have now verified that the problem is feasible for $t \in [0, \theta_3^*]$. Similarly the heat generated $[\theta_3^*, \theta_4^*]$ will be the same as when translated to $[D - (\theta_4^* - \theta_3^*), D]$ and also the temperature at $D - (\theta_4^* - \theta_3^*)$ is lower that the original problem since we allowed more time for it to cool down. Hence, the temperature constraint is also feasible. Additionally, the objective function is the same. Hence, the two problems are equivalent. ■

The advantage of considering problem (30) is that we have eliminated two variables from the original problem (5). Hereafter, we will only consider problem (30).

A. Characterization of the Optimal Solution

In this section, we provide the properties of the optimal of (30). This problem is not convex due to the presence of the variables $\{\theta_i\}$ in the integration limits. The main challenge besides the non-convexity of this problem is the non-uniqueness of the global optimal solution. In some cases, we can show that there exists an infinite number of global optimal solutions. A simple example is the case when the temperature constraint is never tight. Hence, in what follows we provide sufficient conditions for the optimality of the solution.

First, we assume that the optimal value for $\{\theta_i^*\}$s are known. Fixing the $\{\theta_i^*\}$s yields a convex optimization problem in $p(t)$. Hence, KKT conditions are now necessary and sufficient. The Lagrangian of the problem is:

$$\begin{aligned} \mathcal{L} = & -\int_0^{\theta_1^*} \frac{1}{2}\log(1+p(\tau))d\tau - \int_{\theta_2^*}^{D} \frac{1}{2}\log(1+p(\tau))d\tau \\ & + \lambda\left(\int_0^{\theta_1^*} (\epsilon_p + p(\tau))d\tau + \int_{\theta_2^*}^{D} (\epsilon_p + p(\tau))d\tau - E\right) \\ & + \int_0^D \beta(t)\left[\int_0^t a e^{b\tau} p(\tau)d\tau - T_\delta e^{bt}\right] dt \end{aligned} \tag{31}$$

which yields KKT optimality conditions:

$$p(t) = \left[\frac{1}{\lambda + e^{bt}\int_t^D \beta(\tau)d\tau} - 1\right]^+, \forall t \in [0, \theta_1^*] \cup [\theta_2^*, D] \tag{32}$$

Next, we study the properties of the optimal solution. We first state the lemma indicating the non-increasing property of the power. The proof follows as [4, Lemma 2].

Lemma 11 *The optimal power allocations is monotonically non-increasing in the durations $(0, \theta_1^*)$ and (θ_2^*, D).*

Next, we show that if the temperature constraint is *never tight*, or equivalently, the temperature constraint is removed, then we get back to the formulation proposed in [21] which yields constant transmission power. We also highlight the fact that the number of solutions can be infinite.

Lemma 12 *If there is no temperature constraint (or equivalently, the temperature constraint is never tight), then the optimal power, $p^*(t)$, is constant and $\theta_2 = D$ achieves the optimal solution. However, the solution is not unique.*

Proof: When the temperature constraint is never tight, from slackness we have $\lambda(t) = 0, \ \forall t$. Hence, using this along with

(31) we have:

$$p^*(t) = \frac{1}{\lambda} - 1, \quad \forall t \in [0, \theta_1^*] \cup [\theta_2^*, D] \tag{33}$$

which proves that the power is constant. Then, if $\theta_2^* < D$, we can define a new policy,

$$\tilde{p}(t) = \begin{cases} p^*(t), \ \forall t \in [0, \theta_1^*] \\ p^*(t + \theta_2^* - \theta_1^*), \quad \forall t \in [\theta_1^*, D - (\theta_2^* - \theta_1^*)] \end{cases} \tag{34}$$

This new policy is then feasible since the temperature constraint is not active. Hence, $\theta_2^* = D$ is feasible and gives the optimal solution.

Now, for the non-uniqueness, note that if we have an optimal solution which is $p^*(t) = c$ for $t \in [0, \theta_1^*]$, then for any $\delta \in [0, \theta_1^*]$, $p^*(t) = c$ for $t \in [0, \theta_1^* - \delta] \cup [D - \delta, D]$ is also an optimal solution. This follows since, for all these values of δ, we still have the same value for the objective function. ■

Lemma 13 *Assume that we fix θ_2 to a value and solve for the optimal value of $\theta_1 < \theta_2$. Then, if the resultant optimal power is constant, then: 1) the power level in both slots is equal, 2) the energy constraint will be satisfied with equality, and 3) if this obtained transmission duration $(\theta_1 + D - \theta_2)$ is equal to the optimal duration with no temperature constraint, then the obtained $(\theta_1, \theta_2, p(t))$ is the optimal solution for this problem.*

Proof: Since the power is constant, this means that the temperature constraint can be tight at most on an interval of zero measure, i.e., only at θ_1, D. Hence, we have $\beta(t) = \beta(\theta_1)\delta(t - \theta_1) + \beta(D)\delta(t - D)$. Since the power is constant, from (32), this implies that $\beta(\theta_1) = \beta(D) = 0$. Hence, the power in both slots are equal, and equal to

$$p(t) = \frac{1}{\lambda} - 1, \forall t \in [0, \theta_1] \cup [\theta_2, D] \tag{35}$$

Since $p(t)$ should be finite, we must have $\lambda \neq 0$. Hence, from complementary slackness, the energy constraint must be satisfied with equality. Since the energy constraint is satisfied with equality, power is constant and the duration $\theta_1 + D - \theta_2$ is the same as with no temperature constraint, we obtain a solution equal to the unconstrained solution. Since the unconstrained problem forms an upper bound to our temperature constrained problem, this is the optimal solution. ■

Hence, if we restrict $\theta_2 = D$ and solve problem (30), if the solution results in an inactive temperature constraint, then solving the original problem (30) optimally without this restriction gives the same value and the power is constant in both cases.

The next lemma states that if we restricted our solution to $\theta_2 = D$, to obtain the optimal θ_1^*, if the temperature constraint is tight for a non-zero measure, then the obtained solution in this case is strictly sub-optimal than allowing $\theta_2 < D$.

Lemma 14 *It cannot happen that the temperature constraint is active for an interval of non-zero measure and $\theta_2^* = D$, while $\theta_1^* < D$.*

Proof: Define $t' = \arg\min\{t \in [0, \theta_1^*] : T(t) = T_c\}$, which is the first instant at which the temperature touches T_c. From [4, Lemma 6], we have that $p^*(t) = \frac{T_\delta b}{a}, \forall t \in [t', \theta_1^*]$. This also implies that the power was monotone decreasing before t', since if it was constant and equal to $\frac{T_\delta b}{a}$, it would not have touched T_c. We then proceed to the proof by contradiction. Assume the statement is not true, then consider a modified policy as follows:

$$\tilde{p}_\delta(t) = \begin{cases} p^*(t), & \forall t \in [0, \delta] \\ p^*(t + \theta_1^* - D), & \forall t \in [D - \theta_1^* + \delta, D] \\ 0, & \text{otherwise} \end{cases} \tag{36}$$

In this policy, we transfer all but δ part of the power to the end of the duration. This policy will give the same optimal value. However, since the temperature constraint was originally tight for an interval, the power would have been monotone decreasing for at least an interval. Then, we can take a small enough interval $[0, \delta]$ in $\tilde{p}(t)$ and replace it by its average, i.e.,

$$\hat{p}_\delta(t) = \begin{cases} \frac{\int_0^\delta \tilde{p}_\delta(t)}{\delta}, & \forall t \in [0, \delta] \\ \tilde{p}_\delta(t), & \text{otherwise} \end{cases} \tag{37}$$

For small enough δ, this will result in a feasible policy with a strictly higher objective function, since we strictly decreased the temperature at the point $D - \theta_1^* + \delta$, so we had room to equalize the power more. This contradicts the optimality of the original policy. ■

B. Solving the Problem for Fixed $\{\theta_i\}$

We will discuss how to obtain the optimal solution for $\{\theta_i\}$ and $p(t)$. In general, we may need to perform line search over all the possible values of θ_1, θ_2. However, with the aid of the previously derived lemmas, we may be able to reduce this search significantly. In the following, we first state how to solve the problem for a fixed $\{\theta_i\}$, then discuss how to search for these optimal $\{\theta_i\}$.

1) Case: $\theta_1 = \theta_2$: In this case, we have the transmitter is on throughout the interval $[0, D]$. In this case, the optimization problem can be rewritten as:

$$\begin{aligned} \max_{p(t), \{\theta_i\}} \quad & \int_0^D \frac{1}{2} \log(1 + p(\tau)) d\tau \\ \text{s.t.} \quad & \int_0^D p(\tau) d\tau \leq E - \epsilon_p D \\ & T(t) \leq T_c \end{aligned} \tag{38}$$

This is the same problem as the single energy arrival case in [4] but with a modified energy arrival equal to $E - \epsilon_p D$. Hence, the solution can be obtained as in [4]. Note that this case will happen only if the energy is large enough to overcome the power needed for processing cost.

2) $0 < \theta_1$ and $\theta_2 = D$: Obtaining the solution for this case is similar to the previous case, however, here the deadline will be θ_1 instead, and the modified energy is equal to $E - \theta_1 \epsilon_p$.

3) $0<\theta_1<\theta_2<D$: In this case, the problem in (30) can be equivalently written as:

$$\begin{aligned} \max_{p(t),\alpha} \quad & \int_0^{\theta_1} \frac{1}{2}\log(1+p(\tau))d\tau + \int_{\theta_2}^{D} \frac{1}{2}\log(1+p(\tau))d\tau \\ \text{s.t.} \quad & \int_0^{\theta_1} (\epsilon_p + p(\tau))d\tau \leq \alpha E \\ & \int_{\theta_2}^{D} (\epsilon_p + p(\tau))d\tau \leq (1-\alpha)E \\ & T(t) \leq T_c, \quad p(t)\geq 0, \quad \alpha\in[0,1] \end{aligned} \tag{39}$$

For each fixed value of α, the above problem breaks down into two single epoch temperature constrained problem as in [4]. However, the rise in temperature in the first epoch due to $[0,\theta_1]$ should be taken into consideration while solving $[\theta_2, D]$. Hence, finding the optimal $\alpha\in[0,1]$ solves the problem.

C. Solving for the Optimal $\{\theta_i\}$

We note that the problem is not jointly convex, hence using the KKTs may lead to a local optimal solution. Thus, one optimal way for determining the solution is to search over θ_1, θ_2; however, due to the previously derived properties, we can limit the computation complexity significantly. We assume without loss of generality that there always exists an optimal policy for which $\theta_1^* > 0$. This follows since we can always shift the transmission to the beginning without violating the constraints, and with the same objective function.

We now present our approach to determine the optimal $\{\theta_i\}$. First, we begin by assuming $\theta_2 = D$ and solve for the optimal $\theta_1 \in [0,D)$, which can be done using line search on $[0,\theta_2]$. If for the optimal θ_1, the optimal power policy is constant, then we terminate the algorithm and this is the optimal solution. Otherwise, if the power is decreasing or if the temperature constraint is tight for an interval of non-zero measure, then according to Lemma 14, this implies that there has to be another phase of transmission, i.e., $\theta_2 = D$ cannot be optimal. Hence, we can decrease θ_2 gradually and obtain the corresponding optimal θ_1. If it happens that we get to a constant power allocation of a duration equal to the unconstrained problem, then by Lemma 13, this is an optimal solution, and the search is terminated. Otherwise, we will have to continue searching and then take the highest optimal value recorded and its corresponding $\{\theta_i\}$.

V. Temperature Increase Due to Energy Harvesting: Problem in (7)

In this section, we focus on the throughput maximization problem in (7) with temperature increase due to the energy harvesting process. Note that this problem is a direct generalization of the problem considered in [4]. In particular, the transmitter is allowed to determine the amount of harvested energy by determining α_i while this is not allowed in [4].

The problem in (7) is convex and the Lagrangian is:

$$\begin{aligned} \mathcal{L} = & \int_0^D \frac{1}{2}\log\left(1+p(t)\right)dt - \int_0^D \beta(t)\Bigg(\int_0^t ae^{b\tau}p(\tau)d\tau \\ & + \sum_{i=0}^{h(t)} e^{bs_i}\epsilon_h \alpha_i E_i u(t-s_i) - T_\delta e^{bt}\Bigg)dt \\ & - \int_0^D \lambda(t)\left(\int_0^t p(\tau)d\tau - \sum_{i=0}^{h(t)}\alpha_i E_i\right)dt \end{aligned} \tag{40}$$

The KKT optimality conditions are:

$$\frac{1}{1+p(t)} - e^{bt}\int_t^D \beta(\tau)d\tau - \int_t^D \lambda(\tau)d\tau = 0 \tag{41}$$

which gives

$$p(t) = \left[\frac{1}{\int_t^D \lambda(\tau)d\tau + e^{bt}\int_t^D \beta(\tau)d\tau} - 1\right]^+ \tag{42}$$

We also have the following condition due to the derivative with respect to α_i:

$$\sum_{k=i}^{N} E_k\left(-\int_0^D \beta(t)e^{bs_k}\epsilon_h u(t-s_k)dt + \int_0^D \lambda(t)dt\right) = 0 \tag{43}$$

whenever $0<\alpha_i^*<1$. If $\alpha_i^* = 1$, then the left hand side in (43) is non-negative and if $\alpha_i^* = 0$, it is non-positive.

We first note that the transmitter has to harvest the energy that will be utilized and energy is never wasted.

Lemma 15 *At $t=D$, we have $\int_0^D p(\tau)d\tau = \sum_{i=0}^N \alpha_i E_i$.*

Proof: Assume $\int_0^D p(\tau)d\tau < \sum_{i=0}^N \alpha_i E_i$ and let $i^* = \max\{i : \alpha_i > 0\}$. Then, α_{i^*} can be replaced with $\tilde{\alpha}_{i^*} < \alpha_{i^*}$ so that $\int_0^D p(\tau)d\tau = \sum_{i=0}^{i^*-1}\alpha_i E_i + \tilde{\alpha}_{i^*}E_{i^*}$. This replacement yields a lower temperature increase and a larger set for feasible power policies $p(t)$ and, therefore, yields larger throughput. ■

We note that despite no energy waste property, the temperature constraint may or may not be tight at $t=D$. Next, we specialize in the solution for the single energy arrival case.

A. Single Energy Arrival

In the single energy arrival case, since non-zero energy is needed to perform transmission, $\alpha^* > 0$. Therefore, the constraint in (43) reduces to the following:

$$-\int_0^D \beta(t)\epsilon_h dt + \int_0^D \lambda(t)dt \geq 0 \tag{44}$$

with equality whenever $\alpha^* < 1$. For fixed α_i, the problem is identical to that in [4] with an arbitrary initial temperature. Therefore, the properties identified in [4] for the single energy arrival case hold here as well in the current setting. Still, there are additional properties that arise due to the fact that the transmitter is allowed to determine the amount of harvested energy. We first note the following:

Lemma 16 *If $\epsilon_h \geq \frac{a}{bD}$ or $E \leq \frac{T_\delta bD}{a}$, then $p(t) = \frac{\alpha^* E}{D}$.*

Proof: If $E \leq \frac{T_\delta bD}{a}$, then $\alpha E \leq \frac{T_\delta bD}{a}$. This follows immediately irrespective of α and we select $\alpha^* = \min\left\{\frac{T_\delta}{\epsilon_h E}, 1\right\}$ so that $\epsilon_h \alpha E \leq T_\delta$. By [4, Lemma 5], $T(t) \leq T_c$ if $p(t) \leq \frac{T_\delta b}{a}$ for all $t \in [0, D]$. Therefore, $p(t) = \frac{\alpha E}{D}$ yields $T(t) \leq T_c$ and hence is optimal. Now, assume $\epsilon_h \geq \frac{a}{bD}$. Since the temperature increase due to harvested energy at $t = 0$ is $\epsilon_h \alpha E \leq T_\delta$, we have $\frac{\alpha^* E}{D} \leq \frac{T_\delta b}{a}$. By [4, Lemma 5], $T(t) \leq T_c$ and therefore, $p(t) = \frac{\alpha^* E}{D}$ is optimal. ■

We note that for $E \leq E_{critical} = \frac{bT_\delta D e^{bD}}{a(e^{bD}-1)}$, optimal power policy is the constant power policy for $\epsilon_h = 0$, see [4]. Next, we extend this property.

Lemma 17 *If $E \leq E_{critical}$ and $\epsilon_h \leq \frac{T_\delta}{E} - \frac{a}{bD}\left(1 - e^{-bD}\right)$ then $p(t) = \frac{E}{D}$ is optimal.*

Proof: We first note that $T(t)$ expression under the constant power policy $p(t) = \frac{\alpha E}{D}$ is:

$$T(t) = T_e + \frac{a}{b}\frac{\alpha E}{D} + \alpha E\left(\epsilon_h - \frac{a}{bD}\right)e^{-bt} \quad (45)$$

Note that when $0 \leq \epsilon_h < \frac{a}{bD}$, $T(t)$ in (45) is monotone increasing. Therefore, in this case, it suffices to guarantee that $T(t) \leq T_c$ at $t = D$. If $E \leq E_{critical}$ and $\epsilon_h \leq \frac{T_\delta}{E} - \frac{a}{bD}\left(1 - e^{-bD}\right) \leq \frac{a}{bD}$ then $T(D) \leq T_c$ when $p(t) = \frac{E}{D}$. If $\alpha = 1$ yields $T(D) \leq T_c$, then for $\lambda(t) = \tilde{\lambda}\delta(t - D)$ and $\beta(t) = 0$, (44) holds and hence $\alpha^* = 1$ and $p(t) = \frac{E}{D}$ is optimal. If $\epsilon_h > \frac{a}{bD}$, then $T(t)$ in (45) is monotone decreasing and hence it suffices to guarantee $T(0) = T_e + \epsilon_h E \leq T_c$. For $\epsilon_h \leq \frac{T_\delta}{E} - \frac{a}{bD}\left(1 - e^{-bD}\right)$, we have $\epsilon_h E \leq T_\delta$, and therefore, $\alpha^* = 1$ and $p(t) = \frac{E}{D}$ is optimal. ■

Lemma 18 *For $\frac{T_\delta bD}{a} \leq E \leq E_{critical}$ and $\frac{T_\delta}{E} - \frac{a}{bD}\left(1 - e^{-bD}\right) < \epsilon_h < \frac{a}{bD}$, optimal $p(t)$ cannot be constant.*

Proof: For $\frac{T_\delta bD}{a} \leq E \leq E_{critical}$ and $\frac{T_\delta}{E} - \frac{a}{bD}\left(1 - e^{-bD}\right) < \epsilon_h < \frac{a}{bD}$, $T(t)$ is monotone increasing and $T(D) > T_c$ if $p(t) = \frac{E}{D}$. Hence, if $p(t) = \frac{\alpha E}{D}$, then $\alpha < 1$ is necessary for $T(D) \leq T_c$, and therefore, (44) has to be satisfied with equality if optimal $p(t)$ is constant. This, in turn, means $\int_0^D \beta(\tau)d\tau > 0$. However, the only possible solution is $\beta(t) = \tilde{\beta}\delta(t - D)$ with $\tilde{\beta} > 0$, and from (42), $p(t)$ cannot be constant in this case. ■

We observe that if $E > E_{critical}$, then constant power policy $p(t) = \frac{\alpha^* E}{D}$ is optimal only for $\epsilon_h \geq \frac{a}{bD}$. In this case, as ϵ_h increases from 0 to $\frac{a}{bD}$, the length of the time interval in which the optimal power policy remains constant also increases. We illustrate the variation of the optimal policy with the coefficient ϵ_h when $E > E_{criticial}$ in Fig. 2.

VI. Numerical Results

In this section, we present numerical examples to illustrate our results. We take the environment temperature as $T_e = 37$.

A. Temperature Dependent Energy Leakage

In this section, we present numerical results for the problem in (4). We set $a = b = 0.1$. We first study the single energy arrival case. In this case, as proved in Lemma 1, the

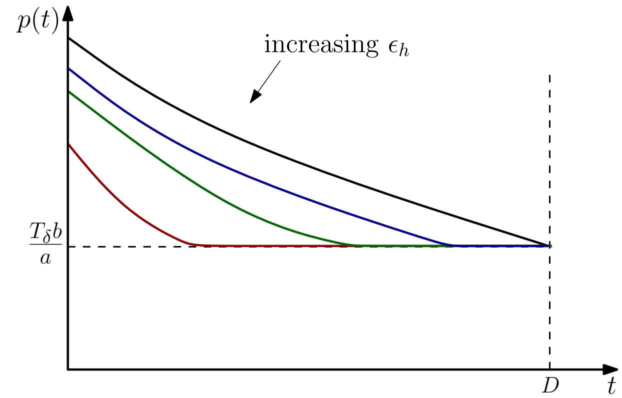

Fig. 2. Optimal power policy with increasing ϵ_h when $E > E_{critical}$.

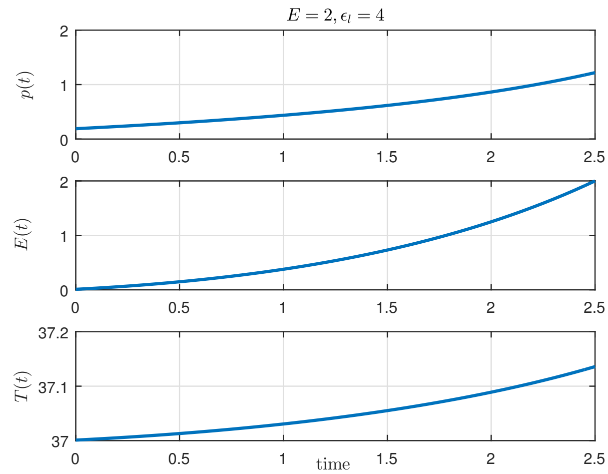

Fig. 3. Optimal policy for the single energy arrival with temperature dependent energy leakage.

transmit power is strictly increasing as shown in Figs. 3 and 4. However, in Fig. 4, we note that since the energy is small and leakage cost is high, the transmitter remains silent at the beginning of the transmission, and begins transmission only at $t = 0.5$. The temperature is increasing as shown in Lemma 2. The battery is empty only at the end.

Then, we study the multiple energy arrival case. Fig. 5 shows that the power and temperature are strictly increasing. In Fig. 5, there are two positive jumps, each at an energy arrival instant. Hence, energy of each slot is used individually, and no energy is transferred between epochs.

B. Non-zero Processing Power

In this section, we present numerical results for the problem in (5). We set $a = 0.1$, $b = 0.3$, $\epsilon_p = 20$ and $D = 2$. We first study the setting in Fig. 6. The optimal value in this setting is equal to 0.77. As shown in the figure, the temperature constraint is tight at the end of transmission in each duration, hence power is decreasing in both epochs. In the middle, when the transmitter is silent, the temperature drops to create a margin for the second transmission epoch. If we do not allow splitting the transmission into two epochs, i.e., $\theta_2 = D$, then

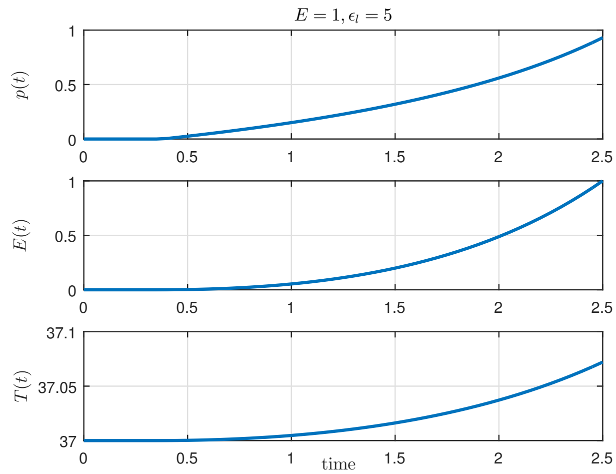

Fig. 4. Illustration of the impact of a large leakage coefficient on the optimal policy in the single epoch case.

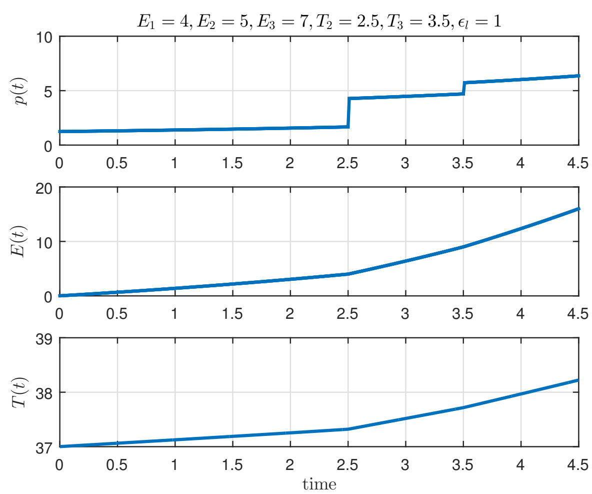

Fig. 5. Optimal policy for three energy arrivals with temperature dependent energy leakage.

Fig. 7 shows the optimal solution. The optimal value in this case is 0.7 which is strictly less than the two epoch case.

Then, we study another case in Fig. 8. In this case, the optimal transmission power is constant. The optimal value in this case is equal to 0.82. Also, it is equal to the solution when the temperature constraint is removed. If we restrict the system to only one epoch as in Fig. 9, then we obtain strictly less optimal value which is 0.78, as this forces the temperature constraint to be tight and the power to be decreasing, and hence, giving less throughput.

C. Temperature Increase Due to Energy Harvesting

In this section, we present numerical results for the problem in (7). We set $a = 0.1$ and $b = 0.3$. In Fig. 10, we show the optimal power in the single energy arrival case. The power is monotonically decreasing, and all the admitted energy is consumed by the end of the deadline. In Fig. 11, we show the multiple energy arrival case, where the power is decreasing in

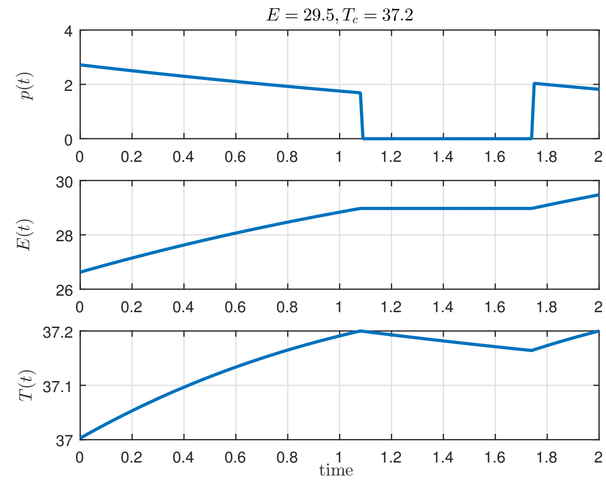

Fig. 6. Considering both θ_1 and θ_2 with processing cost.

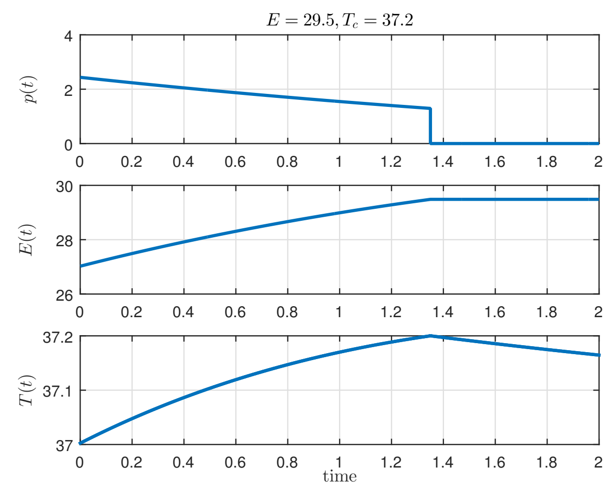

Fig. 7. Considering only θ_1 and setting $\theta_2 = D$ with processing cost.

the first epoch and the temperature is also decreasing in order to give more temperature room for the second epoch.

In both single and multiple energy arrival cases, we note that there is a positive temperature jump at the instants of the energy arrivals. This is due to the immediate heat generated by the admitted energy at these instants due to the energy harvesting process.

References

[1] D. Forte and A. Srivastava. Thermal aware sensor scheduling for distributed estimation. *ACM Trans. on Sensor Networks*, 9(4):53:1–53:31, July 2013.

[2] Q. Tang, N. Tummala, S.K.S Gupta, and L. Schwiebert. Communication scheduling to minimize thermal effects of implanted biosensor networks in homogeneous tissue. *IEEE Trans. on Biomed. Eng.*, 52(7):1285–1294, July 2005.

[3] Y. Liu, R. P. Dick, L. Shang, and H. Yang. Accurate temperature-dependent integrated circuit leakage power estimation is easy. In *Conference on Design, automation and test in Europe*, April 2007.

[4] O. Ozel, S. Ulukus, and P. Grover. Optimal scheduling for energy harvesting transmitters under temperature constraints. In *IEEE ISIT*, June 2015.

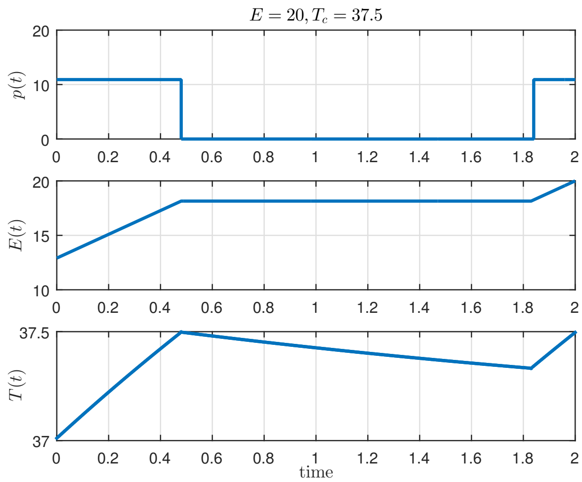

Fig. 8. Considering both θ_1 and θ_2 with processing cost.

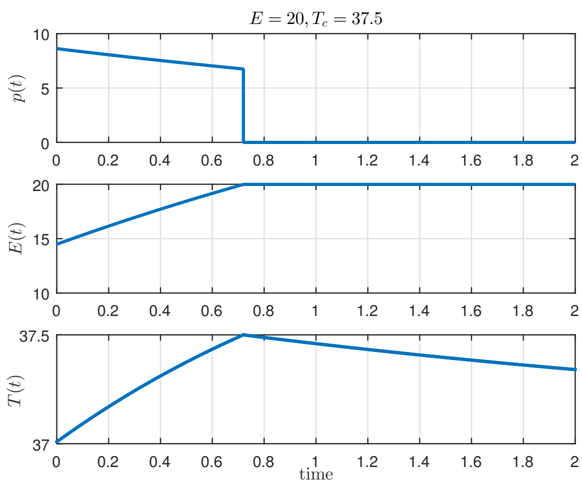

Fig. 9. Considering only θ_1 and setting $\theta_2 = D$ with processing cost.

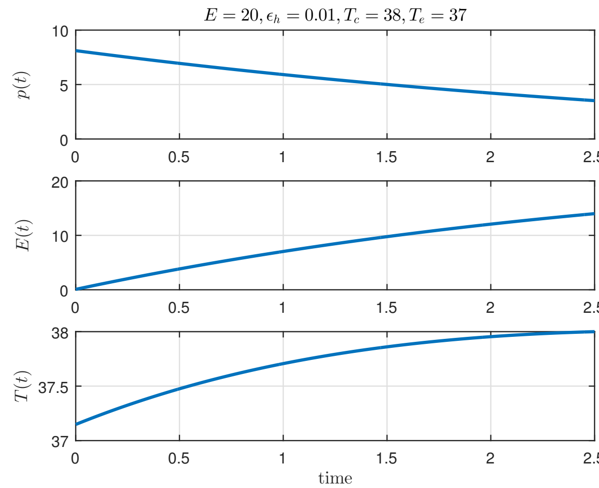

Fig. 10. Temperature increase due to energy harvesting: single epoch.

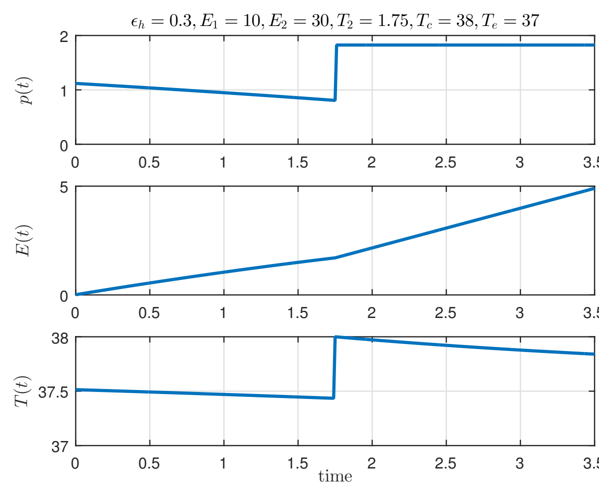

Fig. 11. Temperature increase due to energy harvesting: multiple epochs.

[5] J. Yang and S. Ulukus. Optimal packet scheduling in an energy harvesting communication system. *IEEE Trans. Comm.*, 60(1):220–230, January 2012.

[6] K. Tutuncuoglu and A. Yener. Optimum transmission policies for battery limited energy harvesting nodes. *IEEE Trans. Wireless Comm.*, 11(3):1180–1189, March 2012.

[7] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener. Transmission with energy harvesting nodes in fading wireless channels: Optimal policies. *IEEE JSAC*, 29(8):1732–1743, September 2011.

[8] C. K. Ho and R. Zhang. Optimal energy allocation for wireless communications with energy harvesting constraints. *IEEE Trans. Signal Proc.*, 60(9):4808–4818, September 2012.

[9] J. Yang, O. Ozel, and S. Ulukus. Broadcasting with an energy harvesting rechargeable transmitter. *IEEE Trans. Wireless Comm.*, 11(2):571–583, February 2012.

[10] M. A. Antepli, E. Uysal-Biyikoglu, and H. Erkal. Optimal packet scheduling on an energy harvesting broadcast link. *IEEE JSAC*, 29(8):1721–1731, September 2011.

[11] O. Ozel, J. Yang, and S. Ulukus. Optimal broadcast scheduling for an energy harvesting rechargeable transmitter with a finite capacity battery. *IEEE Trans. Wireless Comm.*, 11(6):2193–2203, June 2012.

[12] J. Yang and S. Ulukus. Optimal packet scheduling in a multiple access channel with energy harvesting transmitters. *Journal of Comm. and Networks*, 14(2):140–150, April 2012.

[13] K. Tutuncuoglu and A. Yener. Sum-rate optimal power policies for energy harvesting transmitters in an interference channel. *Journal of Comm. Networks*, 14(2):151–161, April 2012.

[14] D. Gunduz and B. Devillers. A general framework for the optimization of energy harvesting communication systems with battery imperfections. *Journal of Comm. Networks*, 14(2):130–139, April 2012.

[15] K. Tutuncuoglu, A. Yener, and S. Ulukus. Optimum policies for an energy harvesting transmitter under energy storage losses. *IEEE JSAC*, 33(3):476–481, March 2015.

[16] O. Orhan, D. Gunduz, and E. Erkip. Energy harvesting broadband communication systems with processing energy cost. *IEEE Trans. Wireless Comm.*, 13(11):6095–6107, November 2014.

[17] J. Xu and R. Zhang. Throughput optimal policies for energy harvesting wireless transmitters with non-ideal circuit power. *IEEE JSAC*, 32(2):322–332, February 2014.

[18] O. Ozel, K. Shahzad, and S. Ulukus. Optimal energy allocation for energy harvesting transmitters with hybrid energy storage and processing cost. *IEEE Trans. Signal Proc.*, 62(12):3232–3245, June 2014.

[19] B. Li, L. Peh, and P. Patra. Impact of process and temperature variations on network-on-chip design exploration. In *ACM/IEEE NoC*, April 2008.

[20] N. Kr. Shukla, S. Birla, K. Rathi, R. K. Singh, and M. Pattanaik. Analysis of the effect of temperature and vdd on leakage current in conventional 6 t-sram bit-cell at 90 nm and 65 nm technology. *International Journal of Computer Applications*, 26(10), July 2011.

[21] P. Youssef-Massaad, L. Zheng, and M. Medard. Bursty transmission and glue pouring: on wireless channels with overhead costs. *IEEE Trans. Wireless Comm.*, 7(12):5188–6194, December 2008.