Single-User Channel with Data and Energy Arrivals: Online Policies

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***Abstract*— We consider a single-user channel in which the transmitter is equipped with finite-sized data and energy buffers. The transmitter receives energy and data packets randomly and intermittently over time and stores them in the finite-sized buffers. The arrival amounts are known only causally as they happen. We study the *online* power allocation problem, in which the transmitter relies only on the causal arrival (energy and data) information. We focus on the special case when the energy and data arrivals are fully-correlated. We first study the case when the arrivals are Bernoulli. For this case, we determine the optimal policy. Inspired by this policy and in order to study the case of general fully-correlated arrivals, we propose a structured policy and bound its performance by a multiplicative gap from the optimal. We then show that this policy *is optimal* when the energy arrivals dominate the data arrivals, and is within a constant additive gap from the optimal policy when the data arrivals dominate the energy arrivals.**

I. Introduction

We consider an energy harvesting single-user system where the transmitter receives energy and data packets randomly and intermittently over time, and stores them in finite-sized queues, see Fig. 1. We study the *online* power scheduling problem for this system, where both energy and data arrivals are known only causally at the transmitter. As a first step towards the most general setting, we focus on the case when the energy and data arrivals are fully-correlated. This setting may practically arise when energy and information are simultaneously transferred as in simultaneous wireless energy and information transfer [1]. We characterize the optimal policy in the special case of Bernoulli arrivals. We then propose a structured policy for general arrivals. The proposed policy takes into account the available energy and data at each instant. We show that the performance of this policy is near-optimal for general arrivals.

Energy harvesting in wireless communication systems has attracted significant recent attention. References [2]–[14] study the case when the energy arrivals are known ahead of time (offline), and references [4], [5], [15]–[27] study the case when the energy arrivals are known only causally (online). For the offline case, structured solutions are developed, such as, the geometric algorithms in [2], [3], directional water-filling algorithm in [4], and directional glue pouring algorithm in [11]. For the online case, the optimal solution results in

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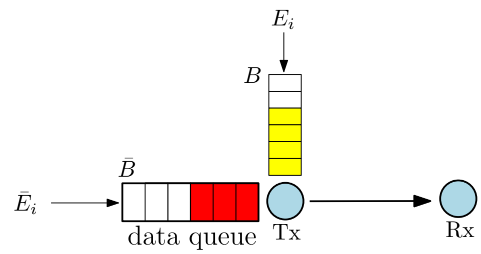

Fig. 1. An energy harvesting single-user transmitter with finite-sized energy and data buffers.

dynamic programming problems, which usually have high computation complexity and little structure.

In this paper, we follow the approach presented in [22] and extended in [23]–[27] to propose a structured near-optimal solution for the online problem. We also show, for some special cases, that the proposed policy is exactly optimal. Our current paper is closely related to [10] in that it extends the presented model to the online setting as developed in [22]. We first study the case of synchronized Bernoulli arrivals, where both data and energy arrivals are either zero or they fill up their corresponding queues simultaneously. We characterize the exactly optimal solution for this case. Then, for the case of fully-correlated general arrivals with the same arrival means as the Bernoulli arrivals, we propose a structured policy. We show that this policy is optimal when the energy arrivals dominate, and it is within a constant additive gap when the data arrivals dominate. In addition, we derive a multiplicative gap result for the performance of the proposed policy in all cases.

II. System Model

The physical layer is a Gaussian single-user channel with noise variance at the receiver equal to unity. The capacity of this channel in slot i is,

$$r_i = \frac{1}{2}\log\left(1+P_i\right) \tag{1}$$

where P_i is the transmit power in slot i. The transmitter is equipped with finite-sized data and energy buffers, with sizes $\bar{B}$ and B, respectively. The battery state b_i evolves as,

$$b_{i+1} = \min\left\{B, b_i - P_i + E_{i+1}\right\} \tag{2}$$

The data queue state, denoted as $\bar{b}_i$, evolves as,

$$\bar{b}_{i+1} = \min\left\{\bar{B}, \bar{b}_i - \frac{1}{2}\log\left(1+P_i\right) + \bar{E}_{i+1}\right\} \tag{3}$$

The transmit power, P_i, is constrained by,

$$P_i \leq \min\left\{b_i, 2^{2\bar{b}_i} - 1\right\} \tag{4}$$

This constraint ensures that the transmission power does not exceed the energy available in the battery and does not attempt to send more data than remaining in the queue.

The objective then is to maximize the long term-average throughput subject to data and energy constraints,

$$\Phi = \lim_{n\to\infty} \max_{P^n \in \mathcal{F}^n} \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} \frac{1}{2}\log(1+P_i)\right] \tag{5}$$

where $\mathcal{F}^n$ is the set of all feasible online policies for n time slots. The aim now is to characterize the optimal power allocation. Under the existence of the optimal Markov policy, the optimal power allocation will be function of the current energy and battery states. We fully characterize the powers in the case of Bernoulli arrivals. For general arrivals, we propose a structured policy and we determine the cases in which it is optimal or near optimal.

III. Optimal Strategy: Bernoulli Arrivals

In this section, we characterize the optimal policy for Bernoulli arrivals. We consider the case when data and energy arrivals are fully-correlated; whenever the energy queue is filled, the data queue is filled also and there is no intermediate values for the energy or data arrivals. In other words, we have $E_i = \alpha\bar{E}_i$, and $\mathbb{P}[E_i = \alpha\bar{E}_i = B] = 1 - \mathbb{P}[E_i = \alpha\bar{E}_i = 0] = p$. From [28, Theorem 3.6.1], and similar to [22], [23], [25], we characterize the optimal powers by solving a modified offline problem with a single arrival. For the case of data and energy arrivals, we obtain the optimal powers by solving,

$$\begin{aligned} \max_{\{P_i\}} \quad & \sum_{i=1}^{\infty} p(1-p)^{i-1}\frac{1}{2}\log\left(1+P_i\right) \\ \text{s.t.} \quad & \sum_{i=1}^{\infty} P_i \leq B, \quad \sum_{i=1}^{\infty}\frac{1}{2}\log(1+P_i) \leq \bar{B} \end{aligned} \tag{6}$$

This is a non-convex problem due to the last constraint which is a non-convex constraint. We transform this problem to an equivalent convex problem by expressing the problem in terms of rates, i.e., we apply the transformation $r_i = \frac{1}{2}\log\left(1+P_i\right)$. The equivalent problem is,

$$\begin{aligned} \max_{\{r_i\}} \quad & \sum_{i=1}^{\infty} p(1-p)^{i-1} r_i \\ \text{s.t.} \quad & \sum_{i=1}^{\infty} 2^{2r_i} - 1 \leq B, \quad \sum_{i=1}^{\infty} r_i \leq \bar{B} \end{aligned} \tag{7}$$

This is a convex optimization problem in r_i which we can solve using the KKTs which are necessary and sufficient. The Lagrangian of this problem is,

$$\mathcal{L} = -\sum_{i=1}^{\infty} p(1-p)^{i-1} r_i + \lambda\left(\sum_{i=1}^{\infty} 2^{2r_i} - 1 - B\right) + \mu\left(\sum_{i=1}^{\infty} r_i - \bar{B}\right) \tag{8}$$

We differentiate with respect to r_i and equate to zero to get,

$$r_i = \frac{1}{2}\log\left(\frac{p(1-p)^{i-1}}{\lambda} - \frac{\mu}{\lambda}\right), \quad i = 1,\ldots,\tilde{N} \tag{9}$$

It is clear that the transmit power decreases in time. The rates should be non-negative. Thus, we need $\frac{p(1-p)^{i-1}}{\lambda} - \frac{\mu}{\lambda} \geq 1$ to be satisfied for $i = 1,\ldots,\tilde{N}$. Hence, it suffices just to ensure that it is satisfied in the last slot, i.e.,

$$p(1-p)^{\tilde{N}-1} \geq \lambda + \mu \tag{10}$$

We can then identify the optimal λ, μ and $\tilde{N}$ by solving (10) along with the total energy and data constraints. This can be done using simple one-dimensional line search.

IV. Near-Optimal Strategy: General Arrivals

In this section, we propose a structured online policy as in [22], [23]–[27]. We first note that the optimal power allocation in (9) follows a fractional policy. The power allocation is controlled by the available data through the Lagrange multiplier μ. Hence, we propose the following fractional policy which is bounded by the available data in the queue,

$$P_i = \min\left\{pb_i, 2^{2\bar{b}_i} - 1\right\} \tag{11}$$

The policy mimics the optimal policy in (9) in that it is fractional when the amount of fractional power is less than the amount needed to transmit the remaining data, or else it is limited by the remaining data. To describe the policy for the Bernoulli energy arrivals, we first define i^* as follows,

$$i^* = \max\left\{i : Bp(1-p)^{i-1} \leq 2^{2\left[\bar{B} - \sum_{k=1}^{i-1}\frac{1}{2}\log\left(1+Bp(1-p)^{k-1}\right)\right]} - 1\right\} \tag{12}$$

This represents the last index at which the policy transmits with a fractional decreasing power. In slot i^*+1, if no new arrival occurs, the transmitter transmits all the remaining data in its buffer. Hence, the allocated power is as follows,

$$P_i = Bp(1-p)^{i-1}, \qquad i = 1,\ldots,i^* \tag{13}$$

$$P_i = 2^{2\left[\bar{B} - \sum_{k=1}^{i-1}\frac{1}{2}\log(1+Bp(1-p)^{k-1})\right]} - 1, \quad i = i^*+1 \tag{14}$$

$$P_i = 0, \qquad i > i^*+1 \tag{15}$$

Note that i^* is a deterministic number which depends only on the system parameters $B, \bar{B}, p$. We define the following random variable, which will be useful later in the analysis,

$$K = \min\{L, i^*\} \tag{16}$$

where L is the time between the Bernoulli arrivals, which is geometrically distributed with parameter p.

In what follows, we begin by deriving a universal upper bound for all online policies with general arrivals. We then study the performance of the policy proposed in (11) under Bernoulli energy arrivals. We first derive a multiplicative lower bound. Then, we study the case when this policy is optimal and the case when it is within a constant additive gap. We then show that the performance of the proposed policy is the worst under Bernoulli arrivals with the same arrival rate, hence, all the lower bounds derived for Bernoulli arrivals are also valid for general arrivals.

A. Upper Bound

In the following lemma, we present a universal upper bound which depends only on the average arrival rates.

Lemma 1 *For an average energy arrival rate of μ_e and an average data arrival rate of μ_d, the throughput of any online policy is upper bounded as,*

$$r_{on} \leq \min\left\{\frac{1}{2}\log\left(1+\mu_e\right), \mu_d\right\} \tag{17}$$

The proof of Lemma 1 follows from the single-user offline upper bound with no data arrival constraints [22] in addition to the data arrival constraint: the transmitter cannot transmit more data on average than the average data arrival. Hence, the upper bound on the rate is the minimum of these two upper bounds.

B. Multiplicative Gap

We now analyze the performance of the proposed policy. We first derive a multiplicative gap for Bernoulli arrivals.

Lemma 2 *The performance of the fractional policy is lower bounded by,*

$$r_{on} \geq \max\{p, c\} \min\left\{\frac{1}{2}\log\left(1+pB\right), \bar{B}p\right\} \tag{18}$$

where c is defined as,

$$c = \frac{1+\max\left\{2^{2\left[\bar{B}-\sum_{k=1}^{\infty}\frac{1}{2}\log(1+Bp(1-p)^{k-1})\right]}-1, 0\right\}}{2-p} - \frac{\max\left\{2^{2\left[\bar{B}-\frac{1}{2}\log(1+Bp)\right]}-1, 0\right\}}{Bp} \tag{19}$$

Proof: We derive two lower bounds and then take the maximum of both. For the case when $i^*=0$, we show later that the policy is optimal. Hence, the multiplicative lower bound is still valid. Thus, we now consider, without loss of generality, the case when $i^* \geq 1$. We first begin with the one with p multiplicative gap in (18). For this case, we have,

$$\begin{aligned} r_{on} =& \sum_{i=1}^{i^*}\frac{1}{2}p(1-p)^{i-1}\log\left(1+Bp(1-p)^{i-1}\right) \\ &+ p(1-p)^{i^*}\left[\bar{B}-\sum_{k=1}^{i^*}\frac{1}{2}\log\left(1+Bp(1-p)^{k-1}\right)\right] && (20) \\ \geq& \sum_{i=1}^{i^*}\frac{1}{2}p(1-p)^{i-1}\log\left(1+Bp(1-p)^{i-1}\right) && (21) \\ \geq& \sum_{i=1}^{i^*}\frac{1}{2}p(1-p)^{2(i-1)}\log\left(1+Bp\right) && (22) \\ =& \left(\frac{1-(1-p)^{2i^*}}{2-p}\right)\frac{1}{2}\log\left(1+Bp\right) && (23) \\ \geq& \left(\frac{1-(1-p)^{2}}{2-p}\right)\frac{1}{2}\log\left(1+Bp\right) && (24) \\ =& p\frac{1}{2}\log\left(1+Bp\right) && (25) \\ \geq& p\min\left\{\frac{1}{2}\log\left(1+Bp\right), \bar{B}p\right\} && (26) \end{aligned}$$

where (21) follows from the positivity of $\bar{B} - \sum_{k=1}^{i^*}\frac{1}{2}\log\left(1+Bp(1-p)^{k-1}\right)$, (22) follows from the monotonicity of the logarithm, and (24) follows by setting $i^*=1$. This proves the first lower bound.

We then derive the other lower bound with c multiplicative gap in (18). We first derive an upper bound on $(1-p)^{i^*}$,

$$\begin{aligned} Bp(1-p)^{i^*} &\leq Bp(1-p)^{i^*-1} && (27) \\ &\leq 2^{2\left[\bar{B}-\sum_{k=1}^{i^*-1}\frac{1}{2}\log(1+Bp(1-p)^{k-1})\right]}-1 && (28) \\ &\leq 2^{2\left[\bar{B}-\frac{1}{2}\log(1+Bp)\right]}-1 && (29) \end{aligned}$$

where (27) follows from monotonicity, (28) follows from the definition of i^*, and (29) follows by considering only the first term in the summation. Hence, we have,

$$(1-p)^{i^*} \leq \frac{2^{2\left[\bar{B}-\frac{1}{2}\log(1+Bp)\right]}-1}{Bp} \tag{30}$$

We also derive a lower bound on $(1-p)^{i^*}$,

$$\begin{aligned} Bp(1-p)^{i^*} &> 2^{2\left[\bar{B}-\sum_{k=1}^{i^*}\frac{1}{2}\log(1+Bp(1-p)^{k-1})\right]}-1 && (31) \\ &> 2^{2\left[\bar{B}-\sum_{k=1}^{\infty}\frac{1}{2}\log(1+Bp(1-p)^{k-1})\right]}-1 && (32) \end{aligned}$$

Hence, we have,

$$(1-p)^{i^*} > \frac{2^{2\left[\bar{B}-\sum_{k=1}^{\infty}\frac{1}{2}\log(1+Bp(1-p)^{k-1})\right]}-1}{Bp} \tag{33}$$

We now derive the c multiplicative lower bound in (18),

$$\begin{aligned} r_{on} =& \sum_{i=1}^{i^*}\frac{1}{2}p(1-p)^{i-1}\log\left(1+Bp(1-p)^{i-1}\right) \\ &+ p(1-p)^{i^*}\left[\bar{B}-\sum_{k=1}^{i^*}\frac{1}{2}\log\left(1+Bp(1-p)^{k-1}\right)\right] && (34) \\ =& \sum_{i=1}^{i^*}\frac{1}{2}\left(p(1-p)^{i-1}-p(1-p)^{i^*}\right)\log\left(1+Bp(1-p)^{i-1}\right) \\ &+ p(1-p)^{i^*}\bar{B} && (35) \\ \geq& \sum_{i=1}^{i^*}\frac{1}{2}\left(p(1-p)^{i-1}-p(1-p)^{i^*}\right)(1-p)^{i-1}\log\left(1+Bp\right) \\ &+ p(1-p)^{i^*}\bar{B} && (36) \end{aligned}$$

$$=\sum_{i=1}^{i^*}\frac{1}{2}\left(p(1-p)^{2(i-1)}-p(1-p)^{i^*+i-1}\right)\log\left(1+Bp\right)$$
$$+p(1-p)^{i^*}\bar{B} \tag{37}$$
$$=w\left(1-(1-p)^{i^*}\right)\frac{1}{2}\log\left(1+Bp\right)+p(1-p)^{i^*}\bar{B} \tag{38}$$

where w is defined as follows:

$$w \triangleq \sum_{i=1}^{i^*}\frac{\left(p(1-p)^{2(i-1)}-p(1-p)^{i^*+i-1}\right)}{\left(1-(1-p)^{i^*}\right)} \tag{39}$$
$$=\frac{\frac{1-(1-p)^{2i^*}}{2-p}-(1-p)^{i^*}\left(1-(1-p)^{i^*}\right)}{\left(1-(1-p)^{i^*}\right)} \tag{40}$$
$$=\frac{1+(1-p)^{i^*}}{2-p}-(1-p)^{i^*} \tag{41}$$

It is clear that $0\leq w\leq 1$. Hence, continuing from (38),

$$r_{on}\geq w\left(1-(1-p)^{i^*}\right)\frac{1}{2}\log\left(1+Bp\right)+wp(1-p)^{i^*}\bar{B} \tag{42}$$
$$\geq w\min\left\{\frac{1}{2}\log\left(1+Bp\right),\bar{B}p\right\} \tag{43}$$

where (43) follows since for any x,y and $\theta\in[0,1]$,

$$\theta x+(1-\theta)y\geq\min\{x,y\} \tag{44}$$

It now remains to lower bound w which follows directly from (30) and (33). ■

C. Optimal Case: Energy Dominant Case

We study here the case when the proposed policy is optimal, which we state in the following lemma.

Lemma 3 *The policy proposed in (11) is optimal when*

$$Bp\geq 2^{2\bar{B}}-1 \tag{45}$$

Proof: When we have $Bp\geq 2^{2\bar{B}}-1$, then

$$P_1=2^{2\bar{B}}-1,\ P_i=0\ \ \forall i>1 \tag{46}$$

Then evaluating the achievable rate explicitly gives,

$$r_{on}=p\bar{B}\geq\min\left\{\frac{1}{2}\log\left(1+pB\right),p\bar{B}\right\} \tag{47}$$

which is exactly equal to the upper bound. Hence, the gap is equal to zero in this case and this policy is optimal. ■

We call this the energy dominant case because the average energy arrival rate, Bp, is larger than the energy needed to transmit a full data buffer, i.e., $Bp\geq 2^{2\bar{B}}-1$.

D. Constant Additive Gap: Data Dominant Case

We now study the case when the proposed policy yields performance within a constant additive gap of the optimal.

Lemma 4 *When $\frac{1}{2}\log\left(1+Bp\right)+\frac{Bp}{2}\leq\bar{B}p$, the performance of the fractional policy is lower bounded by,*

$$r_{on}\geq\min\left\{\frac{1}{2}\log\left(1+pB\right),\bar{B}p\right\}-0.72 \tag{48}$$

Proof: The throughput under the sub-optimal policy,

$$r_{on}\geq\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{K}\frac{1}{2}\log\left(1+Bp(1-p)^{i-1}\right)\right]$$
$$+\mathbb{E}\left[\frac{\mathbb{1}[L>i^*]}{\mathbb{E}[L]}\left[\bar{B}-\sum_{k=1}^{i^*}\frac{1}{2}\log\left(1+Bp(1-p)^{k-1}\right)\right]\right] \tag{49}$$
$$=\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{K}\frac{1}{2}\log\left(1+Bp(1-p)^{i-1}\right)\right]$$
$$+\frac{\mathbb{P}[L>i^*]}{\mathbb{E}[L]}\left[\bar{B}-\sum_{k=1}^{i^*}\frac{1}{2}\log\left(1+Bp(1-p)^{k-1}\right)\right] \tag{50}$$
$$\geq\frac{\mathbb{E}[K]}{\mathbb{E}[L]}\frac{1}{2}\log\left(1+Bp\right)-0.72$$
$$+\frac{\mathbb{P}[L>i^*]}{\mathbb{E}[L]}\left[\bar{B}-\sum_{k=1}^{i^*}\frac{1}{2}\log\left(1+Bp(1-p)^{k-1}\right)\right] \tag{51}$$
$$=(1-(1-p)^{i^*})\frac{1}{2}\log\left(1+Bp\right)-0.72$$
$$+p(1-p)^{i^*}\left[\bar{B}-\sum_{k=1}^{i^*}\frac{1}{2}\log\left(1+Bp(1-p)^{k-1}\right)\right] \tag{52}$$
$$=(1-(1-p)^{i^*})\frac{1}{2}\log\left(1+Bp\right)-0.72+p(1-p)^{i^*}\bar{B}$$
$$-p(1-p)^{i^*}\sum_{k=1}^{i^*}\frac{1}{2}\log\left(1+Bp(1-p)^{k-1}\right) \tag{53}$$
$$\geq(1-(1-p)^{i^*})\frac{1}{2}\log\left(1+Bp\right)-0.72+p(1-p)^{i^*}\bar{B}$$
$$-p(1-p)^{i^*}\sum_{k=1}^{i^*}\frac{1}{2}Bp(1-p)^{k-1} \tag{54}$$
$$\geq(1-(1-p)^{i^*})\frac{1}{2}\log\left(1+Bp\right)-0.72$$
$$+p(1-p)^{i^*}\bar{B}-p(1-p)^{i^*}\frac{B}{2} \tag{55}$$
$$=(1-(1-p)^{i^*})\frac{1}{2}\log\left(1+Bp\right)-0.72$$
$$+p(1-p)^{i^*}\left(\bar{B}-\frac{B}{2}\right) \tag{56}$$
$$\geq\min\left\{\frac{1}{2}\log\left(1+Bp\right),p\left(\bar{B}-\frac{B}{2}\right)\right\}-0.72 \tag{57}$$
$$=\frac{1}{2}\log\left(1+Bp\right)-0.72 \tag{58}$$
$$\geq\min\left\{\frac{1}{2}\log\left(1+Bp\right),pB\right\}-0.72 \tag{59}$$

where (52) follows as in the proof of [23, Lemma 3] and (58) follows if $\frac{1}{2}\log\left(1+Bp\right)\leq p\left(\bar{B}-\frac{B}{2}\right)$. ■

We call this the data dominant case because the average data arrival rate, $\bar{B}p$, is larger than the amount of data that can be transmitted by the average energy arrival in addition to half the average energy arrival rate, i.e., $\frac{1}{2}\log\left(1+Bp\right)+\frac{Bp}{2}\leq\bar{B}p$.

E. General Energy Arrivals

For the general arrival case we have the following lemma.

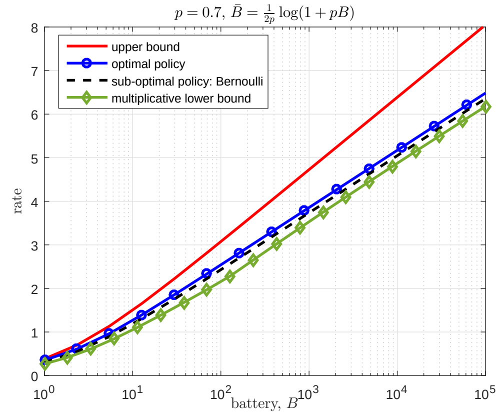

Fig. 2. Illustration of upper bound, optimal policy and the sub-optimal policy. Bernoulli arrivals.

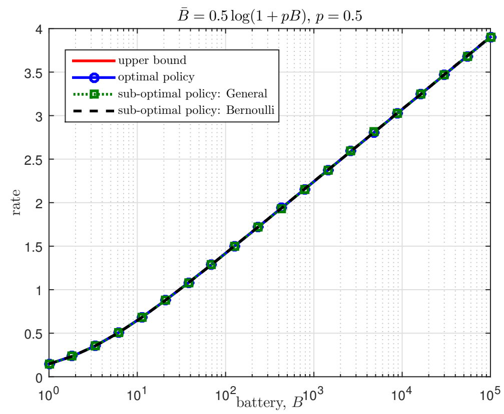

Fig. 3. Illustration of upper bound, optimal policy and the sub-optimal policy. General arrivals: uniform.

Lemma 5 *The performance of the proposed policy under Bernoulli arrivals forms a lower bound on the performance of the proposed policy under general fully-correlated arrival distributions with the same arrival mean.*

Since the minimum of concave functions is concave, the objective function with (11) is concave and the proof of Lemma 5 follows similar to [22].

From Lemma 5, we conclude that all the derived bounds for the fully-correlated Bernoulli arrivals are also valid for the fully-correlated general arrivals with the same arrival rates. Hence, the policy is optimal when the energy is more dominant, in particular, when $Bp \geq 2^{2\bar{B}} - 1$, and is within a constant 0.72 gap when the data is more dominant, in particular, when $\frac{1}{2}\log(1+Bp) + \frac{Bp}{2} \leq \bar{B}p$.

V. Numerical Examples

In this section, we illustrate our results using simple numerical examples. We first show the case when $\bar{B} = \frac{1}{2p}\log(1+Bp)$ in Fig. 2. In this case, we show that the proposed policy performs close to the optimal policy. In addition, the multiplicative lower bound closely lower bounds the performance of the proposed policy. We then show in Fig. 3 that when $\bar{B} = \frac{1}{2}\log(1+Bp)$, the optimal policy and the proposed policies for uniform arrivals are the same.

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