Communicating under Temperature and Energy Harvesting Constraints

Omur Ozel[1], Sennur Ulukus[2], and Pulkit Grover[1]

[1]Department of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA
[2]Department of Electrical and Computer Engineering, University of Maryland, College Park, MD

***Abstract*—Temperature constraints arise naturally in communication scenarios where the act of data transmission causes heat dissipation. We address this problem in point to point communications over an additive white Gaussian noise channel in an information theoretic setting. In the specific scenario, transmitted code symbols cause heat dissipation as an input to a first order discrete time heat circuit and the output of this dynamical system, being the temperature, has to remain below a critical level T_c. Additionally, we allow the transmitter to use an energy harvesting device to power its transmission. We investigate channel capacity for various combinations of peak and average temperature, average power, and energy harvesting constraints on the transmitted code symbols.**

I. Introduction

In many emerging applications, the temperature increase caused by data transmission constrains the communication performance. An instance of this problem is in biomedical wireless sensors implanted in the human body in that the temperature increase due to their communication related operations could be a threat for the metabolism; this possibility has to be addressed in a successful design. For various examples of this issue in biomedical sensor networks, please see [1]–[4]. More generally, temperature increase in a sensor is a threat for the proper operation of the hardware [5], [6] and it has to be dealt with in the architecture and control design for small and large scale computing devices. Therefore, the electric power that feeds the communication circuitry has to be carefully scheduled to avoid heat related damage. In this paper, we consider this problem in an information theoretic setting.

In order to understand the back-off in the optimal information rates due to the temperature sensitivity arising in data transmission, in this paper, we determine performance limits under a safe temperature threshold T_c and analyze the resulting performance with regard to T_c. In addition, we consider data transmission with energy harvesting sensors under such temperature constraints in an information theoretic setting. In this latter model, an energy harvesting transmitter sends messages over a discrete time additive white Gaussian noise (AWGN) channel. We analyze the information theoretic channel capacity for this AWGN channel under energy harvesting as well as temperature constraints.

Data transmission with energy harvesting transmitters has recently been considered in, for example, the references [7]–[10]. These works, specifically, consider data scheduling

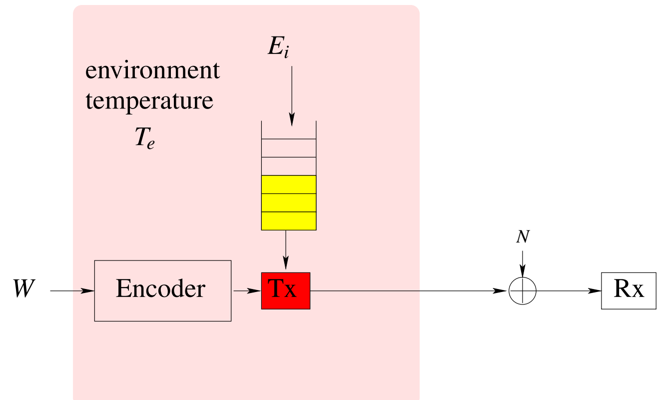

Fig. 1. System model representing an energy harvesting transmitter node placed in an environment that has constant temperature T_e. The transmitter uses harvested energy for data transmission.

problem for optimal transmission policies with an energy harvesting transmitter under offline and online knowledge of energy available for utilization in communication. References [11]–[15] are various works on information theoretic treatment for communications under energy harvesting constraints. The most related work to the current one is [16] where we address a transmission scheduling problem where temperature sensitivity arises due to wireless data transmission in an energy harvesting transmitter. In particular, the temperature model in [16] is based on a view of the heat dynamics as a first order heat circuit driven by the electromagnetic radiation emitted due to the transmit power. Under hard temperature constraints, [16] addresses throughput maximization subject to a limited radiation emitted at the transmitter side.

Motivated by the formulations that appeared in related works [17], [18], and in view of the information theoretic treatments in [11]–[15], in this paper, we consider information theoretic channel capacity under temperature constraints imposed at each channel use in the form of peak temperature constraints and average temperature constraints with additional average power and energy harvesting constraints. We provide an n-letter capacity formula under peak temperature constraints. We show that the capacity under average temperature constraints is identical to the capacity with an effective average power constraint determined by the system parameters. Next, we extend these results to the case where an additional average power constraint is present and to the case where additional energy harvesting constraints are present with unlimited energy storage. In particular, for the cases of peak temperature constraints with an additional average power constraint and with additional energy harvesting constraints, we provide n-letter capacity expressions. For the case of average temperature constraints and average power constraints, we show that the capacity is identical to that with a single average power constraint equal to the same level found before with an additional clipping due to the average power constraint. Finally, for the case of average temperature constraints and energy harvesting constraints with unlimited energy storage, we show that the capacity is identical to the capacity with a single average power constraint clipped by the average recharge rate.

II. Model and Definitions

We consider a transmitter node placed in an environment of temperature T_e as shown in Fig. 1.

A. Channel Model

We consider a scalar AWGN channel characterized by the input X, output Y, additive noise N with normal distribution $\mathcal{N}(0, \sigma^2)$ and a battery. Input and output alphabets are real numbers. In particular, at channel use i, the received signal Y_i, the input X_i, and noise N_i are related as

$$Y_i = X_i + N_i \tag{1}$$

where N_i is independent and identically distributed (i.i.d.) additive Gaussian noise with zero-mean and of variance σ^2, where we take $\sigma^2 = 1$ without loss of generality.

B. Thermal Dynamics

We model the thermal dynamics at the transmitter side as a first order heat circuit as follows:

$$\frac{dT}{dt} = aP(t) - b(T(t) - T_e) \tag{2}$$

where $P(t)$ is the transmit power and $T(t)$ is the temperature at time t. T_e is the temperature of the environment, a and b are nonnegative constants. This thermal model is related to the models used in, e.g., [1], [5], [18] in that the spatial variation in temperature is ignored.

We assume that the system is initially at rest and thus the initial temperature is $T(0) = T_e$. The solution of $T(t)$ for any given $P(t)$ from (2) is [16]:

$$T(t) = T_e + \int_0^t e^{b(\tau - t)} aP(\tau) d\tau \tag{3}$$

Let us now consider a discretization of the temperature in (3) in a slotted (i.e., channel use) setting with slot length is Δ:

$$T_{i+1} = T_e + \sum_{k=1}^{i} e^{b(k-i)\Delta} aP_k \Delta \tag{4}$$

where the index $i \in \{1, 2, \ldots\}$ and $T_1 = T_e$. Here, $P_k = X_k^2$ denotes the energy spent at time index k. The update rule of T_i is governed by the following difference equation:

$$T_{i+1} - T_i = \alpha X_i^2 - \beta(T_i - T_e) \tag{5}$$

where T_e is the environment temperature, $\alpha = a\Delta$ and $\beta = 1 - e^{-b\Delta}$. This follows from an algebraic manipulation of (4):

$$\begin{aligned} T_{i+1} - T_i &= \sum_{k=1}^{i} e^{b(k-i)\Delta} aP_k \Delta - \sum_{k=1}^{i-1} e^{b(k-i+1)\Delta} aP_k \Delta && (6) \\ &= a\Delta P_i - (1 - e^{-b\Delta}) \sum_{k=1}^{i-1} e^{b(k-i+1)\Delta} aP_k \Delta && (7) \\ &= a\Delta P_i - (1 - e^{-b\Delta})(T_i - T_e) && (8) \end{aligned}$$

More generally, the solution to (5) with an arbitrary initial temperature T_1 is:

$$T_{t+1} = \alpha \sum_{k=1}^{t} (1-\beta)^{t-k} X_k^2 + (1-\beta)^t (T_1 - T_e) + T_e \tag{9}$$

In the following, we use the discrete time system defined by (5) and resulting temperature expression in (9). Unless otherwise specified, initial temperature is $T_1 = T_e$.

C. Energy Harvesting Dynamics

As shown in Fig. 1, the transmitter uses energy supplied from a power source that supplies E_i units of energy in the ith channel use where $E_i \geq 0$. In particular, the initial energy available in the battery at time zero is E_0. E_i are independent of the system parameters and actions. $\{E_1, E_2, \ldots, E_n\}$ is the time sequence of supplied energy in n channel uses. E_i is an i.i.d. sequence with average value E_{avg}, i.e., $E[E_i] = E_{avg}$.

In this model, the energy stored and depleted from the battery are for only communication purposes. Existing energy in the battery is retrieved without any loss and the battery capacity is large, i.e., the battery storage limit is $E_{max} = \infty$. Due to the energy causality constraints, we have [11]:

$$\sum_{i=1}^{k} X_i^2 \leq \sum_{i=0}^{k} E_i, \qquad k = 1, \ldots, n \tag{10}$$

Note that the transmitter is allowed to use the energy E_i at the channel use it is received without any delay. Still, any possible delay in the energy arrival does not affect the capacity.

D. Definitions

In this paper, a channel code $\mathcal{C}^n = (n, 2^{nR_n}, \epsilon_n)$ is defined by the code length n, the code size 2^{nR_n} and the probability of error ϵ_n. The messages in the set $\{1, \ldots, 2^{nR_n}\}$ are equally likely. Encoding function is $f_k^n : \{1, \ldots, 2^{nR_n}\} \rightarrow \mathcal{X}$, $k = 1, \ldots, n$ and the decoding function is $\phi^n : \mathcal{Y}^n \rightarrow \{1, \ldots, 2^{nR_n}\}$. Note that whether the encoding and decoding functions could use energy or battery state information does not alter the capacity. Whenever the energy harvesting model is considered, there are two separate error events. The first one is that a codeword does not satisfy the energy harvesting constraints at a particular channel use. The second one is the decoding error at the receiver. Accordingly, the error event is defined as the union of two events: $\varepsilon^n = \varepsilon_1^n \cup \varepsilon_2^n$ where ε_1^n is the energy shortage event, and ε_2^n is the decoding error event. The overall probability of error is $\epsilon_n = \Pr(\varepsilon^n)$.

III. The Case of Temperature Constraints Only

In this section, we focus on the case where the message transmission is carried out only subject to temperature constraints. In particular, we set a critical temperature constraint T_c, with $T_c > T_e$ and define $T_\delta = T_c - T_e$. We consider two possible temperature constraints, details of which follow next.

A. Peak Temperature Constraints

Here, the temperature constraints at each channel use i are peak temperature constraints for any message almost surely:

$$T_i \leq T_c, \quad \forall i, \quad \text{a.s.} \tag{11}$$

where T_i is the temperature at channel use i. We define the feasible set of input probability distributions under temperature constraints using (9) and (11) with $T_1 = T_e$ as

$$\mathcal{F}_n^{peak} = \left\{ p(x^n) : \alpha \sum_{k=1}^{i} (1-\beta)^{k-i} X_k^2 \leq T_\delta, \quad \forall i \right\} \tag{12}$$

Lemma 1 *The channel capacity under peak temperature constraints is*

$$C^{peak} = \lim_{n\to\infty} \max_{p(x^n)\in\mathcal{F}_n^{peak}} \frac{1}{n} I(X^n;Y^n) \tag{13}$$

Proof: The proof follows from steps similar to those in [13, Theorem 2.1]. Let us define $C_n = \max_{p(x^n)\in\mathcal{F}_n^{peak}} I(X^n;Y^n)$. This is the capacity of an n dimensional parallel Gaussian channel where the n dimensional inputs obey the constraint $p(x^n) \in \mathcal{F}_n^{peak}$. This capacity is achieved by allowing the n symbol blocks to be generated independently from the remaining ones. Since this may not always obey the temperature constraints in (11), we have $C^{peak} \leq \frac{1}{n} C_n$. As this bound holds for any n, we have

$$C^{peak} \leq \inf_n \frac{1}{n} C_n \tag{14}$$

Now, we will show that $\inf_n \frac{1}{n} C_n = \lim_{n\to\infty} \frac{1}{n} C_n$. To do so, we note that

$$C_{n+m} = \max_{p(x^{n+m})\in\mathcal{F}_{n+m}^{peak}} I(X^{n+m};Y^{n+m}) \tag{15}$$

$$\leq \max_{p(x^n)\in\mathcal{F}_n^{peak}} I(X^n;Y^n) + \max_{p(x^m)\in\mathcal{F}_m^{peak}} I(X^m;Y^m) \tag{16}$$

$$= C_n + C_m \tag{17}$$

where the inequality in (16) follows from the inclusion relation $\mathcal{F}_{n+m}^{peak} \subset \mathcal{F}_n^{peak} \times \mathcal{F}_m^{peak}$. That is, C_n is a subadditive sequence. Hence, $\inf_n \frac{1}{n} C_n = \lim_{n\to\infty} \frac{1}{n} C_n$ by Fekete's lemma.

Finally, we argue that $\lim_{n\to\infty} \frac{1}{n} C_n$ is achievable. To see this, we need to consider the dependence of $\mathcal{F}_n^{peak}$ on the initial temperature. Note that it is not possible to return the initial temperature state $T_1 = T_e$ once it leaves this state. Still, a sequence of N zero symbol insertions could take the temperature arbitrarily close to the initial temperature state T_e. In particular, after any N zero symbol insertion, the temperature level drops to a level below $T_e + (1-\beta)^N T_\delta$ in view of (9). To use this observation, we denote $\mathcal{F}_n^{peak}(T_e+\eta)$ as the set of feasible input distributions with initial temperature set as $T_1 = T_e + \eta$ for some $\eta > 0$. Once we insert $N_\eta = \log_{1-\beta}\left(\frac{\eta}{T_\delta}\right)$ zero symbols, the temperature drops below $T_e + \eta$. Hence, the transmitter can select a vector with any input distribution in $\mathcal{F}_n^{peak}(T_e+\eta)$ and then insert N_η zero symbols and then independently select another vector with the same input distribution in $\mathcal{F}_n^{peak}(T_e+\eta)$ and this is feasible in terms of the temperature constraints in (11). This scheme achieves for any $\eta > 0$

$$\lim_{n\to\infty} \frac{1}{n+N_\eta} \max_{p(x^n)\in\mathcal{F}_n^{peak}(T_e+\eta)} I(X^n;Y^n)$$

$$= \lim_{n\to\infty} \frac{1}{n} \max_{p(x^n)\in\mathcal{F}_n^{peak}(T_e+\eta)} I(X^n;Y^n) \tag{18}$$

Finally, we note that $\mathcal{F}_n^{peak}(T_e+\eta_1) \subseteq \mathcal{F}_n^{peak}(T_e+\eta_2)$ for any $\eta_1 > \eta_2$. Therefore, due to monotonicity, we have

$$\sup_{\eta>0} \lim_{n\to\infty} \frac{1}{n} \max_{p(x^n)\in\mathcal{F}_n^{peak}(T_e+\eta)} I(X^n;Y^n)$$

$$= \lim_{n\to\infty} \frac{1}{n} \max_{p(x^n)\in\mathcal{F}_n^{peak}(T_e)} I(X^n;Y^n) \tag{19}$$

This completes the proof. ■

B. Average Temperature Constraint

Here, the temperature constraints are imposed at each channel use i as average temperature constraints:

$$\mathbb{E}[T_i] \leq T_c, \quad \forall i \tag{20}$$

where T_i is the temperature at channel use i and $\mathbb{E}[.]$ denotes averaging over all possible messages.

Lemma 2 *The channel capacity under average temperature constraints is*

$$C^{avg} = \frac{1}{2} \log\left(1 + \frac{T_\delta \beta}{\alpha}\right) \tag{21}$$

Proof: To show the converse, we proceed to take the telescoping sum of (5) to get:

$$T_{i+1} - T_1 = \alpha \sum_{k=1}^{i} X_k^2 - \beta \sum_{k=1}^{i} (T_k - T_e) \tag{22}$$

Since $T_1 = T_e$, this gives:

$$\alpha \sum_{k=1}^{i} X_k^2 = \beta \sum_{k=1}^{i} (T_k - T_e) + (T_{i+1} - T_1) \tag{23}$$

and since $T_k \leq T_c$, we get:

$$\alpha \frac{1}{i} \sum_{k=1}^{i} X_k^2 \leq \beta \frac{1}{i} \sum_{k=1}^{i} (T_c - T_e) + \frac{1}{i}(T_c - T_1) \tag{24}$$

That is, if $\mathbb{E}[T_k] \leq T_c$, the sequence has to obey the average power constraint:

$$\limsup_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} \mathbb{E}[X_i^2] \leq \frac{T_\delta \beta}{\alpha} \tag{25}$$

This proves the converse.

Next, to achieve the stated capacity under average temperature constraints, consider the sequence of codes with code length n and rate R_n such that for $k = 1, \ldots, n$, $f_k^n(m)$ comes from independent realizations of identically Gaussian distributed random variables of zero mean and variance $\frac{T_\delta\beta}{\alpha} - \kappa$ for all $m \in \{1, 2, \ldots, 2^{nR}\}$ where $\kappa > 0$ can be selected arbitrarily small. As n grows large, by law of large numbers, we have $\frac{1}{2^{nR}}\sum_{m=1}^{2^{nR}} X_k^2(m) \to \frac{T_\delta\beta}{\alpha} - \kappa$. However, replacing the instantaneous power X_k^2 in (4) with $\frac{1}{2^{nR}}\sum_{m=1}^{2^{nR}} X_k^2(m)$, and observing that simply letting $\frac{1}{2^{nR}}\sum_{m=1}^{2^{nR}} X_i^2(m) = \frac{T_\delta\beta}{\alpha}$ makes the temperature below the critical level T_c, we conclude that the temperature at each channel use k averaged over the equally likely messages remains below T_c as n grows to infinity. To prove the temperature feasibility, we note from (9) with $T_1 = T_e$ that:

$$\mathbb{E}[T_{i+1}] = T_e + \alpha\sum_{k=1}^{i}(1-\beta)^{k-i}\mathbb{E}[X_k^2], \tag{26}$$

$$= T_e + \alpha\sum_{k=1}^{i}(1-\beta)^{k-i}\frac{T_\delta\beta}{\alpha}, \tag{27}$$

$$\leq T_e + T_\delta\beta\sum_{k=0}^{\infty}(1-\beta)^{k-i} \tag{28}$$

$$= T_c \tag{29}$$

The achievable rate for this scheme is

$$\frac{1}{n}I(X^n;Y^n) = \frac{1}{n}\sum_{j=1}^{n} I(X_j;Y_j) \tag{30}$$

$$= \frac{1}{2}\log\left(1 + \left(\frac{T_\delta\beta}{\alpha} - \kappa\right)\right) \tag{31}$$

Letting $\kappa \to 0$ proves the achievability. ■

We note that the above achievable scheme is obtained without using causal or noncausal information of the temperature level at any specific channel use. The deterministic dynamical system model obviates the need to receive feedback about the temperature. We also note that $\frac{T_\delta\beta}{\alpha}$ plays the role of an effective average power constraint.

IV. Temperature Constraints and An Average Power Constraint

In this section, we consider the case where both temperature constraints and an average power constraint P_{avg} are imposed on the transmitted code symbol sequence.

A. Peak Temperature Constraints

At each channel use i, the temperature constraints are peak temperature constraints for any message almost surely $T_i \leq T_c$ where T_i is the temperature at channel use i. Additionally, the transmitted code symbol is average power constrained: $\frac{1}{n}\sum_{i=1}^{n} X_i^2 \leq P_{avg}$. Let us define the feasible input distributions under these constraints:

$$\mathcal{F}_n^{peak+avg} = \left\{ p(x^n) : \alpha\sum_{k=1}^{i}(1-\beta)^{k-i}X_k^2 \leq T_\delta, \quad \forall i \right.$$
$$\left. \frac{1}{n}\sum_{i=1}^{n} X_i^2 \leq P_{avg} \right\} \tag{32}$$

Lemma 3 *The channel capacity under peak temperature constraints with an additional average power constraint P_{avg} is*

$$C^{peak+avg} = \lim_{n\to\infty} \max_{p(x^n)\in\mathcal{F}_n^{peak+avg}} \frac{1}{n}I(X^n;Y^n)$$

Proof: The proof follows from steps similar to those used in the proof of Lemma 1. Details are left for a longer version of this work. ■

B. Average Temperature Constraints

In this subsection, the temperature constraints are imposed at each channel use i as average temperature constraints averaged over all possible messages:

$$\mathbb{E}[T_i] \leq T_c, \quad \forall i \tag{33}$$

where T_i is the temperature at channel use i.

Lemma 4 *The channel capacity under average temperature constraints with an additional average power constraint P_{avg} is*

$$C^{avg+avg} = \frac{1}{2}\log\left(1 + \min\left\{\frac{T_\delta\beta}{\alpha}, P_{avg}\right\}\right) \tag{34}$$

Proof: The converse follows from the converse in Lemma 2 and that average power constrained capacity is $\frac{1}{2}\log(1 + P_{avg})$.

To prove the achievability, if $\min\{\frac{T_\delta\beta}{\alpha}, P_{avg}\} = \frac{T_\delta\beta}{\alpha}$, then the achievability scheme in Lemma 2 is also average power constrained. Otherwise, if $\min\{\frac{T_\delta\beta}{\alpha}, P_{avg}\} = P_{avg}$, using an i.i.d. Gaussian codebook with zero mean and average power $P_{avg} - \kappa$ satisfies the average power constraint and the temperature constraints as well in the limit as n grows to infinity. ■

V. Temperature and Energy Harvesting Constraints

In this section, we focus on the case where both energy harvesting and temperature constraints are imposed on the transmitted code symbol sequence.

A. Peak Temperature Constraints

The temperature constraints at each channel use i are peak temperature constraints for any message almost surely: $T_i \leq T_c$ where T_i is the temperature at channel use i. Additionally, the transmitted code symbols are constrained due to energy harvesting as in (10).

Lemma 5 *The channel capacity under peak temperature constraints and energy harvesting constraints is*

$$C^{peak+EH} = \lim_{n\to\infty} \max_{p(x^n)\in\mathcal{F}_n^{peak+avg}} \frac{1}{n} I(X^n;Y^n) \qquad (35)$$

where the average power constraint in $\mathcal{F}_n^{peak+avg}$ is the average recharge rate E_{avg}.

Proof: The proof follows from similar steps to those used in Lemma 1. Details are left for a longer version of this work. ■

B. Average Temperature Constraints

Here, the temperature constraints are imposed at each channel use i as average temperature constraints averaged over all messages and the energy arrivals E^i until channel use i:

$$\mathbb{E}[T_i] \leq T_c, \quad \forall i \qquad (36)$$

where T_i is the temperature at channel use i.

Lemma 6 *The channel capacity under average temperature constraints with additional energy harvesting constraints is*

$$C^{avg+EH} = \frac{1}{2}\log\left(1 + \min\left\{\frac{T_\delta \beta}{\alpha}, E_{avg}\right\}\right) \qquad (37)$$

Proof: The capacity in Lemma 4 with P_{avg} set to E_{avg} is inherently an upper bound for the capacity with average temperature constraints and energy harvesting constraints.

Achievability follows from, e.g., the save and transmit scheme in [11] where the first $o(n)$ symbols are set equal to the zero symbol and the remaining symbols are selected as i.i.d. Gaussian with zero mean and average power equal to $\min\{\frac{T_\delta\beta}{\alpha}, E_{avg}\} - \kappa$ with $\kappa > 0$. This scheme is feasible in terms of average temperature due to the steps in Lemma 2 and in terms of energy harvesting in view of [11]. ■

VI. Conclusion

We considered channel capacity of an additive Gaussian noise channel with temperature and energy harvesting constraints. In the model, temperature constraints were imposed at each channel use and they could be in the form of peak temperature constraints and average temperature constraints. We showed that the capacity under average temperature constraints is $\frac{1}{2}\log\left(1 + \frac{T_\delta\beta}{\alpha}\right)$, identical to the capacity with an average power constraint determined by system parameters only. We provided an n-letter capacity expression under peak temperature constraints. We generalized these results under additional energy harvesting constraints. We first studied the case where an additional average power constraint is present and later showed that the capacity with energy harvesting constraints and unlimited energy storage is identical to the one with an additional average power constraint equal to the average recharge rate. In particular, for the average temperature constrained case with an additional average power constraint, we showed that the capacity is $\frac{1}{2}\log\left(1 + \min\{\frac{T_\delta\beta}{\alpha}, P_{avg}\}\right)$, i.e., the additional power constraint is a clipping factor. For the average temperature constrained case with additional energy harvesting constraints with unlimited storage, we showed that the clipping factor is the average recharge rate.

References

[1] H. H. Pennes, "Analysis of tissue and arterial blood temperature in the resting human forearm," *Jour. of App. Physiology*, vol. 1, no. 2, pp. 93–122, August 1948.

[2] R. Hongliang and M.-H. Meng, "Rate control to reduce bioeffects in wireless biomedical sensor networks," in *IEEE Int. Conf. on Mobile and Ubiq. Sys.*, July 2006.

[3] S. Ullah, H. Higgins, B. Braem, B. Latre, C. Blondia, I. Moerman, S. Saleem, Z. Rahman, and K. Kwak, "A comprehensive survey of wireless body area networks," *Jour. Medical. Syst.*, vol. 36, no. 3, pp. 1065–1094, June 2012.

[4] Q. Tang, N. Tummala, S. Gupta, and L. Schwiebert, "Communication scheduling to minimize thermal effects of implanted biosensor networks in homogeneous tissue," *IEEE Trans. on Biomed. Eng.*, vol. 52, no. 7, pp. 1285–1294, July 2005.

[5] D. Forte and A. Srivastava, "Thermal aware sensor scheduling for distributed estimation," *ACM Trans. Sensor Networks*, vol. 9, no. 4, pp. 53:1–53:31, July 2013.

[6] R. Rao, S. Vrudhula, and C. Chakrabarti, "An optimal analytical solution for processor speed control with thermal constraints," in *IEEE Int. Symp. on Low Power Elec. and Design*, October 2006.

[7] J. Yang and S. Ulukus, "Optimal packet scheduling in an energy harvesting communication system," *IEEE Trans. Comm.*, vol. 60, no. 1, pp. 220–230, January 2012.

[8] K. Tutuncuoglu and A. Yener, "Optimum transmission policies for battery limited energy harvesting nodes," *IEEE Trans. Wireless Comm.*, vol. 11, no. 3, pp. 1180–1189, March 2012.

[9] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, "Transmission with energy harvesting nodes in fading wireless channels: Optimal policies," *IEEE Jour. on Selected Areas in Commun.*, vol. 29, no. 8, pp. 1732–1743, September 2011.

[10] C. K. Ho and R. Zhang, "Optimal energy allocation for wireless communications with energy harvesting constraints," *IEEE Trans. Sig. Proc.*, vol. 60, no. 9, pp. 4808–4818, September 2012.

[11] O. Ozel and S. Ulukus, "Achieving AWGN capacity under stochastic energy harvesting," *IEEE Trans. on Information Theory*, vol. 58, no. 10, pp. 6471–6483, October 2012.

[12] K. Tutuncuoglu, O. Ozel, A. Yener, and S. Ulukus, "Binary energy harvesting channel with a unit-sized battery," *to appear in IEEE Trans. on Information Theory*, 2017.

[13] V. Jog and V. Anantharam, "A geometric analysis of the awgn channel with a (σ, ρ) power constraint," *IEEE Trans. on Inform. Theory*, vol. 62, no. 8, pp. 4413 –4438, August 2016.

[14] W. Mao and B. Hassibi, "Capacity analysis of discrete energy harvesting channels," *submitted to IEEE Trans. on Information Theory, available at arXiv:1606.08973*, 2016.

[15] D. Shaviv, P.-M. Nguyen, and A. Ozgur, "Capacity of the energy harvesting channel with finite battery," *IEEE Trans. on Information Theory*, vol. 62, no. 11, pp. 6436–6458, November 2016.

[16] O. Ozel, S. Ulukus, and P. Grover, "Energy harvesting transmitters that heat up: Throughput maximization under temperature constraints," *IEEE Trans. on Wireless Comm.*, vol. 15, no. 8, pp. 5540–5552, August 2016.

[17] P. Grover, ""Information-friction" and its implications on minimum energy required for communication," *IEEE Trans. on Inform. Theory*, vol. 61, no. 2, pp. 895–907, February 2015.

[18] T. Koch, A. Lapidoth, and P. P. Sotiriadis, "Channels that heat up," *IEEE Trans. Inform. Theory*, vol. 55, no. 8, pp. 3594–3612, August 2009.