Resource Management for Fading Wireless Channels with Energy Harvesting Nodes

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***Abstract*—Wireless systems comprised of rechargeable nodes have a significantly prolonged lifetime and are sustainable. A distinct characteristic of these systems is the fact that the nodes can harvest energy throughout the duration in which communication takes place. As such, transmission policies of the nodes need to adapt to these harvested energy arrivals. In this paper, we consider optimization of the transmission policy of an energy harvesting transmitter which has a limited battery capacity, communicating in a wireless fading channel. In particular, we identify the optimal offline transmission policies that maximize the number of bits delivered by a deadline, and minimize the transmission completion time of the communication session. We introduce a *directional water-filling* algorithm which provides a simple and concise interpretation of the necessary optimality conditions as well as energy storage capacity and causality. We solve the throughput maximization problem for the fading channel using the directional water-filling algorithm, which simultaneously adapts to the energy harvested as well as the channel variations in time. We then solve the transmission completion time minimization problem by utilizing its equivalence to its throughput maximization counterpart.**

I. Introduction

This paper considers wireless communication using energy harvesting transmitters. In such a scenario, incremental energy is harvested by the transmitter during the course of data transmission from random energy sources. As such, energy becomes available for packet transmission at random times and in random amounts. In addition, the wireless communication channel fluctuates randomly due to fading. These together lead to a need for designing new transmission strategies that can best take advantage of and adapt to the random energy arrivals as well as channel variations in time.

The simplest system model for which this setting leads to new design insights is a wireless link with a rechargeable transmitter, which we consider here. The incoming energy can be stored in the battery of the rechargeable transmitter for future use. However, this battery has finite storage capacity and the transmitter needs to guarantee that there is always sufficient battery space for each energy arrival, otherwise incoming energy cannot be saved and will be wasted. In this setting, we find the optimal transmission scheme that adapts the instantaneous transmit power to the variations in the energy and fade levels.

In this paper, we consider the offline optimization of the energy harvesting transmitter, where we assume that the non-causal information of energy arrivals and fading levels are available at the transmitter. We consider two related optimization problems. The first problem is the maximization of the number of bits transmitted by a deadline T. The second problem is the minimization of the time (or delay) by which the transmission of B bits is completed. We first solve the former in a static channel. The solution calls for a new algorithm, termed *directional water-filling*. Taking into account the causality constraints on the energy usage, the algorithm allows energy flow only to the right, which is emphasized by *right permeable taps* at each energy arrival point. This algorithm serves as a building block for the fading case. We show that a directional water-filling algorithm that adapts to both energy arrivals and channel fade levels is optimal. Next, we consider the second problem, i.e., the minimization of the time by which transmission of B bits is completed. We solve this second problem by mapping it to the first problem by means of the *maximum departure curve*.

Related work includes the approaches on energy efficiency and delay constraints which have received considerable attention in the previous decade [1]–[4]. In [1], [2], energy minimal packet scheduling is solved in a deterministic time constrained system while in [3], long term average energy minimal power control is found in a finite buffer system governed by a continuous time Markov chain. In [4], optimal energy allocation to a fixed number of time slots is derived under time-varying channel gains and with offline and online knowledge of the channel state at the transmitter. In [5], energy management policies which stabilize the data queue are proposed for single-user communication and under a linear approximation, some delay optimality properties are derived. In [6], throughput optimal energy allocation is studied for energy harvesting systems in a time constrained slotted setting. In [7], [8], minimization of transmission completion time is considered in an energy harvesting system and the optimal solution is obtained using a geometric framework. In [9], energy harvesting transmitters with batteries of finite energy storage capacity are considered and the problem of throughput maximization by a deadline is solved in a static channel.

II. System Model

We consider a single-user fading channel with additive Gaussian noise as shown in Fig. 1. The transmitter has two queues, a data queue, where data packets are stored; and an energy queue, where the arriving (harvested) energy is stored.

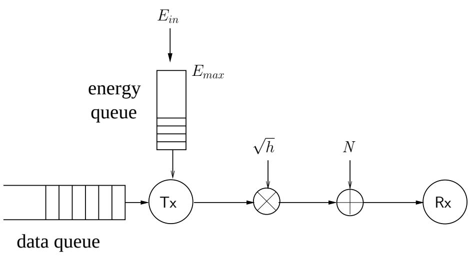

Fig. 1. Additive Gaussian fading channel with an energy harvesting transmitter.

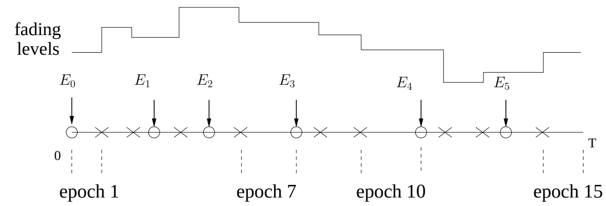

Fig. 2. The system model and epochs under channel fading.

The energy queue, i.e., the battery, can store at most E_{max} units of energy, which is used only for data transmission, i.e., energy required for processing is not considered.

The received signal y is given by

$$y = \sqrt{h}x + n \tag{1}$$

where h is the (squared) fading coefficient, x is the channel input, and n is a Gaussian random noise with zero-mean and unit-variance. Whenever an input signal x is transmitted with power p in an epoch of duration L, $\frac{L}{2}\log(1+hp)$ bits of data is served out from the backlog with the cost of Lp units of energy depletion from the energy queue. The bandwidth is sufficiently wide so that L can take small values and we approximate the slotted system to a continuous time system. Hence, if at time t the transmit power of the signal is $x^2(t) = p(t)$, the instantaneous rate $r(t)$ in bits per channel use is

$$r(t) = \frac{1}{2}\log(1 + h(t)p(t)) \tag{2}$$

We assume that changes in the fading level and energy arrivals occur in countable time instants, which are indexed respectively as $t_1^f, t_2^f, \ldots$ and $t_1^e, t_2^e, \ldots$ with the convention that $t_1^e = t_1^f = 0$. The fading level in $[0, t_1^f)$ is h_1, in $[t_1^f, t_2^f)$ is h_2, and so on. Similarly, E_i units of energy arrives at time t_i^e, and E_0 units of energy is available at time 0. We assume that $E_i \leq E_{max}$ for all i. In the sequel, we will refer to a change in the fading level or in the energy level as an *event* and the time interval between two events as an *epoch*. This model is depicted in Fig. 2. More precisely, epoch i is defined as the time interval $[t_i, t_{i+1})$ where t_i and t_{i+1} are the times at which successive events occur and the length of the epoch is $L_i = t_{i+1} - t_i$. The energy arrival and the channel fade level information are non-causally available to the transmitter before the transmission starts.

A power management policy is denoted as $p(t)$ for $t \in [0, T]$. There are two constraints on $p(t)$, due to energy arrivals at random times, and also due to finite battery storage capacity. Since energy that has not arrived yet cannot be used at the current time, there is a causality constraint on the power management policy as:

$$\int_0^{t_i^e} p(u)du \leq \sum_{j=0}^{i-1} E_j, \quad \forall i \tag{3}$$

where the limit of the integral t_i^e should be interpreted as $t_i^e - \epsilon$, for small enough ϵ. Moreover, due to the finite battery storage capacity, we need to ensure that the energy level in the battery never exceeds E_{max} at the times of energy arrivals. Let $d(t) = \max\{t_i^e : t_i^e \leq t\}$. Then,

$$\sum_{j=0}^{d(t)} E_j - \int_0^t p(u)du \leq E_{max}, \quad \forall t \in [0, T] \tag{4}$$

We emphasize that our system model is continuous rather than slotted. In slotted models, e.g., [5], [6], the energy input-output relationship is written for an entire slot. Such models allow energies larger than E_{max} to enter the battery and be used for transmission in a given single slot. Our continuous system model prohibits such occurrences.

III. Maximizing Throughput in a Static Channel

In this section, we consider maximizing the number of bits delivered by a deadline T, in a non-fading channel with offline knowledge of energy arrivals which occur at times $\{t_1, t_2, \ldots, t_N\}$ in amounts $\{E_1, E_2, \ldots, E_N\}$. The epoch lengths are $L_i = t_i - t_{i-1}$ for $i = 1, \ldots, N$ with $t_0 = 0$, and $L_{N+1} = T - t_N$. There are a total of $N+1$ epochs. This problem was solved in [8], [9] using a geometric framework. Here, we provide an alternative algorithmic solution which will serve as the building block for the solution for the fading channel presented in the next section.

First, we note that the transmit power must be kept constant in each epoch [7], [8], due to the concavity of rate in power. Let us denote the power in epoch i by p_i. The causality constraints in (3) reduce to the following constraints on p_i,

$$\sum_{i=1}^{\ell} L_i p_i \leq \sum_{i=0}^{\ell-1} E_i, \quad \ell = 1, \ldots, N+1 \tag{5}$$

Moreover, since the energy level in the battery is the highest at instants when energy arrives, the battery capacity constraints in (4) reduce to a countable number of constraints, as follows

$$\sum_{i=0}^{\ell} E_i - \sum_{i=1}^{\ell} L_i p_i \leq E_{max}, \quad \ell = 1, \ldots, N \tag{6}$$

Note that since $E_0 > 0$, there is no incentive to make $p_i = 0$ for any i. Hence, $p_i > 0$ is necessary for optimality.

The optimization is subject to causality constraints on the harvested energy, and the finite storage constraint on the rechargeable battery. The optimization problem is:

$$\max_{p_i \geq 0} \quad \sum_{i=1}^{N+1} \frac{L_i}{2} \log(1+p_i) \tag{7}$$

$$\text{s.t.} \quad \sum_{i=1}^{\ell} L_i p_i \leq \sum_{i=0}^{\ell-1} E_i, \quad \ell = 1, \ldots, N+1 \tag{8}$$

$$\sum_{i=0}^{\ell} E_i - \sum_{i=1}^{\ell} L_i p_i \leq E_{max}, \quad \ell = 1, \ldots, N \tag{9}$$

We note that the constraint in (8) must be satisfied with equality for $\ell = N+1$, otherwise, we can always increase some p_i without conflicting any other constraints, increasing the resulting number of bits transmitted.

The objective function in (7) is concave in the vector of powers and the constraints are convex. Hence the above optimization problem has a unique maximizer. We define the following Lagrangian function [10] for any $\lambda_i \geq 0$ and $\mu_i \geq 0$,

$$\mathcal{L} = \sum_{i=1}^{N+1} \frac{L_i}{2} \log(1+p_i) - \sum_{j=1}^{N+1} \lambda_j \left(\sum_{i=1}^{j} L_i p_i - \sum_{i=0}^{j-1} E_i \right) - \sum_{j=1}^{N} \mu_j \left(\sum_{i=0}^{j} E_i - \sum_{i=1}^{j} L_i p_i - E_{max} \right) \tag{10}$$

Lagrange multipliers $\{\lambda_i\}$ are associated with constraints in (8) and $\{\mu_i\}$ are associated with (9). Additional complimentary slackness conditions are as follows,

$$\lambda_j \left(\sum_{i=1}^{j} L_i p_i - \sum_{i=0}^{j-1} E_i \right) = 0, \quad j = 1, \ldots, N \tag{11}$$

$$\mu_j \left(\sum_{i=0}^{j} E_i - \sum_{i=1}^{j} L_i p_i - E_{max} \right) = 0, \quad j = 1, \ldots, N \tag{12}$$

In (11), $j = N+1$ is not included since this constraint is in fact satisfied with equality. Note also that as $p_i > 0$, we did not include any slackness conditions for p_i.

We apply the KKT optimality conditions to this Lagrangian to obtain the optimal power levels p_i^* in terms of the Lagrange multipliers as,

$$p_i^* = \frac{1}{\left(\sum_{j=i}^{N+1} \lambda_j - \sum_{j=i}^{N} \mu_j \right)} - 1, \quad i = 1, \ldots, N \tag{13}$$

and $p_{N+1}^* = \frac{1}{\lambda_{N+1}} - 1$. Note that p_i^* that satisfy $\sum_{i=1}^{N+1} L_i p_i^* = \sum_{i=0}^{N} E_i$ is unique. Based on the expression for p_i^* in terms of the Lagrange multipliers in (13), we have the following observation on the structure of the optimal power allocation.

Theorem 1 *When $E_{max} = \infty$, the optimal power levels is a monotonically increasing sequence: $p_{i+1}^* \geq p_i^*$. Moreover, if for some ℓ, $\sum_{i=1}^{\ell} L_i p_i^* < \sum_{i=0}^{\ell-1} E_i$, then $p_\ell^* = p_{\ell+1}^*$.*

Proof: Since $E_{max} = \infty$, constraints in (9) are satisfied without equality and $\mu_i = 0$ for all i by slackness conditions in (12). From (13), since $\lambda_i \geq 0$, optimum p_i^* are monotonically increasing: $p_{i+1}^* \geq p_i^*$. Moreover, if for some ℓ, $\sum_{i=1}^{\ell} L_i p_i^* < \sum_{i=0}^{\ell-1} E_i$, then $\lambda_\ell = 0$, which means $p_\ell^* = p_{\ell+1}^*$. ■

The monotonicity in Theorem 1 is a result of the fact that energy may be spread from the current time to the future for optimal operation. Whenever a constraint in (8) is not satisfied with equality, it means that some energy is available for use but is not used in the current epoch and is transferred to future epochs. Hence, the optimal power allocation is such that, if some energy is transferred to future epochs, then the power level must remain the same. However, if the optimal power level changes from epoch i to $i+1$, then this change should be in the form of an increase and no energy is transferred for future use. That is, the corresponding constraint in (8) is satisfied with equality.

If E_{max} is finite, its effect on the optimal power allocation is observed through μ_i in (13). In particular, if the constraints in (9) are satisfied without equality, then optimal p_i^* are still monotonically increasing since $\mu_i = 0$. However, as $E_i \leq E_{max}$ for all i, the constraint with the same index in (8) is satisfied without equality whenever a constraint in (9) is satisfied with equality. Hence, a non-zero μ_i and a zero λ_i appear in p_i^* in (13). This implies that the monotonicity of p_i^* may no longer hold. E_{max} constraint restricts power levels to take the same value in adjacent epochs as it constrains the energy that can be transferred from the current epoch to the future epochs. Indeed, from the constraints in (9), the energy that can be transferred from the current, say the ith, to the future epochs is $E_{max} - E_i$. Hence, the power levels are equalized only to the extent that E_{max} constraint allows.

A. Directional Water-Filling Algorithm

We interpret the observed properties of the optimal power allocation as a *directional water-filling* scheme where water is analogous to energy and its level corresponds to transmit power. When E units of energy is filled into a time interval of length L, the water level is $\frac{E}{L}$. When E_{max} is sufficiently large, in a two epoch system if $\frac{E_0}{L_1} > \frac{E_1}{L_2}$, some energy is transferred from epoch 1 to epoch 2 so that the levels are equalized. However, due to energy causality if $\frac{E_0}{L_1} < \frac{E_1}{L_2}$, no energy can flow from right to left and the water levels are not equalized. We implement this using *right permeable* taps, which let energy flow only from left to right. In the water-energy analogy, battery capacity E_{max} forms a physical constraint on the amount of energy, or water, that can be transferred to future epochs. If the equalizing water level requires more than $E_{max} - E_i$ amount of energy to be transferred, then only $E_{max} - E_i$ can be transferred. Because, otherwise, the energy level in the next epoch exceeds E_{max} causing overflow of energy.

IV. Maximizing Throughput in a Fading Channel

We now solve for the offline optimal policy for the fading channel utilizing the insights obtained in the previous section.

The channel state changes M times and energy arrives N times in the duration $[0, T)$. Hence, we have $M+N+1$ epochs. Similar to the non-fading case, the optimal power management strategy is such that the transmit power is constant in each event epoch. We again denote the transmit power in epoch i by p_i, for $i = 1, \ldots, M+N+1$. We define $E_{in}(i)$ as the energy arrived in epoch i. Hence, $E_{in}(i) = E_j$ for some j if event i is an energy arrival and $E_{in}(i) = 0$ if event i is a fade level change. Also, $E_{in}(1) = E_0$. We have causality constraints due to energy arrivals and E_{max} constraints due to finite battery size. Hence, the problem in the fading case is:

$$\max_{p_i \geq 0} \quad \sum_{i=1}^{M+N+1} \frac{L_i}{2} \log\left(1 + h_i p_i\right) \tag{14}$$

$$\text{s.t.} \quad \sum_{i=1}^{\ell} L_i p_i \leq \sum_{i=1}^{\ell} E_{in}(i), \quad \forall \ell \tag{15}$$

$$\sum_{i=1}^{\ell} E_{in}(i) - \sum_{i=1}^{\ell} L_i p_i \leq E_{max}, \quad \forall \ell \tag{16}$$

Note that, as in the non-fading case, the constraint in (15) for $\ell = M+N+1$ must be satisfied with equality, since otherwise, we can always increase one of p_i to increase the throughput.

As in the non-fading case, the objective function in (14) is concave and the constraints are convex. The optimization problem has a unique optimal solution. We define the Lagrangian,

$$\begin{aligned}
\mathcal{L} = & \sum_{i=1}^{M+N+1} \frac{L_i}{2} \log\left(1 + h_i p_i\right) \\
& - \sum_{j=1}^{M+N+1} \lambda_j \left(\sum_{i=1}^{j} L_i p_i - \sum_{i=1}^{j} E_{in}(i) \right) \\
& - \sum_{j=1}^{M+N+1} \mu_j \left(\sum_{i=1}^{j} E_{in}(i) - \sum_{i=1}^{j} L_i p_i - E_{max} \right) \\
& + \sum_{i=1}^{M+N+1} \eta_i p_i
\end{aligned} \tag{17}$$

Note that we have not employed the Lagrange multipliers $\{\eta_i\}$ in the non-fading case, since in that case, we need to have all $p_i > 0$. However, in the fading case, some of the optimal powers can be zero depending on the channel fading state. Associated complimentary slackness conditions are,

$$\lambda_j \left(\sum_{i=1}^{j} L_i p_i - \sum_{i=1}^{j} E_{in}(i) \right) = 0, \quad \forall j \tag{18}$$

$$\mu_j \left(\sum_{i=1}^{j} E_{in}(i) - \sum_{i=1}^{j} L_i p_i - E_{max} \right) = 0, \quad \forall j \tag{19}$$

$$\eta_j p_j = 0, \quad \forall j \tag{20}$$

It follows that the optimal powers are given by

$$p_i^* = \left[\nu_i - \frac{1}{h_i} \right]^+ \tag{21}$$

where the water level in epoch i, ν_i, is

$$\nu_i = \frac{1}{\sum_{j=i}^{M+N+1} \lambda_j - \sum_{j=i}^{M+N+1} \mu_j} \tag{22}$$

We have the following observation for the fading case.

Theorem 2 *When $E_{max} = \infty$, for any epoch i, the optimum water level ν_i is monotonically increasing: $\nu_{i+1} \geq \nu_i$. Moreover, if some energy is transferred from epoch i to $i+1$, then $\nu_i = \nu_{i+1}$.*

Proof: $E_{max} = \infty$ assumption results in $\mu_i = 0$ for all i. From (22), and since $\lambda_i \geq 0$, we have $\nu_{i+1} \geq \nu_i$. If energy is transferred from the ith epoch to $i+1$st epoch, then the ith constraint in (15) is satisfied without equality. This implies, by the slackness conditions in (18), that for those i, we have $\lambda_i = 0$. Hence, by (22), $\nu_i = \nu_{i+1}$. In particular, $\nu_i = \nu_j$ for all epochs i and j that are in between two consecutive energy arrivals as there is no wall between these epochs and injected energy freely spreads into these epochs. ∎

As in the non-fading case, the effect of finite E_{max} is observed via the Lagrange multipliers μ_i. In particular, whenever E_{max} constraint is satisfied with equality, the monotonicity of the water level no longer holds. E_{max} constrains the amount of energy that can be transferred from one epoch to the next. Specifically, the transferred energy cannot be larger than $E_{max} - E_{in}(i)$. Note that this constraint is trivially satisfied for those epochs with $E_{in}(i) = 0$ because $E_{in}(i) < E_{max}$.

A. Directional Water-Filling Algorithm

The directional water-filling algorithm in the fading channel requires walls at the points of energy arrival, with right permeable water taps in each wall which allows at most E_{max} amount of water to flow. The water levels when each right permeable tap is turned on will be found by the directional water-filling algorithm. Optimal power allocation p_i^* is then calculated by plugging the resulting water levels into (21). An example run of the algorithm is shown in Fig. 3, for a case of 12 epochs. Three energy arrivals occur during the course of the transmission, in addition to the energy available at time $t = 0$. We observe that the energy level equalizes in epochs 2, 4, 5, while no power is transmitted in epochs 1 and 3, since the channel gains in these epochs are too low (i.e., $\frac{1}{h_i}$ too high). The energy arriving at the beginning of epoch 6 cannot flow left due to causality constraints, which are enforced by right permeable taps, which allow energy flow only to the right. We observe that the energy equalizes between epochs 8 through 12, however, the excess energy in epochs 6 and 7 cannot flow right, due to the E_{max} constraint enforced by the right permeable tap between epochs 7 and 8.

V. Transmission Completion Time Minimization in a Fading Channel

In contrast to the infinite backlog assumption of the previous sections, we now assume that the transmitter has B bits to be

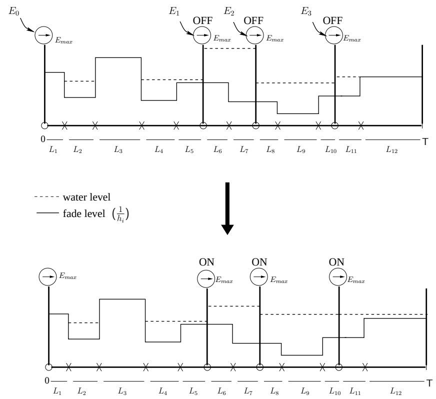

Fig. 3. Directional water-filling with right permeable taps in a fading channel.

communicated to the receiver in the energy harvesting and fading channel setting. Our objective is now to minimize the time necessary to transmit these B bits. The problem is:

$$\min \quad T \tag{23}$$

$$\text{s.t.} \quad \sum_{i=1}^{N+1} \frac{L_i}{2} \log\left(1 + h_i p_i\right) = B \tag{24}$$

$$\sum_{i=1}^{\ell} L_i p_i \leq \sum_{i=1}^{\ell} E_{in}(i), \quad \ell = 1, \ldots, N+1 \tag{25}$$

$$\sum_{i=1}^{\ell} E_{in}(i) - \sum_{i=1}^{\ell} L_i p_i \leq E_{max}, \ell = 1, \ldots, N \tag{26}$$

where $N \triangleq N(T)$ is the number of events in the interval $[0, T]$. Next, we introduce the maximum departure curve which serves to map the transmission completion time minimization problem to the throughput maximization problem.

A. Maximum Departure Curve

Given a deadline T, we define the maximum departure curve $D(T)$ for a given energy arrival and channel fading profile as,

$$D(T) = \max \sum_{i=1}^{N(T)+1} \frac{L_i}{2} \log\left(1 + h_i p_i\right) \tag{27}$$

The maximization in (27) is subject to the energy causality and maximum battery storage capacity constraints in (25) and (26). The maximum departure function $D(T)$ represents the maximum number of bits that can be served out of the backlog by the deadline T given the energy arrival and fading sequences. This is exactly the solution of the problem studied in the previous sections. Some characteristics of the maximum departure curve are stated in the following lemma.

Lemma 1 *The maximum departure curve $D(T)$ is a monotonically increasing and continuous function of T. $D(T)$ is not differentiable at $\{t_i^e\}$ and $\{t_i^f\}$.*

The continuity and monotonicity of $D(T)$ implies that the inverse function of $D(T)$ exists, and that for a closed interval $[a, b]$, $D^{-1}([a, b])$ is also a closed interval. Since $D(T)$ is obtained by the directional water-filling algorithm, the derivative of $D(T)$ has the interpretation of the rate of energy transfer from past into the future at time T, i.e., it is the measure of the tendency of the water to flow right.

B. Solution of the Transmission Completion Time Minimization Problem in a Fading Channel

We now solve the transmission completion time minimization problem stated in (23)-(26). Minimization of the time to complete the transmission of B bits available at the transmitter is closely related with the maximization of the number of bits that can be sent by a deadline. In fact, if the maximum number of bits that can be sent by T is less than B, then it is not possible to complete the transmission of B bits by T. As we state formally below, if T^* is the minimal time to complete the transmission of B bits, then necessarily $B = D(T^*)$. This argument provides a characterization for T^* in terms of the maximum departure curve, as stated in the following theorem.

Theorem 3 *The minimum transmission completion time T^* to transmit B bits is $T^* = \min\{t \in \mathcal{M}_B\}$ where $\mathcal{M}_B = \{t : B = D(t)\}$.*

VI. Conclusions

We developed optimal energy management schemes for energy harvesting systems operating in fading channels, with finite capacity rechargeable batteries. We considered two related problems under offline knowledge of the events: maximizing the number of bits sent by a deadline, and minimizing the time it takes to send a given amount of data. We solved the first problem using a directional water-filling approach. We solved the second problem by mapping it to the first problem via the maximum departure curve function.

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