Online Scheduling for Energy Harvesting Two-Way Channels with Processing Costs

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***Abstract*—We consider an energy harvesting two-way channel with processing costs, where the transmitters incur a processing cost per unit time as long as they are on for transmitting or receiving data. Such processing costs force users to perform bursty transmissions. We develop an *online* power scheduling algorithm where the transmitters determine their transmit powers and burst durations based only on the causal knowledge of the energy arrivals. We consider the case where the users harvest energy from a common energy source, however, the energy intakes of the users are different due to their different battery sizes. Focusing on independent and identically distributed (i.i.d.) energy arrival processes, we determine the *exactly optimum* power schedule for Bernoulli energy arrivals and develop a *nearly optimum* power schedule for general energy arrivals. Our proposed policy is distributed, which users can apply independently with no need for cooperation or coordination between them. We show that the proposed policy is within a constant multiplicative and additive gap from the optimal online policy for all energy arrivals.**

I. Introduction

We consider an energy harvesting two-way channel, Fig. 1, where users harvest energy from a common energy source. The users have finite but arbitrary-sized batteries to save unused energy for future use. Each user is subject to a processing cost which is the cost the user incurs while being on for transmitting or receiving data. The processing costs force users to operate in a bursty mode, where they do not utilize the entire duration available for communication. In this paper, we develop *online* power scheduling algorithms by which users determine transmit powers and burst lengths based only on the *causal* knowledge of the random energy arrival process.

Power control for energy harvesting transmitters has been studied extensively in recent literature. Earlier works have considered *offline* power control, where transmitters know all future energy arrivals a priori and optimize the entire power sequence at once, see e.g., [1]–[21]. Offline power control literature has started with the single-user setting [1]–[4] and extended to multi-user networks [5]–[16]. Of particular relevance to us here are the papers which incorporate processing costs in power optimization [17]–[21]. The presence of processing costs has led to *bursty* transmissions as in the glue-pouring approach in [22]. More recent works have considered *online* power control, where transmitters know only the energy arrivals so far with no knowledge of future energy arrivals [3], [4], [23]–[35].

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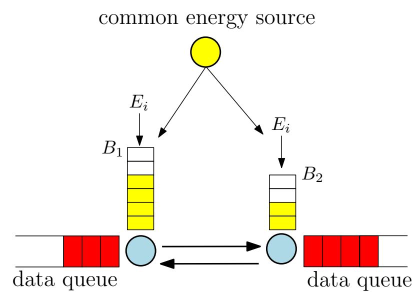

Fig. 1. Energy harvesting two-way channel with a common energy source.

In this paper, we follow the *online* power control technique developed in [31]–[33] for the single-user channel, and extended recently in [34], [35] for broadcast and multiple access channels. In this paper, we extend this line of research to two-way channels and also incorporate processing costs at the same time; the offline version of this problem was considered in [21]. Such extension enables us to understand the effects of processing costs on energy harvesting systems and also the design and interaction of online user power allocations.

As is advocated in this new technique [31]–[33], we start our analysis by considering a special energy arrival process, which is i.i.d. Bernoulli with energy arrival amounts equal either to zero or an amount larger than the sizes of the batteries. Such energy arrival when occurs resets the system and allows for an analytical derivation of the optimum power schedule. We show that the optimum powers of the users decrease over time, and the on-off times of the users are fully synchronized. We show that a burst may occur only in the last slot.

We then extend our analysis to the case of general i.i.d. energy arrivals. For this case, we propose a distributed sub-optimal policy for power and burst duration selection. The policy is fully distributed and can be applied by each user independently without a need for cooperation or coordination. We develop multiplicative and additive lower bounds on the performance of the proposed policy. We show that the proposed sub-optimal policy is near-optimal in that it performs within a constant gap of the optimal policy for all energy arrival processes and sizes of the batteries at the users.

II. System Model

We consider a slotted two-way energy harvesting channel with a common energy harvesting source, see Fig. 1. Transmitter j has a battery of finite size B_j. The energy available in the jth user battery in slot i, b_{ji}, evolves as:

$$b_{j(i+1)} = \min\{B_j, b_{ji} - \theta_{ji}\left(P_{ji} + \epsilon_j\right) + E_{i+1}\} \quad (1)$$

where P_{ji} is the power transmitted by user j in slot i, E_i is the energy harvested in slot i, ϵ_j is the processing cost incurred per unit time for being on, and θ_{ji} is the duration for which user j is on, either transmitting or receiving, in slot i.

The physical layer is Gaussian with sum rate in slot i:

$$r_{1i} + r_{2i} = \frac{\theta_{1i}}{2}\log\left(1 + P_{1i}\right) + \frac{\theta_{2i}}{2}\log\left(1 + P_{2i}\right) \quad (2)$$

where r_{ji} is the rate of user j in slot i. The power and burst of user j, θ_{ji}, P_{ji}, are constrained by the current battery state as $\theta_{ji}P_{ji} + \max\{\theta_{1i}, \theta_{2i}\}\epsilon_j \leq b_{ji}$. The objective of the online scheduling is to obtain the optimal policy which consists of $\{\theta_{1i}, \theta_{2i}, P_{1i}, P_{2i}\}$ to maximize the expected rate. In (2), the $\frac{1}{2}$ factors in front of logs are due to Shannon capacity formula, not due to time-sharing; the system is full-duplex.

In the following, we first consider the case where the energy arrivals, E_i, are i.i.d. Bernoulli random variables with support $\{0, B\}$, and with $\mathbb{P}[E_i = B] = p$, where $B \geq \max\{B_1, B_2\}$, i.e., when an energy comes it fills both batteries completely. For this case, we determine the optimal online policy. Subsequently, we consider the case of general i.i.d. energy arrivals, and propose a distributed near-optimal policy.

III. Optimal Strategy: Case of Bernoulli Arrivals

With Bernoulli energy arrivals, each energy arrival resets the system state; energy arrivals form a *renewal process*. From [36, Theorem 3.6.1], the long-term average throughput is equal to:

$$\lim_{n\to\infty} \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n}(r_{1i} + r_{2i})\right] = \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}(r_{1i} + r_{2i})\right] \quad (3)$$

$$= p\sum_{k=1}^{\infty}p(1-p)^{k-1}\sum_{i=1}^{k}(r_{1i} + r_{2i}) \quad (4)$$

$$= \sum_{i=1}^{\infty}\sum_{k=i}^{\infty}p^2(1-p)^{k-1}(r_{1i} + r_{2i}) \quad (5)$$

$$= \sum_{i=1}^{\infty}p(1-p)^{i-1}(r_{1i} + r_{2i}) \quad (6)$$

where L is the geometric inter-arrival time with $\mathbb{E}[L] = 1/p$. Hence, the online power allocation problem becomes:

$$\begin{aligned} \max_{\{P_{ji}\},\{\theta_{ji}\}} \quad & \sum_{i=1}^{\infty} p(1-p)^{i-1}\left(r_{1i} + r_{2i}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{\infty}(\theta_{1i}P_{1i} + \max\{\theta_{1i}, \theta_{2i}\}\epsilon_1) \leq B_1 \\ & \sum_{i=1}^{\infty}(\theta_{2i}P_{2i} + \max\{\theta_{1i}, \theta_{2i}\}\epsilon_2) \leq B_2 \\ & P_{1i}, P_{2i} \geq 0, \quad 0 \leq \theta_{1i}, \theta_{2i} \leq 1, \quad \forall i \end{aligned} \quad (7)$$

This optimization problem can be viewed as maximizing the *expected* transmitted sum rate before the next energy arrival.

Problem (7) is non-convex. We transform it to an equivalent convex problem by defining $\bar{P}_{ji} = \theta_{ji}P_{ji}$:

$$\begin{aligned} \max \; & \sum_{i=1}^{\infty} p(1-p)^{i-1}\left(\frac{\theta_{1i}}{2}\log\left(1+\frac{\bar{P}_{1i}}{\theta_{1i}}\right) + \frac{\theta_{2i}}{2}\log\left(1+\frac{\bar{P}_{2i}}{\theta_{1i}}\right)\right) \\ \text{s.t.} \; & \sum_{i=1}^{\infty}(\bar{P}_{1i} + \max\{\theta_{1i}, \theta_{2i}\}\epsilon_1) \leq B_1 \\ & \sum_{i=1}^{\infty}(\bar{P}_{2i} + \max\{\theta_{1i}, \theta_{2i}\}\epsilon_2) \leq B_2 \\ & \bar{P}_{1i}, \bar{P}_{2i} \geq 0, \quad 0 \leq \theta_{1i}, \theta_{2i} \leq 1, \quad \forall i \end{aligned} \quad (8)$$

where the maximization is over $\{\bar{P}_{ji}\}, \{\theta_{ji}\}$.

Before proceeding with finding the optimal policy, we state two important observations: First, both users should consume all of their energies in their batteries. If a user does not consume all of its energy, then we can increase its power until all of its energy is used, increasing the objective function. Second, the two users' transmissions should be fully synchronized, i.e.., $\theta_{1i} = \theta_{2i}$, for all i. If for a slot i users are not synchronized, say e.g., $\theta_{1i} < \theta_{2i}$, then we can always increase θ_{1i} to be equal to θ_{2i} without violating the constraints of the problem, while increasing the objective function. Hence, hereafter, we will assume that $\theta_{1i} = \theta_{2i} = \theta_i$ for all i, so that $\max\{\theta_{1i}, \theta_{2i}\} = \theta_i$. In this case, the Lagrangian for (8) is:

$$\begin{aligned} \mathcal{L} = & -\sum_{i=1}^{\infty}p(1-p)^{i-1}\left(\frac{\theta_i}{2}\log\left(1+\frac{\bar{P}_{1i}}{\theta_i}\right) + \frac{\theta_i}{2}\log\left(1+\frac{\bar{P}_{2i}}{\theta_i}\right)\right) \\ & + \lambda_1\left(\sum_{i=1}^{\infty}(\bar{P}_{1i} + \theta_i\epsilon_1) - B_1\right) - \sum_{i=1}^{\infty}\nu_{1i}P_{1i} \\ & + \lambda_2\left(\sum_{i=1}^{\infty}(\bar{P}_{2i} + \theta_i\epsilon_2) - B_2\right) - \sum_{i=1}^{\infty}\nu_{2i}P_{2i} \\ & - \sum_{i=1}^{\infty}\mu_i^l\theta_i - \sum_{i=1}^{\infty}\mu_i^u(1-\theta_i) \end{aligned} \quad (9)$$

From the KKTs, the optimal powers for the slots with $\theta_i > 0$:

$$\frac{\bar{P}_{1i}}{\theta_i} = \left(\frac{p(1-p)^{i-1}}{\lambda_1} - 1\right)^+, \quad \frac{\bar{P}_{2i}}{\theta_i} = \left(\frac{p(1-p)^{i-1}}{\lambda_2} - 1\right)^+ \quad (10)$$

For the slots with $\theta_i = 0$, both powers are zero, i.e., $\bar{P}_{1i} = \bar{P}_{2i} = 0$, as otherwise, any assigned positive power is wasted, since the objective function is zero when $\theta_i = 0$.

From (10), we observe that for slots with $\theta_i > 0$, the powers P_{1i} and P_{2i} are monotonically decreasing in time. In addition, due to the decreasing $p(1-p)^{i-1}$ factors before the log, we can divide the slots into $\{1, \ldots, \tilde{N}\}$ where $\theta_i > 0$, and $\{\tilde{N}+1, \ldots\}$ where $\theta_i = 0$. Furthermore, the transmission duration $\tilde{N}$ is bounded above by the maximum of the user transmission durations without any processing costs (define them as $\tilde{N}_{npc1}$ and $\tilde{N}_{npc2}$), i.e., $\tilde{N} \leq \max\{\tilde{N}_{npc1}, \tilde{N}_{npc2}\}$. This follows as the processing costs reduce the energy available in the battery dedicated for transmission, and hence reduce the effective battery size at both users; it is shown in [34] that the transmission duration is monotone increasing in the battery size.

Similarly, from the optimality conditions, the bursts satisfy:

$$\sum_{j=1}^{2} \log\left(1+\frac{\bar{P}_{ji}}{\theta_i}\right) - \frac{\frac{\bar{P}_{ji}}{\theta_i}}{1+\frac{\bar{P}_{ji}}{\theta_i}} = \frac{\sum_{j=1}^{2}\lambda_j\epsilon_j + \mu_i^u - \mu_i^l}{p(1-p)^{i-1}} \quad (11)$$

substituting (10), we obtain,

$$\sum_{j=1}^{2} \log\left(\frac{p(1-p)^{i-1}}{\lambda_j}\right) = \frac{\sum_{j=1}^{2}\lambda_j(\epsilon_j-1) + \mu_i^u - \mu_i^l}{p(1-p)^{i-1}} + 2 \quad (12)$$

From complementary slackness, if $\theta_i \in (0,1)$, then we have $\mu_i^u = \mu_i^l = 0$. Thus, in this case, (12) becomes:

$$\sum_{j=1}^{2} \left(\log\left(\frac{p(1-p)^{i-1}}{\lambda_j}\right) - 1\right) = \frac{\sum_{j=1}^{2}\lambda_j\left(\epsilon_j - 1\right)}{p(1-p)^{i-1}} \quad (13)$$

The left and right hand sides of (13) are monotone decreasing and increasing, respectively. Hence, (13) can be satisfied at most for one time index, thus the bursty transmission can occur at most in one slot. Due to decreasing $p(1-p)^{i-1}$ multiplying the rate, this bursty transmission can occur only in the last slot.

From the above, the optimal solution is characterized by $\lambda_1, \lambda_2, \tilde{N}, \theta_{\tilde{N}}$. Next, we solve for them. For the complete solution we need to solve (12) along with the total power constraints, which using (10) become:

$$\sum_{i=1}^{\tilde{N}-1}\left[\frac{p(1-p)^{i-1}}{\lambda_1} - 1 + \epsilon_1\right] + \theta_{\tilde{N}}\left[\frac{p(1-p)^{\tilde{N}-1}}{\lambda_1} - 1 + \epsilon_1\right] = B_1 \quad (14)$$

$$\sum_{i=1}^{\tilde{N}-1}\left[\frac{p(1-p)^{i-1}}{\lambda_2} - 1 + \epsilon_2\right] + \theta_{\tilde{N}}\left[\frac{p(1-p)^{\tilde{N}-1}}{\lambda_2} - 1 + \epsilon_2\right] = B_2 \quad (15)$$

Solving (14) and (15) for $\theta_{\tilde{N}}$ we have:

$$\theta_{\tilde{N}} = \frac{B_1 - \sum_{i=1}^{\tilde{N}-1}\left(\frac{p(1-p)^{i-1}}{\lambda_1} - 1 + \epsilon_1\right)}{\frac{p(1-p)^{\tilde{N}-1}}{\lambda_1} - 1 + \epsilon_1} \quad (16)$$

$$= \frac{B_2 - \sum_{i=1}^{\tilde{N}-1}\left(\frac{p(1-p)^{i-1}}{\lambda_2} - 1 + \epsilon_2\right)}{\frac{p(1-p)^{\tilde{N}-1}}{\lambda_2} - 1 + \epsilon_2} \quad (17)$$

We note that (16) and (17) are strictly increasing in λ_1 and λ_2 when the numerators and denominators are non-negative. Hence, for each fixed λ_1 which makes $\theta_{\tilde{N}} \in (0,1)$ there corresponds a unique λ_2 which makes (16) and (17) equal. This in effect makes it easy to search over the pairs $\{\lambda_1, \lambda_2\}$ which equate (16) and (17), using a one dimensional search on either λ_1 or λ_2. We also need to satisfy for $i \in \{1, \dots, \tilde{N}\}$:

$$\lambda_1 \leq p(1-p)^{i-1} \quad (18)$$

$$\lambda_2 \leq p(1-p)^{i-1} \quad (19)$$

$$0 \leq \sum_{j=1}^{2}\left(\log\left(\frac{p(1-p)^{i-1}}{\lambda_j}\right) - 1\right) + \frac{\sum_{j=1}^{2}\lambda_j\left(1-\epsilon_j\right)}{p(1-p)^{i-1}} \quad (20)$$

where (18) and (19) ensure the non-negativity of the power, and (20) guarantees the existence of a non-negative Lagrange multiplier μ_i^u satisfying (12).

Towards this end, next, we present a method to obtain the optimal solution. We first initialize $\tilde{N} = \max\{\tilde{N}_{npc1}, \tilde{N}_{npc2}\}$, where $\tilde{N}_{npcj}$ can be found by solving a single-user problem with no processing costs for user j as in [32]. From this, we obtain $\{\lambda_1, \lambda_2\}$ pairs which equate equations (16) and (17) and make $\theta_{\tilde{N}} \in (0,1)$. Then, we check if any of the obtained pairs satisfies (13), (18), (19) and (20). If yes, then this is the optimal solution. Otherwise, we decrease $\tilde{N}$ by one and repeat this again. If we reach $\tilde{N} = 1$ and no solution is found, then, this implies that $\theta_{\tilde{N}} = 1$. Hence, we solve similarly for the largest integer $\tilde{N}$ and that corresponding λ_1, λ_2 that satisfy:

$$\sum_{i=1}^{\tilde{N}}\left(\frac{p(1-p)^{i-1}}{\lambda_1} - 1 + \epsilon_1\right) = B_1 \quad (21)$$

$$\sum_{i=1}^{\tilde{N}}\left(\frac{p(1-p)^{i-1}}{\lambda_2} - 1 + \epsilon_2\right) = B_2 \quad (22)$$

along with the conditions (18), (19) and (20).

IV. Near-Optimal Strategy: General Arrivals

Now, we consider an arbitrary i.i.d. energy arrival process E_i with average admitted recharge rate $\mu_j = \mathbb{E}[\max\{B_j, E_i\}]$ at the jth user. Although finding the *exactly optimal* policy in this case may not be tractable, we propose a *distributed* sub-optimal policy which we show is near-optimal.

A. Sub-Optimal Policy

We first present our proposed sub-optimal policy for the Bernoulli case and then extend it to the case of general energy arrivals. For Bernoulli energy arrivals, motivated by (10), we assign exponentially decaying total power for each user. In each slot, the users allocate a fraction p of the available energy in the battery then optimize the transmit power and burst duration. Hence, in slot i, the energy allocated for transmission by user j is $B_j p(1-p)^{i-1}$. Then, the users solve the following single-slot optimization problem:

$$\begin{aligned} \max_{\bar{P}_{ji}, \theta_i} \quad & \frac{\theta_i}{2}\log\left(1 + \frac{\bar{P}_{1i}}{\theta_i}\right) + \frac{\theta_i}{2}\log\left(1 + \frac{\bar{P}_{2i}}{\theta_i}\right) \\ \text{s.t.} \quad & \bar{P}_{1i} + \theta_i\epsilon_1 \leq B_1 p(1-p)^{i-1} \\ & \bar{P}_{2i} + \theta_i\epsilon_2 \leq B_2 p(1-p)^{i-1} \\ & \bar{P}_{1i}, \bar{P}_{2i} \geq 0, \quad 0 \leq \theta_i \leq 1 \end{aligned} \quad (23)$$

Since, the first two constraints will be satisfied with equality we have $\bar{P}_{ji} = B_j p(1-p)^{i-1} - \theta_i\epsilon_j$, which reduces (23) to:

$$\begin{aligned} \max_{\theta_i \in [0,1]} \frac{\theta_i}{2}\log\left(1 + \frac{B_1 p(1-p)^{i-1}}{\theta_i} - \epsilon_1\right) \\ + \frac{\theta_i}{2}\log\left(1 + \frac{B_2 p(1-p)^{i-1}}{\theta_i} - \epsilon_2\right) \end{aligned} \quad (24)$$

Similarly, for the case of general energy arrivals, we allocate a fraction $q_j = \frac{\mu_j}{B_j}$ of the battery energy, i.e., $q_j b_{ji}$, and solve:

$$\max_{\theta_i \in [0,1]} \frac{\theta_i}{2}\log\left(1 + \frac{q_1 b_{1i}}{\theta_i} - \epsilon_1\right) + \frac{\theta_i}{2}\log\left(1 + \frac{q_2 b_{2i}}{\theta_i} - \epsilon_2\right) \quad (25)$$

Problems (24) and (25) can be solved by both users independently, because both users know the energy arrival E_i, and they are consuming the power in a deterministic fractional way, hence, both users can track the state of both batteries.

B. An Upper Bound for Online Policies

In the following theorem, we develop an upper bound for all online policies in terms of the average admitted energy.

Theorem 1 *For an average admitted energy μ_j at user j, the achievable rate for any online policy is upper bounded by:*

$$r_{ub} = \max_{\theta\in[0,1]} \frac{\theta}{2}\left(\log\left(1+\frac{\mu_1}{\theta}-\epsilon_1\right)+\log\left(1+\frac{\mu_2}{\theta}-\epsilon_2\right)\right) \quad (26)$$

Proof: We denote the admitted energy arrivals as $\tilde{E}_{ji} = \min\{B_j, E_i\}$. We use the offline achievable rate as an upper bound for the online achievable rate. We consider the following set which is larger than the feasible set of the offline case:

$$\mathcal{F}^n \triangleq \Bigg\{\{\bar{P}_{1i},\bar{P}_{2i},\theta_i\}_{i=1}^n : \frac{1}{m}\sum_{i=1}^{m}\bar{P}_{1i}+\theta_i\epsilon_1 \leq \frac{1}{m}\left(\sum_{i=1}^{m}\tilde{E}_{1i}\right), \frac{1}{m}\sum_{i=1}^{m}\bar{P}_{2i}+\theta_i\epsilon_2 \leq \frac{1}{m}\left(\sum_{i=1}^{m}\tilde{E}_{2i}\right), \forall m=1,\dots,n\Bigg\} \quad (27)$$

which neglects the overflow constraints due to the finite battery [2], [3]. We then consider a bigger feasible set by considering the constraints only when $m = n$ to get:

$$\mathcal{G}^n \triangleq \Bigg\{\{\bar{P}_{1i},\bar{P}_{2i},\theta_i\}_{i=1}^n : \frac{1}{n}\sum_{i=1}^{n}\bar{P}_{1i}+\theta_i\epsilon_1 \leq \frac{1}{n}\left(\sum_{i=1}^{n}\tilde{E}_{1i}\right), \frac{1}{n}\sum_{i=1}^{n}\bar{P}_{2i}+\theta_i\epsilon_2 \leq \frac{1}{n}\left(\sum_{i=1}^{n}\tilde{E}_{2i}\right)\Bigg\} \quad (28)$$

Hence, the online achievable rate is upper bounded by:

$$\lim_{n\to\infty}\max_{\mathcal{G}^n}\frac{1}{n}\sum_{i=1}^{n}\frac{\theta_i}{2}\left(\log\left(1+\frac{\bar{P}_{1i}}{\theta_i}\right)+\log\left(1+\frac{\bar{P}_{2i}}{\theta_i}\right)\right) \quad (29)$$

Since the energies $\tilde{E}_{1i}, \tilde{E}_{2i}$ are i.i.d., from strong law of large numbers, for all $\delta > 0$ there exists an integer N such that for all $n \geq N$, we have $\frac{1}{n}\sum_{i=1}^{n}\tilde{E}_{1i} \leq \mu_1 + \delta$ and $\frac{1}{n}\sum_{i=1}^{n}\tilde{E}_{2i} \leq \mu_2 + \delta$. For large enough n, i.e., $n \geq N$, the constraints in (28) will be:

$$\frac{1}{n}\sum_{i=1}^{n}\bar{P}_{1i}+\theta_i\epsilon_1\leq\mu_1+\delta, \quad \frac{1}{n}\sum_{i=1}^{n}\bar{P}_{2i}+\theta_i\epsilon_2\leq\mu_2+\delta \quad (30)$$

Then, from the joint concavity of the objective function, it is maximizes when all $\theta_i = \theta$ and $\bar{P}_{ji} = \bar{P}_j$. Since this is valid for all $\delta > 0$, we can the take δ to zero, which gives (26). ■

C. A Lower Bound on the Proposed Online Policy

In this section, we derive multiplicative and additive bounds for the performance of the proposed sub-optimal policy. In what follows, we denote the solution of the problems in (24) and (25) for available powers P_1, P_2 as $\theta^*(P_1, P_2)$, i.e., the solutions of (24) and (25) are denoted as $\theta^*(B_1p(1-p)^{i-1}, B_2p(1-p)^{i-1})$ and $\theta^*(q_1b_{1i}, q_2b_{2i})$, respectively.

Lemma 1 *The achievable rate with the proposed fractional policy for any i.i.d. Bernoulli energy arrival process with average admitted energy μ_j at user j is lower bounded as:*

$$r \geq \frac{1}{2}r_{ub} \quad (31)$$

Proof: The first step, (33), for the lower bound follows by using a sub-optimal decreasing burst as $\theta_i = \theta^*(1-p)^{i-1}$, where θ^* is a short notation for $\theta^*(B_1p, B_2p)$:

$$r = \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\left(\log\left(1+\frac{B_1p(1-p)^{i-1}}{\theta_i}-\epsilon_1\right)+\log\left(1+\frac{B_2p(1-p)^{i-1}}{\theta_i}-\epsilon_2\right)\right)\right] \quad (32)$$

$$\geq \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}(1-p)^{i-1}r_{ub}\right] \quad (33)$$

$$= r_{ub}\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}(1-p)^{i-1}\right] \quad (34)$$

$$= r_{ub}\frac{1}{\mathbb{E}[L]}\left[\sum_{L=1}^{\infty}p(1-p)^{L-1}\sum_{i=1}^{L}(1-p)^{i-1}\right] \quad (35)$$

$$= r_{ub}\left[\sum_{L=1}^{\infty}p^2(1-p)^{L-1}\frac{1-(1-p)^L}{p}\right] \quad (36)$$

$$= r_{ub}\left[p\left(\frac{1}{p}-\frac{(1-p)}{2p-p^2}\right)\right] \quad (37)$$

$$= r_{ub}\left(\frac{1}{2-p}\right) \quad (38)$$

$$\geq \frac{1}{2}r_{ub} \quad (39)$$

where (39) follows since $p \geq 0$. ■

Lemma 2 *The achievable rate under the proposed fractional policy for any i.i.d. Bernoulli energy arrival process with average admitted energy μ_j at user j is lower bounded as:*

$$r \geq r_{ub} - 1.44 - \frac{1}{2}\log^+(\epsilon_1) - \frac{1}{2}\log^+(\epsilon_2) \quad (40)$$

where $\log^+(x) = \max\{0, \log(x)\}$.

Proof: The proof technique we use for the case $\epsilon_j \leq 1$ is different than $\epsilon_j > 1$. In what follows, we assume that $\epsilon_1 > 1$ while $\epsilon_2 \leq 1$, however, all other combinations follow similarly. We bound the performance of (32) as follows:

$$r \geq \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\left(\log\left(1+\frac{\frac{B_1p(1-p)^{i-1}}{\theta_i}-\epsilon_1}{\epsilon_1}\right)+\log\left(1+\frac{B_2p(1-p)^{i-1}}{\theta_i}-\epsilon_2\right)\right)\right] \quad (41)$$

$$=\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\left(\log\left(1+\frac{B_1p(1-p)^{i-1}}{\epsilon_1\theta_i}-1\right)\right.\right.$$
$$\left.\left.+\log\left(1+\frac{B_2p(1-p)^{i-1}}{\theta_i}-\epsilon_2\right)\right)\right] \quad (42)$$
$$\geq\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\left(\log\left(\frac{B_1p}{\theta_i}\right)\right.\right.$$
$$\left.\left.+\log\left(1+\frac{B_2p(1-p)^{i-1}}{\theta_i}-\epsilon_2\right)\right)\right]$$
$$-\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\log\left(\epsilon_1\right)\right]$$
$$-\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\log\left(\frac{1}{(1-p)^{i-1}}\right)\right] \quad (43)$$
$$\geq\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\left(\log\left(\frac{B_1p}{\theta_i}\right)\right.\right.$$
$$\left.\left.+\log\left(1+\frac{B_2p(1-p)^{i-1}}{\theta_i}-\epsilon_2\right)\right)\right]-\frac{1}{2}\log\left(\epsilon_1\right)-0.72 \quad (44)$$
$$\geq\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\left(\log\left(1+\frac{B_1p}{\theta_i}-\epsilon_1\right)\right.\right.$$
$$\left.\left.+\log\left(1+\frac{B_2p(1-p)^{i-1}}{\theta_i}-\epsilon_2\right)\right)\right]-\frac{1}{2}\log\left(\epsilon_1\right)-0.72 \quad (45)$$
$$\geq\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\frac{\theta^*}{2}\left(\log\left(1+\frac{B_1p}{\theta^*}-\epsilon_1\right)\right.\right.$$
$$\left.\left.+\log\left(1+\frac{B_2p(1-p)^{i-1}}{\theta^*}-\epsilon_2\right)\right)\right]-\frac{1}{2}\log\left(\epsilon_1\right)-0.72 \quad (46)$$
$$=\frac{\theta^*}{2}\log\left(1+\frac{B_1p}{\theta^*}-\epsilon_1\right)-\frac{1}{2}\log\left(\epsilon_1\right)-0.72$$
$$+\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\frac{\theta^*}{2}\log\left((1-p)^{i-1}\left(\frac{1-\epsilon_2}{(1-p)^{i-1}}+\frac{B_2p}{\theta^*}\right)\right)\right] \quad (47)$$
$$\geq\frac{\theta^*}{2}\log\left(1+\frac{B_1p}{\theta^*}-\epsilon_1\right)-\frac{1}{2}\log\left(\epsilon_1\right)-0.72$$
$$+\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\frac{\theta^*}{2}\log\left((1-p)^{i-1}\left(1-\epsilon_2+\frac{B_2p}{\theta^*}\right)\right)\right] \quad (48)$$
$$\geq\frac{\theta^*}{2}\log\left(1+\frac{B_1p}{\theta^*}-\epsilon_1\right)-\frac{1}{2}\log\left(\epsilon_1\right)-1.44$$
$$+\frac{\theta^*}{2}\log\left(1+\frac{B_2p}{\theta^*}-\epsilon_2\right) \quad (49)$$

where (41) follows as the maximum $\frac{B_1p(1-p)^{i-1}}{\theta_i}-\epsilon_1$ is non-negative, and $\epsilon_1>1$, (43) follows since for any three positive functions $a(x),b(x),c(x)$ we have: $\max_x[a(x)-b(x)-c(x)]\geq\max_x a(x)-\max_x b(x)-\max_x c(x)$, (44) follows by bounding the last term numerically by 0.72, (45) follows since we added $1-\epsilon_1$ which is negative, (49) follows again by numerically bounding $\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\frac{1}{2}\log\left(\frac{1}{(1-p)^{i-1}}\right)\right]$ by 0.72. The θ^* used here is a shorthand for $\theta^*(B_1p,B_2p)$. ■

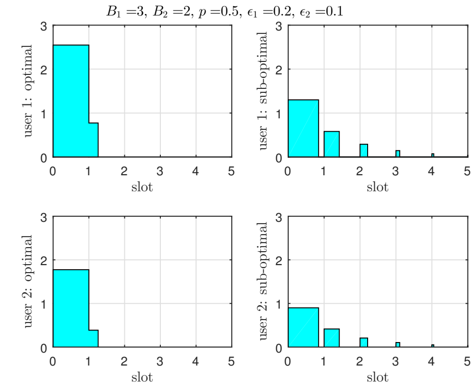

Fig. 2. The optimal and sub-optimal power allocations for Bernoulli.

We next show that i.i.d. Bernoulli energy arrivals give the lowest rate over all i.i.d. energy arrivals with the same mean. The proof follows by the approach in [32, Proposition 4] as

$$f(x_1,x_2)\triangleq$$
$$\max_{\theta_i\in[0,1]}\frac{\theta_i}{2}\left(\log\left(1+\frac{q_1x_1}{\theta_i}-\epsilon_1\right)+\log\left(1+\frac{q_2x_2}{\theta_i}-\epsilon_2\right)\right) \quad (50)$$

is jointly concave in x_1,x_2. The concavity of $f(x_1,x_2)$ follows since it is equivalent to maximizing $\frac{\theta_i}{2}\log\left(1+\frac{\bar{P}_{1i}}{\theta_i}\right)+\frac{\theta_i}{2}\log\left(1+\frac{\bar{P}_{2i}}{\theta_i}\right)$ over the feasible set $\bar{P}_{1i}+\theta_i\epsilon_1\leq q_1x_1$, $\bar{P}_{2i}+\theta_i\epsilon_2\leq q_2x_2$, $0\leq\theta_i\leq 1$, $\bar{P}_{1i},\bar{P}_{2i}\geq 0$. The objective function here is jointly concave $\theta_i,\bar{P}_{1i},\bar{P}_{2i}$ and the constraint set is affine in $x_1,x_2,\theta_i,\bar{P}_{1i},\bar{P}_{2i}$. Then, it follows that $f(x_1,x_2)$ is concave in x_1,x_2; [37, Section 3.2.5]. Also, [32, Lemma 2] can be used as we have a common source.

Lemma 3 *For the proposed fractional policy, any i.i.d. energy arrival process yields an achievable sum rate no less than that of the Bernoulli energy arrivals with the same mean.*

Combining Lemma 1, Lemma 2 and Lemma 3 we have the following theorem for the i.i.d. energy arrival process.

Theorem 2 *The achievable sum rate with the proposed sub-optimal policy for any arbitrary i.i.d. energy arrival process with average admitted energy of μ_j at user j and with $\frac{\mu_1}{B_1}=\frac{\mu_2}{B_2}$ is lower bounded by (31) and (40).*

V. Numerical Results

In this section, we illustrate the obtained results through several numerical examples. We first show the optimal versus proposed sub-optimal power allocation for Bernoulli arrivals in Fig. 2. As we showed, in the optimal power allocation, bursty transmission takes place only in the last slot.

We then compare the performance of the proposed sub-optimal scheme and the optimal policy in Fig. 3. The performance of our proposed policy is close to the optimal. We also show the performance of the sub-optimal policy on a general energy arrival with a continuous uniform distribution

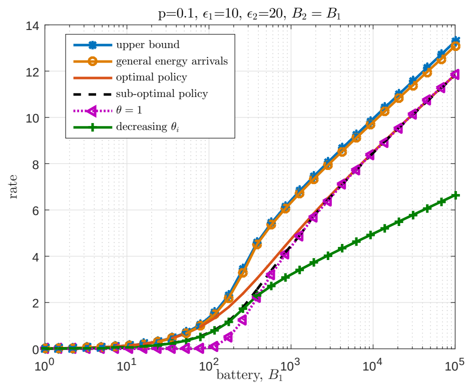

Fig. 3. Performance of Bernoulli and general energy arrivals.

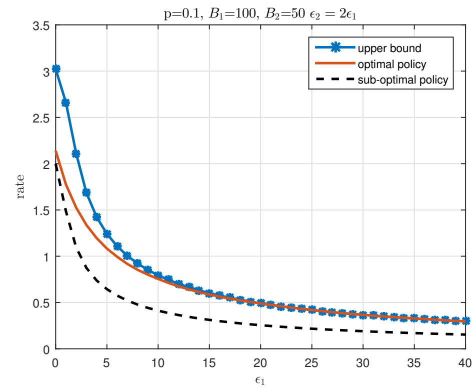

Fig. 4. Performance of Bernoulli energy arrivals versus the processing cost.

with the same mean as Bernoulli. In Fig. 3 we also show the performance of the fractional θ_i scheme which is used in the proof of Lemma 1, and a scheme which always uses $\theta_i = 1$ whenever feasible, i.e., neglects the processing costs. Both perform worse than our proposed policy.

Finally, we show the performance of our scheme versus the processing cost in Fig. 4. We observe that for high processing costs the performance gap is small.

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