Correlated Jamming in Multiple Access Channels[1]

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Abstract —

We investigate the behavior of two users and one jammer in an AWGN channel with and without fading when they participate in a zero-sum mutual information game with the sum capacity as the objective function. We assume that the jammer can eavesdrop the channel and can use the information obtained to perform correlated jamming.

I. Introduction

Correlated jamming, the situation where the jammer has full or partial knowledge about the user signal has been studied in the information-theoretic context under various assumptions [1, 2]. In [1] the best transmitter/jammer strategies are found for an AWGN channel with one user and one jammer who participate in a two person zero-sum game with the mutual information as the objective function. The jammer is power constrained and has full or partial knowledge of the transmitted signal which may be obtained through eavesdropping. In [2], the problem is extended to a single user MIMO fading channel with the assumption that the jammer has full knowledge of the user signal.

We find the optimum strategies towards achieving maximum sum-capacity in a system of two users and one jammer who has full or partial knowledge of the user signals through eavesdropping. We first follow the approach of [1] and show that in a non-fading channel, the power that the jammer allocates for jamming each user's signal is proportional to that user's power. Then we examine single user and multiuser fading channels and show that if the jammer is not aware of the channel states, it would disregard its eavesdropping information and only transmit white Gaussian noise, but if it knew the channel states, it would employ its eavesdropping information to optimally jam the users.

II. System Model

We consider several different settings based on the channel characteristics and the jammer's information. In the absence of fading, the attenuations of the user channels are known to everyone. Therefore, we can assume that the attenuations are scalars, and the AWGN channel with two users and one jammer is modeled as

$$Y = \sqrt{h_1}X_1 + \sqrt{h_2}X_2 + \sqrt{\gamma}J + N \tag{1}$$

where X_i is the i^{th} user's signal, h_i is the attenuation of the i^{th} user's channel, J is the jammer's signal, γ is the attenuation of the jammer's channel and N is a zero-mean Gaussian random variable with variance σ_N^2. To model fading in the received powers, we consider h_i and γ as fading random variables, and to further model the phase of the channel coefficients, we substitute the scalar attenuations $\sqrt{h_i}$ and $\sqrt{\gamma}$, with complex fading random variables H_i for the amplitude fading coefficient of the i^{th} user's channel, and Γ for the amplitude fading coefficient of the jammer's channel

$$Y = H_1X_1 + H_2X_2 + \Gamma J + N \tag{2}$$

The user and jammer power constraints are

$$E[X_1^2] \leq P_1 \tag{3}$$

$$E[X_2^2] \leq P_2 \tag{4}$$

$$E[J^2] \leq P_J \tag{5}$$

Regarding the knowledge of the jammer about the transmitted signals, we analyze both cases of perfect information and imperfect information gained through eavesdropping. In the first case, we assume that the jammer knows the signals of the users perfectly, i.e., it knows X_1 and X_2. In the second case, we assume an AWGN eavesdropping channel for the jammer

$$Y_e = \sqrt{g_1}X_1 + \sqrt{g_2}X_2 + N_e \tag{6}$$

where Y_e is the signal received at the jammer, g_i is the attenuation of the i^{th} user's eavesdropping channel and N_e is a zero-mean Gaussian random variable with variance σ_e^2 which models the AWGN in the eavesdropping channel. To model fading in the received powers, we consider $\sqrt{g_i}$ as real fading variables, and to model fading in the received amplitudes, we substitute them with complex amplitude fading random variables G_i. Various assumptions are made on the amount of information that the parties (users, jammer and receiver) have about the channel fading realization of the eavesdropping and communication channels which are stated at the beginning of each subsection.

III. Correlated Jamming in Non-fading Multiuser AWGN Channel

In this section, we find the best user/jammer strategies when the channels are non-fading, both when the jammer knows the exact user signals, and when it eavesdrops the users' channel to get the user signals.

III.A Jamming with Complete Information

Here the system model is (1) where the attenuations are constant scalars, and X_1 and X_2 are known to the jammer. The jammer and the two users are involved in a zero-sum game with the input/output mutual information $I(Y; X_1, X_2)$ as the objective function. We investigate the existence and uniqueness of a saddle point for this objective function. Using the method in [6] we show that the mutual information is a convex function of the transition probabilities when the input distributions are fixed, and a concave function of each input distribution when the other input distribution and the transition probabilities are fixed.

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Theorem 1 *Let* $(X_1, X_2, Y) \sim p(x_1)p(x_2)p(y|x_1,x_2)$. *The mutual information* $I(X_1, X_2; Y)$ *is a concave function of* $p(x_1)$ *for fixed* $p(x_2)$ *and* $p(y|x_1,x_2)$, *a concave function of* $p(x_2)$ *for fixed* $p(x_1)$ *and* $p(y|x_1,x_2)$ *and a convex function of* $p(y|x_1,x_2)$ *for fixed* $p(x_1)$ *and* $p(x_2)$.

Proof: Follows the proof of [6, Theorem 2.7.4]. □

Due to the convexity/concavity of the mutual information, it has a saddle point which is the solution of the game [7, Theorem 16, p. 75], [8, Proposition 2.6.9]. We show that Gaussian input distribution and linear jamming is a saddle point of the input/output mutual information. Due to the interchangeability property of saddle points [7, Theorem 8, p. 48], if there is any other pair of strategies which is a saddle point as well, it has to result in the same mutual information value as the saddle point corresponding to Gaussian signaling and linear jamming [7, Theorem 7, p. 48].

First, assume that the jamming signal is a linear function of the transmitted signals plus a Gaussian random variable

$$J = \rho_1 X_1 + \rho_2 X_2 + N_J \tag{7}$$

The power constraint on the jammer will force the following condition

$$\rho_1^2 P_1 + \rho_2^2 P_2 + \sigma_{N_J}^2 \leq P_J \tag{8}$$

The output of the channel will be

$$Y = \sqrt{h_1} X_1 + \sqrt{h_2} X_2 + \sqrt{\gamma} J + N \tag{9}$$

$$= (\sqrt{h_1} + \sqrt{\gamma}\rho_1) X_1 + (\sqrt{h_2} + \sqrt{\gamma}\rho_2) X_2 + \sqrt{\gamma} N_J + N \tag{10}$$

From the users' perspective, the channel becomes an AWGN multiple access channel, and therefore the best signalling scheme for the users is Gaussian [6].

Next we should show that if the users perform Gaussian signalling, the best jamming strategy is a linear function of the user signals. We write the input/output mutual information of the channel

$$I(Y; X_1, X_2) = h(X_1, X_2) - h(X_1, X_2|Y) \tag{11}$$

The jammer's strategy can only affect the second term above

$$h(X_1, X_2|Y) = h(X_1 - a_1 Y, X_2 - a_2 Y|Y) \tag{12}$$

$$\leq h(X_1 - a_1 Y, X_2 - a_2 Y) \tag{13}$$

$$\leq \frac{1}{2}\log\left((2\pi e)^2 |\mathbf{\Lambda}|\right) \tag{14}$$

where $\mathbf{\Lambda}$ is the covariance matrix of $(X_1 - a_1 Y, X_2 - a_2 Y)$ and the inequalities hold for arbitrary a_1 and a_2.

Now we show that linear jamming can achieve both inequalities with equality. Given that X_1 and X_2 are i.i.d Gaussian, Y is jointly Gaussian with X_1 and X_2. Choosing $a_1 = E[X_1 Y]/E[Y^2]$ and $a_2 = E[X_2 Y]/E[Y^2]$, makes $X_1 - a_1 Y$ and $X_2 - a_2 Y$ uncorrelated with Y, and both inequalities will hold with equality. Moreover, for these values of a_1 and a_2, $|\mathbf{\Lambda}|$ is only a function of $E[X_1 Y]$ and $E[X_2 Y]$, and therefore, the linear jamming coefficients ρ_1 and ρ_2 can be chosen so as to achieve any $|\mathbf{\Lambda}|$ value which is allowed under the power constraints.

The next step is to find the optimum values of ρ_1 and ρ_2. The sum capacity, which is the maximum $I(X_1, X_2; Y)$ over all input distributions, is

$$C_{sum} = \frac{1}{2}\log\left(1 + \frac{(\sqrt{h_1} + \sqrt{\gamma}\rho_1)^2 P_1 + (\sqrt{h_2} + \sqrt{\gamma}\rho_2)^2 P_2}{\gamma\sigma_{N_J}^2 + \sigma_N^2}\right) \tag{15}$$

This is a monotone function of SNR, therefore the jammer will minimize the SNR value. The jammer is better off using all its available power. Therefore

$$P_J = \rho_1^2 P_1 + \rho_2^2 P_2 + \sigma_{N_J}^2 \tag{16}$$

We have the following minimization

$$\min_{\{\rho_1,\rho_2\}} \quad \frac{(\sqrt{h_1} + \sqrt{\gamma}\rho_1)^2 P_1 + (\sqrt{h_2} + \sqrt{\gamma}\rho_2)^2 P_2}{\gamma\sigma_{N_J}^2 + \sigma_N^2}$$

$$\text{s.t.} \quad \rho_1^2 P_1 + \rho_2^2 P_2 + \sigma_{N_J}^2 = P_J \tag{17}$$

The Karush-Kuhn-Tucker(KKT) necessary conditions are

$$\frac{2\sqrt{\gamma}(\sqrt{h_1} + \sqrt{\gamma}\rho_1)P_1}{\gamma\sigma_{N_J}^2 + \sigma_N^2} + 2\lambda\rho_1 P_1 = 0 \tag{18}$$

$$\frac{2\sqrt{\gamma}(\sqrt{h_2} + \sqrt{\gamma}\rho_2)P_2}{\gamma\sigma_{N_J}^2 + \sigma_N^2} + 2\lambda\rho_2 P_2 = 0 \tag{19}$$

$$-\gamma\frac{(\sqrt{h_1} + \sqrt{\gamma}\rho_1)^2 P_1 + (\sqrt{h_2} + \sqrt{\gamma}\rho_2)^2 P_2}{(\gamma\sigma_{N_J}^2 + \sigma_N^2)^2} + \lambda - \delta = 0 \tag{20}$$

where δ is the complementary slackness variable for $\sigma_{N_J}^2$. The above equations have the following solution

$$\rho_1 = -\sqrt{h_1}\frac{\gamma P_J + \sigma_N^2}{\sqrt{\gamma}(h_1 P_1 + h_2 P_2)} \tag{21}$$

$$\rho_2 = -\sqrt{h_2}\frac{\gamma P_J + \sigma_N^2}{\sqrt{\gamma}(h_1 P_1 + h_2 P_2)} \tag{22}$$

Therefore whenever these values of ρ_1 and ρ_2 are feasible, they characterize the best jammer strategy and the jammer transmits as in (7). We observe that the amount of power the jammer allocates for jamming each user, is proportional to that user's effective received power which is $h_1 P_1$ for the first user and $h_2 P_2$ for the second user. Ultimately, including the limiting feasible values, the optimum jamming coefficients are

$$(\rho_1, \rho_2) = \begin{cases} \left(-\frac{\sqrt{h_1}}{\sqrt{\gamma}}, -\frac{\sqrt{h_2}}{\sqrt{\gamma}}\right) & \text{if} \quad \gamma P_J \geq h_1 P_1 + h_2 P_2 \\ (-\rho\sqrt{h_1}, -\rho\sqrt{h_2}) & \text{if} \quad \gamma P_J < h_1 P_1 + h_2 P_2 \end{cases} \tag{23}$$

where

$$\rho = \min\left\{\sqrt{\frac{P_J}{h_1 P_1 + h_2 P_2}}, \frac{\gamma P_J + \sigma_N^2}{\sqrt{\gamma}(h_1 P_1 + h_2 P_2)}\right\} \tag{24}$$

Figure 1 shows the jammer decision regions when $\gamma = 1$, $P_1 = 10$, $P_2 = 5$, $P_J = 5$ and $\sigma_N^2 = 1$. In region A, $(\rho_1, \rho_2) = (-\frac{\sqrt{h_1}}{\sqrt{\gamma}}, -\frac{\sqrt{h_2}}{\sqrt{\gamma}})$ and the jammer only uses enough power to zero out the transmitted signals. In region B, $(\rho_1, \rho_2) = (-\sqrt{h_1}, -\sqrt{h_2})\sqrt{\frac{P_J}{h_1 P_1 + h_2 P_2}}$ and the jammer uses all its power to cancel the transmitted signals as much as possible. In region C, $(\rho_1, \rho_2) = (-\sqrt{h_1}, -\sqrt{h_2})\frac{\gamma P_J + \sigma_N^2}{\sqrt{\gamma}(h_1 P_1 + h_2 P_2)}$ and the jammer uses part of its power to cancel the transmitted signals, and the rest of its power to add white Gaussian

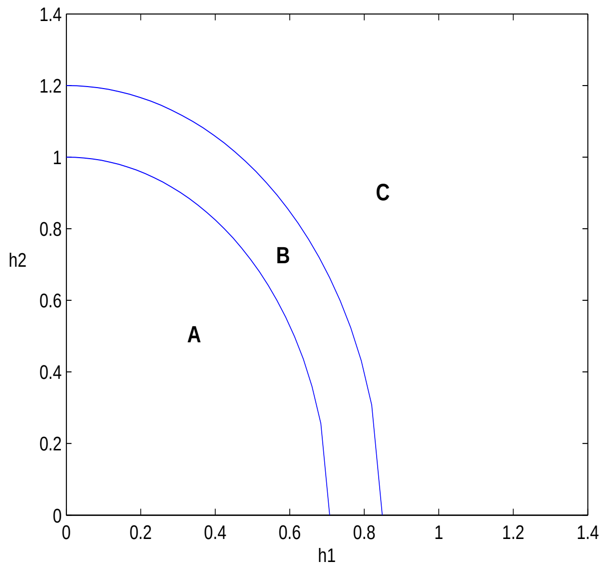

Figure 1: Jamming decision regions when $\gamma = 1$, $P_1 = 10$, $P_2 = 5$, $P_J = 5$ and $\sigma_N^2 = 1$.

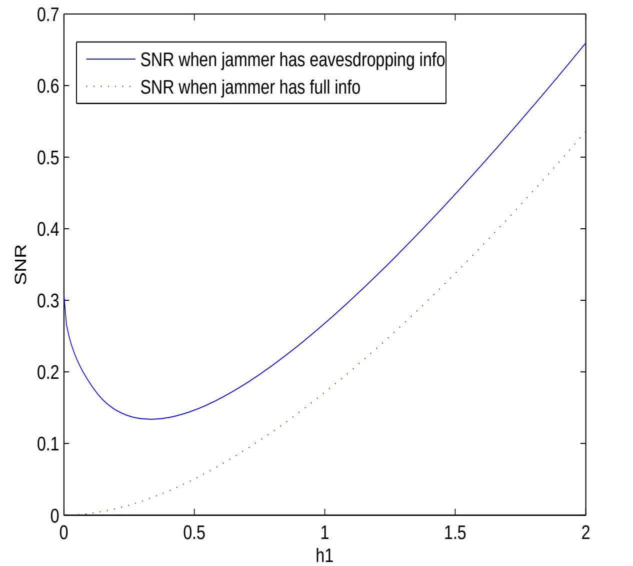

Figure 2: SNR as a function $h1$ when $h_2 = g_1 = g_2 = \gamma = 1$ and the powers are $P_1 = P_2 = P_J = \sigma_N^2 = 1$.

noise to the transmitted signal. Therefore, for low channel coefficients where the effective received powers of the users are small, the optimum jamming strategy is to subtract the user signals as much as possible, while in high channel coefficients, it is not optimal for the jammer to correlate with the user signals with all its power.

III.B Jamming with Eavesdropping Information

Now suppose that the jammer gains information only through an AWGN eavesdropping channel

$$Y_e = \sqrt{g_1}X_1 + \sqrt{g_2}X_2 + N_e \tag{25}$$

Similar to the previous section, it is straightforward that if the jammer is linear, the best signalling is Gaussian. However, when the users employ Gaussian signalling, due to the dependence of X_1 and X_2 through (25), linear jamming may not achieve all $|\mathbf{\Lambda}|$ values that are allowed under the power constraint, and therefore, we cannot show that Gaussian signalling and linear jamming is a saddle point for this mutual information game.

We now derive the jamming coefficient for a suboptimal linear jammer with eavesdropping information. For this jammer, the user signals will be Gaussian. Ignoring the attenuation of the jamming channel, i.e., $\gamma = 1$, the received signal and the jamming signal are

$$J = \rho(\sqrt{g_1}X_1 + \sqrt{g_2}X_2 + N_e) + N_J \tag{26}$$

$$Y = (\sqrt{h_1} + \rho\sqrt{g_1})X_1 + (\sqrt{h_2} + \rho\sqrt{g_2})X_2 + \rho N_e + N_J + N \tag{27}$$

and the jammer's power constraint is

$$\rho^2(g_1P_1 + g_2P_2 + \sigma_{N_e}^2) + \sigma_{N_J}^2 \leq P_J \tag{28}$$

The optimization problem is

$$\min_{\{\rho,\sigma_{N_J}^2\}} \quad \frac{(\sqrt{h_1} + \rho\sqrt{g_1})^2P_1 + (\sqrt{h_2} + \rho\sqrt{g_2})^2P_2}{\rho^2\sigma_{N_e}^2 + \sigma_{N_J}^2 + \sigma_N^2}$$
$$\text{s.t.} \quad \rho^2(g_1P_1 + g_2P_2 + \sigma_{N_e}^2) + \sigma_{N_J}^2 = P_J \tag{29}$$

The KKTs for this problem result in a third degree equation of ρ and can be solved using numerical optimization. Figure 2 shows SNR as a function of one of the channel coefficients h_1, when the other channel coefficients are $h_2 = g_1 = g_2 = \gamma = 1$ and the powers are $P_1 = P_2 = P_J = \sigma_N^2 = 1$. SNR is compared in two scenarios, when the jammer eavesdrops, and when it has full information about the user signals.

IV. Jamming in Fading AWGN Channel

We now investigate the optimum user/jammer strategies when the channels are fading. This section is divided into three parts corresponding to different assumptions: A) no channel state information (CSI) at the transmitter, B) uncorrelated jamming with full CSI at the transmitter, and C) correlated jamming with full CSI at the transmitter. In each part, various assumptions are made on the availability of CSI at the receiver and the jammer.

IV.A No CSI at the Transmitter

When the transmitters do not have the CSI, it is reasonable to assume that the jammer does not have the CSI either. In this case, the jammer's information about the transmitted signals will be irrelevant and therefore, it will not make any difference whether it has perfect or noisy information about the transmitted signals. To see this, assume first that the user channels are fading and the jammer channel is non-fading with $\gamma = 1$. The receiver is assumed to know the fading coefficients but the users and the jammer only know the fading statistics. The received signal is

$$Y = H_1X_1 + H_2X_2 + J + N \tag{30}$$

Here, we assume that all the random variables are complex valued and H_1 and H_2 are circularly symmetric complex Gaussian. Following [2] in finding the saddle point of the mutual information game by conditioning on the fading coefficients,

$$
\begin{aligned}
&I(Y, H_1, H_2; X_1, X_2) \\
&\quad = h(X_1, X_2) - h(X_1, X_2|Y, H_1, H_2) &(31)\\
&\quad = h(X_1, X_2) - h(X_1 - A_1Y, X_2 - A_2Y|Y, H_1, H_2) &(32)
\end{aligned}
$$

where A_1 and A_2 are functions of H_1 and H_2. The jammer's strategy can only affect the second term above

$$
\begin{aligned}
&h(X_1 - A_1Y, X_2 - A_2Y|Y, H_1, H_2) \\
&\quad \leq h(X_1 - A_1Y, X_2 - A_2Y|H_1, H_2) &(33)\\
&\quad \leq \frac{1}{2}\log\left((2\pi e)^2|\mathbf{\Lambda}|\right) &(34)
\end{aligned}
$$

where $\mathbf{\Lambda}$ is the covariance matrix of

$$(X_1 - A_1Y, X_2 - A_2Y|H_1, H_2)$$

A_1 and A_2 can be chosen to make $X_1 - A_1Y$ and $X_2 - A_2Y$ uncorrelated given H_1 and H_2. For the last inequality to hold, Y or equivalently J needs to be jointly Gaussian with X_1 and X_2 given H_1 and H_2, however J, X_1 and X_2 are independent of H_1 and H_2 and therefore, the rest will be identical to Section III.A. The saddle point strategies will be Gaussian signalling and linear jamming. The jamming signal and the received signal will be

$$J = \rho_1X_1 + \rho_2X_2 + N_J \quad (35)$$

$$Y = (H_1 + \rho_1)X_1 + (H_2 + \rho_2)X_2 + N_J + N \quad (36)$$

The last step is to find the best ρ_1 and ρ_2. We need to redefine the objective function of the jammer since the mutual information is a function of the fading coefficients which the jammer is not aware of. Given that the jammer knows the statistics of the fading, its objective is to minimize the mutual information, averaged over the fading random variables,

$$
\begin{aligned}
\min_{\{\rho_1,\rho_2\}} \quad & \frac{1}{2}E\left[\log\left(1 + \frac{|H_1 + \rho_1|^2P_1 + |H_2 + \rho_2|^2P_2}{\sigma_{N_J}^2 + \sigma_N^2}\right)\right] \\
\text{s.t.} \quad & \rho_1^2P_1 + \rho_2^2P_2 + \sigma_{N_J}^2 = P_J \qquad (37)
\end{aligned}
$$

Distributions of H_1 and H_2 are centered around zero, therefore intuitively, shifting them will make their norm larger and hence, we should set $\rho_1 = \rho_2 = 0$. This fact can also be easily derived using [3, Theroem 1]. Therefore, the jammer disregards its complete information, and we conclude that if the jammer's information is noisy, it cannot do any better than what it did when it had noiseless information, and therefore, it should disregard the incomplete information, whether the incompleteness is because of fading or AWGN or both in the jammer's eavesdropping channel.

IV.B Uncorrelated Jamming with CSI at the Transmitter

Capacity of fading channels with CSI both at the transmitter and the receiver has been investigated in [4] and [5], and optimum signalling and power allocation strategies have been derived. We first consider the single-user case and ignore the jamming channel's fading

$$Y = HX + J + N \quad (38)$$

If the CSI is available both at the transmitter and the receiver, the input/output mutual information will be

$$
\begin{aligned}
I(X, H; Y, H) &= h(Y, H) - h(Y, H|X, H) \\
&= h(H) + h(Y|H) - h(Y|X, H) \\
&= h(H) + I(X; Y|H) &(39)
\end{aligned}
$$

For a given fading model, $h(H)$ is constant, therefore we can check the existence of a saddle point for $I(X; Y|H)$ instead of $I(X, H; Y, H)$. Conditioned on H, $I(X; Y|H)$ is a convex function of $f(y|x, h)$ for any fixed conditional input distribution $f(x|h)$, and a concave function of $f(x|h)$ for any fixed conditional transition distribution $f(y|x, h)$. We can say that at each channel state, there is a saddle point which is to employ Gaussian signalling and linear jamming. This specifies the solution to the mutual information sub-game under any given channel state. Moreover, if a saddle point exists over all possible power allocation strategies of the user and the jammer, under user and jammer power constraints, that saddle point power allocation along with the signalling and jamming strategies specified as the solution of the sub-games corresponding to each channel state, will give the overall solution. We proceed with first assuming that there is no CSI at the jammer and then assuming that CSI is available at the jammer.

If the jammer has no information about the fading channel state, just like the non-fading channel with channel coefficients unknown to the jammer studied before, the best strategy for the jammer is to ignore its information about the user's signal and only transmit a white Gaussian noise. The received signal at fading level h is

$$Y = \sqrt{h}X + N_J + N \quad (40)$$

and the ergodic capacity is

$$C = \frac{1}{2}E\left[\log\left(1 + \frac{hP(h)}{\sigma_N^2 + P_J}\right)\right] \quad (41)$$

where $P(h)$ is the user power at fading level h which should satisfy

$$E\left[P(h)\right] = P \quad (42)$$

The best user power allocation is waterfilling over the equivalent parallel AWGN channels [4] with equivalent noise levels $\frac{\sigma_N^2 + P_J}{h}$, i.e.,

$$P(h) = \left(\frac{1}{\lambda} - \frac{\sigma_N^2 + P_J}{h}\right)^+ \quad (43)$$

The corresponding two user system, where the jammer is not aware of the user channel coefficients, is a straightforward extension of the results in [9] where only one user transmits at a time. The jammer will again use all its power to add Gaussian noise.

Next we assume that the uncorrelated jammer has full CSI. The received signal in the single user system is the same as (40). At each channel state, the jammer transmits white Gaussian noise at the power level allocated to that state. The ergodic capacity is

$$C = \frac{1}{2}E\left[\log\left(1 + \frac{hP(h)}{\sigma_N^2 + J(h)}\right)\right] \quad (44)$$

where $J(h)$ is the jammer power at fading level h. The power allocations should satisfy

$$E\left[P(h)\right] = P \quad (45)$$

$$E\left[J(h)\right] = P_J \quad (46)$$

There will exist an optimal solution to this power allocation game if this ergodic capacity has a saddle point in a power allocation pair. We next show the convexity/concavity properties of the ergodic capacity.

Theorem 2 *Let $P(h)$ and $J(h)$ be power allocation functions satisfying corresponding power constraints. The set of such function pairs is a convex set and the ergodic capacity is a concave function of P for fixed J and a convex function of J for fixed P.*

Proof: If two functions P_1 and P_2 both satisfy a power constraint, any convex combination of them will also satisfy the same power constraint and therefore the set of such functions is convex.

Since every term of the ergodic capacity corresponding to a channel state h is concave in $P(h)$ for fixed $J(h)$ and convex in $J(h)$ for fixed $P(h)$, the ergodic capacity is a concave function of P for fixed J and a convex function of J for fixed P. □

Given the convexity/concavity properties of the ergodic capacity and using [7, Theorem 16, p. 75], [8, Proposition 2.6.9], the set of saddle points is compact and nonempty and therefore the mutual information game has a solution. At the game solution, the pair of strategies should satisfy the KKT's of the two optimization problems corresponding to the user and the jammer. The user maximizes (44) subject to (45), while the jammer minimizes (44) subject to (46). Writing the KKTs for each state-allocated user power, we get

$$-\frac{h}{\sigma_N^2 + J(h) + hP(h)} + \lambda - \gamma(h) = 0 \qquad (47)$$

where $\gamma(h)$ is the complementary slackness variable for $P(h)$. Similarly, writing the KKTs for each state-allocated jammer power, we get

$$-\frac{hP(h)}{(\sigma_N^2 + J(h))(\sigma_N^2 + J(h) + hP(h))} + \mu - \delta(h) = 0 \qquad (48)$$

where $\delta(h)$ is the complementary slackness variable for $J(h)$. The optimum strategies should solve (47) and (48) simultaneously. There are four possible cases at each fading level. Case 1: $P(h) > 0$ and $J(h) > 0$, case 2: $P(h) = 0$ and $J(h) > 0$, case 3: $P(h) > 0$ and $J(h) = 0$ and case 4: $P(h) = 0$ and $J(h) = 0$. If $P(h) = 0$, (48) cannot be satisfied unless $\delta(h) > 0$, therefore, case 2 never happens. In case 1, both complementary slackness variables are zero, and (47) and (48) become

$$\lambda = \frac{h}{\sigma_N^2 + J(h) + hP(h)} \qquad (49)$$

$$\mu = \frac{hP(h)}{(\sigma_N^2 + J(h))(\sigma_N^2 + J(h) + hP(h))} \qquad (50)$$

which result in a linear relation between the user and jammer allocated powers

$$\frac{\lambda}{\mu} = \frac{\sigma_N^2 + J(h)}{P(h)} \qquad (51)$$

and solving for the user's power

$$P(h) = \frac{h}{\lambda(h + \frac{\lambda}{\mu})} \qquad (52)$$

which is a monotonically increasing function of the fading variable h. Therefore, at any fading level where the user and jammer powers are nonzero, the jammer's power is a linear function of the user's, and they both allocate more power to better channel states. Case 1 is valid as long as (51) and (52) result in positive $J(h)$, which is for $P(h) > \sigma_N^2\mu/\lambda$ or $h > \sigma_N^2\lambda/(1-\sigma_N^2\mu)$. For $h < \sigma_N^2\lambda/(1-\sigma_N^2\mu)$, $J(h) = 0$ which results in cases 3 and 4 combined. In this case (47) will turn to waterfilling. Therefore the power allocations are

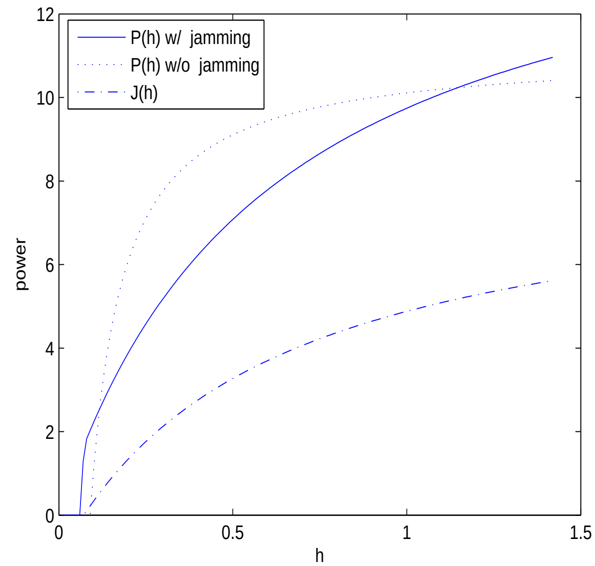

Figure 3: $P(h)$ and $J(h)$ for $E[P(h)] = 10$, $E[J(h)] = 5$ and $\sigma_N^2 = 1$.

$$P(h) = \begin{cases} (\frac{1}{\lambda} - \frac{\sigma_N^2}{h})^+ & \text{if} \quad h < \frac{\sigma_N^2\lambda}{1-\sigma_N^2\mu} \\ \frac{h}{\lambda(h+\frac{\lambda}{\mu})} & \text{if} \quad h \geq \frac{\sigma_N^2\lambda}{1-\sigma_N^2\mu} \end{cases} \qquad (53)$$

$$J(h) = \begin{cases} 0 & \text{if} \quad h < \frac{\sigma_N^2\lambda}{1-\sigma_N^2\mu} \\ \frac{h}{\mu(h+\frac{\lambda}{\mu})} - \sigma_N^2 & \text{if} \quad h \geq \frac{\sigma_N^2\lambda}{1-\sigma_N^2\mu} \end{cases} \qquad (54)$$

Figure 3 shows $P(h)$ and $J(h)$ when $E[P(h)] = 10$, $E[J(h)] = 5$ and $\sigma_N^2 = 1$. Fading is assumed to be Rayleigh with parameter 1. Figure 4 is the same plot when $E[P(h)] = 10$, $E[J(h)] = 1$ and $\sigma_N^2 = 1$. We conclude that both the user and the jammer keep quiet at very low fading levels. Then, as the channel gets better, the user starts transmitting, with more power allocated to better channels, and eventually at even better channels, the jammer starts jamming, again with more power allocated to better channels.

We now discuss the two user system where the jammer is uncorrelated but it has access to CSI. This is a reasonable situation since we can assume that the jammer eavesdrops the downlink communication about the channel state to the users. Following [9] in finding the conditions that the optimum strategies should satisfy, it is straightforward to see that the strategies follow a pattern similar to Figure 5, that is, the users do not transmit simultaneously, no party transmits at very low fading levels, as the channels get better, the user with a relatively better channel transmits, and eventually the jammer starts transmitting at even better channels. The threshold

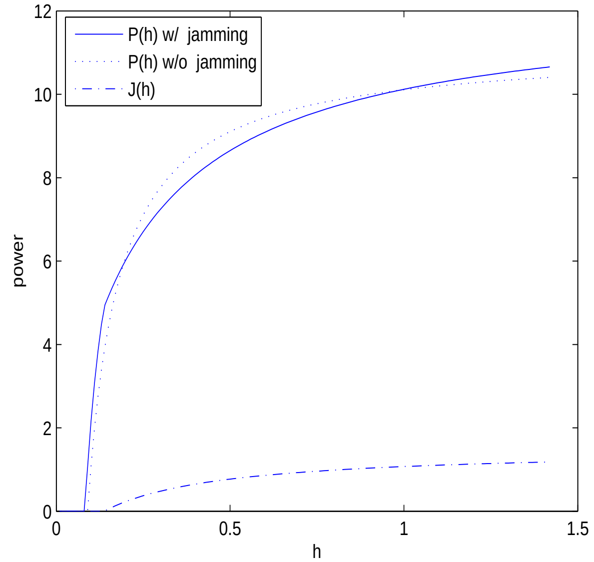

Figure 4: $P(h)$ and $J(h)$ for $E[P(h)] = 10$, $E[J(h)] = 1$ and $\sigma_N^2 = 1$.

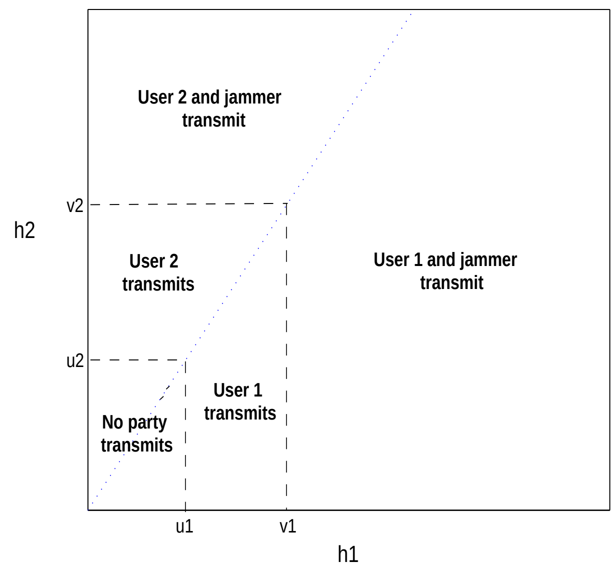

Figure 5: User/Jammer transmission regions in uncorrelated jamming with CSI.

values u1, u2, v1 and v2 are to be chosen to optimize the strategies.

IV.C Correlated Jamming with CSI at the Transmitter

As in the previous sections we start with a single user system. When the jammer knows the channel state and the transmitted signal, the received signal is

$$Y = \left(\sqrt{h} + \rho\left(h, P(h), J(h)\right)\right) X + N_J + N \quad (55)$$

Given a pair of $P(h)$ and $J(h)$ power allocations, the ergodic capacity is

$$C = \frac{1}{2} E\left[\log\left(1 + \frac{\left(\sqrt{h} + \rho\left(h, P(h), J(h)\right)\right)^2 P(h)}{\sigma_N^2 + \sigma_{N_J}^2\left(h, P(h), J(h)\right)}\right)\right] \quad (56)$$

where the jamming coefficient, $\rho\left(h, P(h), J(h)\right)$ and the jammer's additive white noise, $\sigma_{N_J}^2\left(h, P(h), J(h)\right)$, together describe the optimum jamming strategy at each fading level h, as a function of the fading level and the user and jammer state allocated powers $P(h)$ and $J(h)$ as follows [1]

$$\rho\left(h, P(h), J(h)\right) = \begin{cases} -\sqrt{h} & \text{if} \quad J(h) \geq hP(h) \\ -U\sqrt{h} & \text{if} \quad J(h) < hP(h) \end{cases} \quad (57)$$

where

$$U = \min\left\{\sqrt{\frac{J(h)}{hP(h)}}, \frac{J(h) + \sigma_N^2}{hP(h)}\right\} \quad (58)$$

and

$$\sigma_{N_J}^2\left(h, P(h), J(h)\right) = J(h) - \left(\rho\left(h, P(h), J(h)\right)\right) P(h) \quad (59)$$

and $\sigma_{N_J}^2$ is always nonnegative. The power constraints of the user and the jammer are as in (45) and (46). Therefore, the problem is reduced to finding the optimum user and jammer power allocation strategies $P(h)$ and $J(h)$. As opposed to the previous scenarios, since the ergodic capacity here does not have the covexity/concavity properties, the existence of the saddle point, and finding the optimum user and jammer power allocation strategies are open problems, both in this single user system and in a multiuser setting.

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