Adaptive Transmission Policies for Energy Harvesting Wireless Nodes in Fading Channels

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***Abstract*—In this paper, we consider a single-user communication system, where an energy harvesting transmitter communicates with a receiver over a fading wireless channel. We design adaptive transmission policies that adapt to the random energy arrivals at the transmitter and random fluctuations in the channel, in order to maximize the average number of bits transmitted by a finite deadline T. We solve for the optimum transmission scheme using stochastic dynamic programming. This optimal solution does not admit a closed form expression and is computationally expensive. We then propose several suboptimal event based adaptive transmission policies that react to the changes in energy arrivals and fading states. We provide extensive simulation results that compare the performances of the optimal and proposed simpler solutions.**

I. Introduction

We consider a single-user wireless communication system with an energy harvesting transmitter. The transmitter is able to harvest energy from nature to recharge its battery. The energy is modelled to arrive (be harvested) at random times and in random amounts throughout the duration of the communication session. In addition, the wireless communication channel fluctuates randomly due to fading. Our goal is to design adaptive (online) transmission policies that adapt to the random energy arrivals as well as random fluctuations in the channel gain, in order to maximize the average number of bits that can be transmitted reliably by a fixed deadline.

Energy arrivals and channel gains are stochastic processes in time. Naturally, the battery state is causally known by the transmitter. The transmitter has perfect causal knowledge of the fading channel state via a feedback link from the receiver. We also assume that the transmitter has the full knowledge of the statistics of both the energy arrivals and the channel variations. The transmitter uses the knowledge of the current energy and channel states as well as their long-term statistics to adapt its instantaneous transmit power. The transmit power, in turn, determines the instantaneous rate of communication through a rate-power relationship.

In addition, the battery at the transmitter is assumed to have a finite storage capacity. This imposes another constraint on the transmit power policy. Since the battery can store at most a finite amount of energy, excess arriving energy will overflow, causing waste of energy, which could otherwise be used for data transmission. The transmitter must minimize the occurrence of energy overflows as much as possible.

The instantaneous rate is a concave function of the instantaneous power. This implies that, in a static channel, in order to maximize the number of bits transmitted, the energy must be spread as evenly as possible throughout the communication duration, subject to the causality of energy usage with respect to energy arrivals [1], [2]. The finite battery capacity constraint limits the ability of the transmitter spread its energy evenly over time because of the risk of losing energy at the future energy arrivals due to battery overflows. On the other hand, in a fading channel, in order to maximize the number of bits transmitted, an opportunistic transmission policy that exploits the good channel states is needed. We determine the optimal online transmission policy that balances the needs to: spread the energy over time, avoid battery overflows, and exploit channel variations to the advantage of the system.

There has been recent research effort on understanding data transmission with an energy harvesting transmitter that has a rechargeable battery [1]–[10]. In [3], data transmission with energy harvesting sensors is considered, and the optimal online policy for controlling admissions into the data buffer is derived using a dynamic programming framework. In [4], energy management policies which stabilize the data queue are proposed for single-user communication and under a linear approximation, some delay optimality properties are derived. In [5], the optimality of a variant of the back pressure algorithm using energy queues is shown. In [6], throughput optimal energy allocation is studied for energy harvesting systems in a time constrained slotted setting. In [7], information theoretically achievable rates are determined in a single-user energy harvesting communication channel. In [1], [2], minimization of the transmission completion time is considered in an energy harvesting system and the optimal solution is obtained using a geometric framework similar to the calculus approach presented in [11]. In [8], energy harvesting transmitters with batteries of finite energy storage capacity are considered and the problem of throughput maximization by a deadline is solved in a static channel. In [9], offline optimal transmission policies for a single-user energy harvesting transmitter operating in a fading channel are provided.

Energy (or rate) management for maximum rate or minimum energy under delay constraints has been considered in [11]–[17]. In [12], energy minimal offline packet scheduling

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is solved in a delay constrained system with channel fading. In [13], energy minimal rate control in a delay constrained time-varying system is obtained by using continuous time stochastic dynamic programming. In [14], delay constrained capacity of fading channels is found under causal feedback using dynamic programming. In [15], capacity of a two-user fading broadcast channel is determined under stringent delay constraints. In [16], various energy allocation problems in solar powered communication satellites are solved using dynamic programming. In [17], optimal energy allocation to a fixed number of time slots is derived under time-varying channel gains and with offline and online knowledge of the channel state at the transmitter.

In this paper, we study online transmission policies with the objective of maximizing the deadline constrained throughput under channel fluctuations and energy variations, in a continuous time model. We first solve for the optimal online transmission policy by using continuous time stochastic dynamic programming [18]. The optimal policy does not admit a closed form solution and requires excessive computation. Hence, we consider suboptimal solutions that are computationally more tractable. We propose several event-based online (adaptive) algorithms which mimic optimal offline policy found for the deterministic setting through a directional waterfilling algorithm [9], [10]. These algorithms are easier to express in closed form, and they update transmit power only when a change in the fading level or energy arrival occurs. Finally, we provide extensive simulation results to compare the performances of the optimum and proposed simpler transmission policies for practical energy arrival and fading distributions.

II. System Model

We consider a single-user additive Gaussian channel in fading with a causal channel state information (CSI) feedback from the receiver to the transmitter as shown in Fig. 1. The two queues at the transmitter are the data queue where data packets are stored, and the energy queue (battery) where the arriving (harvested) energy is stored. The energy queue can store at most E_{max} units of energy. The harvested energy is used only for data transmission, e.g., energy required for processing is not considered.

The received signal y is given by

$$y = \sqrt{h}x + n \tag{1}$$

where h is the (squared) fading coefficient, x is the channel input, and n is a Gaussian random variable with zero-mean and unit-variance. Whenever an input signal x is transmitted with power p in an epoch of duration L, $\frac{L}{2}\log(1+hp)$ bits of data is served out from the backlog with the cost of Lp units of energy depletion from the energy queue. The bandwidth is sufficiently wide so that L can take small values and we approximate the slotted system as a continuous time system. Hence, if at time t the transmit power of the signal is $x^2(t) = p(t)$, then the instantaneous rate $r(t)$ in bits per channel use

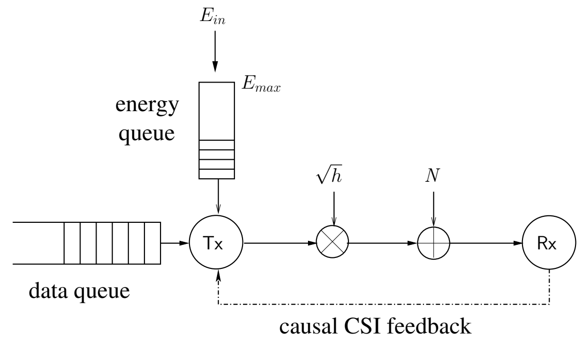

Fig. 1. Additive Gaussian fading channel with an energy harvesting transmitter and causal channel state information (CSI) feedback.

is

$$r(t) = \frac{1}{2}\log(1+h(t)p(t)) \tag{2}$$

The fading level h and energy arrivals are stochastic processes in time that are marked by Poisson counting processes. Hence, the fading level changes and energy arrives for a countable number of times in any time interval. The instants of channel gain changes and energy arrivals are indexed as $t_1^f, t_2^f, \ldots, t_n^f, \ldots$ and $t_1^e, t_2^e, \ldots, t_n^e, \ldots$, respectively, with the convention that $t_1^e = t_1^f = 0$. The fading level in $[0, t_1^f)$ is h_1, in $[t_1^f, t_2^f)$ is h_2, and so on. Similarly, E_i units of energy arrives at time t_i^e, and E_0 units of energy is available at time 0. Hence $\{(t_i^e, E_i)\}_{i=0}^{\infty}$ and $\{(t_i^f, h_i)\}_{i=1}^{\infty}$ completely define the events that take place during the course of data transmission. A sample realization for this model is shown in Fig. 2. In the sequel, we will refer to a change in channel fading level or in energy level as an *event* and the time interval between two events as an *epoch*.

Energy arrival information is available to the transmitter at the time of its occurrence. Moreover, by virtue of the causal feedback link, the transmitter can perfectly track the changes in the fade level. Therefore, at time t all $\{E_i\}$ and $\{h_j\}$ such that $t_i^e < t$ and $t_j^f < t$ are known perfectly by the transmitter. The incoming energy is first buffered in the battery before it is used in data transmission, and the transmitter is allowed to use the battery energy only. Accordingly, we assume $E_i \leq E_{max}$ for all i as otherwise excess energy cannot be accommodated in the battery anyway.

A power management policy is denoted as $p(t)$ for $t \in [0, T]$. There are two constraints on $p(t)$, due to energy arrivals at random times and also due to the finite battery storage capacity. Since energy that has not arrived yet cannot be used at the current time, there is a causality constraint on the power management policy as:

$$\int_0^{t_i^e} p(u)du \leq \sum_{j=0}^{i-1} E_j, \quad \forall i \tag{3}$$

where the limit of the integral t_i^e should be interpreted as $t_i^e - \epsilon$, for small enough ϵ. Moreover, due to the finite battery storage capacity, we need to make sure that energy level in the battery

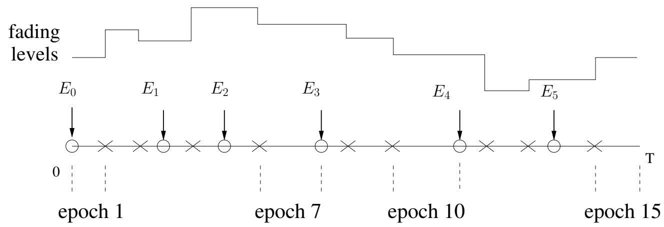

Fig. 2. The system model and epochs under channel fading.

never exceeds E_{max}. Since energy arrives at certain points in time, it is sufficient to ensure that the energy level in the battery never exceeds E_{max} at the times of energy arrivals. Let $d(t) = \max\{t_i^e : t_i^e \le t\}$. Then,

$$\sum_{j=0}^{d(t)} E_j - \int_0^t p(u)du \le E_{max}, \quad \forall t \in [0,T] \tag{4}$$

We emphasize that our system model is continuous rather than slotted. In slotted models, e.g., [5], [6], [16], the energy input-output relationship is written for an entire slot. Such models allow energies larger than E_{max} to enter the battery and be used for transmission in a given single slot. Our continuous system model prohibits such occurrences.

In [9], we considered the setting where the energy arrival and channel fade patterns are known by the transmitter a priori in an offline manner. We have solved for the deadline constrained throughput optimal policy using Lagrangian techniques, and concisely characterized the optimality conditions by an adaptive algorithm termed *directional waterfilling*. In the developed algorithm, the incoming energies are poured over the channel fade pattern from the current time to the future subject to the battery capacity constraint. The transmit power is determined as the water level over the fade level.

In this paper, we first obtain the online optimal transmission policy using dynamic programming. We then develop suboptimal adaptive waterfilling policies that mimic the optimal offline policy in [9]. The developed algorithms determine the transmit power as a function of time using the causal knowledge of the system available at the transmitter, i.e., the instantaneous energy state and fading state.

III. Optimal Online Policy

In this section, we consider the maximization of the average number of bits sent by deadline T when causal information of the energy arrivals and channel fade levels are available at the transmitter as in Fig. 1. Moreover, statistics of the channel state and energy arrival processes are known.

The states of the system are the fade level h and the battery energy e. An online policy, $g(e,h,t)$, denotes the transmit power decided by the transmitter at time t when the states are e and h. We call a policy admissible if g is nonnegative, $g(0,h,t) = 0$ for all h and $t \in [0,T]$ and $e(T) = 0$. Hence, admissible policies guarantee that no transmission can occur if battery energy is zero. In addition, energy left in the battery at the time of deadline is zero so that resources are used fully by the deadline T. The throughput of the policy g in the interval $[t,T]$ is denoted as $J_g(e,h,t)$.

$$J_g(e,h,t) = E\left[\int_t^T \frac{1}{2}\log\left(1 + h(\tau)g(e,h,\tau)\right)d\tau\right] \tag{5}$$

Then, the value function is the supremum over all admissible policies

$$J(e,h,t) = \sup_g J_g \tag{6}$$

Therefore, the optimal online policy $g^*(e,h,t)$ is such that $J(e,h,t=0) = J_{g^*}(e,h,0)$, i.e., it solves the following

$$\max_g E\left[\int_0^T \frac{1}{2}\log\left(1 + h(\tau)g(e,h,\tau)\right)d\tau\right] \tag{7}$$

In order to solve (7), we first consider δ-skeleton of the random processes [13]. For sufficiently small δ, we quantize the time by δ and have the following.

$$\begin{aligned}&\max_g E\left[\int_0^T \frac{1}{2}\log\left(1 + h(\tau)g(e,h,\tau)\right)d\tau\right] \\ &= \max_{g(e,h,0)}\left(\frac{\delta}{2}\log\left(1 + h(0)g(e,h,0)\right) + J(e,h,\delta)\right)\end{aligned} \tag{8}$$

Then, we can recursively solve (8) to obtain $g^*(e,h,t=k\delta)$ for $k = 1,2,\ldots,\lfloor\frac{T}{\delta}\rfloor$. This procedure is the dynamic programming solution for continuous time and yields the optimal online policy [13], [18]. After solving for $g^*(e,h,t)$, the transmitter records this function as a look-up table and at each time t, it receives feedback $h(t)$, senses the battery energy $e(t)$ and transmits with power $g^*(e(t),h(t),t)$.

IV. Suboptimal Online Waterfilling Policies

Due to the *curse of dimensionality* inherent in the dynamic programming solution, it is natural to forgo performance in lieu of less complex online policies. In this section, we propose several suboptimal transmission policies that can somewhat mimic the offline optimal algorithms while being computationally simpler and requiring less statistical knowledge. In particular, we resort to event-based online policies which react to a change in fading level or an energy arrival. Whenever an event is detected, the online policy decides on a new power level. Note that the transmission is subject to the availability of energy and the E_{max} constraint.

A. Constant Water Level Policy

The constant water level policy makes online decisions for the transmit power whenever a change in the fading level is observed through the causal feedback. Assuming that the knowledge of the average recharge rate P is available to the transmitter and that fading density f_h is known, the policy calculates h_0 that solves the following equation.

$$\int_{h_0}^{\infty}\left(\frac{1}{h_0} - \frac{1}{h}\right)f_h(h)dh = P \tag{9}$$

Whenever a change in the fading level occurs, the policy decides on the following power level $p_i = \left(\frac{1}{h_0} - \frac{1}{h_i}\right)^+$. If the energy in the battery is nonzero, transmission with p_i is allowed, otherwise the transmitter becomes silent.

Note that this power control policy is the same as the capacity achieving power control policy in a stationary fading channel [19] with an average power constraint equal to the average recharge rate. In [4], this policy is proved to be stability optimal in the sense that all data queues with stabilizable arrival rates can be stabilized by policies in this form where the power budget is $P - \epsilon$ for some $\epsilon > 0$ sufficiently small. However, for the time constrained setting, this policy is strictly suboptimal as will be verified in the numerical results section. This policy requires the transmitter to know the mean value of the energy arrival process and the full statistics of the channel fading. A channel state information (CSI) feedback is required from the receiver to the transmitter at the times of events only.

B. Energy Adaptive Waterfilling

Another reduced complexity event-based policy is obtained by adapting the water level to the energy level at each event. Again the fading statistics is assumed to be known. Whenever an event occurs, the policy determines a new power level. In particular, the cutoff fade level h_0 is calculated at each energy arrival time as the solution of the following equation

$$\int_{h_0}^{\infty} \left(\frac{1}{h_0} - \frac{1}{h}\right) f(h)dh = E_{current} \tag{10}$$

where $E_{current}$ is the energy level at the time of the event. Then, the transmission power level is determined similarly as $p_i = \left(\frac{1}{h_0} - \frac{1}{h_i}\right)^+$. This policy requires transmitter to know the fading statistics. A CSI feedback is required from the receiver to the transmitter at the times of changes in the channel state.

C. Time-Energy Adaptive Waterfilling

A variant of the energy adaptive waterfilling policy is obtained by adapting the power to the energy level and the remaining time to the deadline. The cutoff fade level h_0 is calculated at each energy arrival time as the solution of the following equation.

$$\int_{h_0}^{\infty} \left(\frac{1}{h_0} - \frac{1}{h}\right) f(h)dh = \frac{E_{current}}{T - t_i^e} \tag{11}$$

Then, the transmission power level is $p_i = \left(\frac{1}{h_0} - \frac{1}{h_i}\right)^+$.

D. Hybrid Adaptive Waterfilling

This adaptive waterfilling policy combines the approaches of constant water level and time-energy adaptive waterfilling policies, hence the name hybrid adaptive waterfilling. The cutoff fade level h_0 is calculated at each energy arrival time as the solution of the following equation.

$$\int_{h_0}^{\infty} \left(\frac{1}{h_0} - \frac{1}{h}\right) f(h)dh = \frac{E_{current}}{T - t_i^e} + P_{avg} \tag{12}$$

Then, the transmission power level is determined as $p_i = \left(\frac{1}{h_0} - \frac{1}{h_i}\right)^+$ whenever $E_{current} > 0$. Fading statistics is required and feedback is necessary only at the times of events.

V. Numerical Results

We consider a fading additive Gaussian channel with bandwidth W where the instantaneous rate is

$$r(t) = W \log\left(1 + h(t)p(t)\right) \tag{13}$$

$h(t)$ is the channel SNR, i.e., the actual channel gain divided by the noise power spectral density multiplied by the bandwidth, and $p(t)$ is the transmit power at time t. Bandwidth is chosen as $W = 1$ MHz for the simulations.

We examine the deadline constrained throughput performances of the optimal online policy and other proposed suboptimal online policies. In particular, we compare the optimal performance with the proposed sub-optimal online policies which are based on waterfilling [19]. The proposed suboptimal online policies use the fading distribution, and react only to the new energy arrivals and fading level changes. These event-based algorithms require less feedback and less computation, however, the fact that they react only to the changes in the fading level and new energy arrivals is a shortcoming of these policies. Since the system is deadline constrained, the policies need to take the remaining time into account yet the proposed policies do not do this optimally. We simulate these policies under various different settings and we observe that the proposed suboptimal policies may perform very well in some cases while not as well in some others.

We perform all simulations for 1000 randomly generated realizations of the channel fade pattern and $\delta = 0.001$ is taken for the calculation of the optimal online policy. The rates of Poisson mark processes for energy arrival and channel fading λ_e and λ_f are assumed to be 1. The unit of λ_e is J/sec and that of λ_f is $1/$sec. Hence, the mean value of the density function f_e is also the average recharge rate and the mean value of f_h is the average fading level. The changes in the fading level occur relatively slowly with respect to the symbol duration.

f_e is set as a non-negative uniform random variable with mean P, and as the energy arrival is assumed to be smaller than E_{max}, we have $2P < E_{max}$. Selection of the E_{max} constraint is just for illustration. In real life, sensors may have batteries of E_{max} on the order of kJ but the battery feeds all circuits in the system. Here, we assume a fictitious battery that carries energy for only communication purposes. Hence, E_{max} on the order of 1 J will be considered. We will examine different fading distributions f_h. In particular, Nakagami distribution with different shape parameter m is considered. We implement the specified fading by sampling its probability density function with a sufficiently large number of points.

In order to assess the performance, we find an upper bound on the performances of the policies by first assuming that the channel fading levels and energy arrivals in the $[0, T]$ interval are known non-causally, and that the total energy that will arrive in $[0, T]$ is available at the transmitter at time $t = 0$. Then, for the water level p_w that is obtained by spreading the total energy to the interval $[0, T]$, with the corresponding fading levels, yield the throughput T^{ub} defined in the following

$$T^{ub} = \frac{W}{T} \sum_{i=1}^{K} l_i \frac{1}{2} \log \left(1 + h_i \left(p_w - \frac{1}{h_i} \right)^+ \right) \quad (14)$$

as an upper bound for the average throughput in the $[0, T]$ interval; here K denotes the number of epochs, l_i denotes the duration of the fade level in the ith epoch.

We start with examining the average throughput of the system under Rayleigh fading with SNR= 0 dB and deadline $T = 10$ sec, $E_{max} = 10$ J as depicted in Fig. 3. We observe that time-energy adaptive waterfilling policy performs quite close to the optimal online policy in the low recharge rate regime. It can be a viable policy to spread the incoming energy when the recharge rate is low; however, its performance saturates as the recharge rate is increased. In this case, the incoming energy cannot be easily accommodated and more and more energy is lost due to overflows. In Fig. 4, we plot the simulation results for the low recharge rate regime with the battery capacity $E_{max} = 1$ J and we observe that hybrid adaptive policy performs better while energy adaptive policy behaves similar to the previous case. Next, we examine the setting with $T = 10$ sec, $E_{max} = 10$ J under Nakagami fading of $m = 3$ (average SNR= 5 dB) and plot the performances in Fig. 5. As a common behavior in these settings, energy adaptive waterfilling performs poorer with respect to the constant water level and time-energy adaptive waterfilling schemes. Since energy adaptive scheme determines the power level considering only the instantaneous energy changes, it cannot take advantage of the battery for saving energy for future use, and hence performs relatively poorly.

We also simulate the degenerate case where the channel is static ($h = 1$). In this case, the suboptimal waterfilling policies behave as follows: whenever a new energy arrival occurs, the constant water level policy decides to transmit with power P_{avg}, the energy adaptive waterfilling policy decides to transmit with power equal to the battery energy, the time energy adaptive policy decides to transmit with power equal to battery energy divided by the time remaining to the deadline, and finally the hybrid adaptive policy decides to transmit with power equal to P_{avg} plus battery energy divided by the time remaining to the deadline. We take $T = 10$ sec and $E_{max} = 1$ J. As the performance plots in Fig. 6 show, the time-energy adaptive waterfilling performs almost optimally under low recharge rate regime while its performance degrades with respect to the optimal online policy as the recharge rate is increased. We observe that hybrid adaptive scheme performs quite close to the optimal.

Finally, we examine the policies under different deadline constraints and present the plots for Nakagami fading distribution with $m = 5$ in Fig. 7. A remarkable result is that as the deadline is increased, stability optimal [4] constant water level policy approaches the optimal online policy. We conclude

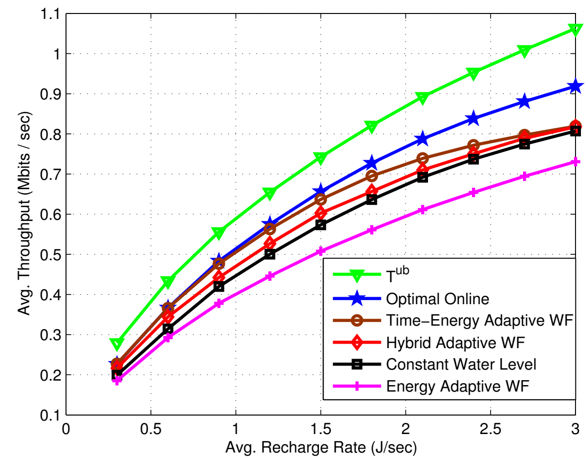

Fig. 3. Performances of the policies for different energy arrival rates under unit mean Rayleigh fading, $T = 10$ sec and $E_{max} = 10$ J.

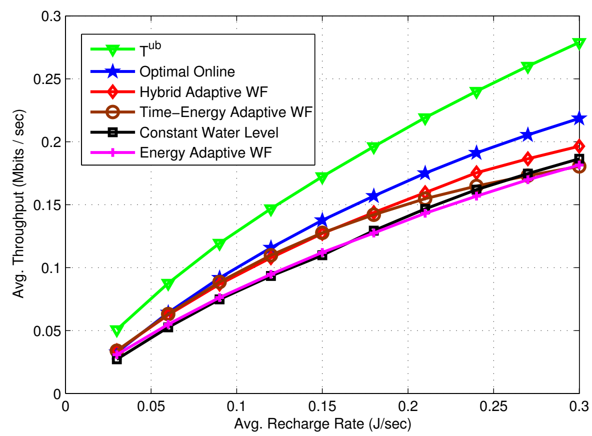

Fig. 4. Performances of the policies for different energy arrival rates under unit mean Rayleigh fading, $T = 10$ sec and $E_{max} = 1$ J.

that the time-awareness of the optimal online policy has less and less importance as the deadline constraint becomes looser. We also observe that the throughput of the energy-adaptive waterfilling policy is roughly a constant regardless of the deadline. This is also true for the upper bound in (14) which is expected as the average is taken over time and realizations and the underlying processes are ergodic. Moreover, the time-energy adaptive policy performs worse as T is increased because energies are spread to very long intervals rendering the transmit power very small and hence energy accumulates in the battery. This leads to significant energy overflows since the battery capacity is limited, and the performance degrades. The hybrid adaptive policy performs right above the constant water level policy and approaches to it as the deadline is increased since the second term $\frac{E_{current}}{T-t_i^e}$ becomes less significant as the deadline is increased.

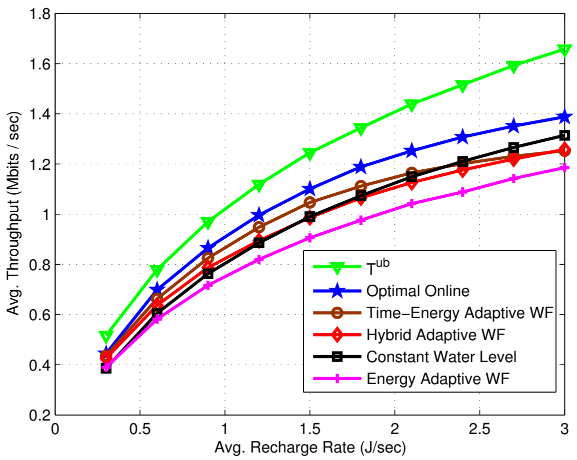

Fig. 5. Performances of the policies for different energy arrival rates under Nakagami fading with $m = 3$, $T = 10$ sec and $E_{max} = 10$ J.

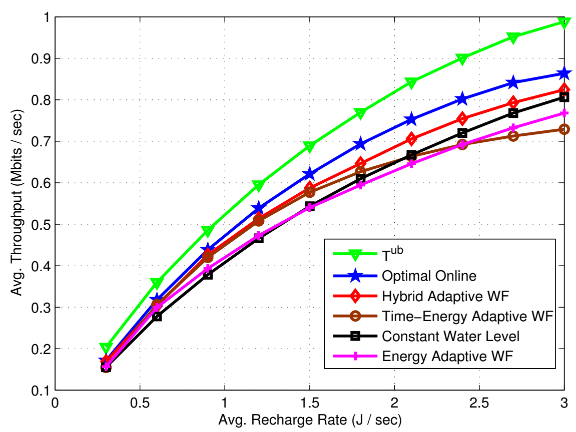

Fig. 6. Performances of the policies for different energy arrival rates under static channel with $T = 10$ sec and $E_{max} = 10$ J.

VI. Conclusions

We developed online energy management schemes for energy harvesting systems operating in fading channels, with finite capacity rechargeable batteries. We first determined the throughput optimal policy for deadline constrained setting under online knowledge of the events using stochastic dynamic programming in continuous time. Next, we provided several suboptimal transmission policies that somewhat mimic offline optimal solution and require less information for processing. Our numerical results show the performances of these algorithms under online knowledge of the events.

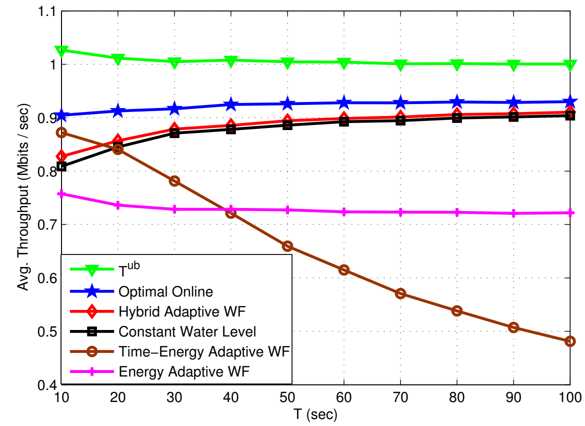

Fig. 7. Performances of the policies with respect to deadline T under Nakagami fading distribution with $m = 3$ and average recharge rate $P = 1$ J/sec and $E_{max} = 10$ J.

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