Optimal Scheduling for Energy Harvesting Transmitters under Temperature Constraints

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***Abstract*—Motivated by damage due to heating in sensor operation, we consider the throughput optimal offline data scheduling problem in an energy harvesting transmitter such that resulting temperature increase remains below a critical level. We model the temperature dynamics of the transmitter as a linear system and determine the optimal transmit power policy under such temperature constraints as well as energy harvesting constraints over an AWGN channel. We first derive the structural properties of the solution for the general case with multiple energy arrivals. We, then, obtain closed form solutions for the case of a single energy arrival. We observe that the optimal power policy is piecewise monotone decreasing with possible jumps at the energy harvesting instants, and remains constant after the temperature reaches the critical level.**

I. Introduction

In many wireless sensor applications, temperature increase caused by sensor operation has to be carefully managed. For example, wireless sensors implanted in the human body have to be designed such that the temperature due to its operation does not cause any threat for the metabolism. A line of medical research started by Pennes in 1948 [1] explores the temperature dynamics due to electromagnetic radiation in conjunction with heat losses to the environment and dissipation of heat in the tissue. In the context of sensors that communicate data, depending on the type of tissue there is a critical temperature T_c over which the tissue is irreversibly harmed and hence data transmission has to be carefully scheduled [2]. This problem also arises in other types of body area sensor networks, see e.g., [3], [4] and references therein. Finally, temperature increase in a sensor is a threat for the proper operation of the hardware [5]. In this regard, the electric power that feeds the amplifier circuitry has to be carefully scheduled. In particular, the temperature should not exceed a critical level T_c so as to avoid permanent damage in the circuit and the environment.

In this paper, we consider data transmission with energy harvesting sensors under such temperature constraints. Data transmission with energy harvesting transmitters has been the topic of recent research [6]–[9]. In particular, throughput maximization under offline and online knowledge of the energy arrivals is considered in these references. In [10]–[14], this problem is investigated under imperfections such as battery energy leakage, charge/discharge inefficiency, and presence of processing cost.

When the sole purpose is to maximize the throughput, the transmitter may generate excessive heat while utilizing the energy resource. In a temperature sensitive application, the heat accumulation caused by the transmission power policy has to be explicitly taken into account. In such a case, heat generated in the transmitter circuitry causes a form of "information-friction" [15]. We study the effect of this "friction" in a deadline constrained communication of an energy harvesting transmitter over an AWGN channel. Our formulation also relates to [16] in that the cumulative effect of heat generated in the hardware affects the communication performance.

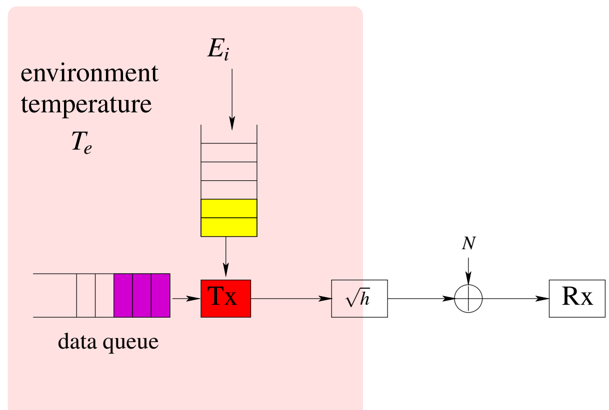

Fig. 1. The model representing an energy harvesting wireless node placed in an environment that has constant temperature T_e.

We determine the throughput optimal offline power scheduling policy under energy harvesting and temperature constraints. Our thermal model is based on a view of the transmitter's circuitry as a linear heat system where transmit power is an input as in [1], [5]. We impose that the temperature does not exceed a critical level T_c. Consequently, we obtain a convex optimization problem. We solve this problem using a Lagrangian framework and KKT optimality conditions. We first derive the structural properties of the solution for the general case of multiple energy arrivals. Then, we obtain closed form solutions under a single energy arrival. For the general case, we observe that the optimal power policy may make jumps at the energy arrival instants, generalizing the optimal policies in [6], [7]. Between energy harvests, the optimal power is monotonically decreasing. For the case of a single energy arrival, we show that the optimal power policy monotonically decreases, corresponding temperature monotonically increases and both remain constant when the critical temperature is reached.

II. The Model

We consider an energy harvesting transmitter node placed in an environment as depicted in Fig. 1. The node harvests energy to run its circuitry and wirelessly send data to a receiver.

A. Channel Model

The received signal Y, the input X, fading level h and noise N are related as $Y = \sqrt{h}X + N$ where N is additive white Gaussian noise with zero-mean and unit-variance. In this paper, channel is non-fading, i.e., $h = 1$. We use a continuous time model: A scheduling interval has a short duration with respect to duration of transmission and we approximate it as $[t, t + dt]$ where dt denotes infinitesimal time. In $[t, t + dt]$, transmitter decides a feasible transmit power level $P(t)$ and $\frac{1}{2}\log(1 + P(t))\, dt$ units of data is sent to the receiver[1].

B. Energy Harvesting Model

As shown in Fig. 2, the initial energy available in the battery at time zero is E_0. Energy arrivals occur at times $\{s_1, s_2, \dots\}$ in amounts $\{E_1, E_2, \dots\}$. We call the time interval between two consecutive energy arrivals an *epoch*. D is the deadline. E_i and s_i are known offline and are not affected by the heat due to transmission. Let $h(t) = \max\{k : s_k < t\}$. Power scheduling policy $P(t)$ is subject to energy causality constraints as:

$$\int_0^t P(\tau)d\tau \leq \sum_{i=0}^{h(t)} E_i, \qquad \forall t \in [0, D] \tag{1}$$

C. Thermal Model

In our thermal model, we use the transmit power as a measure of heat dissipated to the environment. In particular, we model the temperature dynamics of the system as follows:

$$\frac{dT}{dt} = aP(t) - b(T(t) - T_e) + c \tag{2}$$

where $P(t)$ is the transmit power policy and $T(t)$ is the temperature at time t. T_e is the temperature of the environment, a and b are nonnegative constants. c represents the cumulative effect of additional heat sources and sinks and it can take both positive and negative values. In the following, we consider the case of no extra heat source or sink, i.e., $c = 0$.

Our thermal model in (2) is intimately related to that in [5] where hardware heating is modeled as a first order heat circuit. In particular, thermal dynamics of a power controlled transmitter due to its amplifier power consumption (see e.g., [17]) could be modeled as in (2). Our thermal model is also related to the well known Pennes bioheat equation [1]. We assume, for simplicity, that the spatial variation in temperature is not significant and leave the general case as future work.

The solution of $T(t)$ for any given $P(t)$ from (2) with the initial condition of $T(t')$ at time t' is:

$$T(t) = e^{-b(t-t')}\left(\int_{t'}^t e^{b(\tau-t')}\left(aP(\tau) + bT_e\right)d\tau + T(t')\right) \tag{3}$$

[1]To be precise, underlying physical signaling is in discrete time and the scalings in SNR and rate due to bandwidth are inconsequential for the analysis.

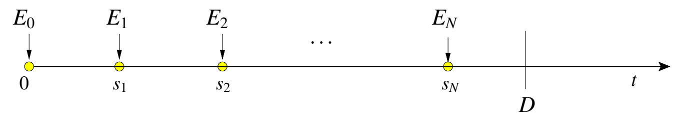

Fig. 2. Energy E_i becomes available for data transmission at time s_i.

Typically, initial temperature is T_e, i.e., initially the temperature is stabilized at the constant environment temperature T_e. Then, by inserting $t' = 0$ and $T(t') = T_e$ in (3), we get:

$$T(t) = e^{-bt}\left(\int_0^t e^{b\tau}\left(aP(\tau) + bT_e\right)d\tau + T_e\right) \tag{4}$$

The temperature should remain below a critical temperature T_c, i.e., $T(t) \leq T_c$, where we assume that $T_c > T_e$. Let us define $T_\delta \triangleq T_c - T_e$. From (4), using $T(t) \leq T_c$, we get the following equivalent condition for the temperature constraint:

$$\int_0^t ae^{b\tau}P(\tau)d\tau \leq T_\delta e^{bt}, \quad \forall t \in [0, D] \tag{5}$$

Note that the temperature constraints in (5) and the energy causality constraints in (1) do not interact.

III. Problem Formulation

Offline throughput maximization problem over the interval $[0, D]$ under energy causality and temperature constraints is:

$$\begin{aligned} \max_{P(t), t\in[0,D]} \quad & \int_0^D \frac{1}{2}\log\left(1 + P(\tau)\right)d\tau \\ \text{s.t.} \quad & \int_0^t ae^{b\tau}P(\tau)d\tau \leq T_\delta e^{bt}, \ \forall t \\ & \int_0^t P(\tau)d\tau \leq \sum_{i=0}^{h(t)} E_i, \qquad \forall t \end{aligned} \tag{6}$$

(6) is a convex optimization problem and the Lagrangian is:

$$\begin{aligned} \mathcal{L} = & \int_0^D \frac{1}{2}\log\left(1 + P(t)\right)dt \\ & - \int_0^D \lambda(t)\left(\int_0^t ae^{b\tau}P(\tau)d\tau - T_\delta e^{bt}\right)dt \\ & - \int_0^D \beta(t)\left(\int_0^t P(\tau)d\tau - \sum_{i=0}^{h(t)} E_i\right)dt \end{aligned} \tag{7}$$

Taking the derivative of the Lagrangian with respect to $P(t)$ and equating to zero:

$$\frac{1}{1 + P(t)} - e^{bt}\int_t^D \lambda(\tau)d\tau - \int_t^D \beta(\tau)d\tau = 0 \tag{8}$$

which gives

$$P(t) = \left[\frac{1}{\int_t^D \beta(\tau)d\tau + e^{bt}\int_t^D \lambda(\tau)d\tau} - 1\right]^+ \tag{9}$$

In addition, the complementary slackness conditions are:

$$\lambda(t)\left(\int_0^t ae^{b\tau}P(\tau)d\tau - T_\delta e^{bt}\right) = 0 \tag{10}$$

$$\beta(t)\left(\int_0^t P(\tau)d\tau - \sum_{i=0}^{h(t)} E_i\right) = 0 \quad (11)$$

We note that (8) and (10)-(11) are necessary and sufficient conditions since the problem is convex. The solution is unique almost everywhere as the objective function is strictly concave.

IV. Properties of an Optimal Policy

In this section, we obtain structural properties of the optimal power scheduling policy. In the following lemmas, $P(t)$ refers to the optimal policy and $T(t)$ is the resulting temperature.

Lemma 1 *At $t = D$, either the temperature constraint or the energy causality constraint or both are tight.*

Proof: If neither of the constraints are tight, then the power policy $P(t)$ could be increased over a set of non-zero Lebesgue measure in the last epoch. This strictly increases the throughput, contradicting the optimality. ■

Lemma 2 *$P(t)$ is piecewise monotone decreasing except possibly at the energy arrival instants.*

Proof: We prove the statement by contradiction. Assume for some $t_1 < t_2$, $P(\tilde{t}_1) < P(\tilde{t}_2)$ for all $\tilde{t}_1$, $\tilde{t}_2$ in a sufficiently small neighborhood around t_1 and t_2, respectively, and that the interval $[t_1, t_2]$ does not contain any energy arrival instant. Define a new power policy $P_{new}(t)$ by replacing $P(\tilde{t}_1)$ with $P(\tilde{t}_2)$, and vice versa. Now, we have $P_{new}(\tilde{t}_1) > P_{new}(\tilde{t}_2)$ for all $\tilde{t}_1$, $\tilde{t}_2$ in corresponding neighborhoods. $P_{new}(t)$ satisfies the energy causality and temperature constraints in (6) as the energy constraint is not tight in this interval and the coefficient that multiplies $P(t)$ in the temperature constraint is strictly monotone increasing in t. In addition, $P_{new}(t)$ yields the same throughput as $P(t)$. The extra temperature margin could be used by decreasing $P_{new}(t)$ around t_1 while increasing it around t_2. This increases the throughput due to the concavity of logarithm. This contradicts the optimality of $P(t)$. The proof holds even when $[t_1, t_2]$ includes an energy arrival instant and if the energy causality constraint is not tight at that instant. ■

Lemma 3 *If there is a discontinuity in $P(t)$, it is a positive jump and it occurs only at the energy arrival instants.*

Proof: Since e^{bt} is a continuous function of t, $\lambda(t) \geq 0$ and $\beta(t) \geq 0$, any jump in $P(t)$ has to be positive due to (9). Any positive jump at instants other than s_k violates monotonicity of $P(t)$ within each epoch due to Lemma 2. ■

Lemma 4 *Let $T(t') = T_c$ for some $t' \in [0, D)$. Then, $P(t' - \epsilon) \geq \frac{T_\delta b}{a}$ for all sufficiently small $\epsilon > 0$.*

Proof: Since $T(t') = T_c$, we have:

$$\int_0^{t'} ae^{b\tau} P(\tau)d\tau = T_\delta e^{bt'} \quad (12)$$

We combine (5) with (12) to get

$$\int_t^{t'} ae^{b\tau} P(\tau)d\tau \geq T_\delta\left(e^{bt'} - e^{bt}\right), \qquad \forall t \in [0, t'] \quad (13)$$

which implies in view of continuity of $P(t)$ proved in Lemma 3 that $P(t' - \epsilon) \geq \frac{T_\delta b}{a}$ for all sufficiently small $\epsilon > 0$. ■

V. Optimal Policy in Single Energy Arrival Case

In this section, we consider a single epoch where E units of energy is available at the transmitter in the beginning.

A. Properties of an Optimal Policy

Lemma 5 *If $0 < P(t) \leq \frac{T_\delta b}{a}$ for $t \in [t_1, D]$ then $P(t)$ is constant over $[t_1, D]$.*

Proof: Assume $P(t)$ is not constant over $[t_1, D]$. Let $E_r = \int_{t_1}^D P(\tau)d\tau > 0$. Define a new policy $P_{new}(t) = \frac{E_r}{D-t_1}$ for $t \in [t_1, D]$ and $P_{new}(t) = P(t)$ otherwise. $P_{new}(t)$ is both energy and temperature feasible. Energy feasibility holds by construction as P_{new} and P have the same energy over $[t_1, D]$. Temperature feasibility also holds: $T(t_1) \leq T_c$ since $P(t)$ is temperature feasible and as $\frac{E_r}{D-t_1} < \frac{T_\delta b}{a}$, we have $T(t) \leq T_c$ for all $t_1 < t < D$ in view of (5). Now, by Jensen's inequality $P_{new}(t)$ achieves strictly larger throughput since log is strictly concave. This contradicts the optimality of $P(t)$. Hence, $P(t) = c > 0$ for $t \in [t_1, D]$. ■

Lemma 6 *Let $t' \in [0, D]$ denote $\min\{t \in [0, D] : T(t) = T_c\}$. If $t' \neq D$, then $P(t) = \frac{T_\delta b}{a}$ for all $t \in [t', D]$.*

Proof: First, assume energy is sufficiently large and initial condition is $T(t') = T_c$. From (3) with $T(t') = T_c$, the constraint $T(t) \leq T_c$ becomes:

$$\int_{t'}^{t} ae^{b\tau} P(\tau)d\tau \leq T_\delta\left(e^{bt} - e^{bt'}\right), \quad t' < t \leq D \quad (14)$$

$P(t) = \frac{T_\delta b}{a}$ satisfies (14) with equality for all t. Due to the concavity of log, $P(t) = \frac{T_\delta b}{a}$ for $t' < t \leq s_k$ is optimal if it is energy feasible. If it is not energy feasible, $P(t) = c < \frac{T_\delta b}{a}$ for $t' < t \leq D$ is optimal. Hence, $P(t) = c \leq \frac{T_\delta b}{a}$ for $t \in [t', D]$.

By Lemma 4, $P(t' - \epsilon) \geq \frac{T_\delta b}{a}$ for all sufficiently small $\epsilon > 0$. Moreover, we have just shown that $P(t) = c \leq \frac{T_\delta b}{a}$ for $t \in [t', D]$. Since there cannot be a negative jump in $P(t)$ by Lemma 3, we get $P(t) = \frac{T_\delta b}{a}$ over the interval $[t', D]$. ■

Lemma 7 *The optimal policy $P(t)$ satisfies:*

$$P(t) \geq \min\left\{\frac{T_\delta b}{a}, \frac{E}{D}\right\}, \quad \forall t \in [0, D] \quad (15)$$

Proof: If temperature constraint is not tight, then the problem reduces to the energy constrained problem in which case $P(t) = \frac{E}{D}$. If temperature constraint is tight, $P(t)$ is monotone decreasing by Lemma 2 and when temperature level reaches T_c, $P(t)$ remains at $\frac{T_\delta b}{a}$ by Lemma 6. Hence, $P(t) \geq \frac{T_\delta b}{a}$. ■

Lemma 8 *In an optimal policy, energy in the battery is non-zero except possibly at $t = D$.*

Proof: By Lemma 7, optimal power is larger than a positive constant. Thus, battery energy does not drop to zero. ■

Lemma 9 *The temperature with the optimal power policy is monotone increasing and concave.*

Proof: If temperature constraint is never tight, the optimal power level is $\frac{E}{D}$ and from (4), temperature is monotone increasing. Now, assume the temperature constraint is tight at $t = D$. By Lemma 7, $P(t) \geq \frac{T_\delta b}{a}$. From (2), we have:

$$\frac{dT}{dt} = aP(t) - b\left(T(t) - T_e\right) \tag{16}$$
$$\geq a\frac{T_\delta b}{a} - b\left(T(t) - T_e\right) \tag{17}$$
$$= b\left(T_c - T(t)\right) \geq 0 \tag{18}$$

As $P(t)$ is monotone decreasing by Lemma 2 and $T(t)$ is monotone increasing, from (16), $\frac{dT}{dt}$ is monotone decreasing, proving the concavity of $T(t)$. ■

B. Optimal Policy

In view of Lemma 8, the energy constraint can be tight only at $t = D$. Therefore, corresponding Lagrange multiplier is a single variable $\beta(t) = \beta\delta(t - D)$. From Lemma 9, $T(t)$ is monotone increasing. Due to Lemma 6, when $T(t)$ reaches T_c, power level has to remain at $\frac{T_\delta a}{b}$. Accordingly, we denote the instant when the temperature reaches T_c as t_0.

1) Sufficiently Large Energy: In this case $\beta = 0$. In view of Lemma 1, the temperature constraint is tight at $t = D$. We also let D be sufficiently large so that there exists $t_0 < D$ such that $T(t_0) = T_c$. For $t \in [0, t_0)$, $T(t) < T_c$ and from (10), $\lambda(t) = 0$. From (9), when $t \in [0, t_0)$ we have $P(t) = Ce^{-bt} - 1$ where $C = \frac{1}{\int_{t_0}^{D} \lambda(\tau)d\tau} > 0$. Since $P(t) \geq \frac{T_\delta b}{a}$ for $t \in [t_0 - \epsilon, t_0]$ due to Lemma 4, and $P(t)$ is continuous due to Lemma 3, it has the following form:

$$P(t) = \left(\left(\frac{T_\delta b}{a} + 1\right)e^{-b(t-t_0)} - 1\right)\left(u(t) - u(t - t_0)\right) + \frac{T_\delta b}{a}u(t - t_0) \tag{19}$$

where $u(t)$ is the unit step function. The following Lagrange multiplier $\lambda(t)$ verifies (19):

$$\lambda(t) = \frac{b}{\left(\frac{T_\delta b}{a} + 1\right)}e^{-b(t-t_0)}u(t - t_0) + \frac{1}{\left(\frac{T_\delta b}{a} + 1\right)}\delta(t - D) \tag{20}$$

Corresponding optimal temperature pattern for $0 \leq t \leq t_0$ is:

$$T(t) = a\left(\frac{T_\delta b}{a} + 1\right)te^{-b(t-t_0)} + \frac{a}{b}e^{-bt} - \frac{a}{b} + T_e \tag{21}$$

and $T(t) = T_c$ for $t_0 \leq t \leq D$. We note that t_0 satisfies:

$$\left(\frac{T_\delta}{a} + \frac{1}{b}\right)e^{bt_0} - \frac{1}{b} = \left(\frac{T_\delta b}{a} + 1\right)t_0 e^{bt_0} \tag{22}$$

so that $T(t_0) = T_c$. Hence, $T(t)$ monotonically increases till it reaches T_c, which is consistent with Lemma 9. If $D < t_0$:

$$P(t) = Ce^{-bt} - 1 \tag{23}$$

where $C = \frac{1}{D}\left(\left(\frac{T_\delta}{a} + \frac{1}{b}\right)e^{bD} - \frac{1}{b}\right)$ and $\lambda(t) = \frac{1}{C}\delta(t - D)$.

2) Energy Limited Case: Note that the optimal power policies in the energy unconstrained cases in (19) and (23) have finite energies. If the available energy E is larger than corresponding energy level in (19) and (23), then the solution is as in (19) and (23). Otherwise, the energy constraint is active and the Lagrange multiplier is $\beta > 0$. From (9), we have:

$$P(t) = \frac{1}{\beta + e^{bt}\int_t^D \lambda(\tau)d\tau} - 1 \tag{24}$$

We first note that there is a critical energy level $E_{critical}$ such that if $E \leq E_{critical}$, then constant power policy $P(t) = \frac{E}{D}$ is optimal. This critical level is:

$$E_{critical} = \frac{bDe^{bD}T_\delta}{a\left(e^{bD} - 1\right)} \tag{25}$$

When $E \leq E_{critical}$, $\lambda(t) = 0$ since temperature constraint is never tight. In this case, $\beta = \frac{1}{\frac{E}{D}+1}$. $E_{critical}$ is the maximum energy level for which a constant power level is optimal. If $P(t) = \frac{E_{critical}}{D}$, $T(t)$ is monotone increasing over $[0, D]$ and reaches T_c at $t = D$. If $E > E_{critical}$, the constant power level $\frac{E_{critical}}{D}$ does not satisfy the temperature constraint. In the following, we consider $E > E_{critical}$.

Due to (10), $\lambda(t) = 0$ for $t \in [0, t_0)$ and from (9), we get:

$$P(t) = \frac{1}{\beta + Ce^{bt}} - 1 \tag{26}$$

where $C = \int_{t_0}^{D} \lambda(\tau)d\tau > 0$. Additionally, $P(t) = \frac{T_\delta b}{a}$ for the remaining portion of the epoch in view of Lemma 6. t_0 is such that for $t > t_0$, $P(t) = \frac{T_\delta b}{a}$ and $T(t_0) = T_c$. Since $P(t_0) = \frac{T_\delta b}{a}$ we have:

$$\frac{1}{\beta + Ce^{bt_0}} = \frac{T_\delta b}{a} + 1 \tag{27}$$

Similarly, for $T(t_0) = T_c$, we have:

$$e^{-bt_0}\left(\int_0^{t_0} e^{bt}\left(a\left(\frac{1}{\beta + Ce^{bt}} - 1\right) + bT_e\right)dt + T_e\right) = T_c \tag{28}$$

Finally, the energy constraint has to be satisfied at $t = D$:

$$\int_0^{t_0}\left(\frac{1}{\beta + Ce^{bt}} - 1\right)dt + \frac{T_\delta b}{a}\left(D - t_0\right) = E \tag{29}$$

If there exists $t_0 \leq D$ for (27)-(29), then $P(t)$ is:

$$P(t) = \left(\frac{1}{\beta + Ce^{bt}} - 1\right)\left(u(t) - u(t - t_0)\right) + \frac{T_\delta b}{a}u(t - t_0) \tag{30}$$

In this case, corresponding Lagrange multiplier is:

$$\lambda(t) = bCe^{-b(t-t_0)}u(t - t_0) + C\delta(t - D) \tag{31}$$

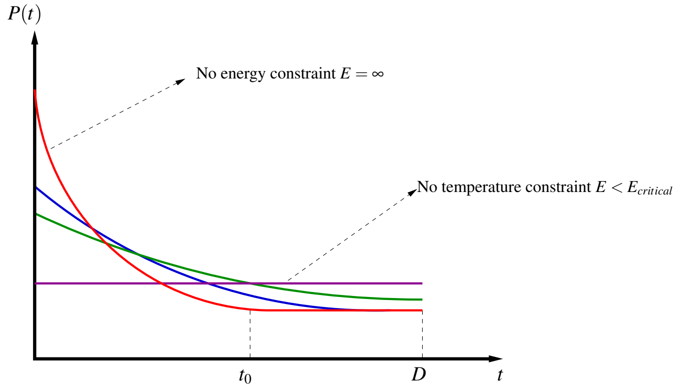

Fig. 3. Variation of the optimal policy with respect to energy constraint E.

Otherwise, $P(t)$ is as in (26) for $t \in [0, D]$ where β and C have to satisfy:

$$e^{-bD}\left(\int_0^D e^{bt}\left(a\left(\frac{1}{\beta + Ce^{bt}} - 1\right) + bT_e\right)dt + T_e\right) = T_c \tag{32}$$

$$\int_0^D \left(\frac{1}{\beta + Ce^{bt}} - 1\right)dt = E \tag{33}$$

Corresponding Lagrange multiplier is $\lambda(t) = C\delta(t - D)$.

Depending on the energy E and the deadline D, the optimal power scheduling policy $P(t)$ varies according to the plots in Fig. 3. For small E, a constant power policy is optimal. For moderate and large E, the optimal power policy is exponentially decreasing and may hit the power level $\frac{T_\delta b}{a}$.

We provide a numerical example in Fig. 4 where we set $a = 0.1$, $b = 0.3$, $T_e = 37$ and $T_c = 38$. Therefore, the critical power level is $\frac{T_\delta b}{a} = 3$. We set the deadline to $D = 3.5$ and the energy limit to $E = 17.71$. In this case, we calculate $t_0 = 3.2$ as the solution of (27)-(29). Power level drops to $\frac{T_\delta b}{a}$ at time t_0 and remains constant afterwards.

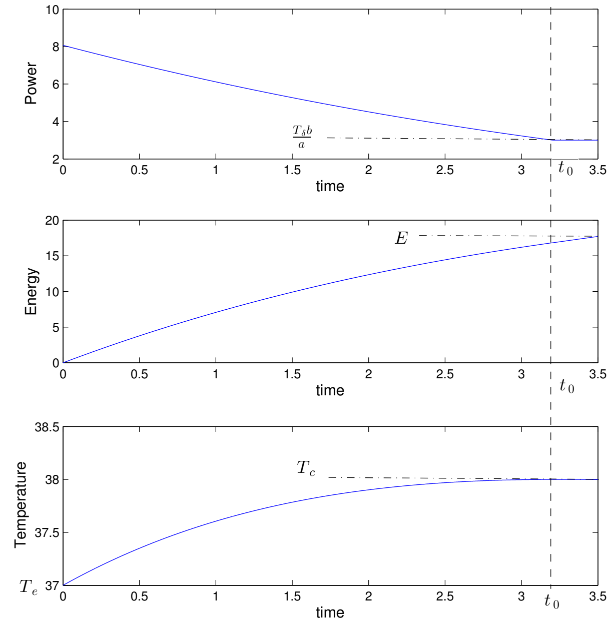

Fig. 4. Plots for limited energy $E = 17.71$ and $D = 3.5$.

VI. Conclusion

We considered throughput maximization for energy harvesting transmitters over AWGN channel under temperature constraints. We used a linear system model for the heat dynamics and determined the throughput optimal power policy. We observed that the optimal power policy is piecewise monotone decreasing with possible jumps at the energy harvesting instants, whereas the temperature is monotonically increasing.

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