Wireless Information and Energy Transfer under Temperature Constraints

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***Abstract*—Wireless energy transfer causes heating in the surrounding tissue due to radiation. We address this issue in a joint wireless information and energy transfer setting. We consider the optimal joint wireless information and energy transfer subject to a peak temperature constraint at the receiver side in a deadline constrained frequency selective additive white Gaussian noise channel. We model the temperature dynamics at the receiving end as a linear heat circuit and determine optimal transmission policies. We investigate optimal policies for various operating points on the boundary of the tradeoff region.**

I. Introduction

The damage due to temperature increase caused by sensor operation is a major concern in the context of wireless sensor networking. Sensors have to guarantee safety in that the temperature due to their operations does not cause any threat for the environment or their proper operation. One of the most prominent examples for such damage is due to radiation in the recharging phase of biomedical wireless nodes implanted in the human body. For various instances of this issue, see [1]–[6]. In particular, Pennes' work in 1948 [1] addresses this issue and explores the temperature dynamics due to electromagnetic radiation and the resulting heat dissipation in the tissue. This problem arises in many types of body area sensor networks [2], [3] where depending on the type of tissue and material properties of the sensor node, data transmission has to be scheduled according to temperature sensitivity [4]. Specifically, in implantable systems, e.g., retinal or cochlear implants, where temperature increase is a strict constraint, it is possible that current standards [5], [6] are not sufficiently safe to avoid tissue scarring. Further, when there is only one transmitting and receiving coil, joint information and power transfer can outperform splitting between individually optimized strategies, thereby dissipating less heat. Thus, we examine the problem of joint information and power transfer.

In general, there is a fundamental tradeoff between information and energy transfer using an electrical signal through a channel [7], [8]. This tradeoff is studied for an amplitude constrained additive Gaussian channel in [7] and for a wireless frequency selective channel in [8]. Following the initial works [7], [8], wireless information and energy transfer has been recently studied extensively [9], [10], [11]. In particular, this tradeoff is studied for several multiuser multi-antenna settings with information and energy reception capabilities.

In a related line of research, data transmission with energy harvesting transmitters is considered in [12]–[15]. In these works, optimal transmission policies for an energy harvesting transmitter in an additive white Gaussian noise channel is studied. In our recent works [16], [17], we address temperature sensitivity due to wireless data transmission in an energy harvesting transmitter. In particular, the temperature model in [16] is based on a view of the heat dynamics as a first order circuit driven by the electromagnetic radiation emitted due to the transmit power. There, we impose a hard temperature constraint and consider throughput maximization subject to a limited radiation emitted at the transmitter side. In [17], we consider the effects of temperature dependent leakage and processing cost in energy harvesting communications.

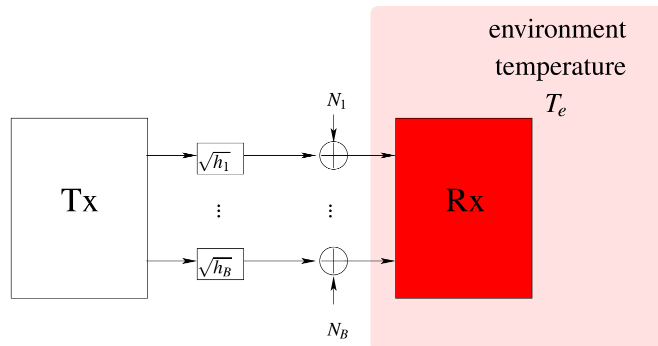

Fig. 1. System model representing an energy harvesting receiver node placed in an environment that has constant temperature T_e. The receiver harvests energy from the incident electromagnetic power in B frequency bands with constant channel gains.

In this paper, we combine the approaches in [8] and [16] and address joint wireless information and energy transfer under a critical temperature constraint T_c at the receiver side. In particular, we consider optimal information and energy transfer subject to a critical temperature constraint T_c at the receiver side over an additive Gaussian noise frequency selective wireless fading channel. The tradeoff between wireless information and energy transfer takes a new form when temperature constraints are present in view of the dynamical system model relating the transmit power policy and temperature.

We determine the optimal tradeoff between wireless information and energy transfer under energy and temperature constraints. We model the thermal dynamics at the receiver side as a linear heat circuit where the incident electric power at the receiver is an input driving the temperature. We impose that the temperature does not exceed a critical level T_c; we obtain a convex optimization problem and its solution by using a Lagrangian framework and KKT optimality conditions. We derive contrasting structural properties of the optimal transmission policy for maximum information transmission and maximum energy transfer. In particular, we show that maximum information transmission requires a smooth power policy whereas maximum energy transmission is enabled by a peaky power policy. We then generalize these results for the whole tradeoff region between information and energy transfer.

II. The Model

We consider a receiver node placed in an environment of temperature T_e as shown in Fig. 1. We investigate the tradeoff between this node's capability to receive information and energy subject to a temperature constraint at the receiver.

A. Wireless Channel

We consider a discrete band frequency selective slow fading wireless channel. There are B parallel fading channels, indexed by $i \in \{1, 2, \dots, B\}$, for information and energy transmission. In each parallel channel, the received signal Y_i, the input X_i, channel gain h_i and noise N_i are related as[1]

$$Y_i = \sqrt{h_i} X_i + N_i$$

where N_i is additive white Gaussian noise with zero-mean and unit-variance. We use a continuous time model: A scheduling interval has a short duration with respect to the length of the whole interval and we approximate it as $[t, t+dt]$ where dt denotes infinitesimal time. The channel gains h_i are distinct and deterministic positive real numbers. In $[t, t+dt]$, transmitter decides a transmit power level $P_i(t)$ for the ith frequency band and $\sum_{i=1}^{B} \frac{1}{2}\log(1 + h_i P_i(t))\, dt$ units of data is sent to the receiver[2]. We consider a communication interval $[0, D]$ over which the fading level h_i remains unchanged.

B. Thermal Dynamics

We model the thermal dynamics at the receiver side as a first order heat circuit as follows:

$$\frac{dT}{dt} = a \sum_{i=1}^{B} h_i P_i(t) - b(T(t) - T_e) \tag{1}$$

where $\sum_{i=1}^{B} h_i P_i(t)$ is the total incident electromagnetic power at the receiver and $T(t)$ is the temperature at the receiver side at time t. T_e is the temperature of the environment, a and b are nonnegative constants. Our thermal model in (1) is a modified version of the well known Pennes' bioheat equation [1] in that the spatial variation in temperature is ignored. In particular, the incident electric power at the receiver side is assumed to be a point source of heat at the location of the receiver that dissipates uniformly to the environment.

Let us take the initial temperature as $T(0) = T_e$. The solution of $T(t)$ for any given $P_i(t)$ from (1) is [16]:

$$T(t) = e^{-bt}\left(\int_0^t e^{b\tau}\left(a\sum_{i=1}^{B} h_i P_i(\tau) + bT_e\right) d\tau + T_e\right) \tag{2}$$

The temperature should remain below a critical level T_c, i.e., $T(t) \leq T_c$, where we assume that $T_c > T_e$. We define $T_\delta \triangleq T_c - T_e$. From (2), using $T(t) \leq T_c$, we get the following equivalent condition for the temperature constraint:

$$\int_0^t a e^{b\tau} \sum_{i=1}^{B} h_i P_i(\tau) d\tau \leq T_\delta e^{bt}, \quad \forall t \in [0, D] \tag{3}$$

We observe in (3) that the cost of spending power increases exponentially in time while the heat margin also increases exponentially in time. This observation will prove to be crucial in the maximum energy transfer problem in particular.

III. Problem Formulation

The transmitter's problem is to choose the power allocation $P_i(t)$ at each frequency band $i \in \{1, 2, \dots, B\}$ over the time interval $t \in [0, D]$ so that the optimal tradeoff between information and energy transfer is achieved while guaranteeing that the temperature at the receiver side is below the critical level T_c. In this regard, we consider the following information rate maximization problem under energy, temperature and received power constraints:

$$\begin{aligned} \max_{P_i(t), t\in[0,D]} \quad & \int_0^D \sum_{i=1}^{B} \frac{1}{2}\log(1 + h_i P_i(\tau))\, d\tau \\ \text{s.t.} \quad & \int_0^t a e^{b\tau} \sum_{i=1}^{B} h_i P_i(\tau) d\tau \leq T_\delta e^{bt}, \ \forall t \\ & \int_0^D \sum_{i=1}^{B} P_i(\tau) d\tau \leq E, \\ & \int_0^D \sum_{i=1}^{B} h_i P_i(\tau) d\tau \geq P_{del} \end{aligned} \tag{4}$$

The Lagrangian for (4) is:

$$\begin{aligned} \mathcal{L} = & \int_0^D \sum_{i=1}^{B} \frac{1}{2}\log(1 + h_i P_i(t))\, dt \\ & - \int_0^D \lambda(t)\left(\int_0^t a e^{b\tau} \sum_{i=1}^{B} h_i P_i(\tau) d\tau - T_\delta e^{bt}\right) dt \\ & - \beta\left(\int_0^D \sum_{i=1}^{B} P_i(\tau) d\tau - E\right) \\ & + \gamma\left(\int_0^D \sum_{i=1}^{B} h_i P_i(\tau) d\tau - P_{del}\right) \end{aligned} \tag{5}$$

[1] Such fading models arise naturally in capacitively and inductively coupled circuits for power and information transfer, and also through multi-path in far-field situations.

[2] Our model is an approximation of the realistic scenario where physical signaling is in discrete time. In each $[t, t+dt]$ interval, sufficiently large number of transmissions occur for reliable transmission. The scalings in SNR and rate due to bandwidth are inconsequential for the reported results.

Taking the derivative of the Lagrangian with respect to $P_i(t)$ and equating to zero:

$$\frac{h_i}{1+h_iP_i(t)} - h_ie^{bt}\int_t^D \lambda(\tau)d\tau - \beta + h_i\gamma = 0 \quad (6)$$

which gives for all $t \in [0, D]$ and for all $i \in \{1, 2, \ldots, B\}$

$$P_i(t) = \left[\frac{1}{\beta - h_i\gamma + h_ie^{bt}\int_t^D \lambda(\tau)d\tau} - \frac{1}{h_i}\right]^+ \quad (7)$$

In addition, the complementary slackness conditions are:

$$\lambda(t)\left(\int_0^t ae^{b\tau}\sum_{i=1}^B h_iP_i(\tau)d\tau - T_\delta e^{bt}\right) = 0 \quad (8)$$

$$\beta\left(\int_0^D \sum_{i=1}^B P_i(\tau)d\tau - E\right) = 0 \quad (9)$$

$$\gamma\left(\int_0^D \sum_{i=1}^B h_iP_i(\tau)d\tau - P_{del}\right) = 0 \quad (10)$$

We note that (6) and (8)-(10) are necessary and sufficient conditions since the problem is convex.

IV. General Properties of Optimal Power Policies

We provide the following results on the general properties of an optimal power policy for an arbitrary feasible value of delivered energy P_{del}. We refer to $P_i(t)$ as the optimal policy and $T(t)$ as the resulting temperature where the dependence on P_{del} is implicit. Some of these results are extensions of those in [16] and their proofs are skipped.

Lemma 1 *At $t = D$, either the temperature constraint or the energy causality constraint or both are tight.*

Lemma 2 *$P_i(t)$ is a monotone decreasing function of t.*

Proof: The proof follows from a similar argument to that in [16, Lemma 6]. In particular, let us consider any interval $[t_1, t_2]$ where $P_i(t)$ is monotone increasing. When $P_i(t)$ is updated as a constant over $[t_1, t_2]$ with the same energy, this still satisfies the temperature constraint due to a modified argument to that in [16, Lemma 6] and yields higher information transfer due to the concavity of rate-power relation. ■

Lemma 3 *$P_i(t)$ is a continuous function of t in the interval $(0, D)$.*

Proof: Since e^{bt} is a continuous function of t and $\lambda(t) \geq 0$, any jump in $P(t)$ has to be positive due to (7). Any positive jump violates monotonicity of $P_i(t)$ within each epoch due to Lemma 2. ■

Lemma 4 *Let $T(t') = T_c$ for some $t' \in [0, D)$. Then, $\sum_{i=1}^B h_iP_i(t'-\epsilon) \geq \frac{T_\delta b}{a}$ for all sufficiently small $\epsilon > 0$.*

Lemma 5 *If $0 < \sum_{i=1}^B h_iP_i(t) \leq \frac{T_\delta b}{a}$ for $t \in [t_1, D]$, then $\sum_{i=1}^B h_iP_i(t)$ is constant over $[t_1, D]$.*

Lemma 6 *Let $t' \in [0, D]$ denote $\min\{t \in [0, D] : T(t) = T_c\}$. If $t' \neq D$, then $\sum_{i=1}^B h_iP_i(t) = \frac{T_\delta b}{a}$ for all $t \in [t', D]$.*

Lemma 7 *The optimal policy $P_i(t)$ satisfies:*

$$\sum_{i=1}^B h_iP_i(t) \geq \min\left\{\frac{T_\delta b}{a}, P_{wf}\right\}, \quad \forall t \in [0, D] \quad (11)$$

where $P_{wf} = \sum_{i=1}^B h_iP_i$ corresponds to the solution P_i for maximizing $\sum_{i=1}^B \frac{1}{2}\log(1+h_iP_i)$ subject to $\sum_{i=1}^B P_i = \frac{E}{D}$ and $\sum_{i=1}^B h_iP_i \geq P_{del}$.

Proof: If temperature constraint is not tight, then the problem reduces to the energy constrained problem in which case $P_i(t)$ should be selected as a constant function due to the concavity of the rate-power relation and $\sum_{i=1}^B P_i = \frac{E}{D}$. In this case, the optimal solution satisfies $\sum_{i=1}^B h_iP_i = P_{wf}$ where P_i corresponds to the solution for maximizing $\sum_{i=1}^B \frac{1}{2}\log(1+h_iP_i)$ subject to $\sum_{i=1}^B P_i = \frac{E}{D}$ and $\sum_{i=1}^B h_iP_i \geq P_{del}$. If temperature constraint is tight, $P_i(t)$ is monotone decreasing by Lemma 2 and when temperature level reaches T_c, $\sum_{i=1}^B h_iP_i(t)$ remains at $\frac{T_\delta b}{a}$ by Lemma 6. Hence, $\sum_{i=1}^B h_iP_i(t) \geq \frac{T_\delta b}{a}$. ■

Lemma 8 *In an optimal policy, energy in the battery is nonzero except possibly at $t = D$.*

Lemma 9 *The temperature with the optimal power policy is monotone increasing and concave.*

The following result states that the optimal policy is found by channel inversion when energy is unlimited.

Lemma 10 *If $E = \infty$, in the optimal policy, $h_iP_i(t) = f(t)$ for all i.*

Proof: Assume $E = \infty$ so that the energy constraint is not active. For any interval $t \in [t_1, t_2]$, updating $P_i(t)$ such that $h_i\tilde{P}_i(t) = \frac{1}{B}\sum_{i=1}^B h_iP_i(t)$ increases the information transfer measured by $\int_0^D \sum_{i=1}^B \frac{1}{2}\log(1+h_iP_i(t))\,dt$ while keeping the temperature unchanged. This proves the desired result. ■

In the following two sections, we investigate optimal transmission policies for maximum information transfer and maximum energy transfer separately. Then, we combine the two cases to obtain the whole tradeoff region for joint wireless information and energy transfer.

V. Maximum Information Transfer

In this section, we focus on the maximum information transfer problem without regard to the minimum delivered energy constraint P_{del}. This is a generalization of the problem considered in [16] with parallel fading channels instead of a point-to-point channel.

A. Unlimited Energy

Let us first assume that the available energy is $E = \infty$ so that the only limitation on the power policy is due to the temperature constraint. In this case, we have from (7):

$$P_i(t) = \left[\frac{1}{h_i e^{bt}\int_t^D \lambda(\tau)d\tau} - \frac{1}{h_i}\right]^+ \quad (12)$$

Note that from Lemma 10, we have $h_iP_i(t) = h_jP_j(t)$ for any i, j. From Lemma 9, $T(t)$ is monotone increasing. Due to Lemma 6, when $T(t)$ reaches T_c, power level has to remain at $\frac{T_\delta a}{b}$. Accordingly, we denote the instant when the temperature reaches T_c as t_0. Using similar steps to those in [16, Section V.B.1], $P_i(t)$ has the following form:

$$\begin{aligned} P_i(t) =& \frac{1}{h_iB}\left(\left(\frac{T_\delta b}{a}+1\right)e^{-b(t-t_0)} - 1\right)(u(t) - u(t-t_0)) \\ &+ \frac{T_\delta b}{ah_iB}u(t-t_0) \end{aligned} \quad (13)$$

where $u(t)$ is the unit step function. The Lagrange multiplier $\lambda(t)$ that verifies (13) and the corresponding temperature pattern are identical to those in [16, Eqs. (20) and (21)], respectively. We also note that t_0 is identical to that in [16, Section V.B.1] and it satisfies:

$$\left(\frac{T_\delta}{a} + \frac{1}{b}\right)e^{bt_0} - \frac{1}{b} = \left(\frac{T_\delta b}{a} + 1\right)t_0e^{bt_0} \quad (14)$$

so that $T(t_0) = T_c$. If $D < t_0$:

$$P_i(t) = \frac{C}{h_iB}e^{-bt} - \frac{1}{h_iB} \quad (15)$$

where $C = \frac{1}{D}\left(\left(\frac{T_\delta}{a} + \frac{1}{b}\right)e^{bD} - \frac{1}{b}\right)$.

B. Limited Energy

The power policies in the energy unconstrained cases in (13) and (15) have finite energies. If the available energy E is smaller than the corresponding energy level in (13) and (15), then the energy constraint is active and the Lagrange multiplier is $\beta > 0$. From (7), we have:

$$P_i(t) = \left[\frac{1}{\beta + h_i e^{bt}\int_t^D \lambda(\tau)d\tau} - \frac{1}{h_i}\right]^+ \quad (16)$$

We first note that there is a critical energy level $E_{critical}$ such that if $E \leq E_{critical}$, then the temperature constraint is not tight and the power allocation in each band remains unchanged in time. In this case, the water-filling policy with $\sum_{i=1}^B P_i(t) = \frac{E}{D}$ and $P_i(t) = \left[\frac{1}{\beta} - \frac{1}{h_i}\right]^+$ is optimal. Note that when $E \leq E_{critical}$, $\lambda(t) = 0$ since temperature constraint is never tight.

Let us now consider $E > E_{critical}$. In this case, we have:

$$P_i(t) = \left[\frac{1}{\beta + h_iCe^{bt}} - \frac{1}{h_i}\right]^+ \quad (17)$$

where $C = \int_{t_0}^D \lambda(\tau)d\tau > 0$. Additionally, $P_i(t)$ remains constant for the remaining portion of the epoch. In particular, in view of Lemma 6, $\sum_{i=1}^B h_iP_i(t) = \frac{T_\delta b}{a}$ and $T(t) = T_c$. $\lambda(t)$ is selected such that $e^{bt}\int_t^D \lambda(\tau)d\tau = Ce^{bt_0}$ for $t \in [t_0, D]$. β, C and t_0 are determined so as to satisfy $\int_0^D \sum_{i=1}^B P_i(\tau)d\tau = E$, $\sum_{i=1}^B h_iP_i(t_0) = \frac{T_\delta b}{a}$ and $T(t_0) = T_c$.

VI. Maximum Energy Transfer

In this section, we consider the maximum energy transfer problem without regard to the amount of information transfer. In this case, we have the following optimization problem:

$$\begin{aligned} \max_{P_i(t), t\in[0,D]} \quad & \int_0^D \sum_{i=1}^B h_iP_i(\tau)d\tau \\ \text{s.t.} \quad & \int_0^t ae^{b\tau}\sum_{i=1}^B h_iP_i(\tau)d\tau \leq T_\delta e^{bt}, \ \forall t. \\ & \int_0^D \sum_{i=1}^B P_i(\tau)d\tau \leq E \end{aligned} \quad (18)$$

which is a linear program and its Lagrangian is

$$\begin{aligned} \mathcal{L} = & \int_0^D \sum_{i=1}^B h_iP_i(\tau)d\tau \\ & - \int_0^D \lambda(t)\left(\int_0^t ae^{b\tau}\sum_{i=1}^B h_iP_i(\tau)d\tau - T_\delta e^{bt}\right)dt \\ & - \beta\left(\int_0^D \sum_{i=1}^B P_i(\tau)d\tau - E\right) \end{aligned} \quad (19)$$

Taking the derivative of the Lagrangian with respect to $P_i(t)$ and equating to zero:

$$h_i - h_ie^{bt}\int_t^D \lambda(\tau)d\tau - \beta = 0 \quad (20)$$

Additionally, the slackness conditions in (8)-(9) have to be satisfied. Note that the solution for the linear program in (18) is not unique in this case. Still, there is an almost surely unique allocation that maximizes the information transfer subject to the maximum energy transfer.

A. Unlimited Energy

Let us first assume that the available energy is $E = \infty$ so that the limitation on the power policy is due only to the temperature constraint. With $\beta = 0$, (20) is equivalent to $\int_t^D \lambda(\tau)d\tau = e^{-bt}$. This is possible only when $\lambda(t) = c_1e^{-bt} - c_2$ for some $c_1, c_2 \geq 0$. In view of the slackness condition in (8) and (9), $T(t) = T_c$ for all $t \in (0, D)$. By inspection, we determine the solution for (20) as

$$\sum_{i=1}^B h_iP_i(t) = \frac{T_\delta}{a}\delta(t) + \frac{T_\delta b}{a} \quad (21)$$

In order to maximize the information transfer subject to the maximum energy delivery constraint, we need to perform the optimization subject to (21). In this case, the solution is obtained by channel inversion due to Lemma 10. In particular, the optimal allocation in this case is

$$P_i^*(t) = \frac{T_\delta}{ah_i B}\delta(t) + \frac{T_\delta b}{ah_i B} \quad (22)$$

B. Limited Energy

Let us now consider the finite energy case. In particular, we assume that $E < \infty$ is sufficiently small so that the energy constraint is tight. We first note that the power allocation has to be in the frequency band with the highest channel gain $i^* = \arg\max_i h_i$.

Lemma 11 *If the energy constraint is tight, then the solution of (18) is such that $P_{i^*}(t) > 0$ and $P_i(t) = 0$ for $i \neq i^*$.*

Proof: Note that it suffices to consider the cases when $\beta > 0$ since $\beta = 0$ is equivalent to the unlimited energy case addressed in the previous subsection. In particular, it is sufficient to take $P_i(t) > 0$ for $i = \arg\max_i h_i$ and $P_i(t) = 0$ otherwise. Whenever $\beta > 0$, (20) is possible if and only if $P_i(t) > 0$ for $i = \arg\max_i h_i$ and $P_i(t) = 0$ otherwise. ■

We note that $P_{i^*}(t)$ is not unique if the objective is only to maximize the energy transfer. Among all $P_{i^*}(t)$ with $\int_0^D P_{i^*}(\tau)d\tau = E$, the one that maximizes information transfer subject to maximum energy transfer is in the following form in view of (7):

$$\begin{aligned} P_i(t) =& \frac{1}{h_{i^*}B}\left(\left(A+1\right)e^{-b(t-t_0)} - 1\right)\left(u(t) - u(t-t_0)\right) \\ &+ \frac{A}{h_{i^*}B}u(t-t_0) \end{aligned} \quad (23)$$

where $u(t)$ is the unit step function. The parameters A and t_0 in (23) are determined such that $\int_0^D P_{i^*}(\tau)d\tau = E$ and $T(t_0) = T_c$. There is a critical energy level $E_{critical}$ such that if $E < E_{critical}$, then $P_{i^*}(t) = \frac{E}{D}$ is optimal in terms of information transfer subject to maximum energy transfer and energy constraint. This critical level is:

$$E_{critical} = \frac{bDe^{bD}T_\delta}{ah_{i^*}\left(e^{bD}-1\right)} \quad (24)$$

VII. Joint Information and Energy Transfer

In this section, we generalize the results in the previous two sections for an arbitrary operating point on the boundary of the tradeoff region between information and energy transfer. In particular, we assume that the minimum energy delivery constraint P_{del} is in between the energy delivered by the optimal policies determined in Sections V and VI.

A. Unlimited Energy

We first assume that the energy available at the transmitter is unlimited, i.e., $E = \infty$. Due to (7), we have:

$$P_i(t) = \left[\frac{1}{-h_i\gamma + h_i e^{bt}\int_t^D \lambda(\tau)d\tau} - \frac{1}{h_i}\right]^+ \quad (25)$$

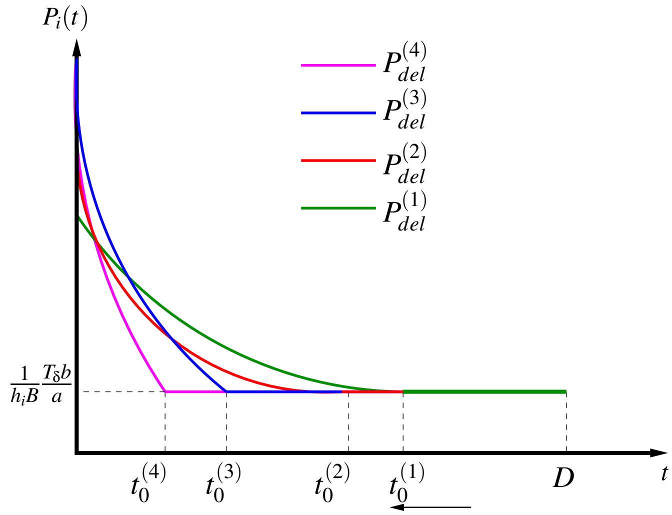

Fig. 2. The optimal policy with respect to the minimum delivered energy P_{del} for unlimited E. As P_{del} is increased from $P_{del}^{(1)}$ to $P_{del}^{(4)}$, t_0 decreases from $t_0^{(1)}$ to $t_0^{(4)}$ and approaches zero; the optimal policy approaches an impulse at $t = 0$ and $\frac{1}{h_i B}\frac{T_\delta b}{a}$ otherwise.

From Lemma 10, we have $h_i P_i(t) = h_j P_j(t)$ for all $i, j \in \{1, \ldots, B\}$. In view of the general properties in Lemmas 6-9, we have for $t \in [0, t_0]$:

$$P_i(t) = \left[\frac{1}{-h_i\gamma + h_i Ce^{bt}} - \frac{1}{h_i}\right]^+ \quad (26)$$

where $C = \int_{t_0}^D \lambda(\tau)d\tau > 0$. Additionally, $P_i(t)$ remains constant for the remaining portion of the epoch. In particular, in view of Lemma 6, $\sum_{i=1}^B h_i P_i(t) = \frac{T_\delta b}{a}$ and $T(t) = T_c$. $\lambda(t)$ is selected such that $e^{bt}\int_t^D \lambda(\tau)d\tau = Ce^{bt_0}$ for $t \in [t_0, D]$. γ, C and t_0 are determined so as to satisfy $\int_0^D \sum_{i=1}^B h_i P_i(\tau)d\tau = P_{del}$, $\sum_{i=1}^B h_i P_i(t_0) = \frac{T_\delta b}{a}$ and $T(t_0) = T_c$.

We illustrate the optimal policy with respect to the minimum delivered energy P_{del} in Fig. 2. We observe that as P_{del} is increased, t_0 approaches zero indicating that the energy transfer has to be conducted quickly in order to create less temperature increase. Indeed, the optimal policy approaches the maximum energy transfer policy in (22) with an impulse at $t = 0$ and $\frac{T_\delta b}{a}$ otherwise.

B. Limited Energy

The power policies in the previous subsection have finite energies. If the available energy E is smaller than the corresponding energy levels, then the energy constraint is active and the Lagrange multiplier is $\beta > 0$. From (7), we have:

$$P_i(t) = \left[\frac{1}{\beta + h_i e^{bt}\int_t^D \lambda(\tau)d\tau} - \frac{1}{h_i}\right]^+ \quad (27)$$

As in the previous two sections on maximum information and energy transfer, there is a critical energy level $E_{critical}$ such that if $E \leq E_{critical}$, then the power allocation in each band is constant throughout time and the temperature constraint

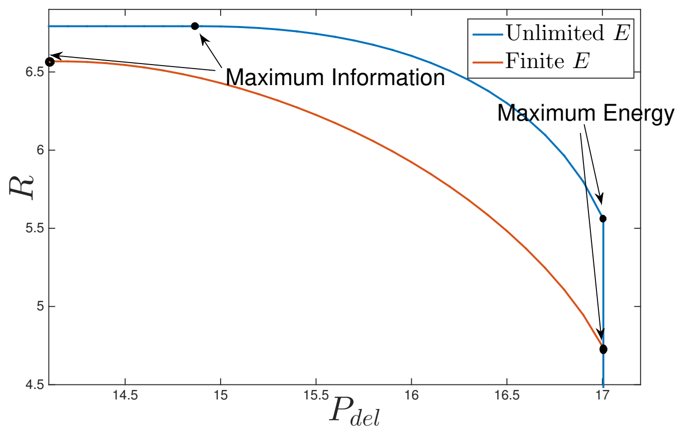

Fig. 3. The tradeoff curves for $D = 2.5$ with unlimited and finite energy limits.

is not tight. In particular, the policy with $\sum_{i=1}^{B} P_i(t) = \frac{E}{D}$, $\sum_{i=1}^{B} h_i P_i(t) = \frac{P_{del}}{D}$ and $P_i(t) = \left[\frac{1}{\beta - h_i\gamma} - \frac{1}{h_i}\right]^+$ is optimal. This policy is identical to that determined in [8] that achieves the tradeoff between wireless information and energy transfer. Note that when $E \leq E_{critical}$, $\lambda(t) = 0$ since temperature constraint is never tight.

We now consider $E > E_{critical}$. In this case, we have:

$$P_i(t) = \left[\frac{1}{\beta - h_i\gamma + h_i C e^{bt}} - \frac{1}{h_i}\right]^+ \tag{28}$$

where $C = \int_{t_0}^{D} \lambda(\tau)d\tau > 0$. Additionally, $P_i(t)$ remains constant for the remaining portion of the epoch. In particular, in view of Lemma 6, $\sum_{i=1}^{B} h_i P_i(t) = \frac{T_\delta b}{a}$ and $T(t) = T_c$. $\lambda(t)$ is selected such that $e^{bt} \int_t^D \lambda(\tau)d\tau = Ce^{bt_0}$ for $t \in [t_0, D]$. β, C and t_0 are determined so as to satisfy $\int_0^D \sum_{i=1}^{B} P_i(\tau)d\tau = E$, $\sum_{i=1}^{B} h_i P_i(t_0) = \frac{T_\delta b}{a}$ and $T(t_0) = T_c$.

VIII. Numerical Results

In this section, we provide numerical results to illustrate the tradeoff between wireless information and energy transfer under a critical temperature constraint. We provide a numerical example where we set $a = 0.1$, $b = 0.3$, $T_e = 37$ and $T_c = 38$. The wireless channel has $B = 2$ bands with channel gains $h_1 = 0.5$ and $h_2 = 0.9$. We set the deadline to $D = 2.5$. In Fig. 3, we show the resulting tradeoff curve for unlimited energy and finite energy with $E = 20$. In this figure, we denote the total information sent over $[0, D]$ by $R = \int_0^D \sum_{i=1}^{B} \frac{1}{2} \log\left(1 + h_i P_i(\tau)\right) d\tau$. We observe the drop in the achievable tradeoff due to the finite energy limitation.

IX. Conclusion

We considered optimal joint wireless information and energy transfer in a deadline constrained frequency selective fading additive white Gaussian noise channel under a safe temperature guarantee at the receiving end. We determined transmit power policies for the optimal operating points in the tradeoff region. We derived structural properties and obtained closed form solutions for various optimal operating points on the boundary of the tradeoff curve. When energy is unlimited, we showed that the optimal power policy has an exponentially decreasing form in time and the allocation over bands is determined by channel inversion. In the maximum energy transfer extreme, we showed that the optimal power policy is peaky in that the energy transfer is performed at $t = 0$ instant to the extent possible subject to the temperature constraint and then the temperature is kept constant throughout the operation. We generalized our results for the whole tradeoff region between wireless information and energy transfer. These results reveal contrasting behaviors for the optimal transmit power policies for maximum information and energy transfer and promises new directions of research to understand the relations among energy, information and delay under temperature sensitivity constraints.

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