Energy Harvesting Cooperative Multiple Access Channel with Decoding Costs

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***Abstract*— We consider an energy harvesting cooperative multiple access channel (MAC) with decoding costs. In this setting, users cooperate at the physical layer (data cooperation) in order to increase the achievable rates. Data cooperation comes at the expense of decoding costs: each user spends some amount of its harvested energy to decode the message of the other user, before forwarding both messages to the receiver. The decoding power spent is an increasing convex function of the incoming message rate. We characterize the optimal power scheduling policies that achieve the boundary of the maximum departure region subject to energy causality constraints and decoding costs by using a generalized water-filling algorithm.**

I. Introduction

Scheduling in energy harvesting communication systems has received considerable attention recently. References [1]–[4] consider a single-user channel and design optimal power policies that maximize the throughput or minimize the transmission completion time under various assumptions on the battery size and channel fading. References [5]–[7] extend these results to multi-user settings. While most of the energy harvesting literature focuses on transmitter-size energy harvesting, recent work [8]–[10] considers receiver-side energy harvesting, where harvested energy is used for decoding incoming data.

In this paper, we consider an energy harvesting cooperative MAC, see Fig. 1. In a cooperative MAC, users decode the signals transmitted by the other user to form common information, and cooperatively send the previously established common information to the receiver to achieve beamforming gains [11]. This model has the unique property that the transmitters act as receivers as well, where transmission power and decoding costs simultaneously reflect on the total energy budget of each node. The energy harvesting cooperative MAC is considered in [12] for data cooperation only, and extended in [13] to the case of joint data and energy cooperation, without taking into account the decoding costs incurred at the nodes, and significant gains in departure regions are demonstrated.

The goal of this paper is to incorporate the decoding cost of cooperation into the cooperative MAC model, and investigate the gains from cooperation in a more realistic setup. To this end, we model the decoding power as an increasing convex function in the incoming rate [8]–[10], and in particular, we focus on exponential decoding functions [14], [15]. We characterize the optimal offline power scheduling policies that maximize the departure region by a given deadline subject to energy causality constraints and decoding costs.

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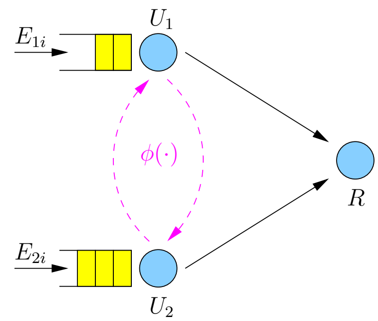

Fig. 1. Energy harvesting cooperative MAC with decoding costs.

II. System Model and Problem Formulation

We consider a time-slotted system, where energies arrive in amounts of E_{1i} and E_{2i} at the first and the second user, respectively, in slot i. The energy arriving at each user can be used for transmission, decoding, or can be saved in a battery to be used in future slots. The users communicate with the receiver over a Gaussian MAC, with a noise variance $\sigma^2 > 1$ at the receiver. They also overhear each other's transmission over stronger links: the channels between the users are assumed to be Gaussian with unit-variance. In order to make use of the overheard information, the messages are transmitted to the receiver using block Markov superposition coding [11]. Users 1 and 2 create common information using powers p_{12} and p_{21}, and convey the created common information to the receiver using powers p_{u1} and p_{u2}. Since user-receiver links are weaker than user-user links, direct transmission is not considered [16].

For a given power policy $(p_{12}, p_{21}, p_{u1}, p_{u2})$, a rate pair (r_1, r_2) belongs to the achievable rate region of the cooperative MAC, denoted by $\mathcal{F}_{CMAC}\left(p_{12}, p_{21}, p_{u1}, p_{u2}\right)$, if [11]

$$r_1 \leq \frac{1}{2}\log\left(1+p_{12}\right) \tag{1}$$

$$r_2 \leq \frac{1}{2}\log\left(1+p_{21}\right) \tag{2}$$

$$r_1 + r_2 \leq \frac{1}{2}\log\left(\frac{S}{\sigma^2}\right) \tag{3}$$

where $S \triangleq \sigma^2 + p_{12} + p_{21} + \left(\sqrt{p_{u1}} + \sqrt{p_{u2}}\right)^2$. Throughout this paper, we denote $g(p) \triangleq \frac{1}{2}\log(1+p)$.

Our goal is to characterize the maximum departure region [5], $\mathcal{F}_{CMAC}$, subject to energy causality constraints and decoding costs at both users. Since $\mathcal{F}_{CMAC}$ is a convex region, its boundary can be characterized by solving the following weighted sum rate maximization problem for all $\mu_1, \mu_2 > 0$,

$$\begin{aligned}
\max_{\substack{\mathbf{r}_1,\mathbf{r}_2,\mathbf{p}_{12},\mathbf{p}_{21}\\ \mathbf{p}_{u1},\mathbf{p}_{u2}}} \quad & \sum_{i=1}^{N} \mu_1 r_{1i} + \mu_2 r_{2i} \\
\text{s.t.} \quad & (r_{1i}, r_{2i}) \in \mathcal{F}_{CMAC}\left(p_{12i}, p_{21i}, p_{u1i}, p_{u2i}\right), \ \forall i \\
& \sum_{i=1}^{k} p_{12i} + p_{u1i} + \phi(r_{2i}) \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k \\
& \sum_{i=1}^{k} p_{21i} + p_{u2i} + \phi(r_{1i}) \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k \\
& r_{1i}, r_{2i}, p_{12i}, p_{21i}, p_{u1i}, p_{u2i} \geq 0
\end{aligned} \tag{4}$$

where $\phi(r)$, an increasing convex function in r, is the decoding power (cost) needed to decode a message of rate r. Therefore, each user needs to adapt its powers (and rates) to both its own and the other user's energy arrivals.

III. Properties of the Optimal Policy

We first show that in the cooperative MAC, the optimal rate allocation (r_1, r_2) can be expressed directly in terms of powers p_{12} and p_{21} used for common message generation.

Lemma 1 *There exists an optimal policy for problem (4) where the two inequalities (1) and (2) hold with equality $\forall i$.*

Proof: Assume that in the optimal policy (1) does not hold with equality for some time slot k. Then, we decrease p_{12k} and increase p_{u1k} by the same amount, until (1) holds with equality. This either increases S_k, or keeps it constant, hence the third inequality still holds. The new power allocation is energy feasible. Since the rate allocation did not change, the newly obtained policy is optimal as well. Similar arguments follow if the second inequality does not hold with equality. ∎

We remark here that in the cooperative MAC with no decoding costs [12], the optimal policy is to send at a rate pair so that (3) as well holds with equality, or else the rates can be improved [12, Lemma 2]. However this is not necessarily true in the presence of decoding costs, as increasing one of the user's rate comes at the expense of decreasing the other user's rate, as some of the power used for transmission needs to be shifted to decoding at the cooperative partner.

In the sequel, we focus on the case of exponential decoding costs. Specifically, we set $\phi = a \cdot g^{-1}$, for some decoding power factor $a > 0$ [17]. By Lemma 1, the problem can now be written only in terms of the powers as

$$\begin{aligned}
\max_{\mathbf{p}_{12},\mathbf{p}_{21},\mathbf{p}_{u1},\mathbf{p}_{u2}} \quad & \sum_{i=1}^{N} \mu_1 g(p_{12i}) + \mu_2 g(p_{21i}) \\
\text{s.t.} \quad & g(p_{12i}) + g(p_{21i}) \leq \frac{1}{2}\log\left(\frac{S_i}{\sigma^2}\right), \quad \forall i \\
& \sum_{i=1}^{k} p_{12i} + p_{u1i} + a p_{21i} \leq \sum_{i=1}^{k} E_{1i}, \quad \forall k \\
& \sum_{i=1}^{k} p_{21i} + p_{u2i} + a p_{12i} \leq \sum_{i=1}^{k} E_{2i}, \quad \forall k \\
& p_{12i}, p_{21i}, p_{u1i}, p_{u2i} \geq 0
\end{aligned} \tag{5}$$

which is not a convex optimization problem due to the first set of constraints. Next, we characterize the (local) optimality conditions for problem (5). The Lagrangian is

$$\begin{aligned}
\mathcal{L} = & -\left(\sum_{i=1}^{N} \mu_1 g(p_{12i}) + \mu_2 g(p_{21i})\right) \\
& + \sum_{i=1}^{N} \lambda_i \left(g(p_{12i}) + g(p_{21i}) - \frac{1}{2}\log\left(\frac{S_i}{\sigma^2}\right)\right) \\
& + \sum_{k=1}^{N} \gamma_{1k} \left(\sum_{i=1}^{k} p_{12i} + p_{u1i} + a p_{21i} - \sum_{i=1}^{k} E_{1i}\right) \\
& + \sum_{k=1}^{N} \gamma_{2k} \left(\sum_{i=1}^{k} p_{21i} + p_{u2i} + a p_{12i} - \sum_{i=1}^{k} E_{2i}\right) \\
& - \left(\sum_{i=1}^{N} \omega_{1i} p_{u1i} + \omega_{2i} p_{u2i} + \eta_{1i} p_{12i} + \eta_{2i} p_{21i}\right)
\end{aligned} \tag{6}$$

where $\{\lambda_i, \gamma_{1i}, \gamma_{2i}, \eta_{1i}, \eta_{2i}, \omega_{1i}, \omega_{2i}\}$ are non-negative Lagrange multipliers. Differentiating with respect to the powers and equating to zero we get the following KKT conditions

$$\sum_{k=i}^{N} \gamma_{1k} + a\gamma_{2k} = \frac{\mu_1 - \lambda_i}{1 + p_{12i}} + \frac{\lambda_i}{S_i} + \eta_{1i} \tag{7}$$

$$\sum_{k=i}^{N} \gamma_{2k} + a\gamma_{1k} = \frac{\mu_2 - \lambda_i}{1 + p_{21i}} + \frac{\lambda_i}{S_i} + \eta_{2i} \tag{8}$$

$$\sum_{k=i}^{N} \gamma_{1k} = \left(1 + \sqrt{\frac{p_{u2i}}{p_{u1i}}}\right)\frac{\lambda_i}{S_i} + \omega_{1i} \tag{9}$$

$$\sum_{k=i}^{N} \gamma_{2k} = \left(1 + \sqrt{\frac{p_{u1i}}{p_{u2i}}}\right)\frac{\lambda_i}{S_i} + \omega_{2i} \tag{10}$$

along with the complementary slackness conditions

$$\lambda_i \left(g(p_{12i}) + g(p_{21i}) - \frac{1}{2}\log\left(\frac{S_i}{\sigma^2}\right)\right) = 0, \quad \forall i \tag{11}$$

$$\gamma_{1k}\left(\sum_{i=1}^{k} p_{12i} + p_{u1i} + a p_{21i} - \sum_{i=1}^{k} E_{1i}\right) = 0, \quad \forall k \tag{12}$$

$$\gamma_{2k}\left(\sum_{i=1}^{k} p_{21i} + p_{u2i} + a p_{12i} - \sum_{i=1}^{k} E_{2i}\right) = 0, \quad \forall k \tag{13}$$

$$\eta_{1i} p_{12i} = 0, \ \eta_{2i} p_{21i} = 0, \quad \forall i \tag{14}$$

$$\omega_{1i} p_{u1i} = 0, \ \omega_{2i} p_{u2i} = 0, \quad \forall i \tag{15}$$

Note that, for the derivatives in (9) and (10) to be well defined, the cooperative powers p_{u1i}, p_{u2i} must be non-zero; otherwise the problem formulation needs to be revisited. Since the case where the users do not send any cooperative codewords occurs very rarely in practice, in this work, we focus only on policies where p_{u1i} and p_{u2i} are positive, i.e., $\omega_{1i} = \omega_{2i} = 0$. We have the following claim regarding the optimal value of λ_i.

Lemma 2 *The optimal λ_i satisfies $\lambda_i \leq \max\{\mu_1, \mu_2\}$.*

Proof: First, note that by concavity of the objective function, it is sub-optimal to move all the energy in slot i forward to future slots. This means that either p_{12i} or p_{21i} is strictly positive for any i. By complementary slackness, this means that either $\eta_{1i} = 0$ or $\eta_{2i} = 0$. Without loss of generality, assume $\eta_{1i} = 0$ for some i. Substituting (9) and (10) into (7), we get

$$\left(1+\sqrt{\frac{p_{u2i}}{p_{u1i}}}\right)\frac{\lambda_i}{S_i} + \left(1+\sqrt{\frac{p_{u1i}}{p_{u2i}}}\right)\frac{a\lambda_i}{S_i} = \frac{\mu_1-\lambda_i}{1+p_{12i}} + \frac{\lambda_i}{S_i} \tag{16}$$

Observe that we always have

$$\left(1+\sqrt{\frac{p_{u2i}}{p_{u1i}}}\right)\frac{\lambda_i}{S_i} \geq \frac{\lambda_i}{S_i} \tag{17}$$

and hence, to satisfy (16) we need to have

$$0 \leq \left(1+\sqrt{\frac{p_{u1i}}{p_{u2i}}}\right)\frac{a\lambda_i}{S_i} \leq \frac{\mu_1-\lambda_i}{1+p_{12i}} \tag{18}$$

which leads to $\lambda_i \leq \mu_1 \leq \max\{\mu_1, \mu_2\}$. ■

Note that if $\lambda_i > \mu_1$ for some i, then we must have $\eta_{1i} > 0$ so that (16) is satisfied (after adding η_{1i} to its right hand side). We will use this observation later in the upcoming proofs. The next lemma shows that we can overcome the non-convexity issue of problem (5) by using its relation to problem (4).

Lemma 3 *Any local optimal point for problem (5) is also a local optimal point for problem (4).*

Proof: We prove the lemma by showing that any primal and dual variables satisfying the KKT conditions for problem (5) correspond to those satisfying the KKT conditions for problem (4). The KKT optimality conditions for (4) are

$$\lambda_{1i} + \lambda_{12i} = \mu_1 + \nu_{1i} \tag{19}$$

$$\lambda_{2i} + \lambda_{12i} = \mu_2 + \nu_{2i} \tag{20}$$

$$\sum_{k=i}^{N} \gamma_{1k} + a\gamma_{2k} = \frac{\lambda_{1i}}{1+p_{12i}} + \frac{\lambda_{12i}}{S_i} + \eta_{1i} \tag{21}$$

$$\sum_{k=i}^{N} \gamma_{2k} + a\gamma_{1k} = \frac{\lambda_{2i}}{1+p_{21i}} + \frac{\lambda_{12i}}{S_i} + \eta_{2i} \tag{22}$$

$$\sum_{k=i}^{N} \gamma_{1k} = \left(1+\sqrt{\frac{p_{u2i}}{p_{u1i}}}\right)\frac{\lambda_{12i}}{S_i} \tag{23}$$

$$\sum_{k=i}^{N} \gamma_{2k} = \left(1+\sqrt{\frac{p_{u1i}}{p_{u2i}}}\right)\frac{\lambda_{12i}}{S_i} \tag{24}$$

along with the complementary slackness conditions

$$\lambda_{1i}\left(r_{1i} - g(p_{12i})\right) = 0, \quad \forall i \tag{25}$$

$$\lambda_{2i}\left(r_{2i} - g(p_{21i})\right) = 0, \quad \forall i \tag{26}$$

$$\lambda_{12i}\left(r_{1i} + r_{2i} - \frac{1}{2}\log\left(\frac{S_i}{\sigma^2}\right)\right) = 0, \quad \forall i \tag{27}$$

$$\gamma_{1k}\left(\sum_{i=1}^{k} p_{12i} + p_{u1i} + ap_{21i} - \sum_{i=1}^{k} E_{1i}\right) = 0, \quad \forall k \tag{28}$$

$$\gamma_{2k}\left(\sum_{i=1}^{k} p_{21i} + p_{u2i} + ap_{12i} - \sum_{i=1}^{k} E_{2i}\right) = 0, \quad \forall k \tag{29}$$

$$\eta_{1i}p_{12i} = 0, \ \eta_{2i}p_{21i} = 0, \quad \forall i \tag{30}$$

$$\nu_{1i}r_{1i} = 0, \ \nu_{2i}r_{2i} = 0, \quad \forall i \tag{31}$$

Now, consider a KKT point for problem (5), i.e., some feasible primal and dual variables $\{\tilde{p}_{jki}, \tilde{p}_{uji}, \tilde{\gamma}_{ji}, \tilde{\lambda}_i, \tilde{\eta}_{ji}\}$, $j,k \in \{1,2\}$, $j \neq k$, satisfying (7)-(14). We then assign the following values for the variables of problem (4)

$$p_{12i} = \tilde{p}_{12i}, \ p_{21i} = \tilde{p}_{21i}, \ p_{u1i} = \tilde{p}_{u1i}, \ p_{u2i} = \tilde{p}_{u2i} \tag{32}$$

$$r_{1i} = \log\left(1+\tilde{p}_{12i}\right), \ r_{2i} = \log\left(1+\tilde{p}_{21i}\right) \tag{33}$$

$$\gamma_{1i} = \tilde{\gamma}_{1i}, \ \gamma_{2i} = \tilde{\gamma}_{2i} \tag{34}$$

$$\lambda_{12i} = \tilde{\lambda}_i, \ \lambda_{1i} = \left(\mu_1 - \tilde{\lambda}_i\right)^+, \ \lambda_{2i} = \left(\mu_2 - \tilde{\lambda}_i\right)^+ \tag{35}$$

$$\nu_{1i} = \left(\tilde{\lambda}_i - \mu_1\right)^+, \ \nu_{2i} = \left(\tilde{\lambda}_i - \mu_2\right)^+ \tag{36}$$

$$\eta_{1i} = \tilde{\eta}_{1i} + \left(\mu_1 - \tilde{\lambda}_i\right)^-, \ \eta_{2i} = \tilde{\eta}_{2i} + \left(\mu_2 - \tilde{\lambda}_i\right)^- \tag{37}$$

where $(\cdot)^+ = \max\{0,\cdot\}$ and $(\cdot)^- = \min\{0,\cdot\}$. Using the observation stated right after Lemma 2, we can directly verify that (19)-(31) are satisfied using the above assignments. ■

We note that problem (4) is convex, and thus its KKT conditions are also sufficient for optimality [18]. Therefore, by Lemma 3, we can optimally solve problem (4) by characterizing the KKT points of problem (5), which we focus on in the remainder of this paper.

A power allocation policy which uses all available energy by the end of the transmission is called an *energy consuming policy*. The next lemma shows that, it is sufficient to restrict our attention to energy consuming policies.

Lemma 4 *There exists an optimal policy for problem (5) where both users exhaust all their energies, in transmission and decoding, by the end of communication.*

Proof: Let one of the users, say user 1, have some leftover energy at the end of transmission. Then, we can increase p_{u1N} until user 1's energy is exhausted. This is feasible, as it increases the right hand side of (3), and does not change the rates, and therefore, is optimal. ■

Note that (3) is a constraint on the total data rate. When it holds with equality, the users send at the maximum allowed data rate. We call such policies *data consuming policies*. The next lemma shows that it is sufficient to restrict our attention to policies that are data consuming in the last slot.

Lemma 5 *There exists an optimal policy for problem (5) that is data consuming in the last time slot.*

Proof: If (3) is not tight in slot N, then we can decrease, say, p_{u1N} until the data consumption constraint holds with equality in time slot N. This is energy feasible, and does not change the rates, and therefore, is optimal. ■

IV. Single Energy Arrival

In this section, we consider the case where each user harvests only one packet of energy. By Lemma 4, both users consume all the available energy, i.e., we have

$$p_{12} + ap_{21} + p_{u1} = E_1, \quad p_{21} + ap_{12} + p_{u2} = E_2 \tag{38}$$

We now solve the above equations for p_{12} and p_{21} in terms of the cooperative powers p_{u1} and p_{u2}, and substitute back in problem (5) for the $N = 1$ case to get the following reduced problem in terms of the cooperative powers[1]

$$\begin{aligned} \max_{p_{u1},p_{u2}} \quad & \mu_1 g\left(\frac{E_1 - aE_2}{1-a^2} - \frac{p_{u1} - ap_{u2}}{1-a^2}\right) \\ & + \mu_2 g\left(\frac{E_2 - aE_1}{1-a^2} - \frac{p_{u2} - ap_{u1}}{1-a^2}\right) \\ \text{s.t.} \quad & g\left(\frac{E_1 - aE_2}{1-a^2} - \frac{p_{u1} - ap_{u2}}{1-a^2}\right) \\ & + g\left(\frac{E_2 - aE_1}{1-a^2} - \frac{p_{u2} - ap_{u1}}{1-a^2}\right) \le \frac{1}{2}\log\left(\frac{S_u}{\sigma^2}\right) \\ & 0 \le p_{u1} \le E_1, \quad 0 \le p_{u2} \le E_2 \\ & a(E_2 - p_{u2}) \le E_1 - p_{u1} \le \frac{E_2 - p_{u2}}{a} \end{aligned} \tag{39}$$

where the last constraint assures the non-negativity of p_{12} and p_{21}, and the term S_u is given by

$$S_u \triangleq \sigma^2 + \frac{E_1 + E_2 + ap_{u1} + ap_{u2} + 2(1+a)\sqrt{p_{u1}p_{u2}}}{1+a} \tag{40}$$

We solve the above problem over two stages as follows.

Stage 1: First, we solve a relaxed problem by ignoring the data consumption constraint. Note that the relaxed problem is a convex problem. To solve it, we further note that, if the last constraint in problem (39) is not binding, i.e., if both p_{12} and p_{21} are strictly positive, then by taking derivative of the objective function with respect to the cooperative powers, the solution of the relaxed problem is found by solving the following two linear equations in (p_{u1}, p_{u2})

$$\left(\frac{1}{a\mu_2} + \frac{a}{\mu_1}\right)p_{u2} - \left(\frac{1}{\mu_2} + \frac{1}{\mu_1}\right)p_{u1} = c_1 \tag{41}$$

$$\left(\frac{1}{\mu_2} + \frac{1}{\mu_1}\right)p_{u2} - \left(\frac{a}{\mu_2} + \frac{1}{a\mu_1}\right)p_{u1} = c_2 \tag{42}$$

where the constants c_1 and c_2 are given by

$$c_1 = \frac{1 - a^2 + E_2 - aE_1}{a\mu_2} - \frac{1 - a^2 + E_1 - aE_2}{\mu_1} \tag{43}$$

$$c_2 = \frac{1 - a^2 + E_2 - aE_1}{\mu_2} - \frac{1 - a^2 + E_1 - aE_2}{a\mu_1} \tag{44}$$

[1]Without loss of generality, we focus on the case $a < 1$ throughout this work. Similar analysis follows for the case $a \ge 1$.

If (41)-(42) admit a solution, $(\tilde{p}_{u1}, \tilde{p}_{u2})$, not satisfying the last constraint in (39), then by the concavity of the objective function, the solution is given by projecting $(\tilde{p}_{u1}, \tilde{p}_{u2})$ onto this last constraint set, which will make one of the constraint's inequalities hold with equality. Substituting this into the objective function, the relaxed problem in this case gets simplified to a one-variable convex optimization problem that can be solved by first derivative analysis over the feasible region. We denote the solution of the relaxed problem by $(\bar{p}_{u1}, \bar{p}_{u2})$.

Stage 2: We now check whether $(\bar{p}_{u1}, \bar{p}_{u2})$ satisfies the data consumption constraint. Denote the left hand side of the constraint by $G(\bar{p}_{u1}, \bar{p}_{u2})$ and let $\bar{S}_u = S_u\big|_{(\bar{p}_{u1}, \bar{p}_{u2})}$. If the constraint is not satisfied, then we have

$$G(\bar{p}_{u1}, \bar{p}_{u2}) > \frac{1}{2}\log\left(\frac{\bar{S}_u}{\sigma^2}\right) \tag{45}$$

Hence, the goal now is to find the closest point (p^*_{u1}, p^*_{u2}) to $(\bar{p}_{u1}, \bar{p}_{u2})$ such that $G(p^*_{u1}, p^*_{u2}) = \frac{1}{2}\log\left(S^*_u/\sigma^2\right)$. Towards that end, we note that $\frac{1}{2}\log\left(S_u/\sigma^2\right)$ is increasing in (p_{u1}, p_{u2}), and that $G(E_1, E_2) = 0$. By the concavity of G, the two functions $G(p_{u1}, p_{u2})$ and $\frac{1}{2}\log\left(S_u/\sigma^2\right)$ are guaranteed to intersect at some point $(p^*_{u1}, p^*_{u2}) > (\bar{p}_{u1}, \bar{p}_{u2})$. The optimal (p^*_{u1}, p^*_{u2}) is the pair at which the intersection of the two functions yields the maximum value for the objective function.

This concludes our discussion on the single energy arrival scenario. In the next section, we use this result to extend the analysis to the general multiple energy arrival scenario.

V. Multiple Energy Arrivals

We present an iterative generalized water-filling algorithm that optimally solves problem (5) for general N. We need to determine the optimal energy distribution among the slots for each user. We first initialize the energy state vectors $\boldsymbol{S}_1 = \boldsymbol{E}_1$ and $\boldsymbol{S}_2 = \boldsymbol{E}_2$ and solve for each slot i independently using the results of the previous section with energies S_{1i} and S_{2i}. Next, given the powers in each slot, we determine λ_i by solving (16) if $p_{12i} > 0$ (and if $p_{21i} > 0$ we solve a similar equation with appropriate coefficients). Next, we solve equations (7)-(10) for all the remaining Lagrange multipliers treating $\sum_{k=i}^{N}\gamma_{1k}$ and $\sum_{k=i}^{N}\gamma_{2k}$ as variables of their own, because we are solving for each slot independently. Let us define

$$\kappa_{1i} \triangleq \frac{1}{\sum_{k=i}^{N}\gamma_{1k} + a\gamma_{2k}}, \quad \kappa_{2i} \triangleq \frac{1}{\sum_{k=i}^{N}\gamma_{2k} + a\gamma_{1k}} \tag{46}$$

We can compute $\{\kappa_{1i}, \kappa_{2i}\}_{i=1}^{N}$ given the initialization policy. We interpret these terms as *generalized water levels* to be equalized to the extent possible among the slots. We have the following lemma regarding their optimal values.

Lemma 6 *The optimal generalized water levels $\{\kappa^*_{1i}, \kappa^*_{2i}\}$ for problem (5) are non-decreasing, and increase synchronously. The latter event occurs only if at least one user consumes its energy in transmission and decoding.*

Proof: The first part follows by noting that due to the non-negativity of the Lagrange multipliers $\{\gamma_{1i}, \gamma_{2i}\}$, the denomi-

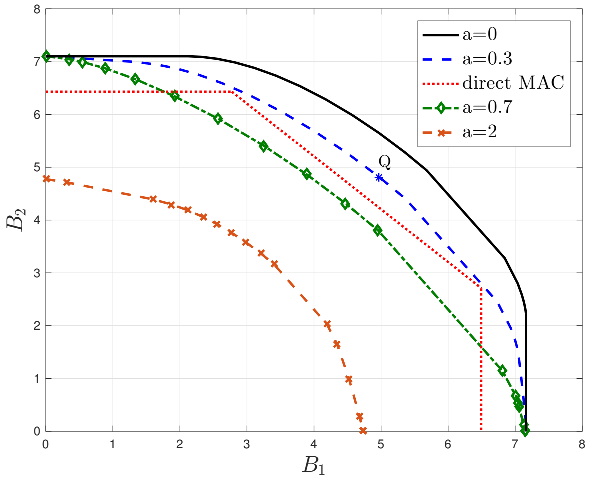

Fig. 2. Departure regions for different values of the decoding cost parameter.

nators of the water levels in (46) are non-increasing. For the second part, since $a > 0$, *both* denominators decrease from slot i to slot $i+1$ iff at least $\gamma_{1i} > 0$ or $\gamma_{2i} > 0$. This makes both water levels increase synchronously. Finally, by complementary slackness, if we have $\gamma_{ji} > 0$, then user j consumes its energy in slot i, $j = 1, 2$. ■

Next, we check if the obtained water levels satisfy the conditions of the previous lemma. If not, then some energy needs to flow forward until they satisfy these conditions. However, due to the decoding costs, energy transfer from one user affects both water levels, and therefore both users' powers. Hence, we keep record of how much energy is transferred forward at each user by, e.g., putting measuring meters in between the slots of each user [19]. We start by updating slots 1 and 2, followed by slots 2 and 3, and so on. If at a given two slots $(i, i+1)$ we have $\kappa_{1i} > \kappa_{1(i+1)}$ or $\kappa_{2i} > \kappa_{2(i+1)}$ then energy flows from slot i to $i+1$ from either one or both users until the water levels are equalized. We keep iterating until the conditions of Lemma 6 are satisfied for all the slots. During the iterations, energy can be drawn back, using the values stored in the meters, if this increases the objective function. Iterations converge to a KKT point of problem (5), which is, by Lemma 3, a KKT point of problem (4), and thereby the optimal solution.

VI. Numerical Results

In this section, we present some simple numerical examples. We consider a five slot system with energies $\mathbf{E}_1 = [5, 1, 6, 2, 2]$ and $\mathbf{E}_2 = [2, 3, 4, 3, 4]$ at the first and the second user, respectively. The receiver noise variance is set to $\sigma^2 = 1.2$.

We solve the problem with different values of decoding costs and plot $B_j = \sum_{i=1}^{N} r_{ji}$, the number of total departed bits for user j, in Fig. 2. For reference, we plot the case $a = 0$ studied in [12] that provides the largest departure region, and also the non-cooperative (direct) MAC departure region studied in [5]. We observe that the departure region shrinks as we increase the decoding cost. With $a = 0.3$, the region is still completely outside the non-cooperative MAC region, showing the advantage of data cooperation. For the case $a = 0.7$, the regions intersect, and not all operating points are better than the non-cooperative MAC. Finally, for a relatively large $a = 2$, the departure region is completely inside the non-cooperative MAC region, showing that the users achieve higher rates if they do not cooperate due to the high decoding costs they incur. Therefore, the results show that it is not always better to perform data cooperation, but rather it depends on how much energy each user spends to decode the other user's message.

We also compute the optimal generalized water levels for a particular operating point: Q in Fig. 2 for the case of $a = 0.3$ with $\mu_1 = \mu_2 = 1$. Iterations converge to: $\boldsymbol{\kappa}_1^* = [4.1, 16.3, 17.5, 17.5, 30.7]$ and $\boldsymbol{\kappa}_2^* = [3.1, 6.6, 7.3, 7.3, 9.2]$. We see that the water levels are non-decreasing, and increase simultaneously, as stated in Lemma 6.

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