Online Scheduling for Energy Harvesting Broadcast Channels with Finite Battery

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***Abstract*— We consider *online* transmission scheduling for an energy harvesting broadcast channel with a finite-sized battery. The energy harvests are independent and identically distributed (i.i.d.) in time, and the transmitter gets to know them only causally as they happen. We first consider the case of Bernoulli energy arrivals, and determine the *optimum online* strategy that allocates power over time and between users optimally. We then consider the case of general i.i.d. energy arrivals, and propose a sub-optimum strategy coined fractional power constant cut-off (FPCC) policy. We develop a lower bound for the performance of the proposed FPCC policy, and a universal upper bound for the capacity region of the energy harvesting broadcast channel. We show that the proposed FPCC policy is *near-optimal* in that it yields rates that are within a constant gap from the optimum online policy, for all system parameters.**

I. Introduction

We consider an energy harvesting broadcast channel, Fig. 1, with a finite-sized battery, which is charged continually by an exogenous i.i.d. energy harvesting process. The transmitter needs to determine a *transmission policy* by choosing a transmission power and dividing it between the users, to maximize the throughput. We consider the *online* setting where the transmitter gets to know the energy arrivals only causally.

Energy harvesting communication has been the subject of intense research recently. *Offline* power scheduling, where all energy arrivals are known non-causally ahead of time, has been studied in many different settings, starting with the single-user setting [1]–[4] and addressing multi-user systems [5]–[16]. In contrast, *online* power scheduling, where energy harvests are known only causally, has been considered in fewer works and mostly for single-user systems so far [3], [4], [17]–[28]. In this case, there is a difficulty that arises due to the uncertainty about the future energy arrivals and the finiteness of the battery size. In most cases, the online problem formulation results in algorithms relying mainly on dynamic programming.

More recently, [26]–[28] developed a unique framework to study online power scheduling in energy harvesting systems. In particular, [26], [27] consider a single-user channel with i.i.d. energy arrivals. They first study a special energy arrival process which is an i.i.d. Bernoulli random process. For this case, they develop an *optimal* online power control policy exploiting the *renewal* property of energy arrivals. They, then, propose a sub-optimal power allocation method and prove it to be within a constant gap away from the optimal for general i.i.d. energy arrivals. Our current paper may be viewed as extending the offline broadcast setting of [5], [7] to the case of online broadcast setting; or as extending the online single-user setting of [26], [27] to the case of online broadcast setting.

This work was supported by NSF Grants CNS 13-14733, CCF 14-22111, CCF 14-22129, and CNS 15-26608.

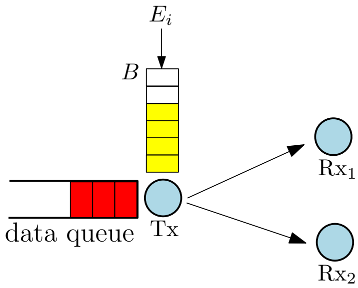

Fig. 1. System model: an energy harvesting broadcast channel.

In this paper, we first consider a special i.i.d. Bernoulli energy arrival process. For this case, we solve for the *exactly optimum* online power scheduling policy. We show that the optimal total transmit power is decreasing in time, and there exists a cut-off level below which all power is allocated to serve the stronger user, and only the power above which is allocated to serve the weaker user. Unlike [5], [7], the optimum total transmit power is not equal to the single-user power, as the online single-user power depends on the receiver noise variance. We determine the optimum online strategy to achieve any point on the boundary of the capacity region. We show that, when both users are served, stronger user is served for a duration no less than the weaker user is served. We show that whenever the stronger user's power allocation is decreasing, the weaker user's power allocation is zero; and whenever the stronger user's power allocation is equal to the cut-off power, the weaker user's power allocation is decreasing.

Next, inspired by the optimum solution for Bernoulli arrivals, we propose a universal sub-optimal strategy for general i.i.d. energy arrivals: fractional power constant cut-off (FPCC) policy, which depends only on the average recharge rate. In FPCC, the transmitter uses a sub-optimal fractional power policy, but allocates it optimally between the users. We develop a lower bound for its performance, and a universal upper bound for the online capacity region. We show that the FPCC scheme is *near-optimal* in that it yields rates that are within a constant gap from the developed upper bound, and thus, from the actual capacity region, for all system parameters.

II. System Model

We consider a two-user energy harvesting broadcast channel, see Fig. 1. The transmitter has a battery of size B. The time is slotted. At each time slot i, E_i units of energy enters the battery, where E_i is an i.i.d. process. The energy available in the battery at time i as b_i. The battery energy level evolves as $b_i = \min\{B, b_{i-1} - P_{i-1} + E_i\}$, where P_{i-1} is the energy of the symbol transmitted in slot $i-1$, and is limited by the amount of energy available in the battery, i.e., $P_{i-1} \leq b_{i-1}$.

The underlying physical layer is a Gaussian broadcast channel, where receiver k has Gaussian noise with variance σ_k^2. Without loss of generality, let $\sigma_1^2 < \sigma_2^2$. The capacity region of this channel in slot i is (see e.g., [5]–[7]):

$$r_{1i} \leq \frac{1}{2}\log\left(1+\frac{\alpha_i P_i}{\sigma_1^2}\right), \quad r_{2i} \leq \frac{1}{2}\log\left(1+\frac{(1-\alpha_i)P_i}{\alpha_i P_i+\sigma_2^2}\right) \quad (1)$$

The boundary of the capacity region is traced by sweeping α_i in $[0,1]$. On the boundary, the transmitted signal is Gaussian with power P_i, where $\alpha_i P_i$ portion of this power is allocated to serve the data of the stronger user, and $(1-\alpha_i)P_i$ is allocated to serve the data of the weaker user. On the boundary of the capacity region where both inequalities in (1) are satisfied with equality, we can write P_i in terms of the rates r_{1i} and r_{2i} as:

$$P_i = \sigma_1^2 e^{2(r_{1i}+r_{2i})} + (\sigma_2^2 - \sigma_1^2)e^{2r_{2i}} - \sigma_2^2 \triangleq g(r_{1i}, r_{2i}) \quad (2)$$

Therefore, $g(r_{1i}, r_{2i})$ is the minimum total power needed to serve users with rates r_{1i} and r_{2i}.

While we will eventually consider an arbitrary i.i.d. energy arrival process E_i in Section IV, initially, we will consider a special i.i.d. energy arrival process in Section III, which is Bernoulli with a particular support set, in particular, $E_i = 0$ with probability $1-p$, and $E_i = B$ with probability p.

III. Optimal Strategy: Case of Bernoulli Arrivals

Since broadcast rate region is convex, we characterize it by determining its tangent lines. Thus, we consider all weighted sum rates of the form $\mu_1 r_1 + \mu_2 r_2$, where μ_1, μ_2 are both in $[0,1]$. The long-term average weighted sum rate is equal to:

$$\lim_{n\to\infty} \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n}(\mu_1 r_{1i} + \mu_2 r_{2i})\right] \quad (3)$$

$$= \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}(\mu_1 r_{1i} + \mu_2 r_{2i})\right] \quad (4)$$

$$= p\sum_{k=1}^{\infty} p(1-p)^{k-1}\sum_{i=1}^{k}(\mu_1 r_{1i} + \mu_2 r_{2i}) \quad (5)$$

$$= \sum_{i=1}^{\infty}\sum_{k=i}^{\infty} p^2(1-p)^{k-1}(\mu_1 r_{1i} + \mu_2 r_{2i}) \quad (6)$$

$$= \sum_{i=1}^{\infty} p(1-p)^{i-1}(\mu_1 r_{1i} + \mu_2 r_{2i}) \quad (7)$$

where L is the inter-arrival time, which is geometric with parameter p. The first equality follows since a non-zero energy arrival resets the system and forms a *renewal*. Thus, we can apply [29, Theorem 3.6.1]. We used $\mathbb{E}[L] = 1/p$ in (5). Hence, the rate allocation problem becomes:

$$\begin{aligned} \max_{\{r_{1i}, r_{2i}\}} \quad & \sum_{i=1}^{\infty} p(1-p)^{i-1}(\mu_1 r_{1i} + \mu_2 r_{2i}) \\ \text{s.t.} \quad & \sum_{i=1}^{\infty} g(r_{1i}, r_{2i}) \leq B, \quad r_{1i}, r_{2i} \geq 0, \quad \forall i \end{aligned} \quad (8)$$

which is a problem only in terms of the rates. In essence, this optimization problem aims to maximize the expected transmitted weighted rate before the next energy arrival.

Here, μ_1 and μ_2 determine the point on the boundary of the capacity region, and the power schedule that achieves it. First, we consider the case where one of the μ_i is zero, say $\mu_2 = 0$. This reduces our broadcast setting into the single-user setting of [26], [27]. In this case, the optimum power is,

$$P_i = \frac{p(1-p)^{i-1}}{\lambda} - \sigma_1^2, \qquad i = 1, \dots, \tilde{N} \quad (9)$$

where $\tilde{N}$ is the last slot where the power is positive, and λ is found from the total power constraint. As noted in [26], [27], the optimum power decreases in time. We also note that it depends on the noise variance, thus, is non-universal, and that the transmission duration $\tilde{N}$ decreases with the noise variance.

Next, we consider the general case $\mu_1 > 0$, $\mu_2 > 0$. The Lagrangian for the problem in (8) is,

$$\begin{aligned} \mathcal{L} = &-\sum_{i=1}^{\infty} p(1-p)^{i-1}(\mu_1 r_{1i} + \mu_2 r_{2i}) - \sum_{i=1}^{\infty}(\nu_{1i} r_{1i} + \nu_{2i} r_{2i}) \\ &+ \lambda\left(\sum_{i=1}^{\infty}\sigma_1^2 e^{2(r_{1i}+r_{2i})} + (\sigma_2^2 - \sigma_1^2)e^{2r_{2i}} - \sigma_2^2 - B\right) \end{aligned} \quad (10)$$

From the KKT optimality conditions together with (2),

$$\begin{aligned} g(r_{1i}, r_{2i}) &= \frac{\mu_2 p(1-p)^{i-1} + \nu_{2i}}{\lambda} - \sigma_2^2 && (11) \\ &\geq \sigma_1^2 e^{2(r_{1i}+r_{2i})} - \sigma_1^2 && (12) \\ &= \frac{\mu_1 p(1-p)^{i-1} + \nu_{1i}}{\lambda} - \sigma_1^2 && (13) \\ &\geq \frac{\mu_1 p(1-p)^{i-1}}{\lambda} - \sigma_1^2 && (14) \end{aligned}$$

where (12) is satisfied with equality when $r_{2i} = 0$, and (14) is satisfied with equality when $r_{1i} > 0$. Thus, when $r_{2i} > 0$,

$$g(r_{1i}, r_{2i}) = \frac{\mu_2 p(1-p)^{i-1}}{\lambda} - \sigma_2^2 > \frac{\mu_1 p(1-p)^{i-1}}{\lambda} - \sigma_1^2 \quad (15)$$

While, when $r_{2i} = 0$ (which implies that $r_{1i} > 0$),

$$g(r_{1i}, r_{2i}) = \frac{\mu_1 p(1-p)^{i-1}}{\lambda} - \sigma_1^2 > \frac{\mu_2 p(1-p)^{i-1}}{\lambda} - \sigma_2^2 \quad (16)$$

Therefore, we have

$$g(r_{1i}, r_{2i}) = \max\left\{\frac{\mu_2 p(1-p)^{i-1}}{\lambda} - \sigma_2^2, \frac{\mu_1 p(1-p)^{i-1}}{\lambda} - \sigma_1^2\right\} \quad (17)$$

Hence, we obtain the general form of the optimum $g(r_{1i}, r_{2i})$, which is the optimum total transmit power, P_i^*.

Next, we solve for the components of the optimum transmit power allocated to serving the two users. From the degradedness of the channel, if $\mu_1 \geq \mu_2$ then $r_{2i}=0$, and $P_{1i}=P_i^*$ and $P_{2i}=0$. When $\mu_1<\mu_2$, the problem in the ith slot is,

$$\max_{\{r_{1i},r_{2i}\}} \quad p(1-p)^{i-1}(\mu_1 r_{1i}+\mu_2 r_{2i})$$
$$\text{s.t.} \quad g(r_{1i},r_{2i}) \leq P_i^*, \quad r_{1i},r_{2i}\geq 0 \tag{18}$$

Using the KKT optimality conditions, we define

$$P_c=\left(\frac{\mu_1\sigma_2^2-\mu_2\sigma_1^2}{\mu_2-\mu_1}\right)^+ \tag{19}$$

and user powers are: $P_{1i}=\min\{P_c,P_i^*\}$, $P_{2i}=P_i^*-P_{1i}$. Here, P_c is the *cut-off* power level, which determines the maximum possible power to allocate to user 1. From (19), if $\mu_2 \geq \frac{\sigma_2^2}{\sigma_1^2}\mu_1$, then $P_c=0$, and hence $P_{1i}=0$ and $P_{2i}=P_i^*$.

From the development in this section, we make the following conclusions: First, the optimum total transmit power, P_i^*, which is given by (17) is decreasing in time, as it is the maximum of two decreasing sequences. Second, P_{1i} is either equal to P_c if $P_i^*\geq P_c$ and thus is constant, or is equal to P_i^* if $P_i^*<P_c$ and thus is decreasing. Hence, P_{1i} is decreasing. Third, P_{2i} is either decreasing or equal to zero; it is decreasing when $P_{1i}=P_c$, and is equal to zero when $P_{1i}=P_i^*$. Note that, when the stronger user's power allocation is strictly decreasing, i.e., when $P_{1i}=P_i^*$, this happens towards the end of the transmission, and during this time the weaker user's power allocation is zero. Hence, there exist numbers $\tilde{M}$ and $\tilde{N}$ with $\tilde{M}<\tilde{N}$ such that the powers allocated to both users are positive for slots $i=1,\ldots,\tilde{M}$ and the power allocated only for the first user is positive for slots $i=\tilde{M}+1,\ldots,\tilde{N}$.

We now give the explicit solution for the optimum power schedule when $\mu_1<\mu_2\leq\mu_1\frac{\sigma_2^2}{\sigma_1^2}$. We need to solve for $\tilde{M}$, $\tilde{N}$ and λ. From (15)-(17), P_i equals the first term in (17) for $i=1,\ldots,\tilde{M}$, and equals the second term for $i=\tilde{M}+1,\ldots,\tilde{N}$. Inserting these into the total power constraint

$$\lambda=\frac{\mu_2-(\mu_2-\mu_1)(1-p)^{\tilde{M}}-\mu_1(1-p)^{\tilde{N}}}{B+\tilde{N}\sigma_1^2+\tilde{M}(\sigma_2^2-\sigma_1^2)} \tag{20}$$

In addition, $\tilde{M}\leq\tilde{N}$ are the smallest integers such that

$$\mu_1 p(1-p)^{\tilde{N}}<\sigma_1^2\lambda, \qquad \frac{\mu_2 p(1-p)^{\tilde{M}}}{\lambda}-\sigma_2^2<P_c \tag{21}$$

Then, (20)-(21) give us the solution which is unique.

IV. Near-Optimal Strategy: General Arrivals

Let $E_i\leq B$ be an arbitrary i.i.d. energy arrival process with average recharge rate $\mathbb{E}[E_i]=\mu$. In this case, finding the *optimal* transmission scheme analytically seems intractable. Nevertheless, we determine a *near-optimal* online policy.

A. Fractional Power Constant Cut-Off (FPCC) Policy

We first define the proposed sub-optimal online policy for Bernoulli energy arrivals, and then generalize it to general energy arrivals. We note that for Bernoulli energy arrivals, the optimal total transmit power allocated decreases exponentially over time. As in [26], [27], this motivates us to construct a fractional total power policy over time. In each slot we allocate a p fraction of the available energy for transmission, i.e., $P_i=Bp(1-p)^{i-1}$ in slot i, which is different from the optimum described in (17), but preserves the *fractional* structure. Next, we follow the exact optimum partition of this sub-optimal total transmit power in all slots among the two users as in around (19). That is, we allocate $P_{1i}=\min\{P_c,Bp(1-p)^{i-1}\}$ for user 1, and $P_{2i}=Bp(1-p)^{i-1}-P_{1i}$ for user 2. For general energy arrivals, we allocate a fraction $q=\mu/B$ of the available energy in the battery for transmission, i.e., $P_i=qb_i$. Then, we partition that power between the two users optimally: $P_{1i}=\min\{P_c,qb_i\}$ for user 1, and $P_{2i}=qb_i-P_{1i}$ for user 2.

B. A Lower Bound on the Proposed Online Policy

We first develop a lower bound for the proposed FPCC policy for the case of Bernoulli arrivals. Let us define a deterministic integer i^* as $i^*\triangleq\max\{i\in\mathbb{N}:P_c\leq Bp(1-p)^{i-1}\}$. If $P_c\leq pB$, then, i^* represents the last slot until which the stronger user's power share is P_c; after i^*, the stronger user gets the entire power. We further define a random variable K as $K\triangleq\min\{i^*,L\}$, where L is a geometric random variable.

Lemma 1 *The achievable rate region with FPCC for any i.i.d. Bernoulli energy arrival process is lower bounded as,*

$$r_1\geq\frac{1}{2}\log\left(1+\frac{\alpha\mu}{\sigma_1^2}\right)-c_1,\ \ r_2\geq\frac{1}{2}\log\left(1+\frac{(1-\alpha)\mu}{\alpha\mu+\sigma_2^2}\right)-c_2 \tag{22}$$

for some $\alpha\in[0,1]$*, where* $c_1=0.72$ *and* $c_2=0.99$*.*

Proof: Due to the sub-optimal fractional power allocation in FPCC, i.e., $P_{1i}=\min\{P_c,Bp(1-p)^{i-1}\}$, if $P_c>Bp$, then $P_c>Bp(1-p)^{i-1}$ for all i, and the stronger user gets all the power, and the system reduces to a single-user system. Using (19), this happens when $\mu_1\geq\frac{\sigma_1^2+Bp}{\sigma_2^2+Bp}\mu_2$. Therefore, we only consider the remaining case, which is $\frac{\sigma_2^2+Bp}{\sigma_1^2+Bp}\mu_1<\mu_2<\frac{\sigma_2^2}{\sigma_1^2}\mu_1$. First, we consider the first user's rate,

$$\begin{aligned} r_1=&\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{K}\frac{1}{2}\log\left(1+\frac{P_c}{\sigma_1^2}\right)\right]\\ &+\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=K+1}^{L}\frac{1}{2}\log\left(1+\frac{Bp(1-p)^{i-1}}{\sigma_1^2}\right)\right] \end{aligned}\tag{23}$$

$$\begin{aligned} \geq&\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{K}\frac{1}{2}\log\left(1+\frac{P_c}{\sigma_1^2}\right)\right]\\ &+\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=K+1}^{L}\frac{1}{2}\log\left(1+\frac{Bp}{\sigma_1^2}\right)+\frac{i-1}{2}\log(1-p)\right] \end{aligned}\tag{24}$$

$$\geq\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\frac{1}{2}\log\left(1+\frac{P_c}{\sigma_1^2}\right)+\sum_{i=1}^{L}\frac{i-1}{2}\log(1-p)\right] \tag{25}$$

$$=\frac{1}{2}\log\left(1+\frac{P_c}{\sigma_1^2}\right)+\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\frac{L(L-1)}{4}\log(1-p)\right] \tag{26}$$

$$\geq\frac{1}{2}\log\left(1+\frac{P_c}{\sigma_1^2}\right)-0.72 \tag{27}$$

where (24) follows because $\log(a+x)$ is monotone in x, (25) follows since $\log(1-p)$ is negative and since $P_c \leq Bp$, and (27) follows by bounding the last term numerically as in [27].

Next, we consider the second user's rate,

$$r_2 = \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{K}\frac{1}{2}\log\left(1+\frac{Bp(1-p)^{i-1}-P_c}{P_c+\sigma_2^2}\right)\right] \quad (28)$$

$$\geq \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{K}\frac{1}{2}\log\left(\frac{(1-p)^{i-1}\left(Bp+\sigma_2^2\right)}{P_c+\sigma_2^2}\right)\right] \quad (29)$$

$$\geq \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{K}\frac{1}{2}\log\left(\frac{Bp+\sigma_2^2}{P_c+\sigma_2^2}\right)+\sum_{i=1}^{L}\frac{i-1}{2}\log(1-p)\right] \quad (30)$$

$$\geq \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{K}\frac{1}{2}\log\left(\frac{Bp+\sigma_2^2}{P_c+\sigma_2^2}\right)\right]-0.72 \quad (31)$$

$$\geq \left(1-\frac{P_c}{pB}\right)\frac{1}{2}\log\left(1+\frac{Bp-P_c}{P_c+\sigma_2^2}\right)-0.72 \quad (32)$$

where (29) follows because $\log(a+x)$ is monotone in x, (30) follows since $\log(1-p)$ is negative, (31) follows by bounding the last term numerically as in [27], and (32) follows since we can show that $\frac{\mathbb{E}[K]}{\mathbb{E}[L]}$ is lower bounded by $\left(1-\frac{P_c}{pB}\right)$.

Since $P_c < Bp$, by substituting $P_c = \alpha pB$ with some $\alpha \in [0,1]$, and denoting $\mu = Bp$, from (27) and (32), we obtain the result for the first user, while for the second user, we have $r_2 \geq \frac{(1-\alpha)}{2}\log\left(1+\frac{(1-\alpha)\mu}{\alpha\mu+\sigma_2^2}\right)-0.72$. Next, we develop a further lower bound for this as follows: since $\frac{\alpha}{2}\log\left(1+\frac{(1-\alpha)\mu}{\alpha\mu+\sigma_2^2}\right)$ increases in μ, we upper bound it with $\frac{\alpha}{2}\log_2\left(\frac{1}{\alpha}\right)$, and upper bound it numerically by 0.265. This gives the result. ■

We next show that i.i.d. Bernoulli energy arrivals give the lowest rate over all i.i.d. energy arrivals with the same mean. The proof follows from the concavity of optimum total rate in [7, Lemma 2] and the approach in [27, Proposition 4].

Lemma 2 *For the FPCC policy, any i.i.d. energy arrival process yields an achievable rate region no smaller than that of the Bernoulli energy arrivals with the same mean.*

Finally, we give a universal lower bound for the proposed FPCC policy under any i.i.d. energy arrival process in the following theorem. The proof of Theorem 1 follows from combining Lemma 1 and Lemma 2.

Theorem 1 *The achievable rate region with FPCC for any arbitrary i.i.d. energy arrival process is lower bounded as,*

$$r_1 \geq \frac{1}{2}\log\left(1+\frac{\alpha\mu}{\sigma_1^2}\right)-c_1,\ \ r_2 \geq \frac{1}{2}\log\left(1+\frac{(1-\alpha)\mu}{\alpha\mu+\sigma_2^2}\right)-c_2 \quad (33)$$

for some $\alpha \in [0,1]$, where $c_1 = 0.72$ and $c_2 = 0.99$.

C. An Upper Bound for Online Policies

Here, we develop an upper bound for the performance of all online scheduling algorithms only in terms of the average recharge rate. The proof follows from upper bounding the online capacity region by the offline case, and letting $B \to \infty$.

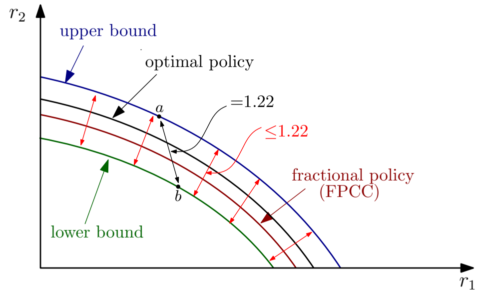

Fig. 2. Illustration of the developed bounds for the broadcast channel.

Theorem 2 *The achievable rate region is upper bounded as,*

$$r_1 \leq \frac{1}{2}\log\left(1+\frac{\alpha\mu}{\sigma_1^2}\right),\ \ r_2 \leq \frac{1}{2}\log\left(1+\frac{(1-\alpha)\mu}{\alpha\mu+\sigma_2^2}\right) \quad (34)$$

for $\alpha \in [0,1]$, where $\mu = \mathbb{E}[E_i]$ is the average recharge rate.

We show the relations developed in Fig. 2. The Euclidean distance between any two points with the same α on the upper and lower bounds is equal to $\sqrt{0.72^2+0.99^2} = 1.22$. Since the distance between the two points with the same α can be no less than the distance between the two bounds, the distance between the two bounds is less than or equal to 1.22. Hence, combining Theorem 1 and Theorem 2, we conclude that the proposed online policy FPCC yields rates which are within a constant gap from the universal upper bound.

V. Numerical Results

We illustrate the results obtained using several numerical examples. We first consider the broadcast channel with i.i.d. Bernoulli arrivals, and find the optimum power allocations: P_i, P_{1i} and P_{2i}. We let $\sigma_1^2 = 1$ and $\sigma_2^2 = 2$, and $B = 2$ and $p = 0.1$. In Figs. 3, we plot the optimum power allocations for a point on the boundary of the capacity region corresponding to $\mu_1 = 1$, $\mu_2 = 1.8$. We note that when the first user's power is constant, the second user's power is decreasing; and when the first user's power is decreasing, the second user's power is zero. In this example, $\tilde{M} = 3$ and $\tilde{N} = 5$.

Next, we consider the achievable rate as a function of the battery size B for a fixed $p = 0.1$ for i.i.d. Bernoulli arrivals. In Fig. 4, we plot weighted rates for $\mu_1 = 1$, $\mu_2 = 1.5$ for the optimal online solution and the sub-optimal FPCC scheme together with the upper bound. We observe that FPCC performs close to the optimal online. In Fig. 5, we plot the entire achievable rate and upper bound regions for the i.i.d. Bernoulli energy arrivals with $B = 5$, $p = 0.5$, and $\sigma_1^2 = 1$, $\sigma_2^2 = 5$. The arrows denote the movement of achievable rate pairs from the optimal policy to FPCC.

Finally, we consider an example of general i.i.d. energy arrivals by considering a (continuous) uniform probability distribution for the energy arrivals in $[0, B]$. Therefore, the average recharge rate is $\mu = B/2$. In Fig. 6, we plot the achievable rate regions with FPCC for this uniform energy arrivals. We also show the achievable rate region with a corresponding Bernoulli arrivals; for this case energies arrive in amounts 0 and B with probabilities $p = 0.5$ and $1-p = 0.5$.

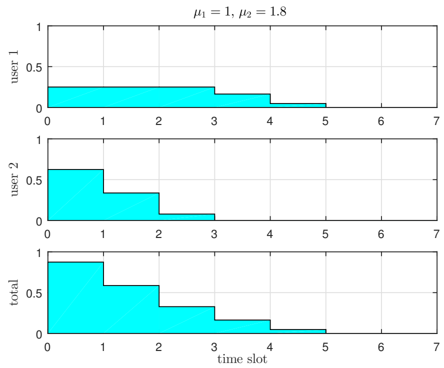

Fig. 3. Optimum online power allocation for i.i.d. Bernoulli arrivals.

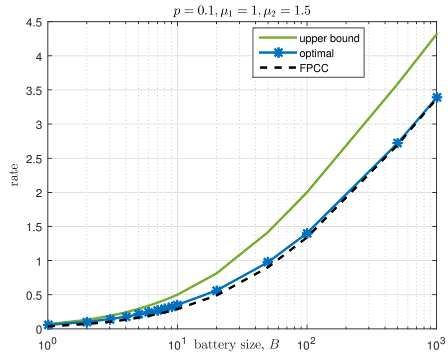

Fig. 4. Achievable weighted rate for i.i.d. Bernoulli arrivals.

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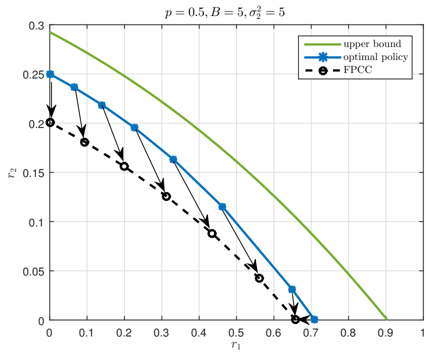

Fig. 5. Achievable rate region for i.i.d. Bernoulli arrivals.

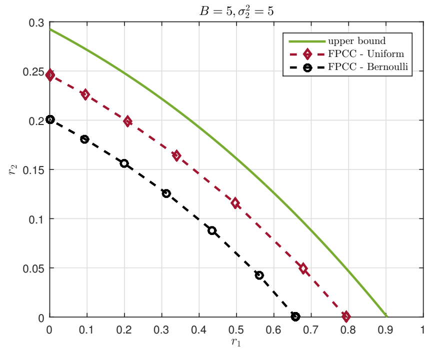

Fig. 6. Achievable rate region for i.i.d. uniform and Bernoulli arrivals.

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