Age of Information in G/G/1/1 Systems

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***Abstract*—We consider a single server communication setting where the interarrival times of data updates at the source node and the service times to the destination node are arbitrarily distributed. We consider two service discipline models. In the first model, if a new update arrives when the service is busy, it is blocked; in the second model, a new update preempts the current update in service. For both models, we derive exact expressions for the age of information metric with no restriction on the distributions of interarrival and service times. In addition, we derive upper bounds that are easier to calculate than the exact expressions. In the case with blocking, we also derive a second upper bound by utilizing stochastic ordering if the interarrival and service times have log-concave distribution.**

I. Introduction

No matter how important information might be, there is a duration of time after which information loses its freshness. Especially in today's world of immensely interactive everything, information ages fast. Hence, in recent years, researchers have begun to consider the age of information (AoI) in addition to the value of information. Age of anything can be defined as the duration between the time of birth and the current time. This definition is sufficiently broad to cover almost all communication scenarios. However, most of the AoI literature so far has considered queueing systems with well-behaved distributions. In this paper, we take this a step forward and apply the AoI viewpoint to more general queueing distributions, and hence to more general communication scenarios.

The first papers that consider the AoI in a communication setting are [1], [2], and [3]. Reference [1] assumes First Come First Served (FCFS) systems and calculates the average AoI expressions for M/M/1, M/D/1 and D/M/1 queues, reference [2] assumes Last Come First Served (LCFS) systems with and without preemption and calculates the average AoI expression for M/M/1 queues, reference [3] assumes multi-source FCFS systems with M/M/1 queues, and reference [4] provides a more detailed analysis. Starting with these works, there has been a growing interest in AoI analysis. For example, reference [5] considers a packet management approach for M/M/1/1 and M/M/1/2 queues. Reference [6] calculates the AoI for an M/G/1/1 queue and finds the optimum arrival rate to minimize age.

While the literature on calculating age expressions for different queueing models expands, another line of research applies the AoI approach to energy harvesting problems. The goal is to find the optimum update generation policy that minimizes age, given the service time distribution. In [7], the authors show the existence of an optimal stationary deterministic update generation policy when the service time process is a stationary and ergodic Markov chain. Application of AoI to offline energy harvesting is considered in [8]–[10], and online energy harvesting is considered in [11]–[14].

In this paper, our goal is to analyze AoI for general communication scenarios, i.e., general interarrival and service time distributions. Using queueing theory terminology, our model corresponds to a G/G/1/1 system. An example of such a G/G/1/1 system is the multicast problem in [15], where a new update is generated when a percentage of the destinations has received the current update, and service time to each destination is a shifted exponential random variable. Although an exact expression for their model is derived in [15], in general, calculating an exact age expression for non-exponential interarrival times is difficult. For example, [16] considers a two-stage multicast extension of [15], where only an upper bound is derived for the age of the second stage nodes. In this paper, we derive exact age expressions when the distribution of service times is arbitrary but known. In addition, we also derive upper bounds to the AoI that may be easier to use for further system design and optimization. For instance, if one designs age-minimizing policies using our upper bounds, the resulting age will be an achievable age.

We consider two service disciplines. The first one is called G/G/1/1 *with blocking*, where a new arrival is blocked if the server is busy. This model is also used in [5] for an M/M/1/1 system and in [6] for an M/G/1/1 system. Here, we do not restrict ourselves to exponential interarrival times or exponential service times. We derive an exact expression and two upper bounds for our first service model. While the first upper bound does not have any restrictions, the second upper bound requires the interarrival and service times to have log-concave probability density functions [17]. Many distributions, including exponential, Rayleigh, Erlang, and gamma, that appear in arrival processes, have log-concave probability densities [18].

Our second service discipline model is called G/G/1/1 *with preemption in service*, where a new arrival preempts any ongoing service. This model is used in [1] and [4] for an M/M/1/1 system and in [6] for an M/G/1/1 system. The exact expression and upper bound that we derive for this model does not have any restrictions. As numerical examples, we simulate our system models with general distributions, and compare the simulated age values to calculated exact age and upper bound expressions. We observe that for most of the parameter range, upper bounds are close to the exact age values.

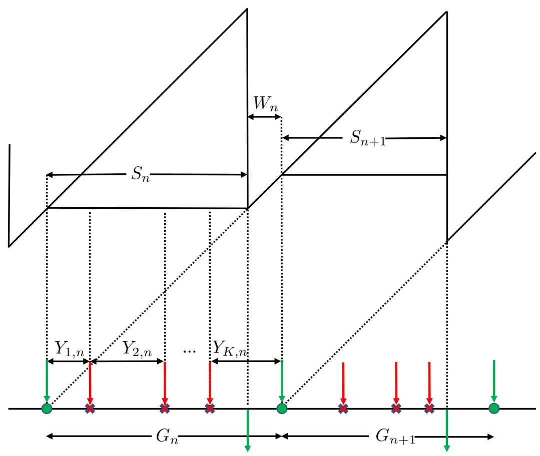


Fig. 1: Age curves for G/G/1/1 with blocking model.

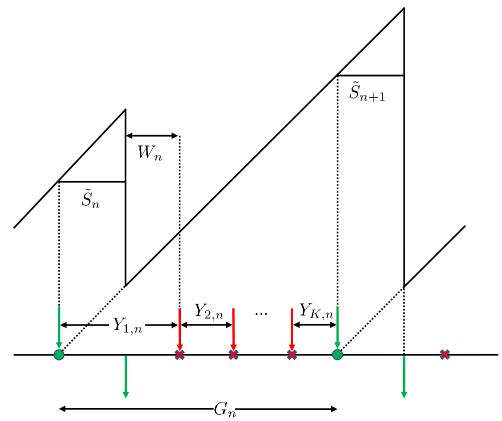


Fig. 2: Age curves for G/G/1/1 with preemption in service.

II. System Model

We consider a communication scenario where the data arrive at the source according to an arrival process with independent and identically distributed (i.i.d.) interarrival times Y_n. The source transmits the data through a single server. Time duration of service is modeled as a random process with i.i.d. service times S_n. Interarrival times, Y_n, and service times, S_n, are independent. We specify general probability distributions for the interarrival times and service times.

In Figs. 1 and 2, arrows correspond to packet arrivals at the source. The interval where the system is idle (no packets in the service or in the queue) is denoted by W_n, and the service time is denoted by S_n.

A. Blocking

In this model, if an update arrives while the server is busy, it is blocked. If an update arrives while the server is idle, service is initiated immediately. We refer to those arrivals that initiate a service as the successful arrivals. In Fig. 1, successful arrivals for this model are shown with circles, while unsuccessful arrivals are shown with crosses. After a service, S_n, is completed, a successful update departs the server. Service idle time, W_n, is the time between a departure of a successful update and the arrival of the next update. Interarrival times between consecutive successful updates, $G_n = S_n + W_n$, are called effective interarrival times. It is important to note that, the effective interarrival time, G_n, can be written as a random sum of random numbers, $G_n = \sum_{k=1}^{K} Y_{k,n}$. Note that K is an integer random variable that describes the total number of arrivals before the next successful arrival. Probability mass function of K can be written as

$$\Pr(K = k) = \Pr\left(\sum_{j=1}^{k-1} Y_{j,n} \leq S_n < \sum_{j=1}^{k} Y_{j,n}\right). \quad (1)$$

B. Preemption in service

In this model, if an update arrives while the server is idle, the service is initiated immediately. If an update arrives while the server is busy, the packet being served is terminated and the new update is pushed to the server. A successful arrival is the one that can finish the service. In Fig. 2, successful arrivals for this model are shown with circles, while unsuccessful arrivals are shown with crosses. Since the service time of a successful arrival needs to be smaller than the interarrival time, the time that a successful arrival stays in service, $\tilde{S}_n$, is less than the service time of the server, i.e., $\tilde{S}_n = S_n | S_n < Y_{1,n}$.

Similar to the blocking model, interarrival times between successful updates are called effective interarrival times, G_n, which can be written as a random sum of random numbers, $G_n = \sum_{k=1}^{K} Y_{k,n}$. Unlike the blocking model, here the waiting time depends only on the current interarrival time, $W_n = Y_{1,n} - \tilde{S}_n$. Although K in the blocking discipline does not follow a specific distribution, K in the preemption in service discipline is a geometric random variable. An effective interarrival time is the sum of a successful arrival with probability $p = \Pr(Y_{1,n} > S_n)$, and $K-1$ unsuccessful arrivals, all with the same probability $1-p$.

III. G/G/1/1 with Blocking

For G/G/1/1 with blocking discipline, average age can be written as the difference of the areas of two triangles, divided by the expected value of the effective interarrival time. From Fig. 1, we have

$$\Delta^b_{\text{G/G}} = \frac{E[(G_n + S_{n+1})^2] - E[(S_{n+1})^2]}{2E[G_n]} \quad (2)$$

$$= \frac{E[G^2]}{2E[G]} + E[S], \quad (3)$$

where S_{n+1} is independent of G_n, and time indices are dropped. For most general arrival and service time models, it is not easy to calculate the first and second moments of effective interarrival times. In the following, we first derive an exact expression for (3) that depends only on the general distributions of interarrival times, Y, and service times, S. Then, we also derive upper bounds to (3) that is easier to calculate.

A. Age for General Interarrival and Service Times

In this section, we start with deriving an exact age expression for the blocking discipline.

Theorem 1 *Consider a G/G/1/1 system with blocking discipline, where Y_n are i.i.d. interarrival times with a general distribution and S_n are i.i.d. service times with a general distribution. The average age of an information update in this system is*

$$\Delta^b_{G/G} = \frac{E[Y^2]}{2E[Y]} + \frac{\sum_{k=1}^{\infty} E[A_k \bar{F}_S(A_k)]}{1 + \sum_{k=1}^{\infty} E[\bar{F}_S(A_k)]} + E[S] \quad (4)$$

where $A_k = \sum_{j=1}^{k} Y_j$, and $\bar{F}_S$ is the complementary cdf of S.

Proof: Remember from Section II-A that effective interarrival times, G_n, can be written as random sums of random numbers. Although K is not independent of all Y_j, it is possible to calculate the expected value of G using Wald's equation [19, Theorem 3.3.2] as $E[G] = E[K]E[Y]$.

Next, we derive an expression for the second moment of effective interarrival times, $E[G^2]$. Let us first define the indicator function as

$$I_k = \begin{cases} 1 & \text{if } k \leq K \\ 0 & \text{if } k > K. \end{cases} \quad (5)$$

Now, we have

$$E\left[\left(\sum_{k=1}^{K} Y_k\right)^2\right] = E\left[\left(\sum_{k=1}^{\infty} Y_k I_k\right)^2\right] \quad (6)$$

$$= \sum_{k=1}^{\infty} E[Y_k^2 I_k] + 2\sum_{k=2}^{\infty}\sum_{l=1}^{k-1} E[Y_k I_k Y_l I_l]. \quad (7)$$

Note that, $I_k = 1$ if and only if, we have not stopped after successively observing $Y_1, \ldots, Y_{k-1}$. Therefore, I_k is determined by $Y_1, \ldots, Y_{k-1}$, and is thus independent of Y_k. We have $E[Y_k^2 I_k] = E[Y_k^2]E[I_k]$, and $E[Y_k I_k Y_l I_l] = E[Y_k]E[I_k Y_l I_l]$, for $l < k$. Now, (7) becomes

$$E\left[G^2\right] = E\left[Y^2\right]\sum_{k=1}^{\infty} E[I_k] + 2E[Y]\sum_{k=2}^{\infty}\sum_{l=1}^{k-1} E[I_k Y_l I_l]. \quad (8)$$

Using (5), we have $\sum_{k=1}^{\infty} E[I_k] = E[K]$. Next, let us calculate

$$\sum_{l=1}^{k-1} E[I_k Y_l I_l] = \sum_{l=1}^{k-1} E[Y_l | I_k = 1]\Pr(I_k = 1) \quad (9)$$

$$= E\left[\sum_{l=1}^{k-1} Y_l \middle| I_k = 1\right]\Pr\left(I_k = 1\right) \quad (10)$$

$$= E\left[\sum_{l=1}^{k-1} Y_l \middle| \sum_{l=1}^{k-1} Y_l < S\right]\Pr\left(\sum_{l=1}^{k-1} Y_l < S\right) \quad (11)$$

where we used the fact that $I_l = 1$ for $l < k$ and given $I_k = 1$. Note that the condition $I_k = 1$ and $\sum_{l=1}^{k-1} Y_l < S$ are equivalent for blocking service discipline. Next, let us denote $A_{k-1} = \sum_{l=1}^{k-1} Y_l$. Then using Bayes' rule, we can calculate for any k that

$$E\left[A_k | A_k < S\right] = \int_0^{\infty} a \frac{\Pr(A_k < S | A_k = a)}{\Pr(A_k < S)} f_{A_k}(a) da \quad (12)$$

$$= \frac{E[A_k \bar{F}_S(A_k)]}{\Pr(A_k < S)} \quad (13)$$

where $\Pr(A_k < S | A_k = a) = \Pr(S > a) = \bar{F}_S(a)$. Now, (8) becomes

$$E[G^2] = E[Y^2]E[K] + 2E[Y]\sum_{k=2}^{\infty} E[A_{k-1}\bar{F}_S(A_{k-1})] \quad (14)$$

Finally, noting that $E[K] = \sum_{k=1}^{\infty} \Pr(S > A_{k-1})$, and inserting (14) in (3), we have (4). ■

The exact expression in (4) requires a calculation of an infinite sum. In order to reduce the complexity of calculation, we derive the following upper bound.

Corollary 1 *Consider a G/G/1/1 system with blocking discipline, where Y_n are interarrival times and S_n are service times. The average age of this system is upper bounded by*

$$\Delta^b_{G/G} \leq \frac{E[Y^2]}{2E[Y]} + E[Y]\left(\frac{E[K^2]}{2E[K]} - \frac{1}{2}\right) + E[S] \quad (15)$$

The proof of Corollary 1 follows by noting that

$$E\left[\sum_{l=1}^{k-1} Y_l \middle| \sum_{l=1}^{k-1} Y_l < S\right] \leq E\left[\sum_{l=1}^{k-1} Y_l\right], \quad (16)$$

and $\sum_{k=1}^{\infty}(k-1)E[I_k] = \frac{1}{2}E[K(K-1)]$. In our system model, the number of terms in the random sum, G, depends on the summands, Y_k. Under this system model, the upper bound in (15) can only be achieved with deterministic interarrival times, Y_k. On the other hand, when (16) is applied to (14), we get

$$E[G^2] \leq E[Y^2]E[K] + (E[Y])^2(E[K^2] - E[K]) \quad (17)$$

where the right hand side is equal to the second moment of a random sum when the number of terms in the sum is independent of the summands [20].

It is important to note that Corollary 1 reduces the complexity of calculation. For exponential service times, we have a closed form expression for the upper bound to the average age. It can be shown that for exponentially distributed service times with parameter μ, K is a geometric random variable with $p = 1 - E[e^{-\mu Y}]$. Then, the upper bound to the age of G/M/1/1 systems can be written as

$$\Delta^b_{\text{G/M}} \leq \frac{E[Y^2]}{2E[Y]} + E[Y]\left(\frac{1}{1 - E[e^{-\mu Y}]} - 1\right) + \frac{1}{\mu}. \quad (18)$$

Since all the components in (18) are known, one can use this upper bound to design age-minimal policies for communication systems with exponential service time and without a restriction on the arrival process. We remark that the resulting age of such an optimization is guaranteed to be achieved.

When the interarrival times are also exponentially distributed with rate parameter λ, (18) becomes

$$\Delta^b_{\text{M/M}} \leq \frac{1}{\lambda} + \frac{2}{\mu} \tag{19}$$

where the age of M/M/1/1 queues is known to be $\Delta_{\text{MM}} = \frac{1}{\lambda} + \frac{2}{\mu} - \frac{1}{\lambda+\mu}$ [5].

B. Age for Log-Concave Interarrival and Service Times

For some communication models, it might not be possible to calculate the moments of K that is needed in (15) for general interarrival times. For such cases, we derive another upper bound that does not include those moments. This bound requires the interarrival and service times to have log-concave probability density [17].

Theorem 2 *Consider a LC/LC/1/1 system with blocking discipline, where Y_n are i.i.d. interarrival times with a log-concave distribution, and S_n are i.i.d. service times with a log-concave distribution. Also consider an M/LC/1/1 system that is formed by replacing the interarrival times of LC/LC/1/1 system with exponentially distributed interarrival times, Y^e_n, where $E[Y^e] = E[Y]$. Then, the average age of the LC/LC/1/1 system, $\Delta^b_{LC/LC}$ is upper bounded by the average age of M/LC/1/1 system, $\Delta^b_{M/LC}$.*

Proof: In this proof, we need several definitions and results from total positivity theory [18] and stochastic ordering [17], which are summarized in [21]. First, we show that $G = \sum_{k=1}^{K} Y_k$ has a log-concave density when interarrival time distribution is log-concave [21].

Next, we write the waiting time as $W = G - S | G > S$. For a given $S = s$, we note that the random variable $W|S = s$ follows the residual life distribution of G. Therefore, when we take the expected value of $W|S = s$ over G, we obtain the mean residual life function of G, $m_G(s)$. Since G is log-concave, we know that $m_G(s)$ is a non-increasing function of s and $\text{Cov}(W, S) \leq 0$.

Let us consider a random variable $\bar{S}$ that is i.i.d. with S and independent of W. Using the fact that $\text{Cov}(W, S) \leq 0$, one can show that $E[(W + S)^2] \leq E[(W + \bar{S})^2]$. Now, (3) can be upper bounded as

$$\Delta^b_{\text{LC/LC}} \leq \frac{E[(W + \bar{S})^2]}{2E[W + \bar{S}]} + E[\bar{S}]. \tag{20}$$

where W and $\bar{S}$ are independent.

Next, we use the fact that $W|S = s$ follows the residual life distribution of G. From [18, page 182], we know that the residual life distribution of a log-concave distribution is log-concave as well. Noting also that S is also log-concave, we conclude that W is log-concave from [22, Lemma 2]. Therefore, W is smaller (in the convex order) than the exponential random variable with the same mean, W^e, i.e., $W \leq_{\text{cx}} W^e$. Equivalently, we have $W \leq_{\text{hmrl}} W^e$. Since W is log-concave and therefore has decreasing mean residual life, we have $E[W] \leq E[Y]$, or equivalently, $E[W^e] \leq E[Y^e]$.

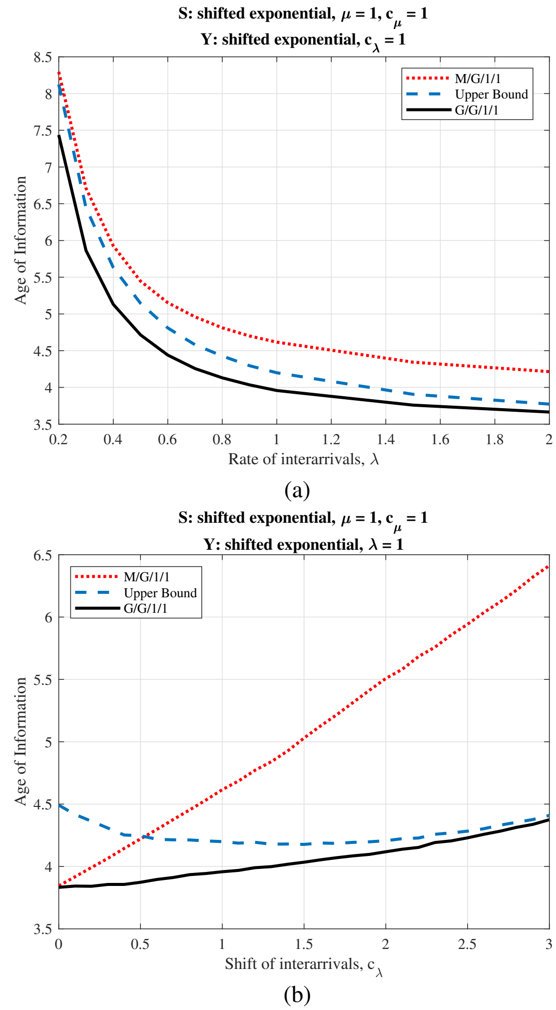


Fig. 3: Average AoI for G/G/1/1 with blocking.

We also conclude that $W^e \leq_{\text{hmrl}} Y^e$. Moreoever, since $\bar{S}$ is independent of W and Y^e, we have $W + \bar{S} \leq_{\text{hmrl}} Y^e + \bar{S}$. Finally, using [17, eqn. (2.B.5)], we have

$$\frac{E[(W + \bar{S})^2]}{E[W + \bar{S}]} \leq \frac{E[(Y^e + \bar{S})^2]}{E[Y^e + \bar{S}]}, \tag{21}$$

which directly implies $\Delta^b_{\text{LC/LC}} \leq \Delta^b_{\text{M/LC}}$. ■

In order to observe the tightness of our upper bounds, we simulate an example G/G/1/1 system, calculate its age and compare it to the proposed upper bounds. As in [15], we consider shifted exponential interarrival times with rate parameter λ and shift parameter c_λ; and shifted exponential service times with rate parameter μ and shift parameter c_μ. In Fig. 3, we observe that age decreases with the rate parameter and increases with the shift parameter of the interarrival distribution. The distance between the first proposed upper bound and the exact age for G/G/1/1 seems to be bounded and small for both curves. When the shift of interarrival times is zero, G/G/1/1 reduces to M/G/1/1. As the shift of interarrival times increase, upper bound converges to the exact age for

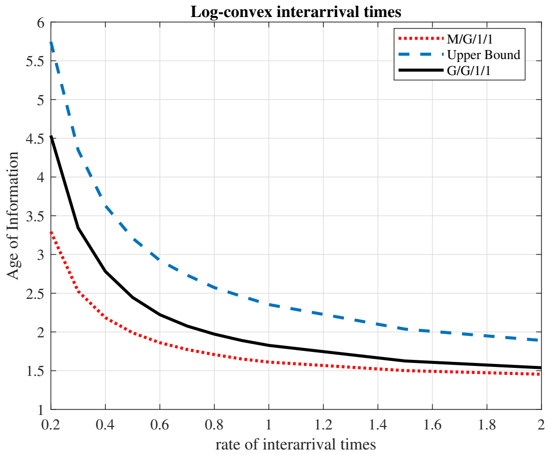


Fig. 4: Log-convex interarrival times for G/G/1/1 with blocking.

G/G/1/1. The reason for this is that as c_λ increases, coefficient of variation of the interarrival times decreases, and therefore interarrival times appraoches to a deterministic value.

In Fig. 4, we consider the effect of having log-convex interarrival times. We observe that while the expression in Theorem 1 is still a valid upper bound, the age for M/G/1 systems becomes a lower bound to age for G/G/1 systems. In fact, this can be easily proved by slightly modifying the proof of Theorem 2.

IV. G/G/1/1 with Preemption in Service

Similar to the case with blocking discipline, for G/G/1/1 with preemption in service discipline as well, average age can be written as the difference of the areas of two triangles, divided by the expected length of the effective interarrival time. From Fig. 2, we have

$$\Delta^p_{\text{G/G}} = \frac{E[(G_n + \tilde{S}_{n+1})^2] - E[(\tilde{S}_{n+1})^2]}{2E[G_n]} \tag{22}$$

$$= \frac{E[G^2]}{2E[G]} + E[\tilde{S}] \tag{23}$$

where $\tilde{S}_{n+1} = \{S|S < Y\}_{n+1}$ is independent of G_n, and time indices are dropped.

It is important to note that the random variable G in this model is not the same G as in blocking model. The difference can be observed from Figs. 1 and 2 by noting the change in scale for S_n. In the following, we derive an exact closed form expression for (23), which is valid for general interarrival and service times. Unlike the expression in blocking model, the expression here does not require the calculation of the average number of arrivals between two consecutive effective interarrivals.

Theorem 3 *Consider a G/G/1/1 system with preemption in service discipline, where Y_n are i.i.d. interarrival times with a general distribution and S_n are i.i.d. service times with a general distribution. The average age of an information update in this system is*

$$\Delta^p_{G/G} = \frac{E[Y^2]}{2E[Y]} + \frac{E[Y\bar{F}_S(Y)]}{1 - E[\bar{F}_S(Y)]} + E[\tilde{S}] \tag{24}$$

where $\tilde{S} = S|S < Y$.

Proof: We know from our system model that $G = \sum_{k=1}^K Y_k$ is a random sum of random numbers, where K is a geometric random variable. From Wald's equation [19, Theorem 3.3.2], we have $E[G] = E[K]E[Y]$. Next, we derive an expression for the second moment of effective interarrival times, $E[G^2]$. Let us first use the indicator function in (5) and the expansion of $E[G^2]$ in (7). Similar to blocking in service discipline, I_k is independent of Y_k. Let us consider

$$\sum_{l=1}^{k-1} E[I_k Y_l I_l] = \sum_{l=1}^{k-1} E[Y_l | I_k = 1]\Pr(I_k = 1) \tag{25}$$

$$= (k-1)E[Y|Y < S]E[I_k] \tag{26}$$

where we used the fact that conditions $I_k = 1$ and $Y_l < S$ are equivalent for preemption in service discipline and for $l < k$. It can be shown that $\sum_{k=1}^\infty E[I_k] = E[K]$ and $\sum_{k=1}^\infty E[I_k] \sum_{l=1}^{k-1} E[I_l] = \frac{1}{2}E[K(K-1)]$. We have

$$E[G^2] = E[Y^2]E[K] + E[Y]E[Y|Y < S]E[K(K-1)]. \tag{27}$$

Next, using Bayes' rule, we can calculate that

$$E[Y|Y < S] = \int_0^\infty y \frac{\Pr(Y < S|Y = y)}{\Pr(Y < S)} f_Y(y)dy \tag{28}$$

$$= \frac{E[Y\bar{F}_S(Y)]}{1-p}. \tag{29}$$

where, $\Pr(Y < S|Y = y) = \Pr(S > y) = \bar{F}_S(y)$. Now, the average age can be written as

$$\Delta_{\text{GG}} = \frac{E[Y^2]}{2E[Y]} + E[Y\bar{F}_S(Y)]\frac{E[K(K-1)]}{2E[K](1-p)} + E[\tilde{S}]. \tag{30}$$

Since K is geometric with $p = 1 - E[\bar{F}_S(Y)]$, we have (24). ■

An easier to calculate upper bound is given in the following corollary.

Corollary 2 *Consider a G/G/1/1 system with preemption in service, where Y_n are interarrival times and S_n are service times. The average age of this system is always upper bounded by*

$$\Delta^p_{G/G} \leq \frac{E[Y^2]}{2E[Y]} + \frac{E[Y](1 - E[\bar{F}_S(Y)])}{1 - E[\bar{F}_S(Y)]} + E[\tilde{S}] \tag{31}$$

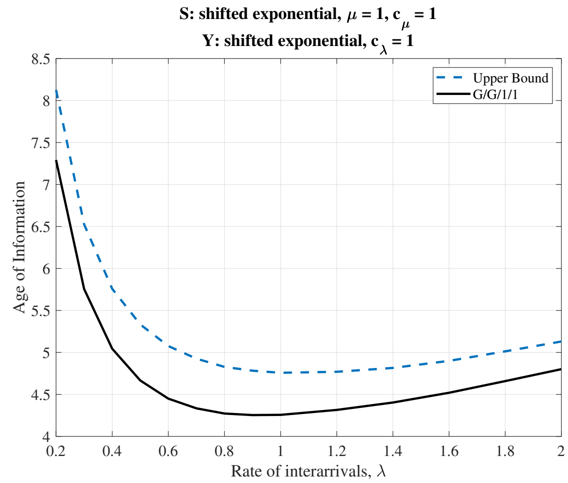
The proof of Corollary 2 follows by noting that $E[Y|Y < S] \leq E[Y]$. The upper bound in (31) is achieved when K is independent of Y_k. An example of this is the multicast model in [15], where random sum parameter K is independent of Y_k.

In order to observe the tightness of the bound in Corollary 2 for a general case, we simulate the same G/G/1/1 system as in the case with blocking discipline, calculate its age using Theorem 3 and compare it to the upper bound in Corollary 2. In Fig. 5, we observe that the difference between the exact age and the upper bound is bounded and small. We also observe that the difference between the exact age and the upper bound depends on the interarrival and service time distributions. The upper bound is tighter for uniform interarrival time and Rayleigh service time distributions than it is for shifted exponential interarrival and service times.

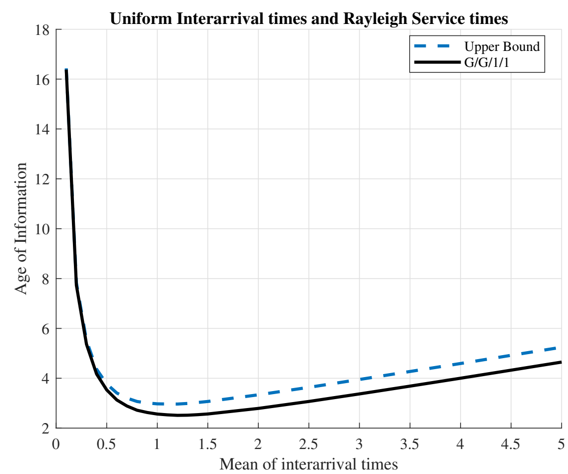
In addition, age curve with respect to the rate parameter of the interarrival times is not monotonic. When λ is very large, in other words when the interarrivals are too frequent, preemption starts to overload the system. Time duration between two successive successful interarrivals gets larger, and hence age increases. This observation for G/G/1/1 systems with preemption in service differs significantly from M/M/1/1 systems with preemption in service, where age is monotonically decreasing in λ [4]. In addition, minimum age for G/G/1/1 systems over the rate parameter is smaller in blocking scenario than it is in preemption in service scenario. However, we know from [4] and [5] that the opposite is true for M/M/1/1 systems. These observations reassure our initial motivation to consider the AoI for G/G/1 systems, as they can behave much more differently than M/M/1 systems.

V. Conclusions

Most real world applications require non-exponential interarrival and service time distributions. This paper is an attempt to extend AoI approach to more practical communication scenarios. We derived exact expressions for and upper bounds to AoI for two service disciplines. We observed that the upper bounds are in general close to exact average age. Designing general communication systems with respect to these upper bounds will result in achievable age values.



(a)



(b)

Fig. 5: Average AoI for G/G/1/1 with preemption in service.

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