The Binary Energy Harvesting Channel with On-Off Fading

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***Abstract*—A noiseless binary energy harvesting channel with on-off fading is considered. When causal fading state information is available at the transmitter only, an equivalent timing channel with additive geometric noise and noise information known at the transmitter is obtained. In this channel, the transmitter's strategy is a stopping rule with respect to the channel fade levels given the message and the additive noise. Next, capacity when energy arrival information is available at the receiver and capacity when both energy arrival and fading information are available at the receiver are obtained. Additionally, several achievable schemes are proposed and evaluated.**

I. Introduction

We consider a noiseless binary channel with an energy harvesting transmitter with unit battery. The channel is on-off fading as shown in Fig. 1, that is, the channel nulls out the transmitted symbol if it is in the off state. The energy of each transmitted symbol is constrained by the energy available in the battery in that channel use. The energy arrival and fading processes are independent and identically distributed (i.i.d.) in time and independent of the message. The transmitter has causal knowledge of the channel fading state. This model is a generalization of the model in [1], [2] with the presence of on-off fading. In [1], binary energy harvesting channel is introduced and achievable strategies based on a timing channel are proposed along with upper bounds. In [2], bounds and schemes are improved.

Related work to this paper includes [3]–[9] in addition to [1], [2]. Reference [3] has considered the discrete memorlyess energy harvesting channel with finite battery and obtained n-letter expressions. References [4]–[7] consider the Gaussian energy harvesting channel. When the battery size is unlimited, [4] has shown that the capacity is equal to the capacity of the same system with an average power constraint equal to the average recharge rate. On the other hand, when the battery size is zero, [5] has shown that the capacity is achieved by using Shannon strategies [10]. More recently, [6] has characterized the capacity in the Gaussian setting for the finite battery regime when energy arrivals are deterministic and [7] has determined the capacity within a certain gap when energy arrivals are binary. Other recent works include capacity in the presence of receiver side battery [8] and energy arrival [9] information.

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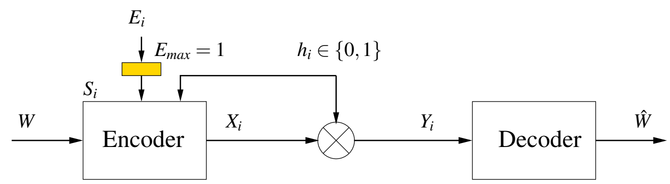

Fig. 1. The binary energy harvesting channel with on-off fading. Fading state information is causally available at the transmitter only.

In this paper, we build on [1], [2] and study the binary energy harvesting channel with on-off fading. In essence, on-off fading introduces the need to balance the impact of the channel impairment with that of the potential energy shortage. On the one extreme when channel is non-fading, optimal coding is performed by i.i.d. Shannon strategies in a timing channel to combat the effect of the timing noise related to the randomness in the battery energy level [1]. On the other extreme when energy is always available, optimal coding is by i.i.d. Shannon strategies in the classical state dependent channel [10]. The pattern dependence in a timing channel conflicts with the memoryless Shannon strategies in the classical state dependent channel. We address this inherent conflict to incorporate the effects of channel fading and energy uncertainty.

Since the fading state information is available at the transmitter causally, the decision to transmit an energy carrying symbol is taken at a channel use when fading state is on. We show that this channel is equivalent to a timing channel [11] with additive geometric noise and noise information at the transmitter. In this timing channel, the transmitter's strategy is a stopping time[1] with respect to the channel fade levels given the message and the additive noise. We determine the capacity of the equivalent timing channel using an auxiliary random variable. Next, we determine the capacity when energy arrival information is available at the receiver and the capacity when both energy arrival and fading information are available at the receiver. In addition, we propose a finite cardinality timing based achievable scheme and compare its performance with rates achievable by Shannon strategies in the classical setting [3]. Through numerical evaluations, we provide performance comparisons and discuss the value of side information in various energy and fading regimes.

[1]See [12, Definition 1] for the definition of a stopping time.

II. The Channel Model

We consider an energy harvesting transmitter, with i.i.d. binary energy arrivals, i.e., $E_i \in \{0,1\}$, $i = 1, 2, \ldots$ with $\Pr[E_i = 1] = q$. The battery in the transmitter can store at most one unit of energy. Input alphabet is also binary, i.e., $X_i \in \{0,1\}$. Symbol 1 requires one unit energy while symbol 0 needs no energy. Underlying physical channel is noiseless with a multiplicative fading $h_i \in \{0,1\}$. That is, the input X_i and received signal Y_i are related as:

$$Y_i = h_i X_i \tag{1}$$

where $h_i \in \{0,1\}$ is i.i.d. Bernoulli process with $\Pr[h_i = 1] = p_{on}$. Channel state information h_i and the energy arrival are available at the transmitter causally. In Section III, we assume that the receiver has no state information. In Section IV, the receiver has the energy arrival information while in Section V, the receiver has both energy arrival and fading state information.

At each channel use, the transmitter can harvest energy and transmit a symbol. The order of harvesting and transmission in a channel use is as follows: S_i denotes the energy available in the battery at channel use i. The transmitter observes the available battery energy S_i and the channel fade level h_i before transmitting a symbol X_i. The energy of this symbol is constrained by the battery energy. If $S_i = 0$, only 0 is feasible. If $S_i = 1$, both symbols are feasible. In this regard, X_i is a function of the past and current states such that it is feasible with respect to S_i: $X_i = f_i(W, h^i, S^i)$. After sending the symbol, the transmitter harvests energy. Incoming energy E_i is first stored in the battery, if there is space, before it is used for transmission. Next battery state is expressed as:

$$S_{i+1} = \min\{S_i - X_i + E_i, 1\} \tag{2}$$

We determine the channel capacity in the timing domain [1].

III. The Channel Capacity

The receiver can decode by calculating the length of intervals between two 1s in the received sequence, as in [1]. Since the transmitter has causal fading state information, it puts symbol 1 when fading coefficient is 1. In particular, the transmitter observes the time length Z_i between the events 1 is transmitted and next unit of energy arrives. During this time, battery is empty and the channel input is 0. After the energy arrival, transmitter observes the fading levels and decides to put the 1 symbol at the V_ith channel use. This yields:

$$T_i = V_i + Z_i \tag{3}$$

where Z_i is additive geometric noise with support set $\{0, 1, \ldots\}$ and parameter q, representing the waiting time of the transmitter for energy to enter the system. V_i represents the time the transmitter decides to put the energy carrying symbol. We denote the sequence of fading over the V_i portion of the ith channel use as $\boldsymbol{G}_i = \{h_k,\ k = \sum_{j=1}^{i-1} t_j + z_i + 1, \ldots, \sum_{j=1}^{i} t_j\}$. The variables that define a channel use in this equivalent timing channel are shown in Fig. 2.

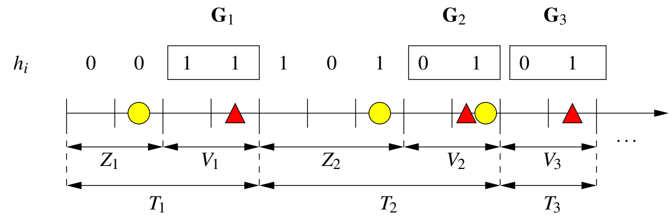

Fig. 2. The variables V_i, Z_i, $\boldsymbol{G}_i$ and T_i in the timing channel.

In the equivalent channel, the channel input is V and the channel output is T. Similar to the channel model in [1], additive noise Z is available at the transmitter before deciding V; that is, Z is a channel state which is causally available to the transmitter. Note that, irrespective of fading state, channel input is zero when battery is empty. In addition to the model in [1], the input V is determined based on the causal observation of the fading process after Z is obtained. That is, V is a deterministic stopping time with respect to the fading process given Z. For completeness, we provide a simplified definition of a deterministic stopping time, c.f. [12, Definition 1]:

Definition 1 *An integer random variable R is a stopping time with respect to a random process $\{L_i\}$ if the event $\{R = r\}$ is determined by the random variables $L_1, L_2, \ldots, L_r$. The stopping time R is called deterministic if $Pr[R = r | L_1 = \ell_1, L_2 = \ell_2, \ldots, L_r = \ell_r] \in \{0,1\}$.*

Due to the causal observation of the fading pattern and the noise in the timing channel (3), the transmitter decides whether $V = v$ based on $h_{z+1}, h_{z+2}, \ldots, h_{z+v} \in \boldsymbol{G}$ given $Z = z$. Therefore, the channel input V in the timing channel (3) is a stopping time with respect to the fading process $\{h_i\}$ given Z. Indeed, it could be taken as a deterministic stopping time without losing optimality since extra randomness in encoding does not increase achievable rate.

As the transmitter has causal fading state information, energy carrying symbol 1 is released from the energy queue only when channel fading is in the on state. Hence, capacities of the channel described in Section II and the timing channel are equal; see also [1, Lemma 1]. This follows from the fact that the encoders and decoders use different representations of the same object (except when the battery is empty) and the rates in the channels are defined according to the time cost of codewords. Since the channel output is 0 whenever the battery is empty and since fading process is i.i.d., use of fading state information when battery is empty does not increase capacity.

Lemma 1 *The timing channel capacity with additive causally known noise and causally observed fading pattern is equal to the capacity of the classical state dependent channel.*

Note that the channel index in which energy carrying 1 symbol is released is determined based on the progression of the fading process after 1 unit energy arrives, whereas in [1] this decision is taken as soon as 1 unit energy arrives. In the following theorem, we generalize [1, Theorem 1] and provide a characterization of the capacity.

Theorem 1 *The timing channel capacity is:*

$$C = \max_{p(u),v(u,z,\boldsymbol{g})} \frac{I(U;T)}{E[T]} \quad (4)$$

where $v(u,z,\boldsymbol{g})$ is a deterministic stopping time with respect to the fading process for fixed u, z.

Proof: Let W be uniform over $\{1,\ldots,M\}$. Transmitter has W, T^{i-1}, Z^i, $\boldsymbol{G}^{i-1}$ and it causally observes the fading process to determine V_i. We define $U_i = (W, T^{i-1}, Z^{i-1}, \boldsymbol{G}^{i-1})$. Note that U_i is independent of Z_i and the fading process in the ith channel use and that V_i is a stopping time with respect to the fading given Z_i and U_i. We have:

$$\begin{aligned}
\log(M) - H(W|T^m) &= H(W) - H(W|T^m) && (5)\\
&= I(W;T^m) && (6)\\
&= \sum_{i=1}^m I(W;T_i|T^{i-1}) && (7)\\
&\leq \sum_{i=1}^m I(W,T^{i-1};T_i) && (8)\\
&\leq \sum_{i=1}^m I(W,T^{i-1},Z^{i-1},\boldsymbol{G}^{i-1};T_i) && (9)\\
&\leq \frac{n}{\sum_{i=1}^m E[T_i]} \sum_{i=1}^m I(U_i;T_i) && (10)\\
&\leq n \max_{p(u),v(u,z,\boldsymbol{g})} \frac{I(U;T)}{E[T]} = nC && (11)
\end{aligned}$$

where (8) and (9) follow from the nonnegativity of mutual information and (10) holds due to the fact that $\sum_{i=1}^m E[T_i] \leq n$. Finally, (11) follows from the facts that U_i is independent of Z_i and the fading process in the ith channel use and $\frac{\sum_i a_i}{\sum_i b_i} \leq \max_i \frac{a_i}{b_i}$ for $a_i, b_i > 0$. Here, $v(u,z,\boldsymbol{g})$ denotes the deterministic stopping time of the transmitter given u and z. $H(W|T^m) \to 0$ by Fano's inequality. Thus, $R = \frac{\log(M)}{n} \leq C$.

To achieve C, generate 2^{nC} sequences of realizations of U_i such that $\sum_{i=1}^m E[T_i] \leq n$. Each time Z_i is observed, the stopping rule is set as $v(u,z,\boldsymbol{g})$. Given u and z, transmitter releases 1 based on $v(u,z,\boldsymbol{g})$ as it causally observes the fading. This way, we obtain m channel uses in the timing channel. As m grows, probability of error goes to zero by using a joint typicality decoder in the timing channel. ■

IV. Energy Arrival Information at the Receiver

In this section, we consider the case when energy arrival information is available at both sides while fading is available only at the transmitter.

Theorem 2 *The channel capacity with causal energy arrival information at the transmitter and the receiver is:*

$$C^{ESI} = \max_{p\in[0,p_{on}]} \frac{\frac{h(p)}{p} - \frac{h(p_{on})}{p_{on}}}{\frac{1}{p} + \frac{1}{q} - 1} \quad (12)$$

Proof: Let W be uniform over $\{1,\ldots,M\}$. Define $U_i = (W, V^{i-1}, Z^i, \boldsymbol{G}^i, E^{(\sum_{j=1}^i T_j)-1})$. Note that U_i is independent of the fading process in ith timing channel use and that V_i is a stopping time with respect to the fading process given U_i.

$$\begin{aligned}
\log(M) &- H(W|Y^n, E^n)\\
&= H(W) - H(W|T^m, E^n) && (13)\\
&= I(W;E^n) + I(W;T^m|E^n) && (14)\\
&= I(W;V^m|E^n) && (15)\\
&= \sum_{i=1}^m I(W;V_i|V^{i-1}, E^n) && (16)\\
&= \sum_{i=1}^m I(W;V_i|V^{i-1}, E^{(\sum_{j=1}^i T_j)-1}) && (17)\\
&\leq \sum_{i=1}^m I(W, V^{i-1}, Z^i, \boldsymbol{G}^i, E^{(\sum_{j=1}^i T_j)-1}; V_i) && (18)\\
&\leq \frac{n}{\sum_{i=1}^m E[T_i]} \sum_{i=1}^m I(U_i;V_i) && (19)\\
&\leq n \max_{p(u),v(u,\boldsymbol{g})} \frac{I(U;V)}{E[T]} && (20)
\end{aligned}$$

where (15) follows from independence of E^n from W and that T^m is equivalently represented as V^m given E^n, (17) follows from the fact that V_i is independent of energy arrivals that occur after $\sum_{j=1}^i T_j$th channel use, (18) follows from nonnegativity of mutual information, and (19) holds since $\sum_{i=1}^m E[T_i] \leq n$. Finally, (20) follows from $\frac{\sum_i a_i}{\sum_i b_i} \leq \max_i \frac{a_i}{b_i}$ for $a_i, b_i > 0$. As m gets larger, $H(W|T^m) \to 0$ by Fano's inequality and therefore $R = \frac{\log(M)}{n} \leq \sup_{p(u),v(u,\boldsymbol{g})} \frac{I(U;V)}{E[T]}$.

Next, we claim that $H(V|U) \geq \frac{h(p_{on})}{p_{on}}$. To prove this claim, we note that for fixed $U = u$, V is a stopping time with respect to the fading process h_i and if $V = v$ then $h_v = 1$. Hence, for any such V, there exists a sequence of stopping rules $\tilde{V}_n \in \{1,\ldots\}$ with respect to fading process h_i such that the transmitter releases 1 symbol at the $\sum_{i=1}^n \tilde{V}_i + 1$st channel use if the channel fading is on. Since $P[V < \infty|U = u] = 1$, number of trials has to be infinite; that is, $\tilde{V}_n$ is an infinite sequence and 1 symbol is released the first time the channel fade level in the $\sum_{i=1}^n \tilde{V}_i + 1$st channel use turns on. Let n^* denote the first time $h_{\sum_{i=1}^n \tilde{V}_i + 1} = 1$. We have:

$$P(V = v|U = u) = \sum_{n^*} p_{on} (1 - p_{on})^{n^*-1} P(\tilde{V}_1^{n^*} = \tilde{v}_1^{n^*}) \quad (21)$$

where $v = \sum_{i=1}^{n^*} \tilde{v}_i + 1$. Note that for any fixed v, the number of trials n^* is determined by the realizations of the stopping times $\tilde{V}_i$. Conditioned on the realizations of $\tilde{V}_i = \tilde{v}_i$, V has a geometric distribution over the integers $\sum_{i=1}^n \tilde{v}_i + 1$. As conditioning reduces entropy, we have for given $U = u$:

$$\begin{aligned}
H(V|U = u) &\geq H(V|\{\tilde{V}_i\}_{i=1}^\infty, U = u) && (22)\\
&= \frac{h(p_{on})}{p_{on}} && (23)
\end{aligned}$$

By taking expectation over all u, we get $H(V|U) \geq \frac{h(p_{on})}{p_{on}}$.

From (20), we have:

$$C^{ESI} \leq \max_{p(u),v(u,\boldsymbol{g})} \frac{H(V)-H(V|U)}{E[V]+E[Z]} \quad (24)$$

$$\leq \max_{p(u),v(u,\boldsymbol{g})} \frac{H(V)-\frac{h(p_{on})}{p_{on}}}{E[V]+E[Z]} \quad (25)$$

$$\leq \max_{p\in[0,p_{on}]} \frac{\frac{h(p)}{p}-\frac{h(p_{on})}{p_{on}}}{\frac{1}{p}+\frac{1}{q}-1} \quad (26)$$

where (25) follows from $H(V|U) \geq \frac{h(p_{on})}{p_{on}}$, and (26) follows from the fact that $H(V)$ is maximized by a geometric distribution among all distributions with the same $E[V]$. Finally, $E[V] \geq \frac{1}{p_{on}}$ as channel fading has to be in its on state when 1 is sent.

The rate on the right hand side of (26) is achievable by encoding over the interval with battery state equal to 1 by using a Shannon strategy in the classical state dependent channel. Note that the receiver can track the battery state as E_i is given to it. In $\frac{E[V]}{E[V]+E[Z]}$ fraction of time, optimal coding is done over the channel $p(y|u) = p_{on}\delta(y-u) + (1-p_{on})\delta(y)$ with transmission probability p_{tx}. The following rate is achievable:

$$\max_{p_{tx}\in[0,1]} \frac{E[V]}{E[V]+E[Z]} I(U;Y)$$

$$= \max_{p_{tx}\in[0,1]} \frac{\frac{1}{p_{tx}p_{on}}}{\frac{1}{p_{tx}p_{on}}+\frac{1}{q}-1} \left(H(p_{tx}p_{on}) - p_{tx}H(p_{on})\right) \quad (27)$$

$$= \max_{p\in[0,p_{on}]} \frac{\frac{H(p)}{p}-\frac{H(p_{on})}{p_{on}}}{\frac{1}{p}+\frac{1}{q}-1} \quad (28)$$

which is equal to the right hand side in (26). ■

V. Fading State and Energy Arrival Information at the Receiver

In this section, we focus on the case of fading state and energy arrival information available at both transmitter and receiver. We will show that in this case, it suffices to encode over the channel indices with on fading state only and this gives the following modified timing channel:

$$\tilde{T}_i = \tilde{V}_i + \tilde{Z}_i \quad (29)$$

where $\tilde{T}_i$ is the number of channel uses between the ith and the $i+1$st energy carrying 1 symbols with on fading state. $\tilde{Z}_i$ is the number of channel uses transmitter waits for energy arrival with on fading state and $\tilde{V}_i$ is the corresponding channel input. $\tilde{Z}_i$ is i.i.d. with $E[\tilde{Z}_i] = p_{on}\left(\frac{1}{q}-1\right)$. In addition, we have:

$$\sum_{i=1}^{m} E[\tilde{T}_i] \leq np_{on} \quad (30)$$

By encoding and decoding over the modified timing channel in (29), we obtain the capacity with the energy and fading state information available at the receiver as follows:

Theorem 3 *The channel capacity with causal energy arrival and fading information at the transmitter and the receiver is:*

$$C^{EFSI} = \max_{p\in[0,1]} \frac{h(p)}{\frac{1}{p_{on}}+\frac{p(1-q)}{q}} \quad (31)$$

where $h(p)$ is the binary entropy function.

Proof: Let the message W be uniform over $\{1,\ldots,M\}$. The received sequence Y^n, h^n, E^n is equivalently represented as $\tilde{V}^m, h^n, E^n$ in view of the fact that one can uniquely construct $\tilde{V}^m, h^n, E^n$ from Y^n, h^n, E^n and vice versa.

$$\begin{aligned} \log(M) - H(W|Y^n,h^n,E^n) & \\ &= H(W) - H(W|Y^n,h^n,E^n) \quad (32)\\ &= I(W;\tilde{V}^m,h^n,E^n) \quad (33)\\ &= I(W;\tilde{V}^m|h^n,E^n) \quad (34)\\ &\leq H(\tilde{V}^m) \quad (35)\\ &\leq \sum_{i=1}^{m} H(\tilde{V}_i) \quad (36)\\ &\leq \frac{np_{on}}{\sum_{i=1}^{m} E[\tilde{T}_i]} \sum_{i=1}^{m} H(\tilde{V}_i) \quad (37)\\ &\leq np_{on} \max_{p(\tilde{v})} \frac{H(\tilde{V})}{E[\tilde{T}]} = nC^{EFSI} \quad (38) \end{aligned}$$

where (34) follows from independence of the message W and h^n, E^n, (35) holds due to non-negativity of entropy and the fact that conditioning reduces entropy, (37) follows from (30), and (38) follows from the fact that $\frac{\sum_i a_i}{\sum_i b_i} \leq \max_i \frac{a_i}{b_i}$ for $a_i, b_i > 0$. Achievability of C^{EFSI} directly follows from encoding and decoding $\tilde{V}$ over the modified timing channel in (29). Finally, the expression in (31) follows from the fact that entropy of V is maximized by a geometric distribution among all distributions with the same $E[V]$. ■

VI. Numerical Results

In this section, we numerically evaluate the achievable rates and capacities obtained in previous sections. We propose a finite cardinality achievable scheme and consider capacities in the extreme cases of infinite and zero energy storage.

A. A Finite Cardinality Achievable Scheme

We propose a class of achievable schemes that use a finite cardinality auxiliary variable U. In particular, we design an encoding function $f(u,z)$ and the attempts to send the energy carrying symbol are performed at every channel use after $f(u,z)$. This scheme corresponds to an achievable scheme in the following timing channel:

$$T = V + Z + Z_h \quad (39)$$

where Z_h is geometrically distributed over the set $\{0,1,2,\ldots\}$ with parameter p_{on} and the transmitter has perfect knowledge of only Z. Note that this scheme is an extension of the schemes for non-fading channel considered in [1], [2]. In particular,

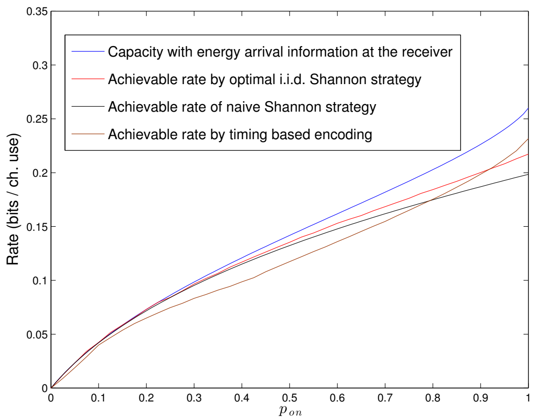

Fig. 3. Achievable rates and capacities versus the fading probability for fixed energy arrival rate $q = 0.1$.

we consider the following encoding function as in the hybrid scheme in [2]:

$$V = \begin{cases} U - Z + 1, & U \geq Z \\ (U - Z \bmod N) + 1, & U < Z \end{cases} \tag{40}$$

The rates achievable with this scheme are calculated by searching for N and $p_U(u)$ that maximize $R = \frac{I(U;T)}{\mathbb{E}[T]}$. We note that this achievable scheme does not make the best use of the channel fading information as the stopping rule $v(u, z, \boldsymbol{g})$ cannot fully adapt to the memoryless fading process.

B. Capacities with Infinite and Zero Batteries

When there is zero battery to save energy in the transmitter and the incoming energy can be used only in that channel use, we obtain a new on-off fading channel with on probability qp_{on} and fading state information perfectly available at the transmitter only. In this case, the capacity is [5]:

$$C_{ZS} = \max_{p_T(t)} I(T;Y) \tag{41}$$

where $p(y|t) = qp_{on}\delta(t-y) + (1 - qp_{on})\delta(y)$.

When there is infinite-sized battery at the transmitter, the capacity is achieved by a Shannon strategy with respect to the channel fading only and an average constraint on the number of 1s put to the channel [4]:

$$C_{IS} = \max_{p_T(t): E[T] \leq \frac{q}{p_{on}}} I(T;Y) \tag{42}$$

where $p(y|t) = p_{on}\delta(y-t) + (1-p_{on})\delta(y)$.

In Fig. 3, we plot achievable rates and capacities with respect to the fading probability p_{on} for $q = 0.1$. We also plot achievable rates by Shannon strategies [3]. We observe that the achievable rate by the proposed timing based encoding scheme is below the achievable rate by naïve and optimal i.i.d. Shannon strategies for moderate p_{on} values. In contrast, the achievable rate by timing based encoding scheme outperforms the other achievable schemes as the channel gets closer to the nonfading channel, corroborating the finding in [2]. We also observe that the capacity with energy arrival information is very close to the achievable rates without energy arrival information at the receiver for moderate to small p_{on}. Hence, the value of energy arrival information at the receiver is becoming less as p_{on} becomes smaller.

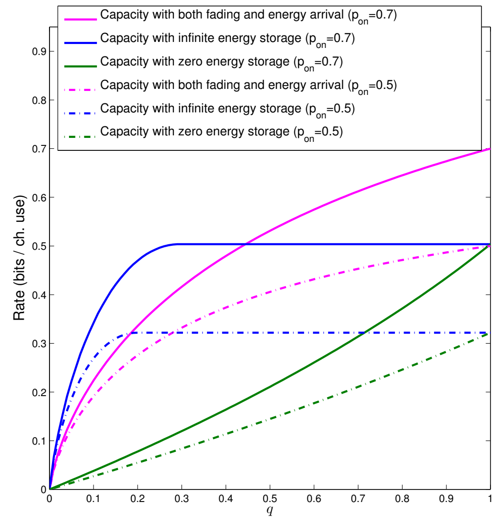

Fig. 4. Channel capacities versus the energy arrival rate.

In Fig. 4, we provide comparison of channel capacities under zero and infinite energy storage and the channel capacity with energy and fading state information at the transmitter with respect to the energy arrival rate. We provide this comparison for $p_{on} = 0.5$ and $p_{on} = 0.7$. We observe that the capacity with infinite energy storage is surpassed by the capacity with energy arrival and fading information at both sides as q increases. This suggests that the value of energy storage space diminishes with respect to the value of fading state information as the energy arrival probability increases.

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