# Age of Information in Two-Hop Multicast Networks

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***Abstract*—We consider the age of information in a two-hop multicast network where there is a single source node sending time-sensitive updates to $n^2$ end nodes through $n$ middle nodes. In the first hop, the source node sends updates to $n$ middle nodes, and in the second hop each middle node relays the update packets that it receives to $n$ end users that are connected to it. We study the age of information experienced by the end nodes, and in particular, its scaling as a function of $n$. We show that, using an earliest $k$ transmission scheme, the age of information at the end nodes can be made a constant independent of $n$. In particular, the source node transmits each update packet to the earliest $k_1$ of the $n$ middle nodes, and each middle node that receives the update relays it to the earliest $k_2$ out of $n$ end nodes that are connected to it. We determine the optimum $k_1$ and $k_2$ stopping values for arbitrary shifted exponential link delays.**

## I. Introduction

Recently, with the increase in the number of communication network applications requiring real-time status information, timeliness of the received messages has become a critical and desirable feature for networks. Such applications include sensor networks measuring ambient temperature [1], autonomous vehicular networks where instantaneous vehicle information including velocity, position and acceleration is needed [2] and news reports from Twitter. In all these applications, information loses its value as it becomes stale.

This motivates the study of age of information, which is a metric measuring the freshness of the received information. A typical model to study age of information includes a source which acquires time-stamped status updates from a physical phenomenon. These updates are transmitted over the network to the receiver(s) and the age of information in this network, or simply the age, is the time elapsed since the most recent update at the receiver was generated at the transmitter. In other words, at time $t$, age $\Delta(t)$ of a packet which was generated at time $u(t)$ is $\Delta(t) = t - u(t)$.

Most of the existing work focuses on age analysis in a queuing-theoretic setting. References [3]–[5] study the age under various arrival and service profiles. Reference [6] investigates packet management strategies including blocking and preemption for M/G/1/1 queues. Reference [7] studies multi-hop networks in which update packets are relayed from one node to another. Another line of research studies the age under the energy harvesting setting [8]–[17].

Considering dense IoT deployments and the increase in the number of users in networks supplying time-sensitive information, the scalability of age as a function of the number of nodes has become a critical issue. To this end, we need to study how the age performance of the network changes with growing network size. Reference [18] studies a mobile social network with a single service provider and $n$ communicating users, and shows that under Poisson contact processes among users and uniform rate allocation from the service provider, the average age of the content at the users grows logarithmically in $n$. In contrast, reference [19] observes that in a single-hop multicast network appropriate stopping threshold $k$ can prevent information staleness as the network grows.

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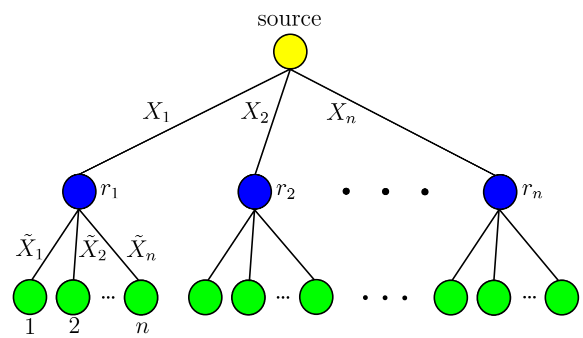


Fig. 1. System model.

Motivated by this observation in [19], we study the scalability of the age in a two-hop multicast network (see Fig. 1) using similar threshold ideas. Extending the results of [19], we first analyze the single-hop problem with exogenous arrivals where the source directly communicates with the end users but cannot generate the updates itself. We then characterize the age for the two-hop case using our single-hop with exogenous arrivals result as a building block. We show that for this two-hop multicast network under i.i.d. shifted-exponential link delays and stopping thresholds $k_1$ and $k_2$ at each hop, an upper bound on the average age can be obtained. Through this upper bound, we show that the average age is limited by a constant as $n$ increases. We determine the optimal stopping thresholds for each stage, $k_1$ and $k_2$, that minimize the average age for arbitrary shifted exponential link delays.

## II. System Model and Age Metric

We consider a two-hop system (see Fig. 1), where in the first hop a single source node broadcasts time-stamped updates to $n$ middle nodes using $n$ links with i.i.d. random delays, and in the second hop, each middle node relays the update packets it receives to $n$ further nodes that are connected to it. An update takes $X$ time to reach from the source node to the mid-level nodes and $\tilde{X}$ time to reach from middle nodes to the end nodes where $X$ and $\tilde{X}$ are shifted exponential random variables with parameters $(\lambda, c)$ and $(\tilde{\lambda}, \tilde{c})$, respectively, where $c$ and $\tilde{c}$ are positive constants.

Age is measured for each of the $n^2$ end nodes and for node $i$ at time $t$ age is the random process $\Delta_i(t) = t - u_i(t)$ where $u_i(t)$ is the time-stamp of the most recent update at that node. When the source node sends out update $j$, it waits for the acknowledgment from the earliest $k_1$ of $n$ middle nodes. After it receives all $k_1$ acknowledgement signals, we say that update $j$ has been completed and the source node generates update $j+1$. At this time, transmissions of the remaining $n-k_1$ packets are terminated. In the second hop, these earliest $k_1$ nodes that received update $j$ start transmitting this update to their end nodes and they stop whenever $k_2$ of their end nodes receive the current update. When the middle nodes finish transmitting the current update to $k_2$ of their children nodes, they wait for the next update delivery. Middle nodes implement a blocking scheme when they are busy transmitting to the end nodes, i.e., they discard arriving packets when they are not idle.

The metric we use, time averaged age, is given by

$$\Delta = \lim_{\tau\to\infty} \frac{1}{\tau} \int_0^\tau \Delta(t)dt \tag{1}$$

where $\Delta(t)$ is the instantaneous age of the last successfully received update as defined above. We will use a graphical argument similar to [19] to derive the average age at an individual end node. Since all link delays are i.i.d. for all nodes and packets, each node $i$ experiences statistically identical age processes and will have the same average age. Therefore, it suffices to focus on a single end user for the age analysis.

## III. Building Block

We first note that at the mid-level what we essentially have is $n$ parent nodes each tied to $n$ subnodes. Therefore, each middle node and its subnodes correspond to the single-hop network analyzed in [19] with one difference: middle-level nodes cannot generate update packets. They can only relay packets sent from the source node. Thus, in this section, we first analyze a single-hop network in which update packets arrive exogenously with a given rate $\mu$. Then, using this network as a building block we analyze the two-hop network described in Section II. We have i.i.d. shifted exponential service time between the source and each of its $n$ subnodes as in [19]. Similarly, transmission of the current update stops when $k$ out of $n$ nodes receive the update. Thus, in this section, we extend the result from [19] to exogenous arrivals, and determine a $k$ threshold which depends on $\lambda$, $c$ and $\mu$.

When the system is empty, a newly received update packet goes into service and stays in the system until earliest $k$ of the total $n$ nodes receive the update packet. In the mean time, any packet reaching the source is discarded since the system is full and the blocking scheme is utilized. When the current update reaches $k$ earliest nodes source terminates the remaining $n-k$ transmissions and begins to wait for the next arrival and then repeats the process when the next update packet arrives.

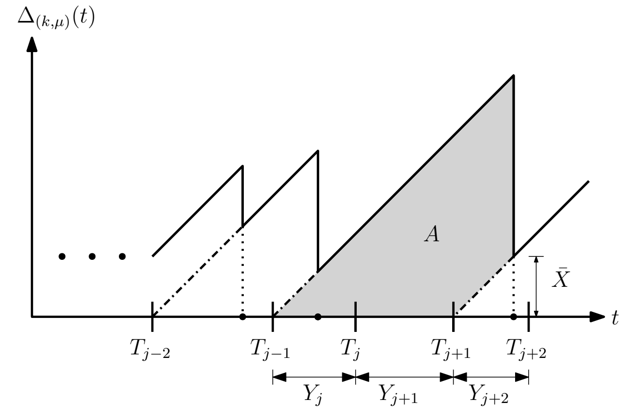


Fig. 2. Sample age evolution $\Delta_{(k,\mu)}(t)$ of an end node. Updates that find the system idle arrive at times $T_i$ at the source. Here, update $j$ arrives at time $T_{j-1}$ and immediately goes into the service. Successful update deliveries are marked with • and in this figure, updates $j-1$, $j$ and $j+2$ are delivered successfully whereas update $j+1$ is terminated.

Since the link delays are i.i.d. end users receive the packet in service with probability $\bar{p} = k/n$. If an end user receives update $j$ and the next one it receives is update $j+M$, then $M$ is geometrically distributed with $\bar{p}$ with moments

$$E[M] = \frac{1}{\bar{p}} = \frac{n}{k}, \qquad E[M^2] = \frac{2-\bar{p}}{\bar{p}^2} = \frac{2n^2}{k^2} - \frac{n}{k} \tag{2}$$

Under this model with i.i.d. link delay $X$, an update takes $X_{k:n}$ units of time to reach $k$ out of $n$ nodes where we denote the $k$th order statistic of random variables $X_1, \ldots, X_n$ as $X_{k:n}$. For shifted exponential random variable $X$, we have

$$E[X_{k:n}] = c + \frac{1}{\lambda}(H_n - H_{n-k}) \tag{3}$$

$$Var[X_{k:n}] = \frac{1}{\lambda^2}(G_n - G_{n-k}) \tag{4}$$

where $H_n = \sum_{j=1}^{n} \frac{1}{j}$ and $G_n = \sum_{j=1}^{n} \frac{1}{j^2}$. Using these,

$$E[X_{k:n}^2] = c^2 + \frac{2c}{\lambda}(H_n - H_{n-k}) + \frac{1}{\lambda^2}\left((H_n - H_{n-k})^2 + G_n - G_{n-k}\right) \tag{5}$$

Note that after each update is completed, the source waits for a random time before the next update arrives since it cannot create the updates itself. This time is denoted by $Z$. Here, $Z$ is a random variable denoting the residual interarrival time before the next update packet arrives upon completion of the current update packet transmission. Thus, the service interval which is the time between two departures from the source is $Y_j = (X_{k:n})_j + Z_j$ for update $j$. Note that update arrival and transmission processes are independent.

Similar to [19], the average age for the earliest $k$ stopping scheme with exogenous packet arrivals with rate $\mu$ is

$$\Delta_{(k,\mu)} = \frac{E[A]}{E[L]} \tag{6}$$

where $A$ denotes the shaded area in Fig. 2 and $L$ is its length. Since an end node waits for $M$ service intervals between two successive deliveries, we have $L = \sum_{i=1}^{M} Y_i$. Thus, we get $E[L] = E[M]E[Y]$. Inspecting Fig. 2 to calculate $A$, we find $E[A] = \frac{1}{2}(E[M]E[Y^2] + E[Y]^2E[M^2 - M]) + E[M]E[Y]E[\bar{X}]$. Here, $\bar{X}$ denotes the service time of a successful update such that $E[\bar{X}] = E[X_i|i \in \mathcal{K}]$, where $\mathcal{K}$ is the set of earliest $k$ nodes that receive the update. This shows that an arriving update packet to the source does not wait in the system given that it finds the system empty. If the system is busy when it arrives it is discarded. This makes sure that the next update to be sent will be the *freshest* at all times. Using these in (6) we obtain

$$\Delta_{(k,\mu)} = E[\bar{X}] + \frac{E[M^2]}{2E[M]}E[Y] + \frac{Var[Y]}{2E[Y]} \tag{7}$$

Thus, we have the following theorem stating the age of a node.

**Theorem 1** *For the earliest $k$ stopping scheme with exogenous arrivals with rate $\mu$ and blocking, the average age at an individual node is*

$$\begin{aligned}\Delta_{(k,\mu)} =& \frac{1}{k}\sum_{i=1}^{k} E[X_{i:n}] + \frac{2n-k}{2k}(E[X_{k:n}] + E[Z]) \\ &+ \frac{Var[X_{k:n} + Z]}{2(E[X_{k:n}] + E[Z])}\end{aligned} \tag{8}$$

*Here, random variable $Z$ is a function of the arrival rate $\mu$.*

**Proof:** The second and third terms are obtained upon substitution of $Y = X_{k:n} + Z$ and $E[M]$ and $E[M^2]$ expressions are derived above. The first term comes from $E[\bar{X}]$ as

$$E[\bar{X}] = E[X_j|j \in \mathcal{K}] = \sum_{i=1}^{k} E[X_{i:n}]Pr[j = i|j \in \mathcal{K}] \tag{9}$$

Since we have $k$ out of $n$ nodes selected independently and identically in $\mathcal{K}$, $Pr[j = i|j \in \mathcal{K}] = \frac{1}{k}$. ■

When we have general interarrival times as we have in this problem, $X_{k:n}$ and $Z$ may be dependent. However, with exponential interarrivals we can show their independence using the memoryless property as follows.

**Corollary 1** *When the arrival process is Poisson with rate $\mu$, the age of an individual node is*

$$\begin{aligned}\Delta_{(k,\mu)} =& \frac{1}{k}\sum_{i=1}^{k} E[X_{i:n}] + \frac{2n-k}{2k\mu}(\mu E[X_{k:n}] + 1) \\ &+ \frac{\mu Var[X_{k:n}]}{2(\mu E[X_{k:n}] + 1)} + \frac{1}{2(\mu^2 E[X_{k:n}] + \mu)}\end{aligned} \tag{10}$$

**Proof:** When the arrival process is Poisson with $\mu$, the random variable $Z$ which corresponds to the residual interarrival time is exponentially distributed with the same parameter and independent of $X$ due to the memoryless property. Then,

$$Var[Y] = Var[X_{k:n} + Z] = Var[X_{k:n}] + Var[Z] \tag{11}$$

and we plug in $E[Z] = \frac{1}{\mu}$ and $Var[Z] = \frac{1}{\mu^2}$. ■

**Corollary 2** *For large $n$ and $n > k$, set $k = \alpha n$. For shifted exponential $(\lambda, c)$ service times $X$, the average age for the earliest $k$ scheme with exogenous Poisson arrivals with rate $\mu$ can be approximated as*

$$\begin{aligned}\Delta_{(k,\mu)} \approx& \frac{c}{\alpha} + \frac{c}{2} + \frac{1}{\lambda} - \frac{1}{2\lambda}\log(1-\alpha) + \\ &+ \frac{1}{\alpha\mu} - \frac{1}{2\mu} + \frac{1}{2}\left(\mu^2 c - \frac{\mu^2\log(1-\alpha)}{\lambda} + \mu\right)^{-1}\end{aligned} \tag{12}$$

**Proof:** Using the order statistics above,

$$\delta_1 = \frac{1}{k}\sum_{i=1}^{k} E[X_{i:n}] = c + \frac{H_n}{\lambda} - \frac{1}{k\lambda}\sum_{i=1}^{k} H_{n-i} \tag{13}$$

As in [19], we have $\sum_{i=1}^{k} H_{n-i} = \sum_{i=1}^{n-1} H_i - \sum_{i=1}^{n-k-1} H_i$ and the series identity $\sum_{i=1}^{k} H_i = (k+1)(H_{k+1} - 1)$. Using these we get

$$\delta_1 = c + \frac{1}{\lambda} - \frac{n-k}{k\lambda}(H_n - H_{n-k}) \approx c + \frac{1}{\lambda} + \frac{1-\alpha}{\alpha\lambda}\log(1-\alpha) \tag{14}$$

since for large $n$, we have $H_i \approx \log(i) + \gamma$. Also,

$$\begin{aligned}\delta_2 =& \frac{2n-k}{2\mu k}(\mu E[X_{k:n}] + 1) & (15)\\ =& \frac{2n-k}{2\mu k}\left(\mu\left(c + \frac{H_n - H_{n-k}}{\lambda}\right) + 1\right) & (16)\\ \approx& \frac{(2-\alpha)c}{2\alpha} + \frac{\alpha - 2}{2\alpha\lambda}\log(1-\alpha) + \frac{2-\alpha}{2\alpha\mu} & (17)\end{aligned}$$

Next, we note that we have

$$\lim_{n\to\infty} \frac{\mu Var[X_{k:n}]}{2(\mu E[X_{k:n}] + 1)} = 0 \tag{18}$$

We see this from the expected values of order statistics,

$$\frac{\mu Var[X_{k:n}]}{2(\mu E[X_{k:n}] + 1)} = \frac{\mu(G_n - G_{n-k})}{2(\mu\lambda^2 c + \mu\lambda(H_n - H_{n-k}) + \lambda^2)} \tag{19}$$

We know that the sequence $G_n$ converges to $\frac{\pi^2}{6}$. As $n$ increases $G_{n-k} = G_{(1-\alpha)n}$ also goes to the same value making the numerator 0. Thus, as $n$ tends to $\infty$ (18) is achieved. Similarly,

$$\delta_3 = \frac{1}{2(\mu^2 E[X_{k:n}] + \mu)} \approx \frac{1}{2}\left(\mu^2 c - \frac{\mu^2\log(1-\alpha)}{\lambda} + \mu\right)^{-1} \tag{20}$$

Summing $\delta_1$, $\delta_2$, and $\delta_3$ yields the expression. ■

Note that the age expression when $n$ is large is a function of the ratio $\alpha = k/n$ implying that the age still converges to a constant even when the packets arrive exogenously.

Although there is no explicit closed form solution for the optimal $\alpha$, denoted as $\alpha^*$, which minimizes (12) we can calculate it numerically. For instance, when $n = 100$, $\lambda = 1$, $c = 1$ and Poisson arrival rate $\mu = 1$, age minimizing $\alpha^* = 0.84$. This optimal value is higher than that of the original case in which the source itself generates the packets,

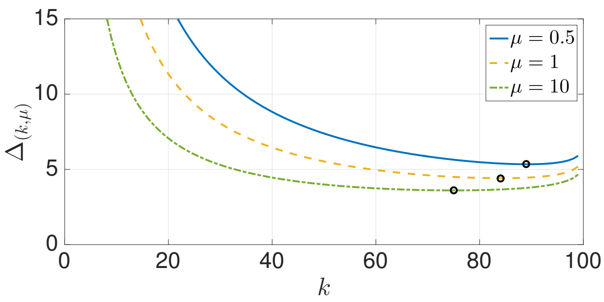


Fig. 3. Average age as a function of stopping threshold $k$ with exogenous Poisson arrivals with rate $\mu$ and shifted exponential service times with $\lambda = 1$ and $c = 1$. $\circ$ marks the minimized average age $\Delta_{(k,\mu)}$.

which is $\alpha^* = 0.73$ found in [19]. We see that when the Poisson arrival rate is increased $\alpha^*$ decreases. This is expected because when the arrival rate is high, the source node prefers to wait for the *freshest* one instead of sending the current update to more and more end users. Similarly, when the arrival rate is low, $\alpha^*$ is higher because in this case source knows that interarrival time is higher so that before it waits for the next packet it wants to update as many end nodes as it can (see Fig. 3). We also note that as we take $\mu \to \infty$ in (12), we get

$$\Delta_{(k)} \approx \frac{c}{\alpha} + \frac{c}{2} + \frac{1}{\lambda} - \frac{1}{2\lambda}\log(1-\alpha) \tag{21}$$

which is the age expression in [19]. Thus, age expression under exogenous Poisson arrivals with rate $\mu$ converges to the case in which source generates the packets itself as $\mu$ tends to $\infty$.

## IV. Two-Hop Network

Using the building block problem solved in the previous section we are now ready to solve our two-hop problem described in Section II. Note that in our model middle nodes cannot generate updates rather they receive them from the source node. Thus, each middle node and its $n$ subnodes can be modeled as in Section III. Since the source node sends updates to the first $k_1$ of its nodes, a middle node receives a certain update packet with probability $p_1 = \frac{k_1}{n}$. Assume it receives update $j$ and the next one it receives is update $j+M_1$. As in the building block problem, this $M_1$ is a geometrically distributed random variable with parameter $p_1$. Since the source generates a new update once it is delivered to $k_1$ middle nodes, the service interval for the first stage is $Y_1 = X_{k_1:n}$.

When a middle node successfully receives a certain update it is delivered to one of its end nodes with probability $p_2 = \frac{k_2}{n}$. Similar to the first stage we have random variable $M_2$ which is geometrically distributed with parameter $p_2$ denoting the number of cycles between successive updates to an end node given that its parent node recieves the updates. Middle nodes transmit the update until it is delivered to $k_2$ of their subnodes. While they are busy transmitting the current update middle nodes discard all other updates reaching them. Upon successful delivery to $k_2$ nodes they wait for some time denoted by random variable $Z$. Thus, the service interval for the second stage is $Y_2 = \tilde{X}_{k_2:n} + Z$.

Note that in this model a successful update reaches an end node without waiting in the system. Thus, the service time of a successful update denoted by $\bar{X}$ is the sum of link delays in each stage and corresponds to the total time spent in the system by that update. Then, the total service time of a successful update delivered to some node $i$ through middle node $l$ is $E[\bar{X}] = E[X_l | l \in \mathcal{K}] + E[\tilde{X}_i | i \in \mathcal{K}_l]$. Here the set $\mathcal{K}$ is the set of first $k_1$ middle nodes that receive the update and the set $\mathcal{K}_l$ defined for each $l$ in $K$ is the set of first $k_2$ end nodes receiving the update. Thus, for an update to reach an end node that end node has to be among the earliest $k_2$ subnodes of its middle node and the corresponding middle node has to be one of the earliest $k_1$ middle nodes. Thus, by using (8), the average age of an end node under this model is given as follows.

**Theorem 2** *For the earliest $k_1$, $k_2$ stopping scheme, the average age at an individual end node is*

$$\begin{aligned}\Delta_{(k_1,k_2)} =& \frac{1}{k_1}\sum_{i=1}^{k_1} E[X_{i:n}] + \frac{1}{k_2}\sum_{i=1}^{k_2} E[\tilde{X}_{i:n}] \\ &+ \frac{2n-k_2}{2k_2}(E[\tilde{X}_{k_2:n}] + E[Z]) \\ &+ \frac{Var[\tilde{X}_{k_2:n} + Z]}{2(E[\tilde{X}_{k_2:n}] + E[Z])}\end{aligned} \tag{22}$$

Note that here, random variable $Z$ is a function of $k_1$ and $X_{k_1:n}$. Above result follows from Theorem 1 upon observing that the second stage is the same as the building block problem. However, we now need to take both stages into consideration to determine $\bar{X}$ as explained above.

This theorem is valid for any distribution for $X$ and $\tilde{X}$. A middle node receives an update in every $M_1$ update cycles. Since each update cycle takes $X_{k_1:n}$ units of time, update interarrival time to middle nodes is $\sum_{i=1}^{M_1}(X_{k_1:n})_i$. Thus, mean interarrival time is $E[M_1]E[X_{k_1:n}]$. Random variable $Z$ denotes the residual interarrival time before the next update arrives at the middle node. When we no longer have exponential interarrival times, it is not easy to determine the first and second order statistics of $Z$. However, using [20]–[22], it can be shown that, age of our model $\Delta_{(k_1,k_2)}$, can be upper bounded by the age under exponential interarrivals to middle nodes with the same mean $E[M_1]E[X_{k_1:n}]$, which is denoted by $\Delta'_{(k_1,k_2)}$. The proof that $\Delta'_{(k_1,k_2)}$ forms an upper bound for $\Delta_{(k_1,k_2)}$ is omitted here due to space limitations but it follows from the DMRL (decreasing mean residual life) property of interarrival times and NBUE (new better than used in expectation) property of service times. Thus, since $\Delta_{(k_1,k_2)} \leq \Delta'_{(k_1,k_2)}$ all we need to show is that the latter one is upper bounded by a constant as the network grows.

**Corollary 3** *Assuming exponential interarrivals to middle nodes with mean $E[M_1]E[X_{k_1:n}]$, the average age at an end node under the earliest $k_1$, $k_2$ stopping scheme is*

$$\Delta'_{(k_1,k_2)} = \frac{1}{k_1}\sum_{i=1}^{k_1} E[X_{i:n}] + \frac{1}{k_2}\sum_{i=1}^{k_2} E[\tilde{X}_{i:n}]$$

$$+\frac{2n-k_2}{2k_2}E[\tilde{X}_{k_2:n}]+\frac{2n^2-nk_2}{2k_1k_2}E[X_{k_1:n}]$$
$$+\frac{k_1Var[\tilde{X}_{k_2:n}]}{2(k_1E[\tilde{X}_{k_2:n}]+nE[X_{k_1:n}])}$$
$$+\frac{n^2E[X_{k_1:n}]^2}{2k_1(k_1E[\tilde{X}_{k_2:n}]+nE[X_{k_1:n}])} \quad (23)$$

**Proof:** When the interarrivals are exponential $Z$ is also exponential with mean $E[M_1]E[X_{k_1:n}]$ because of the memoryless property. Then, $Var[Z] = E[M_1]^2E[X_{k_1:n}]^2$ where $M_1$ is geometrically distributed with $p_1 = \frac{k_1}{n}$. Combining these and noting that $Z$ and $\tilde{X}_{k_2:n}$ are independent yields the result. ■

**Corollary 4** *Assuming $n$ is large and $n > k_1$ and $n > k_2$ and letting $k_1 = \alpha_1 n$ and $k_2 = \alpha_2 n$, under exponential interarrival assumption to middle nodes with mean $E[M_1]E[X_{k_1:n}]$, and shifted exponential service times $X$ with $(\lambda, c)$ and $\tilde{X}$ with $(\tilde{\lambda}, \tilde{c})$, the average age for the earliest $k_1, k_2$ scheme can be approximated as*

$$\Delta'_{(k_1,k_2)} \approx \frac{1}{\lambda}+\frac{1}{\tilde{\lambda}}+\frac{\tilde{c}}{\alpha_2}+\frac{\tilde{c}}{2}-\frac{1}{2\tilde{\lambda}}\log(1-\alpha_2)$$
$$+\frac{2-\alpha_2+2\alpha_1\alpha_2}{2\alpha_1\alpha_2}c+\frac{\tilde{\lambda}K_1^2}{2\alpha_1\lambda[\lambda\alpha_1K_2+\tilde{\lambda}K_1]}$$
$$+\frac{3\alpha_2-2\alpha_1\alpha_2-2}{2\alpha_1\alpha_2\lambda}\log(1-\alpha_1) \quad (24)$$

*where $K_1 = (\lambda c - \log(1-\alpha_1))$ and $K_2 = (\tilde{\lambda}\tilde{c} - \log(1-\alpha_2))$.*

With this corollary we showed that $\Delta'_{(k_1,k_2)}$ derived in Corollary 3 is independent of $n$ for large $n$. Since it upper bounds our age expression $\Delta_{(k_1,k_2)}$, we conclude that age under the earliest $k_1$, $k_2$ stopping scheme for two-hop multicast networks is also independent of $n$ for large $n$ and is bounded by a constant as the number of end nodes increases.

## V. Numerical Results

In this section, we provide simple numerical results. In the two-hop network, in order to optimize the age, we need to select appropriate $k_1$ and $k_2$ values, i.e., optimum ratios $\alpha_1^*$ and $\alpha_2^*$. When $\lambda = \tilde{\lambda} = 1$, $c = \tilde{c} = 1$ and $n = 100$, we obtain $\alpha_1^* = 0.62$ and $\alpha_2^* = 0.92$. This shows that when all link delays are statistically identical, to achieve a good age performance, we need to be more aggressive in the second stage than the first stage.

Moreover, we observe that $\alpha_2$ is responsive to the changes in the parameters of the first stage. This is intuitive because as $k_1$ varies, the mean interarrival time of the second stage changes. As shown in Fig. 4, for the same $(\tilde{\lambda}, \tilde{c})$ pair, when the mean interarrival time gets lower by increasing $\lambda$, $\alpha_2^*$ gets lower as well. Knowing that the next update arrival is not far away, middle nodes tend to wait for the next one instead of sending the current packet to more and more end users when the arrivals are frequent. This is exactly what we observed in the building block problem with exogenous update arrivals. Note that in addition to the parameters of the first stage, $\alpha_2$ also depends on those of the second stage.

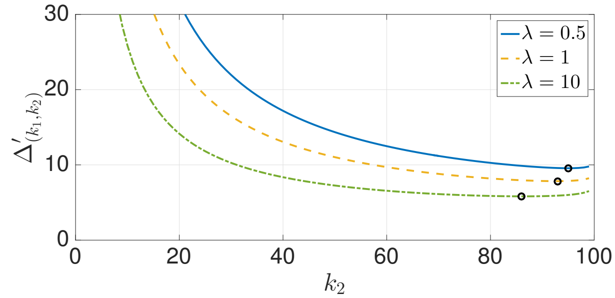


Fig. 4. Under exponential arrivals to middle nodes with mean $E[M_1]E[X_{k_1:n}]$, the average age as a function of stopping threshold $k_2$ with $(\lambda, c)$ and $(\tilde{\lambda}, \tilde{c}) = (1, 1)$ in the first and second stages respectively. ∘ marks the minimized average age $\Delta'_{(k_1,k_2)}$.

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