Improved Capacity Bounds for the Binary Energy Harvesting Channel

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***Abstract*—We consider a binary energy harvesting channel (BEHC) where the encoder has unit energy storage capacity. We first show that an encoding scheme based on block indexing is asymptotically optimal for small energy harvesting rates. We then present a novel upper bounding technique, which upper bounds the rate by lower-bounding the rate of information leakage to the receiver regarding the energy harvesting process. Finally, we propose a timing based hybrid encoding scheme that achieves rates within 0.03 bits/channel use of the upper bound; hence determining the capacity to within 0.03 bits/channel use.**

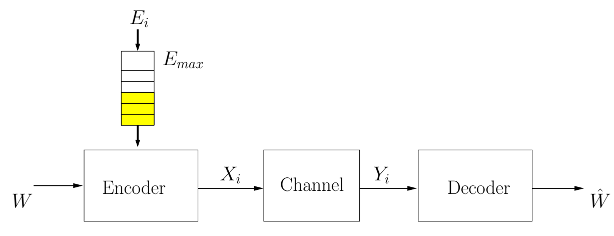

Fig. 1. The binary energy harvesting channel (BEHC) model.

I. Introduction

The binary energy harvesting channel (BEHC) with finite energy storage is introduced in [1], where an energy harvesting encoder with a finite battery communicates with a receiver over a binary channel using only the harvested energy. It was shown in [1] that this channel model is analogous to a finite queue which communicates with the receiver by timing the departures of the arriving packets, where arriving packets are the harvested energy units. Even in its simplest form, i.e., with a unit-size battery and a noiseless binary channel, the exact capacity of this channel is still unknown, as the single-letter capacity expression found in [1] involves an auxiliary variable. In this paper, we present an encoding scheme that is asymptotically optimal at low harvesting rates, develop a tighter upper bound by quantifying and minimizing the information leakage from the transmitter to the receiver regarding the energy harvesting process, and propose a timing based hybrid encoding scheme that yields achievable rates within 0.03 bits/channel use of the upper bound; hence determining the capacity of this channel to within 0.03 bits/channel use.

The capacity of additive white Gaussian noise (AWGN) energy harvesting channel was studied in [2] with an infinite-sized battery, and in [3] with no battery. Reference [2] showed that the capacity with an infinite-sized battery is equal to the capacity of an average power constrained channel with average power constraint equal to the average energy harvesting rate. With no energy storage, the problem is a time-varying stochastic amplitude constrained communication problem with causally known amplitude constraints, which was treated in [3] by combining Shannon strategies for channels with causally known states [4] and Smith's approach to constant amplitude constrained channels [5]. Concurrent to [1], reference [6] considered a finite-storage energy harvesting communication problem and specified a multi-letter capacity expression for a general discrete memoryless channel. This expression required n-letter Shannon strategies, where each channel input depends on the entire energy harvesting history, thus making evaluation difficult. It was conjectured in [6] that instantaneous strategies, where only the current energy state is considered in each channel use, are sufficient to achieve the capacity. A related work where similar discrete channel/discrete energy abstraction is also used for communication can be found in [7].

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II. Channel Model and Previous Results

We consider the binary energy harvesting channel in [1], [6] shown in Fig. 1. The model consists of an energy harvesting encoder which harvests energy into its finite-sized battery, and a conventional decoder. In each channel use, the encoder first sends a binary symbol X_i, which is 0 or 1, subject to the energy available in its battery, and then harvests a unit of energy with probability q and saves it in the battery if there is space. Energy harvests over time are i.i.d. A channel input of $X_i = 1$ consumes one unit of stored energy, while $X_i = 0$ does not require any energy. As in [1], we focus on the case of unit-sized battery, i.e., $E_{max} = 1$. Hence, the encoder can only send a 1 by consuming the unit of energy in its battery. The communication channel is noiseless, i.e., $Y_i = X_i$. Battery state and energy arrivals are naturally causally known to the transmitter, but unknown to the receiver. Battery state determines the set of feasible channel inputs at any given channel use. Even when energy arrivals are i.i.d., the battery state is correlated over time due to energy storage, and is also affected by the past transmitted symbols.

Although the channel is noiseless, finding the capacity of this channel model is difficult, as the battery state is unknown to the receiver, it is correlated over time, and also is affected by the past channel inputs. For the case of $E_{max} = 1$, reference [1] shows the equivalence of this channel model to a timing channel model where the information is conveyed through the distances between consecutive 1s. In particular, [1, Lemma 1] shows that BEHC is equivalent to the timing channel:

$$T_i = V_i + Z_i \tag{1}$$

where $T_i \in \{1, 2, \ldots\}$ is the number of channel uses between the $(i-1)$st and the ith 1s, $Z_i \in \{0, 1, \ldots\}$ is the number of channel uses the encoder has to wait for the next energy arrival, and $V_i \in \{1, 2, \ldots\}$ is the number of channel uses the encoder chooses to wait after receiving the energy, and thus after observing Z_i. Since energy harvests are i.i.d. Bernoulli with q, Z_i are i.i.d. geometric with parameter q, and thus the channel model in (1) is memoryless. Every timing channel use costs T_i channel uses in the BEHC. This equivalence yields the capacity as follows as found in [1, Theorem 1]

$$C = \max_{p(u), v(u,z)} \frac{I(U;T)}{\mathbb{E}[T]} \tag{2}$$

where U is an auxiliary variable with distribution $p(u)$, and $v(u, z)$ is a mapping from the message carrying signal U and state for the timing channel Z which is causally known to the transmitter, to the channel input. This result is a hybrid of Shannon's channel with causal state information [4] and Anantharam-Verdu's bits through queues [8]. However, although single-letterized, the capacity in (2) is still difficult to evaluate due to the infinite cardinalities of U, V and Z.

III. Asymptotically Optimal Encoding

In [1], an achievable scheme based on *block indexing* is proposed for the timing channel representation of the problem. The transmission duration is divided into blocks of length N, and indexed within blocks in mod N. The channel input is chosen in terms of $U \in \{0, 1, \ldots, N-1\}$ and Z as follows

$$V = (U - Z \text{ mod } N) + 1 \tag{3}$$

The achievable rate with the $v(u, z)$ selection in (2) is

$$R_A = \max_N \max_{p(u)} \frac{H(U)}{\mathbb{E}[V] + \mathbb{E}[Z]} \tag{4}$$

which involves optimization with respect to $p(u)$ and the block size N. In addition, the following is an upper bound that was obtained in [1] by giving Z information to the receiver

$$C_{UB} = \max_p \frac{qH(p)}{q + p(1-q)} \tag{5}$$

In the following theorem, we show that this encoding scheme is asymptotically optimal as q goes to zero, i.e., when the harvesting rate is small. We establish this by proposing specific encoding parameters $p(u)$ and N which yield achievable rates that asymptotically equal the upper bound in (5).

Theorem 1 *The encoding scheme for the timing channel with auxiliary $U \in \{0, 1, \ldots, N-1\}$ and the channel input given in (3) is asymptotically optimal as energy harvest rate $q \to 0$,*

$$\lim_{q \to 0} \frac{C_{UB}}{R_A} = 1 \tag{6}$$

Proof: For a given q, the objective in (5) is a continuous, differentiable and concave in p. This can be verified by observing that the second derivative is always negative. Therefore, p^* maximizing (5) satisfies

$$\frac{q(\log(1-p^*) - q\log(p^*))}{(p^* + q - p^*q)^2} = 0 \tag{7}$$

or equivalently for $q > 0$,

$$q = \frac{\log(1-p^*)}{\log(p^*)} \tag{8}$$

As a result of (8), we observe that there exists a feasible $0 < p^* \leq 0.5$ for all $0 < q \leq 1$, and that it satisfies

$$\lim_{q \to 0} p^* = 0 \tag{9}$$

At harvesting rate q, we choose $N = \left\lceil \frac{1}{p^*} \right\rceil$, and U uniformly distributed over $\{0, 1, \ldots, N-1\}$, i.e., $p(u) = 1/N$ for $0 \leq u \leq N-1$. Note that $N \geq 2$. Substituting these selections into (4), the rate achievable with this scheme becomes

$$R_A = \frac{H(U)}{\mathbb{E}[V] + \mathbb{E}[Z]} = \frac{\log(N)}{\frac{N+1}{2} + \frac{1-q}{q}} \geq \frac{q\log(N)}{Nq + 1 - q} \tag{10}$$

where $\mathbb{E}[Z] = (1-q)/q$ and $\mathbb{E}[V] = (N+1)/2$ follows from U being independent of Z and uniform on $\{0, 1, \ldots, N-1\}$. The last term in (10) is increasing in N within the interval $[\frac{1}{p^*}, \lceil \frac{1}{p^*} \rceil]$, which allows us to further lower bound R_A as

$$R_A \geq \frac{q\log(N)}{Nq + 1 - q} \geq \frac{-qp^*\log(p^*)}{q + p^*(1-q)} = \bar{R}_A \tag{11}$$

We are now ready to show (6) as follows:

$$\begin{aligned}
\lim_{q \to 0} \frac{C_{UB}}{R_A} &\leq \lim_{q \to 0} \frac{C_{UB}}{\bar{R}_A} && (12)\\
&= \lim_{q \to 0} \frac{qH(p^*)}{q + p^*(1-q)} \cdot \frac{q + p^*(1-q)}{-qp^*\log(p^*)} && (13)\\
&= 1 + \lim_{p^* \to 0} \frac{(1-p^*)\log(1-p^*)}{p^*\log(p^*)} && (14)\\
&= 1 && (15)
\end{aligned}$$

In addition, $C_{UB} \geq R_A$ by the definition of an upper bound, concluding the proof of the theorem. ■

The asymptotically optimal encoding scheme in Theorem 1 can be interpreted as sending a uniformly distributed symbol in $\{1, \ldots, N\}$ with each harvested energy. In this case, the frame length N poses a trade-off between sending more bits per symbol and vacating the battery quickly to increase chances of capturing more energy. The asymptotically optimal choice for N in the proof of Theorem 1 gives us a provably optimal encoding scheme, which is also simple to implement, for low energy harvesting scenarios.

IV. A Tighter Upper Bound

With every timing symbol T_i conveyed to the receiver, some information about the energy arrival, Z_i, is also *leaked* to the receiver. Since Z and the message-carrying signal U are independent, this implies that some of the information capacity of the binary noiseless channel is occupied by information about the energy arrival sequence. As an insightful example to this phenomenon, we point to [9], which considers communication through a queue with unit packet buffer. In [9], the departures from the buffer are random, and the encoder chooses the arrival sequence to convey its message to the receiver. In a similar manner, we may consider energy harvested from *nature* to be encoded in some way, achieving some positive rate between the nature and the receiver. Clearly, the sum of the nature's rate and the message rate cannot exceed the entropy of the channel output. We make use of this insight to obtain a tractable and tighter upper bound for the capacity of the BEHC.

We begin with the following lemma, which provides an upper bound for the conditional entropy $H(Z|T=t,U=u)$, which is the amount of uncertainty in Z at the receiver upon observing $T=t$ and successfully decoding $U=u$ (message). This bound will be useful in lower bounding the amount of information leaked to the receiver about the Z^n sequence.

Lemma 1 *For the timing channel $T=V+Z$, where Z is geometric with q, and $V=v(U,Z)$ with U independent of Z,*

$$H(Z|T=t,U=u) \le H(Z_t) \tag{16}$$

where Z_t is a truncated geometric random variable distributed on $\{0,1,\ldots,t-1\}$ with parameter q, i.e.,

$$p_{Z_t}(z) = \begin{cases} \frac{q(1-q)^z}{1-(1-q)^t}, & \text{if } z<t \\ 0, & \text{otherwise} \end{cases} \tag{17}$$

Proof: We first examine the joint distribution $p(z,t|u)$. We depict this discrete distribution as a two-dimensional matrix in Fig. 2. First, observe that given z and u, the output of the channel is deterministic, i.e., $T=v(u,z)+z$. Hence, for any given z and u, $p(z,t|u)$ can be non-zero only for a single value of t, and consequently each row of the matrix in Fig. 2 consists of a single non-zero entry. Next, we write

$$p(z) = p(z|u) \tag{18}$$
$$= \sum_{t=1}^{\infty} p(z,t|u) \tag{19}$$
$$= p(z, t=v(u,z)+z|u) \tag{20}$$

where (18) is due to the independence of Z and U. This implies that the single non-zero entry in each row of $p(z,t|u)$ is equal to $p(z)$. Also note that

$$p(z,t|u)=0, \qquad z \ge t \tag{21}$$

by the definition of the channel.

Let $\mathcal{A} \subset \{0,1,...,t-1\}$ be the set of indices $z \in \mathcal{A}$ for which $p(z,t|u)=p(z)$. This allows us to write $p_{\mathcal{A}}(z) = p(z|t,u)$ as

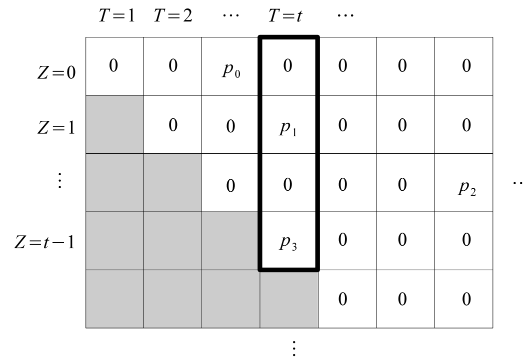

Fig. 2. The joint probability matrix $p(z,t|u)$ for a fixed strategy u. There is only one non-zero term in each row, and the rows sum up to $p(z)$. Shaded area corresponds to $t \le z$, which is not possible by definition. When calculating $H(Z|T=t,U=u)$, only the values in the bold rectangle are relevant.

$$p_{\mathcal{A}}(z) = \frac{p(z,t|u)}{\sum_{t=1}^{\infty} p(z,t|u)} \tag{22}$$
$$= \begin{cases} \frac{q(1-q)^z}{\sum_{a\in\mathcal{A}} q(1-q)^a}, & \text{if } z \in \mathcal{A} \\ 0, & \text{otherwise} \end{cases} \tag{23}$$

We next prove that $H(Z|T=t,U=u)$ is maximized when $\mathcal{A}^* = \{0,1,\ldots,t-1\}$, i.e., when all terms in the bold rectangle in Fig. 2 are non-zero. We do this by showing that the distribution $p_{\mathcal{A}^*}(z)$ is majorized by $p_{\mathcal{A}}(z)$ for all index sets $\mathcal{A} = \{a_0,a_1,\ldots,a_{k-1}\} \subset \{0,1,\ldots,t-1\}$, $k \le t$. Without loss of generality, we assume that $a_0 < a_1 < \ldots < a_{k-1}$, which yields the decreasing ordering

$$p_{\mathcal{A}}(a_0) > p_{\mathcal{A}}(a_1) > \ldots > p_{\mathcal{A}}(a_{k-1}) \tag{24}$$

for all $\mathcal{A}$. For $0 \le n \le k-1$ we write

$$\sum_{i=0}^{n} p_{\mathcal{A}}(a_i) = \frac{\sum_{i=0}^{n} q(1-q)^{a_i}}{\sum_{i=0}^{k-1} q(1-q)^{a_i}} \tag{25}$$
$$\ge \frac{\sum_{i=0}^{n}(1-q)^{a_n+i-n}}{\sum_{i=0}^{n}(1-q)^{a_n+i-n} + \sum_{i=n+1}^{k-1}(1-q)^{a_i}} \tag{26}$$
$$\ge \frac{\sum_{i=0}^{n}(1-q)^{a_n+i-n}}{\sum_{i=0}^{k-1}(1-q)^{a_n+i-n}} \tag{27}$$
$$\ge \frac{\sum_{i=0}^{n}(1-q)^{i}}{\sum_{i=0}^{t-1}(1-q)^{i}} = \sum_{i=0}^{n} p_{\mathcal{A}^*}(i) \tag{28}$$

where we obtain (26) by subtracting

$$\delta_1 = \sum_{i=0}^{n}(1-q)^{a_i} - \sum_{i=0}^{n}(1-q)^{a_n+i-n} \tag{29}$$

from both the numerator and the denominator, and (27) is obtained by adding

$$\delta_2 = \sum_{i=n+1}^{k-1}(1-q)^{a_n+i-n} - \sum_{i=n+1}^{k-1}(1-q)^{a_i} \tag{30}$$

to the denominator. Note that both δ_1 and δ_2 are positive since $a_n - a_i \geq n - i$ for $n \geq i$. Finally, (28) follows from $k \leq t$.

The concavity of $f(x) = -x\log(x)$ and the majorization shown in (25)-(28) implies that $H(Z|T = t, U = u)$ is maximized at $\mathcal{A}^* = \{0, 1, \ldots, t-1\}$. This yields the upper bound $H(Z_t)$, with Z_t in (17); concluding the proof. ■

Next, we present an upper bound for the BEHC by using the result of Lemma 1 in the following theorem.

Theorem 2 *The capacity of the BEHC is upper bounded by*

$$C \leq \max_{p_T(t)\in\mathcal{P}} \frac{H(T) - \sum_{t=1}^{\infty} \frac{H((1-q)^t)}{1-(1-q)^t} p(t)}{\mathbb{E}[T]} \tag{31}$$

where

$$\mathcal{P} = \left\{ p_T(t) \middle| \sum_{t=1}^{n} p(t) \leq 1-(1-q)^n,\ n = 1, 2, \ldots \right\} \tag{32}$$

and $H(a)$ is the binary entropy function.

Proof: Since $T = v(U, Z) + Z$ is a deterministic function of U and Z, we rewrite the numerator of the capacity in (2) as

$$\begin{aligned} I(U;T) &= I(U, Z; T) - I(Z; T|U) && (33)\\ &= H(T) - H(T|U, Z) - I(Z; T|U) && (34)\\ &= H(T) - I(Z; T|U) && (35) \end{aligned}$$

Note that the second term in (35) represents the information leaked to the receiver about the energy harvesting process Z after U is decoded. We lower bound this term as

$$\begin{aligned} I(Z; T|U) &= H(Z|U) - H(Z|T, U) && (36)\\ &= H(Z) - H(Z|T, U) && (37)\\ &= \sum_{t=1}^{\infty}\sum_{u} p(t, u)\left[H(Z) - H(Z|T = t, U = u)\right] && (38)\\ &\geq \sum_{t=1}^{\infty} \left[H(Z) - H(Z_t)\right] \sum_{u} p(t, u) && (39)\\ &= \sum_{t=1}^{\infty} \left[H(Z) - H(Z_t)\right] p(t) && (40) \end{aligned}$$

where (37) follows from the independence of Z and U, and (39) follows from Lemma 1. Substituting (35) and (40) in (2),

$$C \leq \max_{p(u), v(u,z)} \frac{H(T) - \sum_{t=1}^{\infty}[H(Z) - H(Z_t)]p(t)}{\mathbb{E}[T]} \tag{41}$$

The objective in (41) is only a function of $p_T(t)$. Hence, we can perform the maximization over distributions $p_T(t)$ that are achievable by some auxiliary $p_U(u)$ and function $v(U, Z)$. Note that since $T = V + Z$, we have $T > Z$, and therefore

$$\sum_{t=1}^{n} p(t) \leq \sum_{z=0}^{n-1} p(z) = 1 - (1-q)^n, \quad n = 1, 2, \ldots \tag{42}$$

Hence, the set of distributions $\mathcal{P}$ in (32) contains all $p_T(t)$ that can be generated by some $p_U(u)$ and $v(U, Z)$. We note that relaxing the constraint $p_T(t) \in \mathcal{P}$ gives an upper bound as well, though it is strictly looser than that in the theorem. We also note that for the geometrically distributed Z we have,

$$H(Z) - H(Z_t) = \frac{H((1-q)^t)}{1-(1-q)^t} \tag{43}$$

Substituting (42) and (43) in (41), we obtain the upper bound in (31); completing the proof of the theorem. ■

V. Computing the Upper Bound

We can rewrite the maximization problem in (31) as

$$C \leq \max_{\beta} \frac{1}{\beta} \max_{p_T(t)\in\mathcal{P}, \mathbb{E}[T]\leq\beta} H(T) - \sum_{t=1}^{\infty} \Delta_t p(t) \tag{44}$$

where we have defined $\Delta_t = \frac{H((1-q)^t)}{1-(1-q)^t}$. Note that the inner objective is the sum of a concave and a linear function, and the constraints are linear. Hence, the inner problem is convex, for which we write the KKT optimality conditions as

$$p(t) = \exp\left(-\mu t - \Delta_t + \lambda_t - \sum_{n=1}^{t}\gamma_n - \eta - 1\right),\ t = 1, 2, \ldots \tag{45}$$

where $\lambda_t \geq 0$, $\gamma_n \geq 0$, $\mu \geq 0$ and η are the Lagrange multipliers for the constraints $p(t) \geq 0$, $\sum_{t=1}^{n} p(t) \leq 1-(1-q)^n$, $\mathbb{E}[T] \leq \beta$, and $\sum p(t) = 1$, respectively. The complementary slackness and dual feasibility conditions are

$$\lambda_t p(t) = 0, \quad \lambda_t \geq 0 \tag{46}$$

$$\gamma_t\left(\sum_{n=1}^{t} p_T(n) - 1 + (1-q)^t\right) = 0, \quad \gamma_t \geq 0 \tag{47}$$

$$\mu\left(\mathbb{E}[T] - \beta\right) = 0, \quad \mu \geq 0 \tag{48}$$

$$\eta\left(\sum_{n=1}^{\infty} p_T(n) - 1\right) = 0 \tag{49}$$

We note that for t with $p(t) = 0$, we need the exponent term in (45) to go to $-\infty$. In that case, the value of λ_t is redundant in (45). Thus, without loss of generality, we assign $\lambda_t = 0$ for all t, and rewrite the solution as

$$p(t) = A\exp\left(-\mu t - \Delta_t - \sum_{n=1}^{t}\gamma_n\right) \tag{50}$$

where $A = e^{-\eta-1}$. For all $\mu \geq 0$ and γ_i, using (49), we find

$$A = \left(\sum_{t=1}^{\infty} e^{-\mu t - \Delta_t - \sum_{n=1}^{t}\gamma_n}\right)^{-1} \tag{51}$$

The remaining parameters can be searched numerically, observing that due to (47), $\gamma_t > 0$ only when $\sum_{n=1}^{t} p_T(n) = 1-(1-q)^t$. In particular, for each $\mu > 0$, there exists a unique set of multipliers $\{\gamma_t\}$ which satisfies (47) when substituted in (50). Due to (48), each $\mu > 0$ gives the solution of the second maximization in (44) for a specific $\beta = \mathbb{E}[T]$. Hence, we solve (44) by tracing μ in $[0, \infty)$ and finding the corresponding $\{\gamma_t\}$ and β in each case.

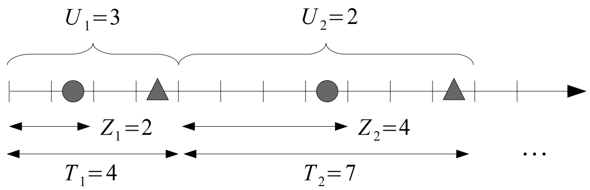

Fig. 3. Encoding in the BEHC for $N = 4$. Circles indicate energy harvests and triangles indicate a transmission of 1 using the harvested energy.

VI. An Improved Encoding Strategy

As a new encoding strategy, we propose choosing $U \in \{0, 1, \ldots\}$ with distribution $p_U(u)$, and calculating the channel input $V = v(U, Z)$ as

$$V = \begin{cases} U - Z + 1, & U \geq Z \\ (U - Z \mod N) + 1, & U < Z \end{cases} \quad (52)$$

The interpretation of this scheme in the BEHC is demonstrated in Fig. 3 for $N = 4$. If possible, the transmitter waits for $V = U - Z + 1$ channel uses, so that $T = U + 1$, and U is decoded perfectly, as in the case for U_1 in the figure. However, if $Z > U$, then the encoder immediately departs the energy within N channel uses, making the battery available for new energy harvests while providing partial information on U via

$$(T - 1) \mod N = U \mod N \quad (53)$$

as seen in the figure for U_2. The rates achievable with this scheme are calculated by searching for N and $p_U(u)$ that maximize $R = I(U; T)/\mathbb{E}[T]$.

Note that this scheme is a hybrid between the block indexing scheme in [1] and choosing T as close to a desired value as possible. Moreover, in this scheme, U is not limited to $\{0, 1, \ldots, N-1\}$. If we restrict the support set of U as $U < N$, then the new achievable scheme reduces to that in [1], and therefore it performs at least as good as the one in [1], as will be verified next.

VII. Numerical Results

Fig. 4 compares the achievable rate (4) and upper bound (5) of [1], the rates achieved by the instantaneous Shannon strategies proposed in [6], and the upper bound and achievable rate found in Sections IV and VI of this paper. The improved upper and lower bounds on the capacity of the BEHC are significantly tighter than previous results. In particular, the proposed upper bound, which considers the information leaked to the receiver regarding the harvesting process, is especially tight for large harvest rates q. The proposed achievable scheme which performs encoding over the equivalent timing channel, performs better than that of [6] with instantaneous Shannon strategies for the zeroth and first order Markov cases. The scheme proposed in [6] only observes the current battery state in each channel use, while allowing a Markovian dependence over time in the strategy. Our results show that at least in the noiseless channel and with unit-sized battery, the history of the battery state that our scheme is able to exploit is helpful.

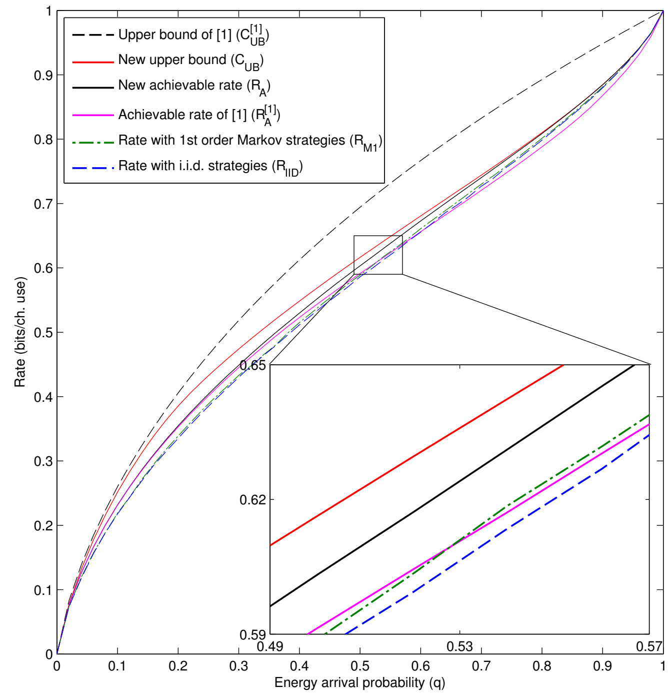

Fig. 4. Upper bounds and achievable rates for the BEHC.

VIII. Conclusion

We considered the noiseless BEHC with unit-sized battery. We presented an achievable scheme which is asymptotically optimal for small energy harvesting rates. We then developed an upper bound by quantifying the information leakage to the receiver regarding the energy harvesting random process at the transmitter and by lower bounding it. Finally, we proposed a new achievable scheme which outperforms the previous achievable scheme in [1], and also the one in [6] for the zeroth and first order Markovian inputs. The proposed achievable scheme achieves rates within 0.03 bits/channel use of the proposed upper bound, therefore, determining the capacity of this channel to within 0.03 bits/channel use.

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