Online Policies for Multiple Access Channel with Common Energy Harvesting Source

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***Abstract*— We consider *online* transmission policies for the two-user multiple access channel, where both users harvest energy from a common source. The transmitters are equipped with arbitrary but finite-sized batteries. The energy harvests are independent and identically distributed (i.i.d.) over time, and synchronized at the two users due to their common source. The transmitters know the energy arrivals only causally. We first consider the special case of Bernoulli energy arrivals. For this case, we determine the optimal policies that achieve the boundary of the capacity region. We show that the optimal power allocation decreases in time, and that the capacity region is a pentagon. We then consider general i.i.d. energy arrivals, and propose a distributed fractional power (DFP) policy. We develop lower and upper bounds on the performance of the proposed DFP policy for general i.i.d. energy arrivals, and show that the proposed DFP is *near-optimal* in that it yields rates which are within a constant gap of the derived lower and upper bounds.**

I. Introduction

We consider a two-user energy harvesting multiple access channel, Fig. 1, where each user has an (arbitrary) finite-sized battery. The transmitters harvest energy from a *common* i.i.d. energy source. Practically, the common energy source can arise when the receiver is the actual energy source, i.e., through wireless energy transfer; and also, when the transmitters are spatially close to each other and are exposed to the same energy source. Mathematically, the common energy source implies synchronous (but non-identical, due to the different battery sizes) recharge processes at the transmitters, enabling analytical tractability. We consider the *online* scheduling problem where the energy arrivals are known only causally.

Scheduling for energy harvesting systems has been considered in *offline* and *online* settings in recent research. When energy arrivals are known non-causally ahead of time, *offline* power allocation has been considered for the single-user [1]–[4] and multi-user [5]–[16] settings. When energy arrivals are known only causally as they happen, *online* power allocation has been considered mostly for single-user settings using dynamic programming, Markov decision processes, stochastic control, learning, Lyapunov techniques [3], [4], [17]–[28].

Our work in this paper is most closely related with [8] and [26]–[28]. Reference [8] develops optimum power allocation schemes for the energy harvesting multiple access channel using generalized directional water-filling techniques.

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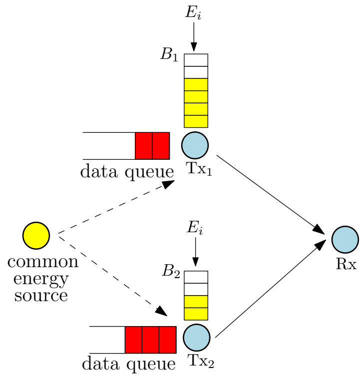

Fig. 1. System model: an energy harvesting multiple access channel model.

References [26]–[28] develop a unique approach to the online power allocation problem in the single-user setting, by first considering a special Bernoulli energy arrival process which provides a *renewal* structure that enables analytical tractability, and then by generalizing it to general i.i.d. arrivals. Our paper may be viewed as extending the offline multiple access setting in [8] to the online case; or as extending the online single-user setting in [26], [27] to the multiple access case.

In this paper, we first consider the case of i.i.d. Bernoulli energy arrivals. For this case, we obtain the jointly *optimum* online power schedules for the users. We show that the optimum transmission powers of both users decrease exponentially in time. We show that the capacity region, which is in general a union of pentagons over power allocation policies, is a single pentagon for Bernoulli arrivals. We show that at the corner points of the pentagon where one of the users gets the single-user rate, the user getting the single-user rate transmits for a shorter (or equal) duration than the other user.

We then consider the case of general i.i.d. energy arrivals. Motivated by the fractional structure of the optimal policies for i.i.d. Bernoulli arrivals, we propose a sub-optimal policy, which is fixed but distributed between the users, coined distributed fractional power (DFP) policy. The fractional policy at each user is different and depends on its battery size. The DFP is *universal* in that, it does not depend on the statistics of the energy arrival process; it depends only on the average recharge rate at each user. We obtain lower and upper bounds on the performance of the proposed DFP, and show that they are within a constant gap from each other, hence, showing that the DFP is within a constant gap from the optimal policy.

II. SYSTEM MODEL

We consider a two-user energy harvesting multiple access channel. User k has a battery of size B_k, see Fig. 1. There is a common energy harvesting source which delivers E_i amount of energy to each user in slot i. The battery state of user k at time i, b_{ki}, evolves as $b_{ki} = \min\{B_k, b_{k(i-1)} - P_{k(i-1)} + E_i\}$. Here, $P_{k(i-1)}$ is the transmit power of user k at time $(i-1)$, which is limited as $P_{k(i-1)} \leq b_{k(i-1)}$.

The physical layer is a Gaussian multiple access channel with noise variance at the receiver equal to σ^2. The capacity region, $\mathcal{C}(P_{1i}, P_{2i})$, of this channel in slot i is (see e.g., [8]):

$$r_{1i} \leq \frac{1}{2}\log\left(1+\frac{P_{1i}}{\sigma^2}\right) \tag{1}$$

$$r_{2i} \leq \frac{1}{2}\log\left(1+\frac{P_{2i}}{\sigma^2}\right) \tag{2}$$

$$r_{1i}+r_{2i} \leq \frac{1}{2}\log\left(1+\frac{P_{1i}+P_{2i}}{\sigma^2}\right) \tag{3}$$

The above capacity region in each slot is a pentagon. The overall capacity region is a union of all possible pentagons corresponding to all feasible power allocations over time, and thus, may no longer be a pentagon [8], as shown in Fig. 2.

We first study the special case of Bernoulli energy arrivals with a particular support set in Section III, and determine the *optimum* power allocation policies. In this case, energy arrivals are such that either no energy arrives, $E_i = 0$ with probability $1-p$, or an energy arrives in the amount of $E_i = B$ with probability p, where $B \geq \max\{B_1, B_2\}$. This means that, when energy arrives, the batteries of both users fill completely. This constitutes a *renewal*, and we can evaluate the expected throughput analytically. Another interpretation of this energy arrival process is that, the two users have *synchronized* energy arrivals that fill their individual batteries completely. In Section IV, we will consider general i.i.d. arrivals, and develop *near-optimal* power allocation policies.

III. OPTIMAL STRATEGY: BERNOULLI ARRIVALS

Since the capacity region of the multiple access channel is convex, we consider weighted sum rates $\mu_1 r_1 + \mu_2 r_2$. Whenever a positive energy arrives a *renewal* occurs. From [29, Theorem 3.6.1], the long-term average throughput is:

$$\lim_{n\to\infty} \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n}(\mu_1 r_{1i}+\mu_2 r_{2i})\right]$$

$$= \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}(\mu_1 r_{1i}+\mu_2 r_{2i})\right] \tag{4}$$

$$= p\sum_{L=1}^{\infty} p(1-p)^{L-1}\sum_{i=1}^{L}(\mu_1 r_{1i}+\mu_2 r_{2i}) \tag{5}$$

$$= \sum_{i=1}^{\infty}\sum_{L=i}^{\infty} p^2(1-p)^{L-1}(\mu_1 r_{1i}+\mu_2 r_{2i}) \tag{6}$$

$$= \sum_{i=1}^{\infty} p(1-p)^{i-1}(\mu_1 r_{1i}+\mu_2 r_{2i}) \tag{7}$$

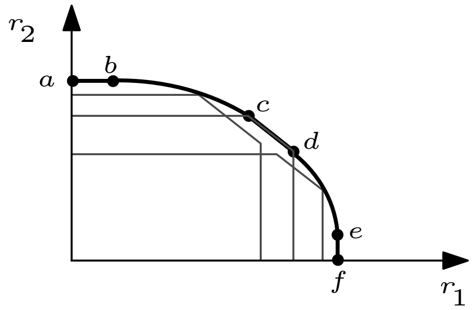

Fig. 2. Capacity region of a multiple access channel.

where L is geometric with parameter p, i.e., $\mathbb{E}[L] = \frac{1}{p}$.

Therefore, we focus on the optimization problem:

$$\begin{aligned} \max_{\{P_{1i},P_{2i}\}} \quad & \sum_{i=1}^{\infty} p(1-p)^{i-1}(\mu_1 r_{1i}+\mu_2 r_{2i}) \\ \text{s.t.} \quad & (r_{1i}, r_{2i}) \in \mathcal{C}(P_{1i}, P_{2i}) \\ & \sum_{i=1}^{\infty} P_{1i} \leq B_1, \ \sum_{i=1}^{\infty} P_{2i} \leq B_2 \end{aligned} \tag{8}$$

This problem in effect maximizes the expected weighted sum rate until the next energy arrival. Point a in Fig. 2 represents the single-user rate for user 2, corresponding to $\mu_1 = 0$, and can be obtained as in [26], [27]. Point b represents the largest rate user 1 gets when user 2 maintains its single-user rate; this point can be obtained by fixing the second user's rate at its single-user rate and maximizing the first user's rate. The line between points c and d represents the sum-rate line where the sum of the two users' rate is constant; these points are obtained by setting $\mu_1 = \mu_2$. The curved part of the capacity region between b and c is obtained by tracing μ_1, μ_2 over $\mu_1 < \mu_2$.

We first consider point a. At this point, $P_{1i} = 0$, and user 2 transmits with its optimum single-user rate [26], [27]:

$$P_{2i}^* = \frac{p(1-p)^{i-1}}{\lambda_2} - \sigma^2, \qquad i = 1, \ldots, \tilde{N}_2 \tag{9}$$

where the optimum power decreases in time and $\tilde{N}_2$ is the last slot where the power is positive; λ_2 in (9) is found by satisfying the total power constraint with equality.

Next, we consider point b. At this point, we maximize the first user's rate, after fixing the power allocation of the second user to its optimal single-user power allocation P_{2i}^*:

$$\begin{aligned} \max_{\{P_{1i}\}} \quad & \sum_{i=1}^{\infty} p(1-p)^{i-1} r_{1i} \\ \text{s.t.} \quad & r_{1i} \in \mathcal{C}(P_{1i}, P_{2i}^*), \quad \sum_{i=1}^{\infty} P_{1i} \leq B_1 \end{aligned} \tag{10}$$

The Lagrangian of this problem is:

$$\begin{aligned} \mathcal{L} = & -\sum_{i=1}^{\infty} p(1-p)^{i-1}\log\left(1+\frac{P_{1i}}{P_{2i}^*+\sigma^2}\right) \\ & + \lambda_1\left(\sum_{i=1}^{\infty} P_{1i} - B_1\right) - \sum_{i=1}^{\infty}\nu_{1i}P_{1i} \end{aligned} \tag{11}$$

The KKT optimality conditions are:

$$P_{1i} = \frac{p(1-p)^{i-1}}{\lambda_1 - \nu_{1i}} - \sigma^2 - P^*_{2i} \tag{12}$$

along with complementary slackness and $\lambda_1, \nu_{1i} \geq 0$.

We prove that at point b user 1 transmits for a duration no shorter than user 2, before proceeding to determine P^*_{1i}.

Lemma 1 *At point b, where user 2 gets single-user capacity, user 1 transmits for a duration no shorter than user 2.*

Proof: We first note that, in slots 1 through $\tilde{N}_2$, where user 2 transmits with power in (9), user 1 experiences a larger coefficient in front of the log term, $p(1-p)^{i-1}$, but also a larger interference power, P^*_{2i}, inside the log term, in earlier slots. The objective in (10) is $p(1-p)^{i-1}\log(1+\frac{P_{1i}}{p(1-p)^{i-1}/\lambda_2})$ in these slots. As this decreases in i, user 1 starts its transmission in slot 1 and always utilizes earlier slots with larger transmit power. Then, the rest of the proof follows by contradiction. Assume user 1 has a transmission duration $\tilde{N}_1 < \tilde{N}_2$. Then,

$$\sum_{i=1}^{\tilde{N}_1} \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) p(1-p)^{i-1} = B_1 \tag{13}$$

Thus, we have $\frac{1}{\lambda_1} - \frac{1}{\lambda_2} > 0$. Next, by assumption, we have $P_{1i} = 0, P_{2i} > 0$ in slot $\tilde{N}_2$. Then, from (12), we have

$$P_{1\tilde{N}_2} = p(1-p)^{\tilde{N}_2-1}\left(\frac{1}{\lambda_1 - \nu_{1\tilde{N}_2}} - \frac{1}{\lambda_2}\right) = 0 \tag{14}$$

However, since we have $\nu_{1\tilde{N}_2} \geq 0$ and $p(1-p)^{\tilde{N}_2-1} > 0$,

$$0 = \left(\frac{1}{\lambda_1 - \nu_{1\tilde{N}_2}} - \frac{1}{\lambda_2}\right) \geq \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) > 0 \tag{15}$$

which is a contradiction. Thus, $\tilde{N}_1 \geq \tilde{N}_2$. ■

Hence, at point b, user 1 transmits for a duration $\tilde{N}_1$ where $\tilde{N}_1 \geq \tilde{N}_2$. At this point, user 2 transmits with its single-user power allocation until $\tilde{N}_2$. Then, for user 1,

$$\sum_{i=1}^{\tilde{N}_2}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) p(1-p)^{i-1} + \sum_{i=\tilde{N}_2+1}^{\tilde{N}_1} \frac{p(1-p)^{i-1}}{\lambda_1} - \sigma^2 = B_1 \tag{16}$$

where λ_2 and $\tilde{N}_2$ are obtained from the second user's single-user power allocation, while λ_1 and $\tilde{N}_1$ are obtained from solving (16) and insuring that the Lagrange multiplier λ_1 is non-negative, i.e., $\tilde{N}_1$ the largest integer satisfying,

$$p(1-p)^{\tilde{N}_1-1} \geq \lambda_1\sigma^2 \tag{17}$$

and $\lambda_2 > \lambda_1$, simultaneously. Solving (16) for λ_1 we have

$$\lambda_1 = \frac{1-(1-p)^{\tilde{N}_1}}{B_1 + (\tilde{N}_1 - \tilde{N}_2)\sigma^2 + \frac{1}{\lambda_2}(1-(1-p)^{\tilde{N}_2})} \tag{18}$$

Therefore, $\tilde{N}_1$ is the largest integer that satisfies (17) when λ_1 in (18) is inserted into (17).

We also note that, at point b, both users' powers are decreasing in time. It is clear that the second user's power is decreasing, as it follows the single-user allocation in (9). For user 1, it is clear from (16) that the power is decreasing for the first $\tilde{N}_2$ slots, and again decreasing from slot $\tilde{N}_2+1$ onwards. Thus, it remains to check the transition from slot $\tilde{N}_2$ to slot $\tilde{N}_2+1$. We have,

$$P_{1\tilde{N}_2} = \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) p(1-p)^{\tilde{N}_2-1} \tag{19}$$

$$\geq \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) p(1-p)^{\tilde{N}_2} \tag{20}$$

$$\geq \frac{1}{\lambda_1} p(1-p)^{\tilde{N}_2} - \sigma^2 = P_{1(\tilde{N}_2+1)} \tag{21}$$

where (20) follows since $(1-p) \leq 1$ and (21) follows since the second user's transmission ends at $\tilde{N}_2$, hence $\frac{p(1-p)^{\tilde{N}_2}}{\lambda_2} < \sigma^2$. Thus, the first user's power is also decreasing throughout its transmission. This concludes the characterization of the optimal policies achieving point b.

Next, we consider sum capacity achieving points between point c and point d. For the sum rate, problem (8) reduces to:

$$\begin{aligned} \max_{\{P_{1i},P_{2i}\}} \quad & \frac{1}{2}\sum_{i=1}^{\infty} p(1-p)^{i-1}\log\left(1+\frac{P_{1i}+P_{2i}}{\sigma^2}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{\infty} P_{1i} \leq B_1, \ \sum_{i=1}^{\infty} P_{2i} \leq B_2 \end{aligned} \tag{22}$$

Consider the *relaxed* problem with a total power constraint:

$$\begin{aligned} \max_{\{P_{1i},P_{2i}\}} \quad & \frac{1}{2}\sum_{i=1}^{\infty} p(1-p)^{i-1}\log\left(1+\frac{P_{1i}+P_{2i}}{\sigma^2}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{\infty} P_{1i} + P_{2i} \leq B_1 + B_2 \end{aligned} \tag{23}$$

First, we remark that problems in (22) and (23) are equivalent: This follows since, any optimal solution of (22) is also feasible in (23) with the same optimum value; and, any optimal solution for (23), $P^*_{1i} + P^*_{2i}$, can be made feasible in (22) by defining $P_{1i} = (P^*_{1i} + P^*_{2i})\frac{B_1}{B_1+B_2}$ and $P_{2i} = (P^*_{1i} + P^*_{2i})\frac{B_2}{B_1+B_2}$, with the same optimum value.

Using this equivalence, we can find the sum-rate optimal policies by first solving a single-user problem with a battery size $B_t = B_1 + B_2$, and then dividing the total power to users in a feasible way. The feasible policy is not unique, and each feasible policy results in a different point on the c-d line.

Next, we characterize the two extreme points of this line: c and d. From the single-user analysis in [26], [27], it follows that the transmission duration $\tilde{N}$ is an increasing function of the battery size, i.e., the larger the battery, the longer the transmission duration will be. Hence, in the optimal solution for (23), $P^*_{1i}+P^*_{2i}$ is positive for a duration $\tilde{N}_s$ which is no less than the durations for the single-user solutions of the users.

We now show that the extreme achievable sum rate optimal point c is actually the point b, i.e., the capacity region for the case of Bernoulli arrivals is a single pentagon. We will show this by showing that, given the optimum total power allocation policy in (23), a feasible distribution can be found such that the single-user capacity for either of the users (we will show for user 2) is achieved. We denote the optimal Lagrange multiplier and the transmission duration for problem (23) by λ_s and $\tilde{N}_s$, respectively. Similarly, we have λ_2 and $\tilde{N}_2$ for the second user single-user power allocation. It is sufficient to show that $\lambda_s \leq \lambda_2$, since it will imply:

$$\left(\frac{p(1-p)^{i-1}}{\lambda_s} - \sigma^2\right) - \left(\frac{p(1-p)^{i-1}}{\lambda_2} - \sigma^2\right) \geq 0 \quad (24)$$

Recall we have $\tilde{N}_s \geq \tilde{N}_2$. First, if $\tilde{N}_s = \tilde{N}_2$, then we have $\sum_{i=1}^{\tilde{N}_2} \frac{p(1-p)^{i-1}}{\lambda_2} - \sigma^2 = B_2 \leq B_1 + B_2 = \sum_{i=1}^{\tilde{N}_s} \frac{p(1-p)^{i-1}}{\lambda_s} - \sigma^2$ which can happen if and only if $\lambda_s \leq \lambda_2$. Next, if $\tilde{N}_s > \tilde{N}_2$, i.e., $\tilde{N}_s - 1 \geq \tilde{N}_2$, then we have,

$$\lambda_s \sigma^2 \leq p(1-p)^{\tilde{N}_s - 1} < p(1-p)^{\tilde{N}_2} < \lambda_2 \sigma^2 \quad (25)$$

implying $\lambda_s < \lambda_2$. In (25) the middle inequality follows from the monotonicity, and the outer inequalities follow since $\lambda_s, \lambda_2, \tilde{N}_s, \tilde{N}_2$ satisfy their optimality conditions. Hence, we have proved the following result.

Lemma 2 *The online capacity region of the multiple access channel with i.i.d. Bernoulli energy arrivals is a pentagon.*

IV. Near-Optimal Strategy: General Arrivals

In this section, we consider general i.i.d. energy arrivals, propose a sub-optimal online policy, develop lower and upper bounds on its performance, and show that it is near-optimal. Let average recharge rate at user k be $\bar{P}_k = \mathbb{E}[\min\{B_k, E_i\}]$.

A. Distributed Fractional Power (DFP) Policy

We first define the sub-optimal online policy for Bernoulli arrivals, and then generalize it to arbitrary arrivals. The optimal powers achieving any point on the capacity of the multiple access channel are exponentially decreasing, hence as in [26], [27], this motivates us for a fractional structure for the sub-optimal policy. Moreover, the capacity region for Bernoulli arrivals is a pentagon, this motivates that the policy need not depend on μ_1, μ_2. For Bernoulli arrivals, each transmitter transmits a fraction p of its available energy. The first user transmits $B_1 p(1-p)^{i-1}$ in slot i, and the second user transmits $B_2 p(1-p)^{i-1}$. For general energy arrivals, user i transmits a fraction of $q_i \triangleq \frac{\bar{P}_i}{B_i}$ of its available energy in its battery.

B. A Lower Bound on the Proposed Online Policy

Lemma 3 *Under the proposed fractional policy, DFP, the achievable rate region is lower bounded as follows:*

$$\frac{1}{2}\log\left(1 + \frac{\bar{P}_k}{\sigma^2}\right) - 0.72 \leq r_k, \qquad k = 1, 2 \quad (26)$$

$$\frac{1}{2}\log\left(1 + \frac{\bar{P}_1 + \bar{P}_2}{\sigma^2}\right) - 0.72 \leq r_1 + r_2 \quad (27)$$

Proof: It is clear that the achievable rate region with the proposed DFP is a pentagon, as it is a single policy which does not depend on μ_1, μ_2. Hence, the whole region is completely characterized by four points, which are shown by a, b, c, d in Fig. 3. Therefore, to lower bound this region it suffices to lower bound the points a, b, c and d. Points a and d are the single-user rates which can be lower bounded as in [27] to obtain $\frac{1}{2}\log\left(1 + \frac{\bar{P}_1}{\sigma^2}\right) - 0.72 \leq r_1$ and $\frac{1}{2}\log\left(1 + \frac{\bar{P}_2}{\sigma^2}\right) - 0.72 \leq r_2$, which are (26). These identify points a', d'. Then, we lower bound the achievable sum rate by noting for the proposed policy: $P_{1i} + P_{2i} = (B_1 + B_2)p(1-p)^{i-1}$. Hence, again using [27], we have (27). This identifies points b', c'. ∎

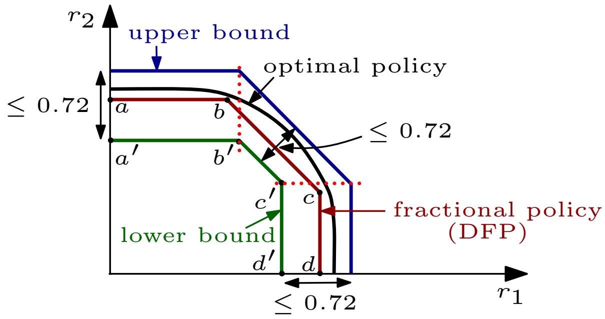

Fig. 3. Relationships between the bounds.

Since the rate region with the DFP policy is a pentagon even for general energy arrivals, to show that Bernoulli arrivals give a lower bound for all other energy arrivals, it suffices to show it only for the expected single-user and sum rates. These follow directly for the single-user rates from [27, Prop. 4]. It also follows for the sum rate, since the expectation is taken over a single random variable which is the common energy arrival process. Hence, [27, Lemma 2] can still be applied and the proof follows similar to the proof of [27, Prop. 4].

Theorem 1 *With the DFP policy, any arbitrary i.i.d. energy arrival process yields an achievable rate region no smaller than the one an i.i.d. Bernoulli process with the same mean.*

C. An Upper Bound for Online Policies

Theorem 2 *The optimal online capacity region for the multiple access channel is upper bounded as follows:*

$$r_k \leq \frac{1}{2}\log\left(1 + \frac{\bar{P}_k}{\sigma^2}\right), \qquad k = 1, 2 \quad (28)$$

$$r_1 + r_2 \leq \frac{1}{2}\log\left(1 + \frac{\bar{P}_1 + \bar{P}_2}{\sigma^2}\right) \quad (29)$$

The proof follows from upper bounding the online capacity region with the offline case, and then making use of the concavity of the rate functions and applying Jensen's inequality, hence favoring constant powers by ignoring some of the energy causality constraints and finite battery sizes.

Finally, combining Theorem 1 and Theorem 2, we conclude that the proposed online DFP policy yields rates which are within a constant gap from the optimal online policy.

V. Numerical Examples

In this section, we illustrate the results obtained through several numerical results. We set $\sigma^2 = 1$. We first consider

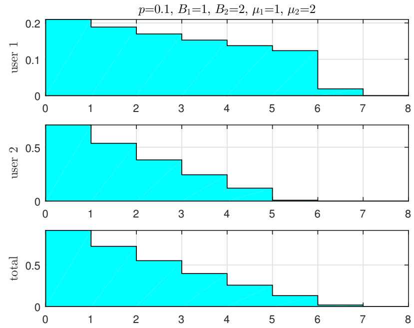

Fig. 4. Optimal powers for Bernoulli arrivals.

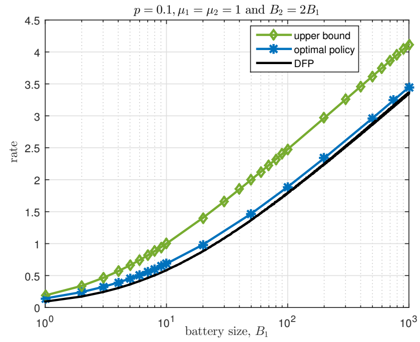

Fig. 5. Sum rate: optimal policy, DFP, and upper bound for Bernoulli arrivals.

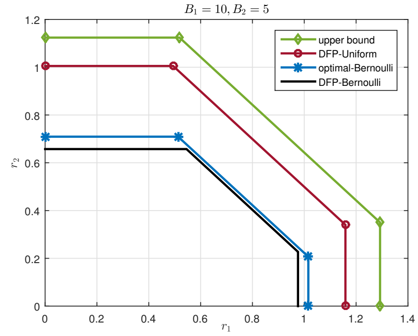

Fig. 6. Achievable rate region for i.i.d. uniform and Bernoulli arrivals.

the case of i.i.d. Bernoulli energy arrivals. For the corner point where user 2 operates at its single-user capacity, we plot the optimal power allocations in Fig. 4, for $p = 0.1, B_1 = 1, B_2 = 2$. As we proved, user 1 transmits for a longer duration than user 2, and user 2 follows its single-user powers allocation.

Next, we show the performance of the proposed DFP policy versus the optimal and the upper bound, in Fig. 5 for Bernoulli arrivals. We observe that DFP performs close to optimal. In Fig. 6, we show the performance of DFP policy for continuous uniform distribution of support $[0, 10]$. We also plot the case for Bernoulli arrivals with the same mean, $B = 10, p = 0.5$.

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