Channel Estimation and Adaptive M-QAM in Cognitive Radio Links

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***Abstract*—Cognitive radios have the ability to sense their RF environment and adapt their transmission parameters to perform optimally in any situation. Part of this involves selecting the best modulation type for a particular channel. In this paper we consider a variable-rate, variable-power, adaptive, M-ary Quadrature Amplitude Modulation (M-QAM) scheme in a single-user communication scenario. The channel between the transmitter and receiver is assumed to be a Rayleigh block-fading channel. Each block is divided into training and data phases. During the training phase, the receiver estimates the channel and feeds the estimate back to the transmitter. During the data phase, the transmitter sends its message by adapting the size of the M-QAM constellation. We first find a closed-form expression that relates the Bit Error Rate (BER) to the constellation size of the M-QAM, and therefore to the data rate of our system. Then, for a given target BER, we maximize the data rate over the training parameters, which are the training signal, the training duration, and the training power. When these optimum parameters are used in a MATLAB implementation, we find that the target BER is matched to within an order of magnitude, and the resulting data rate is close to the theoretical limit.**

I. Introduction

Multi-path fading is a characteristic of many wireless communication channels, and causes randomness in received signals. Information theoretically, taking advantage of the fading, instead of fighting it, has been shown to result in higher bit rates. The more CSI available at the receiver and the transmitter, the better the transmission scheme can be adapted to the channel, and the larger the bit-rates that can be achieved. However, system resources, such as time, bandwidth, and power, should be spent in estimating and feeding back the CSI. Channel-adaptive transmission schemes find applications in cognitive radios, which are able to measure, feed back, and adapt to, the fading channel state. We consider a single-user cognitive radio link, where the transmitter sends training symbols from which the receiver estimates the fading channel state, and uses it in its symbol-detection task. In addition, the receiver feeds the estimated CSI back to the transmitter, which adapts its M-QAM transmission scheme, to achieve larger rates, subject to a BER constraint.

The topic of optimizing for a fading channel using CSI has been widely studied. With perfect CSI at both the transmitter and receiver at the start of each symbol, bit rate is maximized when a variable-power, variable-rate scheme is used [1]. When the transmitter does not have any CSI, this scheme reduces to a constant power allocation scheme over all channel values.

When there is channel estimation error at the receiver, information-theoretic rate/capacity formulas become too complex, and the optimum signaling scheme and the resulting capacity for such channels are unknown. However, a lower bound to the capacity is achieved by Gaussian signaling. While finding the best lower bound, some research assumes the existence of a separate channel for computing channel-estimation parameters that does not consume communications resources [2]–[4]. If the channel estimation process is assumed to consume communication resources, optimizing the channel capacity lower bound involves finding optimal allocation of power between training and data phases, optimal training sequence length, and optimal training symbols [5]–[7].

Another line of research looks at the problem from a communication-theoretic perspective, specifically for variable-rate, variable-power M-QAM scheme when both the transmitter and the receiver have the perfect CSI [8]. Unlike [1] where input signals were chosen from a Gaussian-distributed codebook, [8] chooses input signals from a Gray-encoded M-QAM constellation. For a fixed-target BER, [8] proposes a variable-rate, variable-power communication scheme that achieves rates close to the capacity given in [1].

In this paper, we consider a combination of these works. Our approach is communication-theoretic as in [8]. We assume that there is a feedback link from the receiver to the transmitter, and the transmitter adapts its M-QAM communication scheme based on the CSI feedback received. However, unlike [8] we assume that there is channel estimation error at the receiver, resulting in noisy CSI. As in [5]–[7], we assume that the training phase uses communication resources, and we optimize the training process by choosing the optimum training sequence, optimum training length, and optimum division of power between the training and data transmission phases. However, unlike [5]–[7] we assume there is an instantaneous

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CSI feedback link, and that the transmitter can adapt to the estimated CSI.

II. System Model

We consider a point-to-point channel with single antenna at both ends. The channel between the transmitter and receiver is represented by a circularly-symmetric, complex, Gaussian random variable h, i.e., Rayleigh fading. We consider a block-fading scenario where the channel stays constant for T symbols and changes to an i.i.d. realization during block transitions. The received signal can be represented as

$$y = \sqrt{P}hx + n \tag{1}$$

where n is zero-mean, unit-variance, circularly-symmetric, complex, Gaussian noise. The input signal x has a unit average power constraint, $E[|x|^2] \leq 1$, and P is the average Signal-to-Noise Ratio (SNR).

III. Training Phase

For practical communication scenarios, the channel is estimated at the receiver, and the estimate is fed back to the transmitter. After a delay, the transmitter receives the estimate. For our analysis, we assume that the feedback is instantaneous and error-free[1]. Therefore, we focus on the effect of the channel estimation error.

We let T_t be the training sequence length, and $T_d = T - T_t$ be the data sequence length. Since the channel realization is constant during the entire block, we can equivalently write the input-output relation of the channel during the training phase as

$$\mathbf{y}_t = \sqrt{P_t}h\mathbf{x}_t + \mathbf{n}_t \tag{2}$$

where P_t is the SNR of the training symbol, and length-T_t vectors $\mathbf{y}_t$, $\mathbf{x}_t$ and $\mathbf{n}_t$ represent the received signal, transmitted signal, and noise, respectively. The power constraint for the training input signal becomes $\frac{1}{T_t}\mathbf{x}_t^H\mathbf{x}_t \leq 1$.

The receiver will estimate the channel using the received signal $\mathbf{y}_t$, and the training signal $\mathbf{x}_t$, which will be optimized later. In order to find the optimal linear MMSE estimator, we solve the following optimization problem with $\hat{h} = \mathbf{c}^H\mathbf{y}_t$ as the estimate of h, and $\tilde{h} = h - \hat{h}$ as the estimation error,

$$\min_{\mathbf{c}} E\left[\tilde{h}\tilde{h}^*\right] = \min_{\mathbf{c}} E\left[(h - \mathbf{c}^H\mathbf{y}_t)(h - \mathbf{c}^H\mathbf{y}_t)^*\right]. \tag{3}$$

The solution to this optimization problem is

$$\hat{h} = \frac{\sqrt{P_t}\mathbf{x}_t^H\mathbf{y}_t}{1 + P_t\mathbf{x}_t^H\mathbf{x}_t}. \tag{4}$$

After this estimated value is fed back to the transmitter, the data transmission phase starts.

[1]This assumption is not too unrealistic. Imagine a 10Mbps link separated by 1km. The transmission time for 1000 bytes is 800 μs, while the propagation time is a mere 1 μs. Thus highly-coded, small feedback packets require orders of magnitude less airtime, making them appear nearly instantaneous.

IV. Data Transmission Phase

Although information-theoretic approaches to data-transmission problem are available in the literature [2], [4]–[7], we will focus on a communication-theoretic approach and consider a variable-rate, variable-power, M-QAM scheme. We will further restrict ourselves to a set of square constellations due to their ease of implementation. An example of 64-QAM constellation is shown in Figure 1(a). We consider a square M-QAM constellation as a combination of two $\sqrt{\text{M}}$-PAM constellations, one as the in-phase component and the other as the quadrature component; see Figure 1(b).

In a $\sqrt{\text{M}}$-PAM scheme, for a unit-energy pulse-shaping function, one-dimensional signal points have values $\frac{A_i}{\sqrt{2}}$, where A_i is the amplitude of the signal point, and $\sqrt{2}$ comes from the energy of the carrier signal. The amplitude values can be expressed as

$$A_i = (2i - 1 - \sqrt{M})d$$

where the Euclidean distance between adjacent signal points is $d\sqrt{2}$. Assuming equiprobable signals, the average energy of the in-phase component is

$$\frac{P_d(\hat{h})T_p}{2} = \frac{1}{\sqrt{M}}\sum_{i=1}^{\sqrt{M}}\frac{A_i^2}{2} = \frac{(M-1)d^2}{6} \tag{5}$$

where $\frac{P_d(\hat{h})}{2}$ is the power allocated to the $\sqrt{\text{M}}$-PAM component, and T_p is the period of the pulse shaping function. Without loss of generality, we assume that $T_p = 1$. We conclude that

$$d^2 = \frac{3P_d(\hat{h})}{M-1}. \tag{6}$$

While A_i is transmitted as the in-phase component, A_j is transmitted as the quadrature component of the input signal. After match-filtering and low-band conversion, the complex received signal can be written as

$$y = \frac{1}{\sqrt{2}}A_{ij}h + n \tag{7}$$

where both $A_{ij} = A_i + jA_j$, and $h = h_x + jh_y$ are complex numbers, and the additive noise n is circularly-symmetric, complex Gaussian with independent real and imaginary components. Due to the estimation process, we have $h = \hat{h} + \tilde{h}$, and

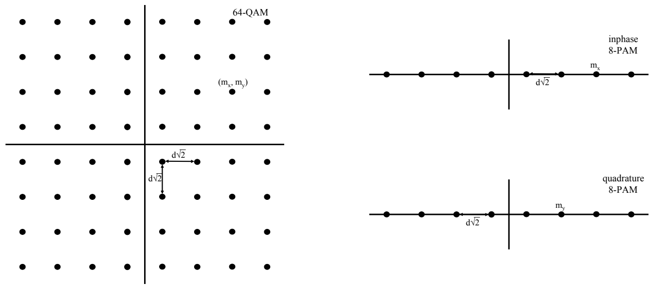
$$y = \frac{1}{\sqrt{2}}A_{ij}\hat{h} + \frac{1}{\sqrt{2}}A_{ij}\tilde{h} + n \tag{8}$$

In our model, the receiver regards $\hat{h}$ to be the actual channel, and $\frac{1}{\sqrt{2}}A_{ij}\tilde{h} + n$ to be the effective noise. It is important to note here that the channel estimation error $\tilde{h}$ contributes to the effective noise.

For an M-QAM scheme, the data rate is

$$R = \frac{T_d}{T}\log_2 M. \tag{9}$$

This rate is parameterized by the transmit power and the BER of the modulation scheme. In order to find the BER of a



(a) (b)

Fig. 1. (a) A 64-QAM constellation. (b) Equivalent in-phase and quadrature 8-PAM constellations.

square M-QAM, we consider it as a combination of two $\sqrt{\text{M}}$-PAM constellations, representing the in-phase and quadrature components. When there is no channel estimation error, the probability of correct signal-point detection is the product of the correct detection probabilities in each component of corresponding $\sqrt{\text{M}}$-PAMs. However, with channel estimation error, in-phase and quadrature component of the noise are correlated. Therefore, the probability of correct detection will be greater than this product. Let us assume that the signal point (i, j) is transmitted, where $1 \leq i \leq \sqrt{M}, 1 \leq j \leq \sqrt{M}$, then the symbol error probability for that signal point is

$$1 - P_{ij} \geq (1 - P_{i|j})(1 - P_{j|i}) \tag{10}$$

$$P_{ij} \lessapprox P_{i|j} + P_{j|i} \tag{11}$$

where $P_{i|j}$ and $P_{j|i}$ are the in-phase and quadrature symbol error probabilities for an $\sqrt{\text{M}}$-PAM scheme. Due to the channel estimation errors, $P_{i|j}$ depends on the signal level in the quadrature component, and $P_{j|i}$ depends on the signal level in the in-phase component. The BER can then be found by averaging (11) over the signal points,

$$BER = \frac{1}{M} \sum_i \sum_j \left(P_{i|j} + P_{j|i} \right). \tag{12}$$

A. Decision Regions with Channel Estimation Error

While calculating $P_{i|j}$ and $P_{j|i}$, we will use the in-phase and quadrature component of the constellation diagram at the receiver. The constellation diagram at the receiver is obtained by multiplying the constellation diagram at the transmitter by $\hat{h}$, resulting in an attenuated and rotated version. Since, the receiver knows $\hat{h}$, and therefore its phase, the rotation can be reversed, as the probability of error is rotation-invariant. Note that although the decision regions are determined to be one half the distance between the signal points when there is perfect channel estimation, they depend on the variance of the effective noise when there is channel estimation error at the receiver.

In order to find the decision regions when there is channel estimation error in the system, we utilize the maximum-likelihood rule which states that (i, j) is decoded if $f(y|m_x, m_y)$ is maximized for $m_x = i, m_y = j$. The decision region corresponding to signal point i for the in-phase component can be found as the region satisfying

$$f(y|i, j, \hat{h}) \geq f(y|i+1, j, \hat{h}) \tag{13}$$

$$f(y|i, j, \hat{h}) \geq f(y|i-1, j, \hat{h}) \tag{14}$$

Unlike the perfect channel estimation case, solving these inequalities is analytically difficult. The right-side and left-side boundaries of the decision region can be found by solving $f(y|i, j, \hat{h}) = f(y|i+1, j, \hat{h})$, and $f(y|i, j, \hat{h}) = f(y|i-1, j, \hat{h})$, respectively. However, since the variances of the effective noises for neighboring signal points are different, there might be some cases where the probability density functions (PDFs) do not intersect between the two neighboring signal points, resulting in empty decision regions. In our simulations, we notice that this problem could arise when the power is extremely high. Therefore, we assume that the PDFs always intersect with their neighbors. As a result, the right-side boundary of the decision region corresponding to i can be found by solving

$$f(y|i, j, \hat{h}) = f(y|i+1, j, \hat{h}) \tag{15}$$

By simplifying the Gaussian PDFs, we have

$$\ln(\tilde{\sigma}_i^2) + \frac{\tilde{n}^2}{\tilde{\sigma}_i^2} = \ln(\tilde{\sigma}_{i+1}^2) + \frac{(\tilde{n} - \sqrt{2}d|\hat{h}|)^2}{\tilde{\sigma}_{i+1}^2} \tag{16}$$

where $\tilde{n} = y - \frac{A_i}{\sqrt{2}}|\hat{h}|$, and $\tilde{\sigma}_i^2 = \frac{1}{2}\left(\frac{A_i^2 + A_j^2}{2}\sigma_{\tilde{h}}^2 + 1\right)$ is the variance of the in-phase component of the effective noise. Although finding a numerical solution to (16) is easy, an analytical solution turns out to be too complex to be used for the rest of the paper. Therefore, we assume that non-canceling terms containing the square of $\tilde{n}$ are negligible, since $\sigma_{\tilde{h}}^2$ is small. Then, we have

$$\tilde{n} = \frac{d|\hat{h}|}{\sqrt{2}} + \frac{\tilde{\sigma}_{i+1}^2}{2\sqrt{2}d|\hat{h}|} \ln\left(\frac{\tilde{\sigma}_{i+1}^2}{\tilde{\sigma}_i^2}\right) \tag{17}$$

$$= \frac{d|\hat{h}|}{\sqrt{2}} + r_{i|j} \triangleq \bar{r}_{i|j} \tag{18}$$

where $r_{i|j}$ is the shift of the decision region on the right edge of signal point i. Similarly, the left edge of the decision region becomes,

$$\tilde{n} = -\left(\frac{d|\hat{h}|}{\sqrt{2}} - \frac{\tilde{\sigma}_{i-1}^2}{2\sqrt{2}d|\hat{h}|} \ln\left(\frac{\tilde{\sigma}_i^2}{\tilde{\sigma}_{i-1}^2}\right)\right) \tag{19}$$

$$= -\left(\frac{d|\hat{h}|}{\sqrt{2}} - l_{i|j}\right) \triangleq -\bar{l}_{i|j} \tag{20}$$

where $l_{i|j}$ is the shift of the decision region on the left edge of signal point i. Combining both edges, we conclude that the decision region corresponding to i is different from the coherent decision region by $r_{i|j}$ on the right boundary and $l_{i|j}$ on the left boundary. Now, we can write the in-phase probability of error as the probability that the real part of the effective noise is in this region, given that (i, j) is transmitted

$$P_{i|j} = Pr\left(-\bar{l}_{i|j} < \tilde{n} < \bar{r}_{i|j} \;\middle|\; \hat{h}, i, j\right) \tag{21}$$

Using the Q-function, we have

$$P_{i|j} = Q\left(\frac{\bar{l}_{i|j}}{\tilde{\sigma}_i}\right) + Q\left(\frac{\bar{r}_{i|j}}{\tilde{\sigma}_i}\right) \quad (22)$$

Since the right-side boundary for i is the left-side boundary for $i+1$, we have the following,

$$\sum_i P_{i|j} = \sum_i 2Q\left(\sqrt{\frac{\bar{r}_{i|j}^2}{\tilde{\sigma}_i^2}}\right) \quad (23)$$

By using the fact that $Q(x)$ is a decreasing function in x, and inserting $\bar{r}_{i|j}$ into (23), we have

$$\sum_i P_{i|j} < \sum_i 2Q\left(\sqrt{\frac{d^2|\hat{h}|^2}{2\tilde{\sigma}_i^2} + \frac{\tilde{\sigma}_{i+1}^2}{2\tilde{\sigma}_i^2}\ln\left(\frac{\tilde{\sigma}_{i+1}^2}{\tilde{\sigma}_i^2}\right)}\right) \quad (24)$$

Note that the average probability of in-phase component error is obtained by averaging the above inequality over j as well. For the range we consider, we can get an approximate upper bound by moving both of the averaging summations inside the Q-function. As a result, the second term inside the Q-function is zero, and we have,

$$P_{in} \lesssim 2Q\left(\sqrt{\frac{d^2|\hat{h}|^2}{P(\hat{h})\sigma_{\hat{h}}^2 + 1}}\right) \quad (25)$$

Finally, by approximating the Q-function by $Q(x) \approx \frac{1}{2}\exp(-x^2)$, and inserting d, we have

$$P_{in} \lesssim \exp\left(\frac{-3P_d(\hat{h})|\hat{h}|^2}{(M-1)(P(\hat{h})\sigma_{\hat{h}}^2 + 1)}\right) \quad (26)$$

Now, the M-QAM probability of error, over in-phase and quadrature components, becomes

$$\text{BER}(\hat{h}) \le P_{in} + P_q \quad (27)$$

$$\lesssim 2\exp\left(\frac{-3P_d(\hat{h})|\hat{h}|^2}{(M-1)(P(\hat{h})\sigma_{\hat{h}}^2 + 1)}\right) \quad (28)$$

This equation says that when M-QAM is used with power $P_d(\hat{h})$ in channel $\hat{h}$, the resulting BER will be smaller than the right hand side of (28). When choosing the size of the constellation, we can equate the target BER to the right hand side of (28), so that the BER of the resulting transmission will be less than the target BER. We rearrange (28) to get the maximum constellation size for a given BER with $K = \frac{-3}{\ln(0.5\text{BER})}$,

$$M(\hat{h}) = 1 + K\frac{P_d(\hat{h})\hat{h}^2}{P_d(\hat{h})\sigma_{\hat{h}}^2 + 1} \quad (29)$$

Since the rate of M-QAM is $R = \frac{T_d}{T}\log_2 M$, we can introduce the following optimization problem,

$$R = \max_{E[P_d(\hat{h})] \le P_d} \frac{T_d}{T} E\left[\log_2\left(1 + K\frac{P_d(\hat{h})\hat{h}^2}{P_d(\hat{h})\sigma_{\hat{h}}^2 + 1}\right)\right]. \quad (30)$$

where the expectation is with respect to the channel estimate.

Although we obtained this formula through a communication-theoretic approach with a variety of approximations, the only difference between (30) and the information theoretic achievable rate which can be obtained using the ideas in [2], [5] is the loss coefficient K in (30).

The maximization in (30) is over the training parameters, i.e., the training symbol $\mathbf{x}_t$, the training power P_t, the training sequence length T_t, and the data power allocation scheme $P_d(\hat{h})$. In order to simplify the analysis, we assume a constant power allocation, $P_d(\hat{h}) = P_d$. Therefore, from (29) we note that our scheme is power-non-adaptive, but M-QAM modulation-adaptive, where the CSI $\hat{h}$ is used to adapt the modulation constellation size, but not the transmit power. In this case, the optimization problem becomes

$$R = \max_{\mathbf{x}_t, P_d, T_d} \frac{T_d}{T} E\left[\log_2\left(1 + K\frac{P_d\sigma_{\hat{h}}^2\bar{h}^2}{P_d\sigma_{\hat{h}}^2 + 1}\right)\right] \quad (31)$$

where $\hat{h} = \sigma_{\hat{h}}\bar{h}$, so that $\bar{h}$ is unit variance. While solving the above optimization problem, we follow [5] closely. First, we find that the training signal $\mathbf{x}_t$ that maximizes the effective SNR, $P_{eff} = \frac{P_d\sigma_{\hat{h}}^2}{P_d\sigma_{\hat{h}}^2+1}$, satisfies $\mathbf{x}_t^H\mathbf{x}_t = T_t$. Therefore, any training signal that has unit-energy is optimum. Next, we optimize the effective SNR over P_d and P_t. Since the total energy is conserved, we have $PT = P_tT_t + P_dT_d$. Let α denote the fraction of energy that is devoted to the data transmission

$$P_dT_d = \alpha PT, \quad P_tT_t = (1-\alpha)PT, \quad 0 < \alpha < 1. \quad (32)$$

Now, we can find the optimum α that maximizes P_{eff} as

$$\alpha = \begin{cases} 1/2, & T_d = 1; \\ \gamma - \sqrt{\gamma(\gamma-1)}, & T_d > 1. \end{cases} \quad (33)$$

Finally, by following [5], we show that the rate is an increasing function of T_d, and hence a decreasing function of T_t. Setting T_t to its minimum value is optimum, which in our case is $T_t = 1$. This might seem counter-intuitive at first. Intuitively, a longer training phase will result in a better channel estimate and therefore a larger achievable rate during the data transmission phase. However, we use channel resources such as time and power during the channel estimation process, which could otherwise be used for data transmission. A longer training phase implies a shorter data transmission phase, as the block length (coherence time) is fixed. A shorter data transmission phase, in turn, implies a smaller achievable rate. Since data transmission length appears as a linear coefficient to the rate and the training length appears inside the logarithm of the achievable rate, using the minimum possible training length makes sense while maximizing the achievable rate.

V. Numerical Results

In this section, we first validate the accuracy of our upper bounds through simulation. We compare the actual probability of error in (23), the first upper bound in (24), the approximate

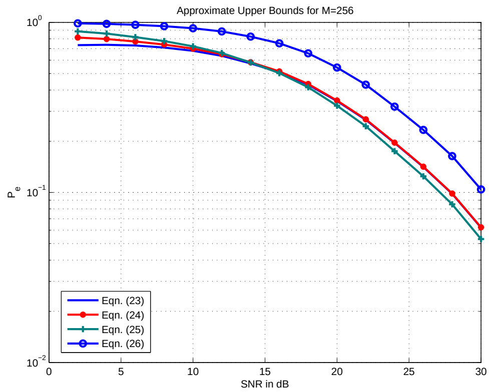


Fig. 2. The actual bit error rate and our upper bound for M=256.

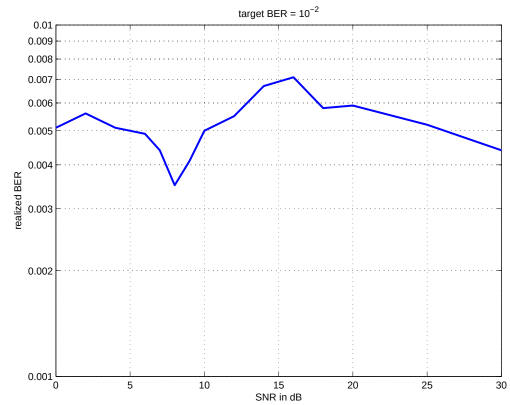


Fig. 3. Realized bit error rate while the target BER $= 10^{-2}$.

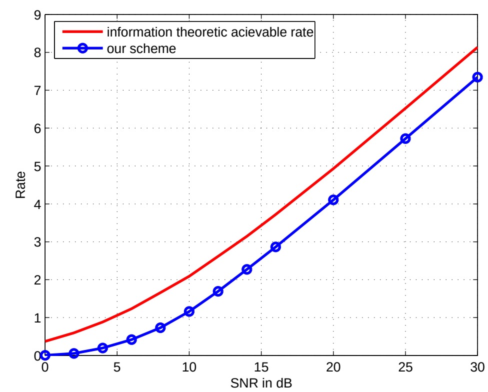


Fig. 4. Achieved rate with our scheme with the target BER $= 10^{-2}$ versus the information theoretic achievable rate.

expression for the upper bound in (25), and the final upper bound that we use in our calculations in (26). We plot these equations in Figure 2 for $M = 256$. For smaller constellation sizes, the first three curves were almost indistinguishable. In Figure 2 we see that (24) is very tight, and especially in high SNR scenarios it is the same as (23). Although (25) is not a strict upper bound, for a system that is dynamic in power allocation and constellation size, its effect will be averaged out. We also see that (26) is a rather loose upper bound to (25), and a strict upper bound for (23). We conclude that equation (26) affects the BER calculations the most.

Next, we analyze the realized BER through a MATLAB implementation of our channel model. We set our target BER to 10^{-2}, and by using the upper bound in (26), our constellation size allocation, and counting the bits that are decoded erroneously, we get the experimental average BER as a function of the SNR as shown in Figure 3. We see that we only overshoot the target BER by a fraction of an order of magnitude for the entire SNR range. By combining this result with Figure 2, we conclude that overshooting is mostly due to the exponential upper bound to the Q-function.

Finally, in Figure 4, we plot data rate that results from the same implementation with the same target BER along with the information-theoretic achievable rate. We see that for a given SNR, our scheme performs within 1 bit of the information-theoretic rate.

VI. Conclusion

In this paper we analyzed the problem of channel estimation for adaptive M-QAM in a Rayleigh block-fading channel. We gave closed-form expressions that provide a tight upper bound on the achievable rate for a system that performs channel estimating during a training phase followed by data transmission. Through simulation we verified the tightness of our bound and showed performance very close to the information-theoretic limit.

The expressions developed in this research can be adapted to a constructive algorithm that allows a cognitive radio to select the optimal M-QAM constellation size for any measured channel state. This will allow policy radios to quickly make decisions about optimal PHY-layer parameters.

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