Probability One Convergence in Joint Stochastic Power Control and Blind MMSE Interference Suppression

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Abstract —

This paper studies the convergence issue in stochastic implementation of standard power control algorithms in wireless communications. It is shown that, under a set of general assumptions, the stochastic power control algorithm converges to the componentwise smallest power vector with probability one. The results are further extended to the joint stochastic power control and receiver optimization problem. Simulation results are given to illustrate the performance of the proposed algorithms in practical systems.

I. Introduction

Since power is an important and limited resource in wireless communication systems, power control algorithms that minimize the transmission power while ensuring the quality of service (QoS) have been widely studied in the literature. Early work on power control viewed the problem as a constrained optimization and found the optimal transmit power for each user by solving the optimization problem directly [1]. Such algorithms are identified as "centralized" since they assume that the optimization problem is solved at a centralized server using the knowledge of some global parameters such as the channel gains of all users to all base stations. When the size of the system increases, computational complexity and acquiring the knowledge about the global parameters become a serious issue. Due to these reasons, many distributed power control algorithms have been developed [2][3]. A framework of *standard* power control is developed in [4] where it was shown that, if the interference function is *standard* then the distributed power control algorithm converges to the componentwise smallest power vector.

The algorithms studied in [4] are *deterministic* in the sense that they require the perfect knowledge of the received interference power. However, in a practical system, the interference power can only be estimated using noisy observations. Stochastic power control that uses noisy interference estimates is first considered in [5]. For conventional matched filter receivers, [5] showed that the stochastic power control converges to the optimal power vector in the Mean Square Error (MSE) sense as long as the step size meets certain requirements. The results are further extended to the case of decision feedback receivers in [6]. The common feature of [5][6] is that, for both cases, the interference is a linear function of the transmit powers.

In this paper, we consider a general power control problem where the deterministic interference function is *standard* and it satisfies the Lipschitz condition defined later. Starting with the deterministic interference function, we define a closely related stochastic version. With an additional set of assumptions on the stochastic interference function, we show that the *standard stochastic power control algorithm* converges to the solution of the deterministic power control problem with probability one, assuming that the step size is decreasing and is decreasing faster than a certain rate.

Due to the NP-hard nature of the optimal CDMA multiuser detection [7], suboptimal detectors that provide reliable decisions and that ensure polynomial complexity have been developed. Among the suboptimal detectors, linear detectors, including the decorrelator and the minimum mean square error (MMSE) detector, posses attractive features [7]. Joint power control and receiver optimization problem is considered in [9]. It is shown that, if the receiver is optimized to be the MMSE receiver at each step of the power control iteration, the resulting interference function is standard, and the power control is convergent. However, in [9], perfect interference estimates are assumed, and the convergence results are shown only for the deterministic power control. Since the MMSE receiver can be implemented in a "blind" manner that requires only single user information [8], joint stochastic power control and blind MMSE interference suppression is of special interest.

In this paper, we formulate and study the stochastic implementation of general standard deterministic power control problems, the special cases of which where the interference function is linear in powers have been studied in [5][6]. After developing the general theory, we then focus on an example of standard power control where interference function is nonlinear in powers: the joint power control and receiver design problem. We show that, if we combine the power control and the blind MMSE multiuser detection in a certain way, the algorithm converges to the optimal power vector and the corresponding MMSE filters with probability one. In addition, a further extension allows the power control and receiver optimization to be performed in parallel, i.e., the power control algorithm does not need to wait for the convergence of the blind MMSE algorithm between consecutive power updates.

II. Standard Power Control

Suppose $\boldsymbol{p}$ is the power vector whose i^{th} component, p_i, is the transmit power of user i. The signal to interference (SIR) requirements of all users can be expressed as

$$\boldsymbol{p} \geq \boldsymbol{I}(\boldsymbol{p}) \tag{1}$$

[1]This work was supported by National Aeronautics and Space Administration award No. NCC8-235, Collaborative Technology Alliance for Communication & Networks sponsored by the U.S. Army Laboratory under Cooperative Agreement DAAD19-01-2-0011, and National Science Foundation grant ANI 02-05330.

where $\boldsymbol{I}(\boldsymbol{p}) = [I_1(\boldsymbol{p}), \ldots, I_K(\boldsymbol{p})]^T$ is the interference function and K is the number of users. Suppose $\boldsymbol{I}(\boldsymbol{p})$ is *standard* as defined by the following [4].

Definition 1: Interference function $\boldsymbol{I}(\boldsymbol{p})$ is standard if for all $\boldsymbol{p} \geq \boldsymbol{0}$ the following properties are satisfied.

- Positivity. $\boldsymbol{I}(\boldsymbol{p}) > 0$.
- Monotonicity. If $\boldsymbol{p} \geq \boldsymbol{p}'$, then $\boldsymbol{I}(\boldsymbol{p}) \geq \boldsymbol{I}(\boldsymbol{p}')$.
- Scalability. For all $\beta > 1$, $\beta \boldsymbol{I}(\boldsymbol{p}) > \boldsymbol{I}(\beta \boldsymbol{p})$.

It is shown in [4] that, the deterministic power control algorithm

$$\boldsymbol{p}(n+1) = \boldsymbol{I}(\boldsymbol{p}(n)) \tag{2}$$

converges to the componentwise smallest power vector $\boldsymbol{p}^*$ which satisfies $\boldsymbol{p} \geq \boldsymbol{I}(\boldsymbol{p})$, and the inequality is indeed satisfied with equality at $\boldsymbol{p}^*$. Furthermore, since $\forall 0 < \alpha \leq 1$, $(1-\alpha)\boldsymbol{p} + \alpha \boldsymbol{I}(\boldsymbol{p})$ is also standard, the power control algorithm

$$\boldsymbol{p}(n+1) = (1-\alpha)\boldsymbol{p}(n) + \alpha \boldsymbol{I}(\boldsymbol{p}(n)) \tag{3}$$

also converges to $\boldsymbol{p}^*$.

The above algorithm is deterministic in the sense that it requires the perfect knowledge of $\boldsymbol{I}(\boldsymbol{p})$. In practical systems, we cannot know $\boldsymbol{I}(\boldsymbol{p})$ perfectly, but we may have a random estimate of $\boldsymbol{I}(\boldsymbol{p})$, denoted as $\tilde{\boldsymbol{I}}(\boldsymbol{p}, \boldsymbol{v})$. Consider now the following stochastic power control algorithm

$$\boldsymbol{p}(n+1) = (1-\alpha(n))\boldsymbol{p}(n) + \alpha(n)\tilde{\boldsymbol{I}}(\boldsymbol{p}(n), \boldsymbol{v}(n)) \tag{4}$$

where $\tilde{\boldsymbol{I}}(\boldsymbol{p}(n), \boldsymbol{v}(n))$ is the noisy estimate of $\boldsymbol{I}(\boldsymbol{p}(n))$ and $\alpha(n)$ is the step size at the n^{th} iteration. We define a *standard stochastic interference* function as:

Definition 2: Stochastic interference function $\tilde{\boldsymbol{I}}(\boldsymbol{p}, \boldsymbol{v})$ is standard if, given $\boldsymbol{p}$ and $\boldsymbol{p} \geq \boldsymbol{0}$, the following properties are satisfied.

- Mean condition. $\tilde{\boldsymbol{I}}(\boldsymbol{p}, \boldsymbol{v}) = \boldsymbol{I}(\boldsymbol{p})$ and $\boldsymbol{I}(\boldsymbol{p})$ is a standard deterministic interference function.
- Lipschitz condition. There exits a constant $K_1 > 0$, such that,
$$\|\boldsymbol{I}(\boldsymbol{p}_1) - \boldsymbol{I}(\boldsymbol{p}_2)\|^2 \leq K_1 \|\boldsymbol{p}_1 - \boldsymbol{p}_2\|^2 \tag{5}$$
- Growing condition. There exits a constant $K_2 > 0$, such that,
$$E[\|\tilde{\boldsymbol{I}}(\boldsymbol{p}, \boldsymbol{v}) - \boldsymbol{I}(\boldsymbol{p})\|^2] \leq K_2(1 + \|\boldsymbol{p}\|^2) \tag{6}$$

Similar to the deterministic power control, when the stochastic interference function is standard as defined in Definition 2, we call (4) the *standard stochastic power control algorithm*. It is easy to verify that, the stochastic power control algorithms that have already been studied in [5] and [6] are standard.

III. Convergence of Standard Stochastic Power Control Algorithms

In this section, we show that the standard stochastic power control algorithm converges to the optimal power vector with probability one, under certain conditions on the step size sequence.

A. Convergence on the mean Ordinary Differential Equation (ODE)

Let us first consider the deterministic power control algorithm in (3). Define a function $V(\boldsymbol{p})$ by

$$\begin{aligned} \nabla_{\boldsymbol{p}} V(\boldsymbol{p}) &= \boldsymbol{p} - \boldsymbol{I}(\boldsymbol{p}) \\ V(\boldsymbol{p}^*) &= 0 \end{aligned} \tag{7}$$

Since $\boldsymbol{I}(\boldsymbol{p})$ is Lipschitz continuous, from (3), we obtain

$$V(\boldsymbol{p}(n+1)) = V(\boldsymbol{p}(n)) - \alpha \|\boldsymbol{p} - \boldsymbol{I}(\boldsymbol{p})\|^2 + O(\alpha^2) \tag{8}$$

Hence, if $\boldsymbol{p}(n) \neq \boldsymbol{p}^*$ and α is small enough, we have $V(\boldsymbol{p}(n+1)) < V(\boldsymbol{p}(n))$. Noting that the power control algorithm (3) converges to $\boldsymbol{p}^*$ from any initial point, $\forall 0 < \alpha \leq 1$, if $\boldsymbol{p}(0) \neq \boldsymbol{p}^*$, we can always find a sequence $\boldsymbol{p}(0), \ldots, \boldsymbol{p}(n), \ldots$ such that $V(\boldsymbol{p}(0)) > \ldots > V(\boldsymbol{p}(n))$ and $\boldsymbol{p}(n) \to \boldsymbol{p}^*$ as $n \to \infty$. Therefore, we have

$$V(\boldsymbol{p}) > 0 \quad \forall \boldsymbol{p} \neq \boldsymbol{p}^* \tag{9}$$

This shows that $V(\boldsymbol{p})$ is a Lyapunov function.

Define the Ordinary Differential Equation (ODE)

$$\frac{d\boldsymbol{p}}{dt} = -[\boldsymbol{p}(t) - \boldsymbol{I}(\boldsymbol{p}(t))] \tag{10}$$

Since (10) minimizes $V(\boldsymbol{p})$, from the above analysis, we can see that $\boldsymbol{p}(t) \to \boldsymbol{p}^*$ as $t \to \infty$.

B. Bounding the power vector

Suppose the power control problem is feasible. Given an arbitrary constant $K_a > 1$, since $\boldsymbol{I}(K_a \boldsymbol{p}^*) < K_a \boldsymbol{p}^*$ with strict inequality, we can always find $0 < \epsilon < 1$, such that

$$\boldsymbol{I}(K_a \boldsymbol{p}^*) \leq K_a(1-\epsilon)\boldsymbol{p}^* \tag{11}$$

Furthermore, $\forall \beta \geq 1$,

$$\boldsymbol{I}(\beta K_a \boldsymbol{p}^*) \leq \beta \boldsymbol{I}(K_a \boldsymbol{p}^*) \leq \beta K_a (1-\epsilon)\boldsymbol{p}^* \tag{12}$$

Now, define

$$\xi_i = \frac{p_i}{p_i^*}, \qquad k = \arg\max_i \xi_i \tag{13}$$

On one hand, if $\xi_k \geq K_a$,

$$\|\boldsymbol{p} - \boldsymbol{I}(\boldsymbol{p})\|^2 \geq (p_k - I_k(\boldsymbol{p}))^2 \geq \epsilon^2 \xi_k^2 {p_k^*}^2 \tag{14}$$

which gives

$$\begin{aligned} \|\boldsymbol{p}\|^2 &\leq \xi_k^2 \|\boldsymbol{p}^*\|^2 \\ &= \frac{\|\boldsymbol{p}^*\|^2}{\epsilon^2 \min_j \{p_j^*\}^2} \xi_k^2 \epsilon^2 \min_j \{p_j^*\}^2 \\ &\leq \frac{\|\boldsymbol{p}^*\|^2}{\epsilon^2 \min_j \{p_j^*\}^2} \|\boldsymbol{p} - \boldsymbol{I}(\boldsymbol{p})\|^2 \end{aligned} \tag{15}$$

On the other hand, if $\xi_k < K_a$, we have

$$\|\boldsymbol{p}\|^2 < K_a^2 \|\boldsymbol{p}^*\|^2 \tag{16}$$

Therefore, combing (15) and (16) gives the following bound.

Bound 1: There exists a constant $K_3 > 0$, such that

$$1 + \|\boldsymbol{p}\|^2 \leq K_3(1 + \|\boldsymbol{p} - \boldsymbol{I}(\boldsymbol{p})\|^2) \tag{17}$$

C. Probability One Convergence

Proposition 1: Suppose the step size satisfies

$$\sum_{n=0}^{\infty} \alpha(n) = \infty, \qquad \sum_{n=0}^{\infty} \alpha(n)^2 < \infty \tag{18}$$

Then the power vector of the standard stochastic power control algorithm (4) converges to $\boldsymbol{p}^*$ with probability one.

Proof: To simplify the notation, we write $\boldsymbol{I}(\boldsymbol{p}(n))$ and $\tilde{\boldsymbol{I}}(\boldsymbol{p}(n))$ as $\boldsymbol{I}(n)$ and $\tilde{\boldsymbol{I}}(n)$, respectively. Since $\boldsymbol{I}(\boldsymbol{p})$ is Lipschitz continuous, from (4), we can find a constant K_4, such that the truncated Taylor expansion on $V(n+1)$ satisfies

$$\begin{aligned} V(n+1) \leq V(n) - \alpha(n)(\boldsymbol{p}(n) - \boldsymbol{I}(n))^T(\boldsymbol{p}(n) - \tilde{\boldsymbol{I}}(n)) \\ + \frac{\alpha(n)^2 K_4}{2} \|\boldsymbol{p}(n) - \tilde{\boldsymbol{I}}(n)\|^2 \end{aligned} \tag{19}$$

Now, defining $E_n[.]$ as the conditional expectation given $\boldsymbol{p}(n)$, we have

$$\begin{aligned} E_n[V(n+1)] &\leq V(n) - \alpha(n)\|\boldsymbol{p}(n) - \boldsymbol{I}(n)\|^2 \\ &+ \frac{\alpha(n)^2 K_4}{2} E_n[\|\boldsymbol{p}(n) - \tilde{\boldsymbol{I}}(n)\|^2] \\ &\leq V(n) - \alpha(n)(1 - \frac{\alpha(n)K_4}{2})\|\boldsymbol{p}(n) - \boldsymbol{I}(n)\|^2 \\ &+ \frac{\alpha(n)^2 K_2 K_4}{2}(1 + \|\boldsymbol{p}(n)\|^2) \end{aligned} \tag{20}$$

Using (17), we obtain

$$\begin{aligned} & E_n[V(n+1)] \leq V(n) \\ & -\alpha(n)(1 - \frac{\alpha(n)K_4(1+K_2K_3)}{2})\|\boldsymbol{p}(n) - \boldsymbol{I}(n)\|^2 \\ & + \frac{\alpha(n)^2 K_2 K_3 K_4}{2} \end{aligned} \tag{21}$$

Let us define

$$\hat{V}(n) = V(n) + \frac{K_2K_3K_4}{2} \sum_{i=n}^{\infty} \alpha(i)^2 \tag{22}$$

Then, from (21), we get

$$\begin{aligned} & E_n[\hat{V}(n+1)] \leq \hat{V}(n) \\ & -\alpha(n)(1 - \frac{\alpha(n)K_4(1+K_2K_3)}{2})\|\boldsymbol{p}(n) - \boldsymbol{I}(n)\|^2 \end{aligned} \tag{23}$$

Since $\alpha(n) \to 0$ when $n \to \infty$, we can assume that there exists a constant N, $\forall n \geq N$, $\alpha(n) \leq \frac{2}{K_4(1+K_2K_3)}$. Therefore

$$E_n[\hat{V}(n+1)] \leq \hat{V}(n) \qquad \forall n \geq N \tag{24}$$

Since $\hat{V}(n) \geq 0$, (24) indicates that $\hat{V}(n)$ is a supermartingale sequence [10]. According to martingale convergence theorem [10], $\hat{V}(n)$ converges to a random variable $\hat{V}_\infty$ with probability one. This yields

$$\begin{aligned} & E[V(N)] + \frac{K_2K_3K_4}{2} \sum_{i=N}^{\infty} \alpha(i)^2 \\ & \geq E[V(N)] + \frac{K_2K_3K_4}{2} \sum_{i=N}^{\infty} \alpha(i)^2 - E[\hat{V}_\infty] \\ & \geq \sum_{i=N}^{\infty} \alpha(i)(1 - \frac{\alpha(i)K_4(1+K_2K_3)}{2}) E[\|\boldsymbol{p}(i) - \boldsymbol{I}(i)\|^2] \end{aligned} \tag{25}$$

Since $E[V(0)] < \infty$ and N is finite, we have $E[V(N)] < \infty$. Together with the assumption that $\sum_N^\infty \alpha(n)^2 < \infty$, the left hand side of (25) is finite. If $\hat{V}_\infty > 0$ has a positive probability, then $\|\boldsymbol{p}(n) - \boldsymbol{I}(n)\|^2 > 0$ has a positive probability. This and $\sum_{n=0}^\infty \alpha(n) = \infty$, $\sum_{n=0}^\infty \alpha(n)^2 < \infty$ lead to a contradiction. Therefore, $\hat{V}_\infty = 0$ and $\boldsymbol{p}(n) \to \boldsymbol{p}^*$ when $n \to \infty$, with probability one. $\diamondsuit$

In the situation when $\alpha(n) \to 0$ but $\sum_{n=0}^\infty \alpha(n)^2 = \infty$, and the situation when the step size is a constant, i.e., $\alpha(n) = \alpha$ for all n, probability one convergence is no longer available. Convergence studies under these conditions for general stochastic approximation algorithms can be found in [11].

IV. Combined Stochastic Power Control and Receiver Optimization

In this section, we consider the stochastic implementation of joint power control and receiver optimization. The deterministic version of this problem was studied in [9]. The problem also serves as an example when the deterministic interference function is not linear in powers.

A. System Model

Consider the uplink of a symbol synchronous wireless CDMA system with a fixed base station assignment of K users to M base stations. The chip matched filter output at the assigned base station of user i can be written as

$$\boldsymbol{z}_i = \sum_{j=1}^{K} \sqrt{p_j}\sqrt{h_{ij}} b_j \boldsymbol{s}_j + \boldsymbol{v}_i \tag{26}$$

where p_j is the transmit power of user j; h_{ij} is the channel gain of user j to the assigned base station of user i; b_j and $\boldsymbol{s}_j$ are the transmitted information bit and the normalized signature sequence of user j, respectively; and $\boldsymbol{v}_i$ is a white Gaussian noise vector with zero mean and $E[\boldsymbol{v}_i \boldsymbol{v}_i^T] = \sigma^2 \boldsymbol{I}$.

Let $\boldsymbol{c}_i$ denote the receiver filter for user i at its assigned base station. The receiver filter output of user i is

$$y_i = \sum_{j=1}^{K} \sqrt{p_j}\sqrt{h_{ij}} (\boldsymbol{c}_i^T \boldsymbol{s}_j) b_j + \boldsymbol{c}_i^T \boldsymbol{v}_i \tag{27}$$

The SIR of user i can be written as

$$\mathrm{SIR}_i = \frac{p_i h_{ii} (\boldsymbol{c}_i^T \boldsymbol{s}_i)^2}{\sum_{j \neq i} p_j h_{ij} (\boldsymbol{c}_i^T \boldsymbol{s}_j)^2 + \sigma^2 (\boldsymbol{c}_i^T \boldsymbol{c}_i)} \tag{28}$$

Suppose that the quality of service requirement for user i is $\mathrm{SIR}_i \geq \gamma_i$. Then the combined power control and receiver filter design problem can be stated as that of finding componentwise smallest power vector that yields $\mathrm{SIR}_i \geq \gamma_i$ [9].

Define

$$I_i(\boldsymbol{p}, \boldsymbol{c}_i) = \frac{\gamma_i}{h_{ii}} \frac{\sum_{j \neq i} p_j h_{ij} (\boldsymbol{c}_i^T \boldsymbol{s}_j)^2 + \sigma^2 (\boldsymbol{c}_i^T \boldsymbol{c}_i)}{(\boldsymbol{c}_i^T \boldsymbol{s}_i)^2} \tag{29}$$

Then the quality of service requirement becomes

$$p_i \geq I_i(\boldsymbol{p}, \boldsymbol{c}_i) \tag{30}$$

Since we want to minimize the powers, $\boldsymbol{c}_i$ should be chosen to minimize $I_i(\boldsymbol{p}, \boldsymbol{c}_i)$.

Define

$$\hat{I}_i(\boldsymbol{p}) = \min_{\boldsymbol{c}_i} I_i(\boldsymbol{p}, \boldsymbol{c}_i) \tag{31}$$

Note that $\boldsymbol{c}_i$ that minimizes $I_i(\boldsymbol{p}, \boldsymbol{c}_i)$ is the scaled version of the well-known MMSE filter

$$\boldsymbol{c}_i^* = k_i \boldsymbol{A}_i^{-1} \boldsymbol{s}_i \tag{32}$$

where $\boldsymbol{A}_i = \sum_{j \neq i} p_j h_{ij} \boldsymbol{s}_j \boldsymbol{s}_j^T + \sigma^2 \boldsymbol{I}$ and $k_i > 0$ is an arbitrary constant. Substituting (32) into (31), we get

$$\hat{I}_i(\boldsymbol{p}) = \frac{\gamma_i}{h_{ii}} \frac{1}{\boldsymbol{s}_i^T \boldsymbol{A}_i^{-1} \boldsymbol{s}_i} \tag{33}$$

B. Combined Stochastic Power Control and Receiver Optimization

According to (4), given $\boldsymbol{c}_i(n)$, the n^{th} iteration of user i in the stochastic power control algorithm is

$$\begin{aligned} p_i(n+1) &= (1-\alpha(n))p_i(n) \\ &+\alpha(n)\frac{\gamma_i}{h_{ii}}\left[(\boldsymbol{c}_i(n)^T \boldsymbol{z}_i(n))^2 - p_i(n)h_{ii}\right] \end{aligned} \tag{34}$$

However, since the transmit powers and channel gains of other users are not available at the receiver of user i, we cannot compute $\boldsymbol{c}_i(n)$ directly using (32). Fortunately, one can obtain the optimal filter, i.e., the MMSE filter for fixed powers, by minimizing the mean output energy (MOE) using the blind adaptive method proposed in [8]. If we require that $\boldsymbol{c}_i^T \boldsymbol{s}_i = 1$, we can rewrite $\boldsymbol{c}_i$ as

$$\boldsymbol{c}_i = \boldsymbol{s}_i + \boldsymbol{x}_i, \qquad \boldsymbol{x}_i^T \boldsymbol{s}_i = 0 \tag{35}$$

The blind MMSE adaptation rule can be summarized as [8]

$$\boldsymbol{c}_i(l+1) = \boldsymbol{c}_i(l) - \mu(l)\boldsymbol{c}_i(l)^T \boldsymbol{z}_i(l)(\boldsymbol{z}_i(l) - \boldsymbol{s}_i^T \boldsymbol{z}_i(l)\boldsymbol{s}_i) \tag{36}$$

where l is the iteration index and $\mu(l)$ is the step size for the l^{th} iteration. It is shown in [12] that, given $\boldsymbol{p}(n)$, if $\mu(l)$ satisfies $\sum_{l=0}^{\infty} \mu(l) = \infty$ and $\sum_{l=0}^{\infty} \mu(l)^2 < \infty$, then (36) converges to $\boldsymbol{c}_i(n)^*$ when $l \to \infty$ with probability one.

However, before performing the n^{th} iteration on the stochastic power control (34), in order to ensure $\boldsymbol{c}_i(n) = \boldsymbol{c}_i(n)^*$, one has to perform infinite number of iterations of (36), which is infeasible for practical systems. When the number of blind MMSE iterations is finite, we can write $\boldsymbol{c}_i(n)$ as

$$\boldsymbol{c}_i(n) = \boldsymbol{c}_i(n)^* + \boldsymbol{w}_i(n) \tag{37}$$

where according to [8], $\boldsymbol{w}_i(n)^T \boldsymbol{s}_i = 0$. Since $E[\|\boldsymbol{w}_i(n)\|^2 | \boldsymbol{p}(n)] \to 0$ when $\mu(l) \to 0$ [8], we can always choose the number of iterations such that

$$E[\|\boldsymbol{w}_i(n)\|^2 | \boldsymbol{p}(n)] \leq \alpha(n) K_5 \tag{38}$$

is statisfied $\forall n$, with $K_5 > 0$ being an arbitrary constant.

C. Probability One Convergence

Proposition 2: In the joint stochastic power control and filter optimization, we apply the blind MMSE method (36) to obtain $\boldsymbol{c}_i(n)$ before performing the n^{th} power control iteration (34). Suppose we control the iteration numbers in the blind MMSE such that (38) is satisfied with probability one. We further assume that the step size satisfies

$$\sum_{n=0}^{\infty} \alpha(n) = \infty, \qquad \sum_{n=0}^{\infty} \alpha(n)^2 < \infty \tag{39}$$

Then the joint power control and filter optimization algorithm converges to $\boldsymbol{p}^*$ and the corresponding $\boldsymbol{c}_i^*$, $\forall i$, with probability one.

Proof: We only need to show the convergence of the power vector to $\boldsymbol{p}^*$ with probability one. Since the interference function $\hat{I}_i(\boldsymbol{p}) = \frac{\gamma_i}{h_{ii}} \frac{1}{\boldsymbol{s}_i^T \boldsymbol{A}_i^{-1} \boldsymbol{s}_i}$ is standard [9], the deterministic power control

$$\boldsymbol{p}(n+1) = (1-\alpha(n))\boldsymbol{p}(n) + \alpha(n)\hat{\boldsymbol{I}}(n) \tag{40}$$

converges to $\boldsymbol{p}^*$ [9].

We first show that $\hat{\boldsymbol{I}}(\boldsymbol{p})$ satisfies the Lipschitz condition.

$$\frac{\partial \hat{I}_i}{\partial p_j} = -\frac{\gamma_i}{h_{ii}} \frac{(\boldsymbol{s}_i^T \boldsymbol{s}_j)^2}{(\boldsymbol{s}_i^T \boldsymbol{A}_i^{-1} \boldsymbol{s}_i)^2} \tag{41}$$

Since

$$\begin{aligned} \frac{1}{\boldsymbol{s}_i^T \boldsymbol{A}_i^{-1} \boldsymbol{s}_i} &= \min_{\boldsymbol{x}_i} (\boldsymbol{s}_i + \boldsymbol{x}_i)^T \boldsymbol{A}_i (\boldsymbol{s}_i + \boldsymbol{x}_i) \\ &\leq \frac{1}{(\hat{\boldsymbol{c}}_i^T \boldsymbol{s}_i)^2} \hat{\boldsymbol{c}}_i^T \boldsymbol{A}_i \hat{\boldsymbol{c}}_i \end{aligned} \tag{42}$$

where $\hat{\boldsymbol{c}}_i$ is the decorrelating filter [7] (according to [7], the right hand side of (42) is not a function of $\boldsymbol{p}$), the right hand side of (41) is bounded.

Next, define

$$\tilde{I}_i(n) = \frac{\gamma_i}{h_{ii}} \left[(\boldsymbol{c}_i(n)^T \boldsymbol{z}_i(n))^2 - p_i(n)h_{ii}\right] \tag{43}$$

Using (26), (37), (38), it can be easily verified that $\tilde{\boldsymbol{I}}_i(n)$ satisfies the growing condition for all n.

Now, define function $V(\boldsymbol{p})$ by $V(\boldsymbol{p}^*) = 0$ and $\nabla_{\boldsymbol{p}} V(\boldsymbol{p}) = \boldsymbol{p} - \hat{\boldsymbol{I}}(\boldsymbol{p})$. Since $\hat{\boldsymbol{I}}(\boldsymbol{p})$ is standard [9], according to the analysis in section III.A, $V(\boldsymbol{p})$ is a Lyapunov function.

Taking the expectation on the truncated Taylor expansion, and noting that $\boldsymbol{A}_i(n)\boldsymbol{c}_i(n)^* = \frac{\boldsymbol{s}_i}{\boldsymbol{s}_i^T \boldsymbol{A}_{i(n)^{-1}} \boldsymbol{s}_i}$, we obtain

$$\begin{aligned} E_n[V(n+1)] &\leq V(n) - \alpha(n)\|\boldsymbol{p}(n) - \hat{\boldsymbol{I}}(n)\|^2 \\ &-2\alpha(n)\sum_i (p_i(n) - \hat{I}_i(n))\left(\frac{\gamma_i}{h_{ii}} E_n\left[\frac{\boldsymbol{w}_i(n)^T \boldsymbol{s}_i}{\boldsymbol{s}_i^T \boldsymbol{A}_i(n)^{-1} \boldsymbol{s}_i}\right]\right) \\ &-\alpha(n)\sum_i (p_i(n) - \hat{I}_i(n))\left(\frac{\gamma_i}{h_{ii}} E_n[\boldsymbol{w}_i(n)^T \boldsymbol{A}_i(n)\boldsymbol{w}_i(n)]\right) \\ &+\frac{\alpha(n)^2 K_4}{2} E_n[\|\boldsymbol{p}(n) - \tilde{\boldsymbol{I}}(n)\|^2] \end{aligned} \tag{44}$$

From (42),

$$\left| E_n\left[\frac{\boldsymbol{w}_i(n)^T \boldsymbol{s}_i}{\boldsymbol{s}_i^T \boldsymbol{A}_i(n)^{-1} \boldsymbol{s}_i}\right] \right| \leq \frac{\hat{\boldsymbol{c}}_i^T \boldsymbol{A}_i(n)\hat{\boldsymbol{c}}_i}{(\hat{\boldsymbol{c}}_i^T \boldsymbol{s}_i)^2} E_n[\|\boldsymbol{w}_i(n)\|] \tag{45}$$

According to (38) and the growing condition, we can find a constant K_6, such that

$$\begin{aligned} &2\alpha(n)\sum_i (p_i(n) - \hat{I}_i(n))\left(\frac{\gamma_i}{h_{ii}} E_n\left[\frac{\boldsymbol{w}_i(n)^T \boldsymbol{s}_i}{\boldsymbol{s}_i^T \boldsymbol{A}_i(n)^{-1} \boldsymbol{s}_i}\right]\right) \\ &+\alpha(n)\sum_i (p_i(n) - \hat{I}_i(n))\left(\frac{\gamma_i}{h_{ii}} E_n[\boldsymbol{w}_i(n)^T \boldsymbol{A}_i(n)\boldsymbol{w}_i(n)]\right) \\ &\leq \alpha(n)^2 K_6(1 + \|\boldsymbol{p}(n)\|^2) \end{aligned} \tag{46}$$

The rest of the proof follows similar to that of proposition 1. ◊

D. Further Extension

Proposition 2 shows the convergence of the joint stochastic power control and filter optimization algorithm under the condition that (38) is satisfied by the filter updates between the power updates. Hence, one may still need to perform a large number of iterations of the blind MMSE (36) between two power control updates (34) in order to ensure that (38) holds. Obviously, when $\alpha(n) \to 0$, the number of steps required on the blind MMSE iteration grows to infinity. Noting that when $\alpha(n)$ is small, $\boldsymbol{c}^*(n+1)$ differs from $\boldsymbol{c}^*(n)$ only slightly, we can initialize the blind MMSE iteration of $\boldsymbol{c}(n+1)$ by $\boldsymbol{c}(n)$. Based on this basic principle, we propose the extended algorithm as follows.

Combined stochastic power control and receiver optimization:

1. Initialize the iteration counter $n = 0$. Initialize $\boldsymbol{p}(0)$, $\boldsymbol{x}_i(0)$.
2. Compute $\boldsymbol{c}_i(n)$ by
$$\boldsymbol{c}_i(n) = \boldsymbol{s}_i + \boldsymbol{x}_i(n) \tag{47}$$
3. $\forall i$, compute $\tilde{I}_i(n)$ via
$$\tilde{I}_i(n) = \frac{\gamma_i}{h_{ii}}\left[(\boldsymbol{c}_i(n)^T\boldsymbol{z}_i(n))^2 - p_i(n)h_{ii}\right] \tag{48}$$
4. $\forall i$, compute $\tilde{\boldsymbol{x}}_i(n)$ via
$$\tilde{\boldsymbol{x}}_i(n) = \boldsymbol{c}_i(n)^T\boldsymbol{z}_i(n)(\boldsymbol{z}_i(n) - \boldsymbol{s}_i^T\boldsymbol{z}_i(n)\boldsymbol{s}_i) \tag{49}$$
5. $\forall i$, update $p_i(n+1)$ and $\boldsymbol{x}_i(n+1)$ by
$$\begin{aligned} p_i(n+1) &= p_i(n) - \alpha_p(n)(p_i(n) - \tilde{I}(n)) \\ \boldsymbol{x}_i(n+1) &= \boldsymbol{x}_i(n) - \alpha_{xi}(n)\tilde{\boldsymbol{x}}_i(n) \end{aligned} \tag{50}$$
where $\alpha_p(n)$, $\alpha_{xi}(n)$ are the step sizes for the power control and blind MMSE updates, respectively.
6. As in [8], perform
$$\boldsymbol{x}_i(n+1) = \boldsymbol{x}_i(n+1) - (\boldsymbol{x}_i(n+1)^T\boldsymbol{s}_i)\boldsymbol{s}_i \tag{51}$$
to ensure that $\boldsymbol{x}_i(n+1)^T\boldsymbol{s}_i = 0$ holds.
7. Stop when the power and filter coefficients converge. Otherwise, let $n = n+1$, and go to step 2. ◇

In the above algorithm, we update the power vector and the filter coefficients in parallel, i.e., the power control algorithm does not wait for the convergence of the blind MMSE. Furthermore, we use different notations on the step sizes of the power control and the blind MMSE to indicate that they are not necessarily the same. However, we should clarify that the convergence proof in proposition 2 does not ensure the convergence of the extended algorithm. Nevertheless, we show via computer simulations that, when the step sizes are small enough, the above algorithm is indeed convergent.

V. Simulation Results

In this section, we present several computer simulations to illustrate the performance of the proposed algorithm.

Example 1: We choose $K = 4$ and the 5-length binary signature sequences are generated randomly as,

$$[\boldsymbol{s}_1, \boldsymbol{s}_2, \boldsymbol{s}_3, \boldsymbol{s}_4] = \frac{1}{\sqrt{5}}\begin{bmatrix} -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \tag{52}$$

For simplicity, we set all channel gains to $h_{ij} = 1$, $\forall i, j$. The SIR targets for the users are chosen arbitrarily at

$$[\gamma_1, \gamma_3, \gamma_3, \gamma_4] = [5.9, 4.9, 7.6, 6.9] \tag{53}$$

And $\sigma^2 = 0.1$. We initialize all user powers at 1, and initialize the filter coefficients to $\boldsymbol{x}_i(0) = \boldsymbol{0}$. The step size is chosen as $\alpha_p(n) = \alpha_{xi}(n) = \frac{10}{10000+n}$ so that the system has a reasonable initial convergence and the step size does not decrease to zero too quickly. Figure 1 shows the convergence of the transmitted power p_i of each user, while Figure 2 shows the convergence of the filter coefficients $\boldsymbol{x}_1$ for user 1. The optimal values of the parameters which are obtained from the deterministic iterations are also provided as horizontal lines in these figures.

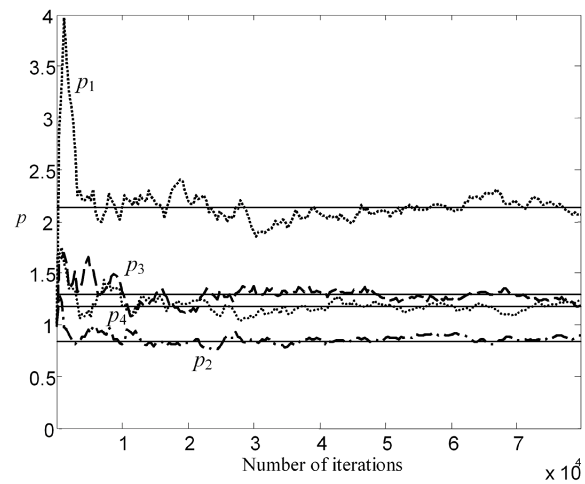

Figure 1: Power convergence, $\sigma^2 = 0.1$

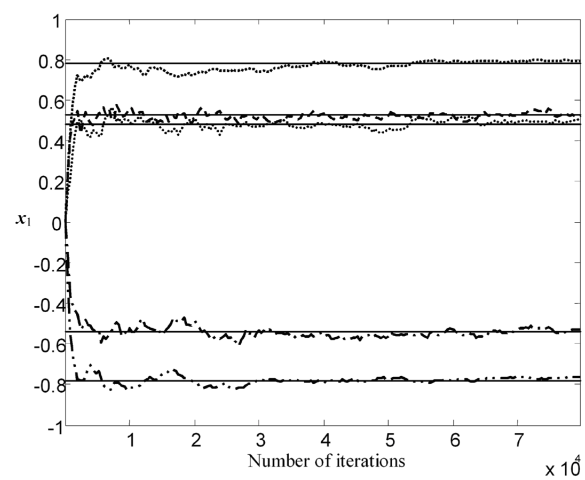

Figure 2: Filter convergence of user 1, $\sigma^2 = 0.1$

Example 2: In this example, we consider a general multicell CDMA system on a rectangular grid. There are 25 base stations with coordinates $(1000i + 500, 1000j + 500)$ for $0 \le i, j \le 4$. We have 400 users, whose positions are randomly and independently generated. Both x and y coordinates of each user are uniformly distributed between $0 \sim 5000$ meters. Figure 3 shows the positions of users and the base stations with symbols $\times$ and o, respectively.

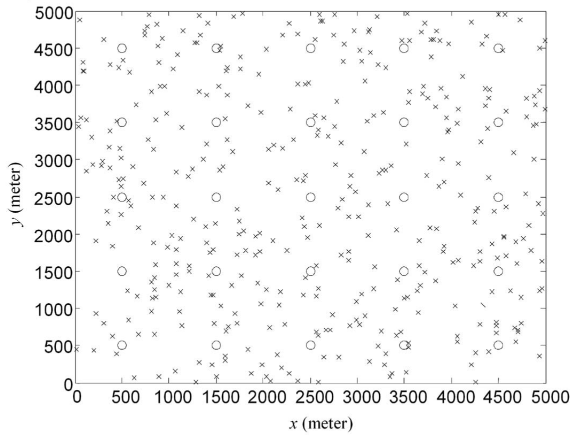

Figure 3: Simulation environments for 400 users. o and $\times$ are base stations and users, respectively.

In this example, each user is assigned to the closest base station. The channel gain of user j to the assigned base station of user i is computed as $h_{ij} = \left(\frac{100}{d_{ij}}\right)^4$, where d_{ij} is the distance between user j and the assigned base station of user i. The target SIR is set at $\gamma = 4$ for all users. All other settings are the same as that in example 1. Figure 4 compares the performances in terms of the average power of the joint power control and blind MMSE and the stochastic power control with matched filter receivers. In addition to the convergence of the two algorithms, we can see that, to achieve the same SIR target, the average power of the MMSE receiver is much lower than the matched filter receiver. Figure 5 shows the average mean square error on the filter coefficients of the joint power control and blind MMSE receiver optimization.

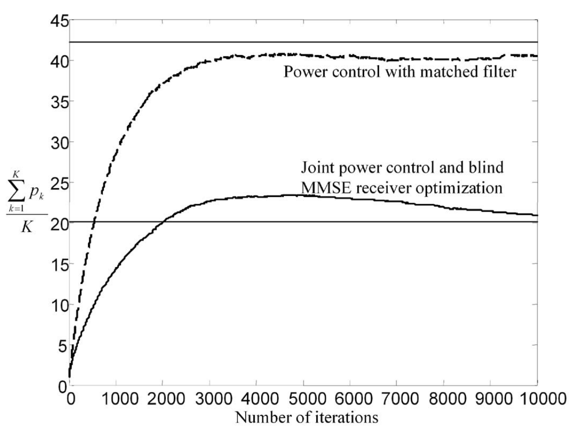

Figure 4: Performance comparison in terms of average power.

VI. Conclusions

The convergence issue of both standard stochastic power control and joint stochastic power and blind MMSE interference suppression is studied. It is shown that, under certain conditions, both algorithms converge to the optimal solutions with probability one. The joint power control and reiver optimization is further extended and simulation results are given to illustrate its performance.

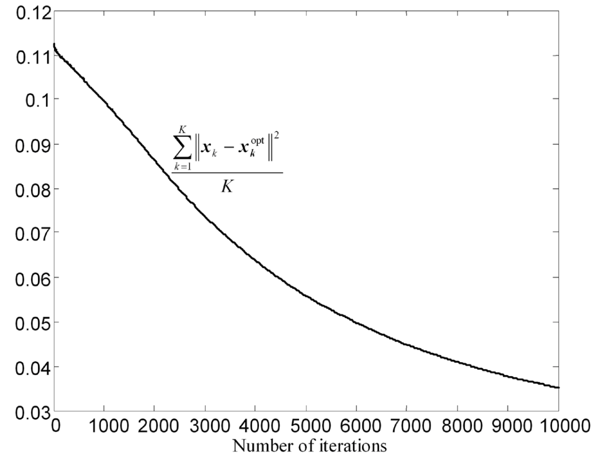

Figure 5: Average mean square error on filter coefficients of the joint power control and blind MMSE interference suppression

Acknowledgments

The authors would like to thank Professor Krishna Pattipati and Dr. Damianos Karakos for their help on stochastic approximation algorithms.

References

[1] S. Grandhi, R. Vijayan, D. Goodman, J. Zander "Centralized Power Control in Cellular Radio Systems", *IEEE Trans. on Vehicular Tech.* , Vol. 42, pp. 466-468, Nov. 1993.

[2] G. Foschini, Z. Miljanic "A Simple Distributed Autonomous Power Control Algorithm and Its Convergence", *IEEE Trans. on Vehicular Tech.* , Vol. 42, pp. 641-646, Nov. 1993.

[3] S. Chen, N. Bambos, G. Pottie "On Distributed Power Control for Radio Networks", *IEEE ICC94*, pp. 1281-1285, May 1994.

[4] R. Yates, "A Framework for Uplink Power Control in Cellular Radio Systems", *IEEE J. on Sel. Areas in Comm.* , Vol. 13, pp. 1341-1347, Sep. 1995.

[5] S. Ulukus, R. Yates "Stochastic Power Control for Cellular Radio Systems", *IEEE Trans. on Comm.* , Vol. 46, pp. 784-798, June 1998.

[6] M. Varanasi, D. Das, "Fast Stochastic Power Control Algorithms for Nonlinear Multiuser Receivers" *IEEE Trans. on Comm.* , Vol. 50, pp. 1817-1827, Nov. 2002.

[7] S. Verdu, "Multiuser Detection", *Cambridge University Press*, New York, 1998.

[8] M. Honig, U. Madhow and S. Verdu, "Blind Adaptive Multiuser Detection", *IEEE Trans. on Info. Theory*, Vol. 41, pp. 944-960, July 1995.

[9] S. Ulukus and R. Yates, "Adaptive Power Control and MMSE Interference Suppression", *ACM Wireless Networks*, Vol. 4, pp. 489-496, Nov. 1998.

[10] S. Karlin and H. Taylor, "A First Course in Stochastic Processes", 2nd edition, *Academic Press*, San Diego, CA, 1975

[11] H. Kushner and G. Yin, "Stochastic Approximation Algorithms and Applications", *Springer*, New York, 1997

[12] L. Gyorfi, "Adaptive Linear Procedures Under General Conditions", *IEEE Trans. on Info. Theory*, Vol. IT-30, pp. 262-267, Mar. 1984.

[0] Any opinions findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Aeronautics and Space Administration and the Army Research Laboratory of the U.S. Government.