Ergodic Capacity Region of Fading Gaussian Multiple Access Channels with Common Data*

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Abstract

A Gaussian multiple access channel with common data is considered. Capacity region when there is no fading is known [1]. We give a characterization of the ergodic capacity region when there is fading, and both the transmitters and the receiver know the channel perfectly. Then, we provide an iterative method for the numerical computation of the ergodic capacity region.

1 Introduction

Correlated data arises naturally in many applications of wireless communications. It arises mainly for three reasons: the observed data is correlated (as in sensor networks) [2–4], the correlated data is created by communication between the transmitters (as in user cooperation diversity) [5], and correlated data results from decoding the data coming from previous stages of a larger network (as in relaying and multihopping in ad-hoc wireless networks) [6–9]. We investigate a multiple access channel (MAC) with correlated data in the sense of Slepian and Wolf [10], which we will call *common data*. The two transmitters each have their individual messages, which will be denoted by W_1 and W_2, respectively. Also, there is a common message W_0, which is known to both transmitters. All three messages are independent. The goal is to determine the rates, R_0, R_1 and R_2, at which all three messages can be decoded with negligible error. The capacity will be a volume in the three dimensional space. This model includes the traditional MAC as a special case, when $R_0 = 0$. It also includes the two-transmitter one-receiver point-to-point system as a special case, when $R_1 = R_2 = 0$, except that we have individual power constraints for the two transmitters instead of a single sum power constraint as in the point-to-point system.

Slepian and Wolf established the capacity region of MAC with common data for discrete memoryless channels in [10]. Prelov and van der Meulen gave the capacity expression for a Gaussian MAC with common data in [1]. We concentrate on the case where there is fading in the channel and obtain a characterization of the ergodic capacity region. Then, we provide an iterative method for the numerical computation of the ergodic capacity region.

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2 System Model

The Gaussian MAC we consider in this paper has two transmitters and one receiver. Without fading, the input and output are related as

$$Y = X_1 + X_2 + Z \tag{1}$$

where Z is a Gaussian random variable with zero-mean and unit-variance. Transmitters 1 and 2 are subject to power constraints $\bar{P}_1$ and $\bar{P}_2$, respectively. We have three independent messages W_0, W_1 and W_2. Transmitter 1 knows W_0 and W_1, and transmitter 2 knows W_0 and W_2. Therefore, X_1 is a function of W_0, W_1, and X_2 is a function of W_0, W_2.

A rate triplet (R_0, R_1, R_2) is *achievable* if there exists a sequence of $((2^{nR_0}\times 2^{nR_1}, 2^{nR_0}\times 2^{nR_2}), n)$ codes with average probability of error approaching zero as n goes to infinity. Here, the probability of error is the probability that any of the three messages is decoded incorrectly. The *capacity region* is the closure of the set of achievable (R_0, R_1, R_2).

With fading, the input and the output are related as

$$Y(k) = \sqrt{H_1(k)}X_1(k) + \sqrt{H_2(k)}X_2(k) + Z(k) \tag{2}$$

where $X_i(k)$ and $H_i(k)$ are the transmitted symbol and fading process of user i and $Z(k)$ is the zero-mean, unit-variance white Gaussian noise sample, at time k. $H_1(k)$ and $H_2(k)$ are jointly stationary and ergodic, and the stationary distribution has continuous density. The user signals are subject to average power constraints of $\bar{P}_1$ and $\bar{P}_2$. We assume that both the transmitters and the receiver know $H_1(k)$ and $H_2(k)$ causally. The *ergodic capacity region* is the closure of the set of achievable rates in this scenario. For notational convenience, let $C(x) = \frac{1}{2}\log(1+x)$.

3 Capacity Region without Fading

The capacity region of the Gaussian MAC with common data is all triplets (R_0, R_1, R_2) [1]

$$R_1 \le C\left(\alpha\bar{P}_1\right) \tag{3}$$

$$R_2 \le C\left(\beta\bar{P}_2\right) \tag{4}$$

$$R_1 + R_2 \le C\left(\alpha\bar{P}_1 + \beta\bar{P}_2\right) \tag{5}$$

$$R_0 + R_1 + R_2 \le C\left(\bar{P}_1 + \bar{P}_2 + 2\sqrt{(1-\alpha)(1-\beta)\bar{P}_1\bar{P}_2}\right) \tag{6}$$

for some α and β such that $0 \le \alpha \le 1$ and $0 \le \beta \le 1$.

An alternative representation of the capacity region is obtained by defining $P_1 = \alpha\bar{P}_1$, $P_2 = \beta\bar{P}_2$. With these definitions, the capacity region is all triplets (R_0, R_1, R_2) such that

$$R_1 \le C\left(P_1\right) \tag{7}$$

$$R_2 \le C\left(P_2\right) \tag{8}$$

$$R_1 + R_2 \le C\left(P_1 + P_2\right) \tag{9}$$

$$R_0 + R_1 + R_2 \le C\left(P_1 + P_2 + P_0\right) \tag{10}$$

for some $0 \le P_1 \le \bar{P}_1$, $0 \le P_2 \le \bar{P}_2$ and $P_0 = \left(\sqrt{\bar{P}_1 - P_1} + \sqrt{\bar{P}_2 - P_2}\right)^2$.

We can interpret the capacity region in the following way. Transmitter 1 spends power P_1 for transmitting its individual message, W_1, and the remaining power, $\bar{P}_1 - P_1$, for transmitting the common message, W_0. Similarly, transmitter 2 spends power P_2 for transmitting its individual message, W_2, and the remaining power, $\bar{P}_2 - P_2$, for transmitting the common message. Since the common message is known to both transmitters, the effective received power for the common message is P_0, which may also be interpreted as the beamforming gain as in a two-transmitter one-receiver point-to-point system.

Yet another way to write the capacity region, which will be useful for the development of the fading case, is the following. The capacity region is all triplets (R_0, R_1, R_2) such that inequalities (7)-(10) hold true for some P_1, P_2, $P_0 \geq 0$, $0 \leq \rho \leq 1$ such that $P_1 + \rho^2 P_0 = \bar{P}_1$ and $P_2 + (1-\rho)^2 P_0 = \bar{P}_2$. This representation of the capacity region can be interpreted as follows: P_1, P_2 and P_0 are the received powers for messages W_1, W_2 and W_0, respectively. In order for the received power for the common message to be P_0, transmitter 1 spends $\rho^2 P_0$ power and transmitter 2 spends $(1-\rho)^2 P_0$ power. Note that the two powers add up to less than P_0 which is to be expected because there is a beamforming gain for the common message. Transmitter 1 spends a total of $P_1 + \rho^2 P_0$ power, and this must equal the power constraint $\bar{P}_1$, and transmitter 2 spends a total of $P_2 + (1-\rho)^2 P_0$ power and this must equal $\bar{P}_2$.

4 Capacity Region in Fading

Consider the system model (2), in the simple case when $H_1(k) = h_1$ and $H_2(k) = h_2$ for all k, using the representation of the capacity region with P_0, P_1, P_2 and ρ, the capacity region is the set of all triplets (R_0, R_1, R_2) such that inequalities (7)-(10) hold true for some P_1, P_2, $P_0 \geq 0$, $0 \leq \rho \leq 1$ such that $\frac{1}{h_1} P_1 + \frac{\rho^2}{h_1} P_0 = \bar{P}_1$ and $\frac{1}{h_2} P_2 + \frac{(1-\rho)^2}{h_2} P_0 = \bar{P}_2$. Here, again, P_1, P_2 and P_0 are all received powers.

Now, we consider the case where the channel is time-varying and both the transmitters and the receiver track the channel perfectly. Let us denote the channel state as a vector $\mathbf{h} = [h_1, h_2]^T$. Let $p_1(\mathbf{h})$, $p_2(\mathbf{h})$ and $p_0(\mathbf{h})$ be three mappings from the channel state space to the received powers in $\mathbb{R}_+$. Also, let us define $\rho(\mathbf{h})$ to be a mapping from the channel state space to [0,1]. Then, heuristically, when the channel state is $\mathbf{h}$, $\frac{p_1(\mathbf{h})}{h_1}$ is the power that transmitter 1 uses for W_1, and $\frac{\rho(\mathbf{h})^2 p_0(\mathbf{h})}{h_1}$ is the power that transmitter 1 uses for W_0. Similarly, $\frac{p_2(\mathbf{h})}{h_2}$ is the power that transmitter 2 uses for W_2, and $\frac{(1-\rho(\mathbf{h}))^2 p_0(\mathbf{h})}{h_2}$ is the power that transmitter 2 uses for W_0. Let $\mathbf{p} = [p_1, p_2, p_0]^T$ be the received power vector and, let $\mathcal{C}_f(\mathbf{p}, \rho)$ be the set of (R_0, R_1, R_2) such that

$$R_1 \leq E\left[C\left(p_1(\mathbf{h})\right)\right] \stackrel{\triangle}{=} f_1(\mathbf{p}, \rho) \tag{11}$$

$$R_2 \leq E[C\left(p_2(\mathbf{h})\right)] \stackrel{\triangle}{=} f_2(\mathbf{p}, \rho) \tag{12}$$

$$R_1 + R_2 \leq E[C\left(p_1(\mathbf{h}) + p_2(\mathbf{h})\right)] \stackrel{\triangle}{=} f_3(\mathbf{p}, \rho) \tag{13}$$

$$R_0 + R_1 + R_2 \leq E[C\left(p_1(\mathbf{h}) + p_2(\mathbf{h}) + p_0(\mathbf{h})\right)] \stackrel{\triangle}{=} f_4(\mathbf{p}, \rho) \tag{14}$$

where the expectation is taken over the joint stationary distribution of the fading states h_1 and h_2.

Theorem 1 *The ergodic capacity region of the fading Gaussian MAC with common data when perfect channel state information is available at the transmitters and the receiver is*

$$\mathcal{C}(\bar{P}_1, \bar{P}_2) = \bigcup_{(p_1, p_2, p_0, \rho) \in \mathcal{F}} \mathcal{C}_f(\mathbf{p}, \rho) \tag{15}$$

where

$$\mathcal{F} = \Bigg\{ (p_1(\mathbf{h}), p_2(\mathbf{h}), p_0(\mathbf{h}), \rho(\mathbf{h})) : p_0(\mathbf{h}), p_1(\mathbf{h}), p_2(\mathbf{h}) \geq 0, 0 \leq \rho(\mathbf{h}) \leq 1,$$
$$E\left[\frac{p_1(\mathbf{h})}{h_1} + \frac{\rho(\mathbf{h})^2 p_0(\mathbf{h})}{h_1}\right] \leq \bar{P}_1, E\left[\frac{p_2(\mathbf{h})}{h_2} + \frac{(1-\rho(\mathbf{h}))^2 p_0(\mathbf{h})}{h_2}\right] \leq \bar{P}_2 \Bigg\} \quad (16)$$

The proof of Theorem 1 is omitted due to space limitations.

To explicitly characterize the capacity region, we solve for the boundary surface of the capacity region. As in [11], the boundary surface of the capacity region $\mathcal{C}(\bar{P}_1, \bar{P}_2)$ is the closure of all points $\mathbf{R}^* = (R_0^*, R_1^*, R_2^*)$ such that $\mathbf{R}^*$ is a solution to the problem

$$\max_{\mathbf{R}} \quad \mu_1 R_1 + \mu_2 R_2 + \mu_0 R_0 \quad \text{subject to } \mathbf{R} \in \mathcal{C}(\bar{P}_1, \bar{P}_2) \quad (17)$$

for some $\mu = [\mu_0, \mu_1, \mu_2]^T \in \mathbb{R}_+^3$. This optimization problem is equivalent to

$$\max_{(\mathbf{R}, \tilde{P}_1, \tilde{P}_2) \in X} \mu_1 R_1 + \mu_2 R_2 + \mu_0 R_0 \quad \text{subject to } \tilde{P}_1 \leq \bar{P}_1, \tilde{P}_2 \leq \bar{P}_2 \quad (18)$$

where

$$X = \{(\mathbf{R}, \tilde{P}_1, \tilde{P}_2) : \tilde{P}_1, \tilde{P}_2 \in \mathbb{R}_+, \mathbf{R} \in \mathcal{C}(\tilde{P}_1, \tilde{P}_2)\} \quad (19)$$

It can be shown that X is a convex set and hence, there exist lagrange multipliers $\lambda = [\lambda_1, \lambda_2]^T \in \mathbb{R}_+^2$ such that $\mathbf{R}^*$ is a solution to the optimization problem

$$\max_{(\mathbf{R}, \tilde{P}_1, \tilde{P}_2) \in X} \mu_1 R_1 + \mu_2 R_2 + \mu_0 R_0 - \lambda_1 \tilde{P}_1 - \lambda_2 \tilde{P}_2 \quad (20)$$

Since $\mathcal{C}(\tilde{P}_1, \tilde{P}_2)$ is a union over $\mathcal{C}_f(\mathbf{p}, \rho)$, we first express $(\mathbf{R}, \tilde{P}_1, \tilde{P}_2)$ in terms of $(\mathbf{p}, \rho)$ and then optimize over $(\mathbf{p}, \rho)$. It can be seen that the capacity region is unchanged if we replace the two power constraint inequalities with equalities in (16). Hence,

$$E\left[\frac{p_1(\mathbf{h})}{h_1} + \frac{\rho(\mathbf{h})^2 p_0(\mathbf{h})}{h_1}\right] = \tilde{P}_1 \quad (21)$$
$$E\left[\frac{p_2(\mathbf{h})}{h_2} + \frac{(1-\rho(\mathbf{h}))^2 p_0(\mathbf{h})}{h_2}\right] = \tilde{P}_2 \quad (22)$$

Instead of considering all $\mathbf{R} \in \mathcal{C}(\tilde{P}_1, \tilde{P}_2)$, it suffices to consider $\mathbf{R} \in \mathcal{C}_f(\mathbf{p}, \rho)$ that maximizes $\mu_0 R_0 + \mu_1 R_1 + \mu_2 R_2$ for each $(\mathbf{p}, \rho)$. Thus, we first focus on the following problem:

$$\max_{\mathbf{R}} \quad \mu_1 R_1 + \mu_2 R_2 + \mu_0 R_0 \quad \text{subject to } \mathbf{R} \in \mathcal{C}_f(\mathbf{p}, \rho) \quad (23)$$

where $\mathcal{C}_f(\mathbf{p}, \rho)$ is a region with shape as in Figure 1. Due to the nature of $\mathcal{C}_f(\mathbf{p}, \rho)$, when $\mu_0 \geq \max(\mu_1, \mu_2)$, point $a = [0, 0, f_4(\mathbf{p}, \rho)]$ achieves the maximum. When $\mu_1 \geq \mu_0 \geq \mu_2$, point $b = [f_1(\mathbf{p}, \rho), 0, f_4(\mathbf{p}, \rho) - f_1(\mathbf{p}, \rho)]$ achieves the maximum. When $\mu_1 \geq \mu_2 \geq \mu_0$, point $c = [f_1(\mathbf{p}, \rho), f_3(\mathbf{p}, \rho) - f_1(\mathbf{p}, \rho), f_4(\mathbf{p}, \rho) - f_3(\mathbf{p}, \rho)]$ achieves the maximum. When $\mu_2 \geq \mu_1 \geq \mu_0$, point $d = [f_3(\mathbf{p}, \rho) - f_2(\mathbf{p}, \rho), f_2(\mathbf{p}, \rho), f_4(\mathbf{p}, \rho) - f_3(\mathbf{p}, \rho)]$ achieves the maximum. When $\mu_2 \geq \mu_0 \geq \mu_1$, point $e = [0, f_2(\mathbf{p}, \rho), f_4(\mathbf{p}, \rho) - f_2(\mathbf{p}, \rho)]$ achieves the maximum. Hence, the optimization problem as defined in (23) is solved and the solution

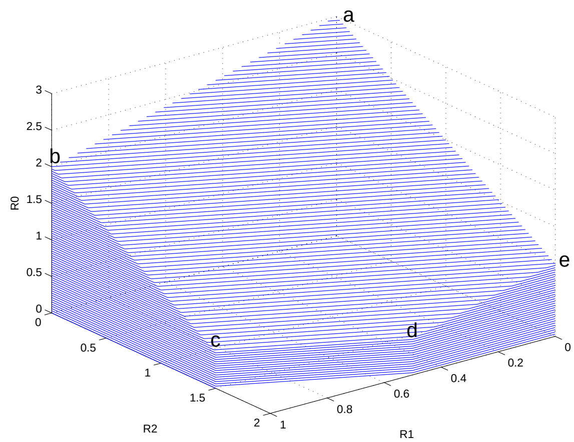


Figure 1: The region $\mathcal{C}_f(\mathbf{p}, \rho)$.

is expressed in terms of $\mathbf{p}, \rho$.

We will consider three cases: 1) $\mu_0 \geq \max(\mu_1, \mu_2)$, 2) $\mu_1 \geq \mu_0 \geq \mu_2$ and 3) $\mu_1 \geq \mu_2 \geq \mu_0$, since $\mu_2 \geq \mu_0 \geq \mu_1$ is the same as case 2) and $\mu_2 \geq \mu_1 \geq \mu_0$ is the same as case 3) by swapping indices 1 and 2.

1) When $\mu_0 \geq \max(\mu_1, \mu_2)$, the optimization problem in (20) is equivalent to

$$\min_{\mathbf{p} \geq 0, 0 \leq \rho \leq 1} E\left[-\mu_0 \log\left(1 + p_1 + p_2 + p_0\right) + \lambda_1 \left(\frac{p_1}{h_1} + \frac{\rho^2}{h_1} p_0\right) + \lambda_2 \left(\frac{p_2}{h_2} + \frac{(1-\rho)^2}{h_2} p_0\right)\right] \quad (24)$$

Since the cost function is an expectation and the probability distributions are nonnegative, it suffices to consider the minimization for a fixed channel state (h_1, h_2), i.e.,

$$\min_{\mathbf{p} \geq 0, 0 \leq \rho \leq 1} \quad -\mu_0 \log\left(1 + p_1 + p_2 + p_0\right) + \lambda_1 \left(\frac{p_1}{h_1} + \frac{\rho^2}{h_1} p_0\right) + \lambda_2 \left(\frac{p_2}{h_2} + \frac{(1-\rho)^2}{h_2} p_0\right) \quad (25)$$

Though the cost function is not convex in $(\mathbf{p}, \rho)$, it is a quadratic function of ρ when $\mathbf{p}$ is fixed. The optimal ρ^* is

$$\rho^*(\mathbf{h}) = \frac{\frac{h_1}{\lambda_1}}{\frac{h_1}{\lambda_1} + \frac{h_2}{\lambda_2}} \quad (26)$$

Since the dependencies of the cost functions on ρ in all three cases are the same, $\rho^*(\mathbf{h})$ is in fact the optimal solution for all three cases. Thus we proceed with ρ^* in place of ρ and the problem becomes convex. We write the Karush-Kuhn-Tucker (KKT) necessary conditions as follows:

$$-\frac{\mu_0}{1 + p_1 + p_2 + p_0} + \frac{1}{\frac{h_1}{\lambda_1}} - \omega_1 = 0 \quad (27)$$

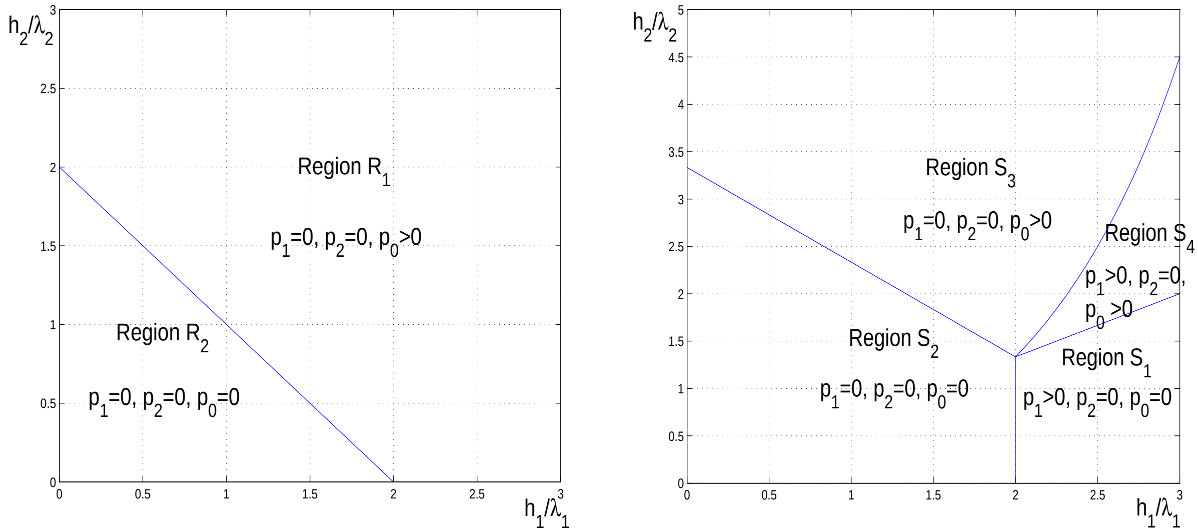
$$-\frac{\mu_0}{1 + p_1 + p_2 + p_0} + \frac{1}{\frac{h_2}{\lambda_2}} - \omega_2 = 0 \quad (28)$$

$$-\frac{\mu_0}{1 + p_1 + p_2 + p_0} + \frac{1}{\frac{h_1}{\lambda_1} + \frac{h_2}{\lambda_2}} - \omega_0 = 0 \quad (29)$$

$$p_1, p_2, p_0, \omega_0, \omega_1, \omega_2 \geq 0 \quad (30)$$

$$\omega_0 p_0 = \omega_1 p_1 = \omega_2 p_2 = 0 \quad (31)$$

where ω_0, ω_1 and ω_2 are the complementary slackness variables. The KKTs have a unique



(a) $\mu_0 \geq \max(\mu_1, \mu_2)$. (b) $\mu_1 \geq \mu_0 \geq \mu_2$.

Figure 2: Power control policies.

global optimum solution. Let us define two regions in $\mathbb{R}_+^2$,

$$\mathcal{R}_1 = \{(x, y) : x + y \geq \frac{1}{\mu_0}\} \tag{32}$$

$$\mathcal{R}_2 = \{(x, y) : x + y < \frac{1}{\mu_0}\} \tag{33}$$

Then, the optimum solution is

$$p_0(\mathbf{h}) = \begin{cases} \mu_0(\frac{h_1}{\lambda_1} + \frac{h_2}{\lambda_2}) - 1, & \text{if } \left(\frac{h_1}{\lambda_1}, \frac{h_2}{\lambda_2}\right) \in \mathcal{R}_1 \\ 0, & \text{if } \left(\frac{h_1}{\lambda_1}, \frac{h_2}{\lambda_2}\right) \in \mathcal{R}_2 \end{cases} \tag{34}$$

$$p_1(\mathbf{h}) = 0 \tag{35}$$

$$p_2(\mathbf{h}) = 0 \tag{36}$$

The transmit powers can be found by dividing these received powers with corresponding channel gains. As can be seen, in the case of $\mu_0 \geq \max(\mu_1, \mu_2)$, the transmitters use their entire power to transmit the common message; they do not allocate any power to transmit their individual messages. When $\frac{h_1}{\lambda_1} + \frac{h_2}{\lambda_2} \geq \frac{1}{\mu_0}$, i.e., the combined channel is good enough, the transmitters transmit the common message using beamforming as if we have a two-transmitter one-receiver point-to-point system. When the channel is poor, i.e., $\frac{h_1}{\lambda_1} + \frac{h_2}{\lambda_2} < \frac{1}{\mu_0}$, the transmitters both keep silent and save their powers for better channel states. This is shown in Figure 2(a).

2) When $\mu_1 \geq \mu_0 \geq \mu_2$, the optimization problem in (20) is equivalent to

$$\min_{\mathbf{p} \geq 0, 0 \leq \rho \leq 1} E\Bigg[-\mu_0 \log(1 + p_1 + p_2 + p_0) - (\mu_1 - \mu_0) \log(1 + p_1) + \lambda_1 \left(\frac{p_1}{h_1} + \frac{\rho^2}{h_1} p_0 \right) + \lambda_2 \left(\frac{p_2}{h_2} + \frac{(1-\rho)^2}{h_2} p_0 \right) \Bigg] \tag{37}$$

Following the same argument as in case 1), let us define four regions in $\mathbb{R}_+^2$,

$$\mathcal{S}_1 = \left\{ (x,y) : x \geq \frac{1}{\mu_1}, \frac{y}{x} < \frac{\mu_1}{\mu_0} - 1 \right\} \tag{38}$$

$$\mathcal{S}_2 = \left\{ (x,y) : x < \frac{1}{\mu_1}, x + y < \frac{1}{\mu_0} \right\} \tag{39}$$

$$\mathcal{S}_3 = \left\{ (x,y) : \frac{1}{x} - \frac{1}{x+y} \geq \mu_1 - \mu_0, x + y \geq \frac{1}{\mu_0} \right\} \tag{40}$$

$$\mathcal{S}_4 = \left\{ (x,y) : \frac{1}{x} - \frac{1}{x+y} < \mu_1 - \mu_0, \frac{y}{x} \geq \frac{\mu_1}{\mu_0} - 1, x + y \geq \frac{1}{\mu_0} \right\} \tag{41}$$

Then, the optimal solution is

$$p_0(\mathbf{h}) = \begin{cases} \mu_0 \left(\frac{h_1}{\lambda_1} + \frac{h_2}{\lambda_2} \right) - 1, & \text{if } \left(\frac{h_1}{\lambda_1}, \frac{h_2}{\lambda_2} \right) \in \mathcal{S}_3 \\ \mu_0 \left(\frac{h_1}{\lambda_1} + \frac{h_2}{\lambda_2} \right) - (\mu_1 - \mu_0) \left(\frac{1}{\frac{h_1}{\lambda_1}} - \frac{1}{\frac{h_1}{\lambda_1} + \frac{h_2}{\lambda_2}} \right)^{-1}, & \text{if } \left(\frac{h_1}{\lambda_1}, \frac{h_2}{\lambda_2} \right) \in \mathcal{S}_4 \\ 0, & \text{otherwise} \end{cases} \tag{42}$$

$$p_1(\mathbf{h}) = \begin{cases} \mu_1 \frac{h_1}{\lambda_1} - 1, & \text{if } \left(\frac{h_1}{\lambda_1}, \frac{h_2}{\lambda_2} \right) \in \mathcal{S}_1 \\ (\mu_1 - \mu_0) \left(\frac{1}{\frac{h_1}{\lambda_1}} - \frac{1}{\frac{h_1}{\lambda_1} + \frac{h_2}{\lambda_2}} \right)^{-1} - 1, & \text{if } \left(\frac{h_1}{\lambda_1}, \frac{h_2}{\lambda_2} \right) \in \mathcal{S}_4 \\ 0, & \text{otherwise} \end{cases} \tag{43}$$

$$p_2(\mathbf{h}) = 0 \tag{44}$$

Again, the transmit powers are found by dividing these with appropriate channel gains. As can be seen, in the case of $\mu_1 \geq \mu_0 \geq \mu_2$, transmitter 2 never uses its power to transmit its individual message. When both channels are poor, no one transmits. When the channel of the first transmitter is much better than that of the second transmitter, transmitter 1 transmits only its individual message and transmitter 2 keeps silent. When the channel of the second transmitter is much better than that of the first transmitter, both transmitters cooperate using beamforming to transmit the common message. When both channels are more or less equally good, both common message and individual message from transmitter 1 are transmitted. These regions are shown in Figure 2(b).

3) When $\mu_1 \geq \mu_2 \geq \mu_0$, the optimization problem in (20) is equivalent to

$$\min_{\mathbf{p} \geq 0, 0 \leq \rho \leq 1} E \Bigg[-\mu_0 \log(1 + p_1 + p_2 + p_0) - (\mu_2 - \mu_0) \log(1 + p_1 + p_2) - (\mu_1 - \mu_2) \log(1 + p_1) + \lambda_1 \left(\frac{p_1}{h_1} + \frac{\rho^2}{h_1} p_0 \right) + \lambda_2 \left(\frac{p_2}{h_2} + \frac{(1-\rho)^2}{h_2} p_0 \right) \Bigg] \tag{45}$$

Let us define eight regions in $\mathbb{R}_+^2$,

$$\mathcal{T}_1 = \left\{ (x,y) : x < \frac{1}{\mu_1}, y < \frac{1}{\mu_2}, x + y < \frac{1}{\mu_0} \right\} \tag{46}$$

$$\mathcal{T}_2 = \left\{ (x,y) : x + y \geq \frac{1}{\mu_0}, \frac{1}{y} - \frac{1}{x+y} \geq \mu_2 - \mu_0, \frac{1}{x} - \frac{1}{x+y} \geq \mu_1 - \mu_0 \right\} \tag{47}$$

$$\mathcal{T}_3 = \left\{ (x,y) : x \geq \frac{1}{\mu_1}, \frac{y}{x} < \min\left(\frac{\mu_1}{\mu_2}, \frac{\mu_1}{\mu_0} - 1 \right) \right\} \tag{48}$$

$$\mathcal{T}_4 = \left\{ (x,y) : y \geq \frac{1}{\mu_2}, \frac{x}{y} < \frac{\mu_2}{\mu_0} - 1, \frac{1}{x} - \frac{1}{y} \geq \mu_1 - \mu_2 \right\} \tag{49}$$

$$\mathcal{T}_5 = \left\{ (x,y) : x + y \geq \frac{1}{\mu_0}, \frac{1}{x} - \frac{1}{x+y} < \mu_1 - \mu_0, \frac{\mu_1}{\mu_0} - 1 \leq \frac{y}{x} < \sqrt{\frac{\mu_1 - \mu_0}{\mu_2 - \mu_0}} \right\} \tag{50}$$

$$\mathcal{T}_6 = \left\{ (x,y) : x + y \geq \frac{1}{\mu_0}, \frac{x}{y} \geq \frac{\mu_2}{\mu_0} - 1, \frac{1}{y} - \frac{1}{x+y} < \mu_2 - \mu_0, \frac{1}{x} - \frac{1}{y} \geq \mu_1 - \mu_2 \right\} \tag{51}$$

$$\mathcal{T}_7 = \left\{ (x,y) : y \geq \frac{1}{\mu_2}, \frac{1}{x} - \frac{1}{y} < \mu_1 - \mu_2, \frac{x}{y} < \min(\frac{\mu_2}{\mu_1}, \frac{\mu_2}{\mu_0} - 1) \right\} \tag{52}$$

$$\mathcal{T}_8 = \Bigg\{ (x,y) : x + y \geq \frac{1}{\mu_0}, \frac{1}{y} - \frac{1}{x+y} < \mu_2 - \mu_0, \frac{1}{x} - \frac{1}{y} < \mu_1 - \mu_2,$$
$$\frac{\mu_2}{\mu_0} - 1 \leq \frac{x}{y} < \min\left(\sqrt{\frac{\mu_2 - \mu_0}{\mu_1 - \mu_0}}, \left(\frac{c + \sqrt{c^2 + 4}}{2} \right)^{-1} \right), \text{ where } c = \frac{\mu_1 - \mu_2}{\mu_0} \Bigg\} \tag{53}$$

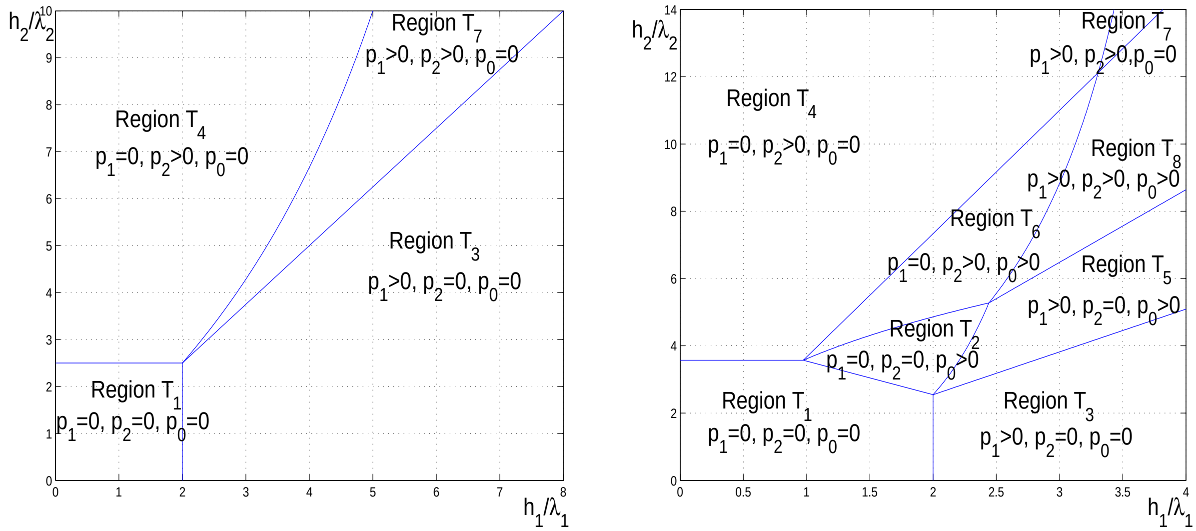
Then, the optimal solution is

$$p_0(\mathbf{h}) = \begin{cases} \mu_0 \left(\frac{h_1}{\lambda_1} + \frac{h_2}{\lambda_2} \right) - 1, & \text{if } \left(\frac{h_1}{\lambda_1}, \frac{h_2}{\lambda_2} \right) \in \mathcal{T}_2 \\ \mu_0 \left(\frac{h_1}{\lambda_1} + \frac{h_2}{\lambda_2} \right) - (\mu_1 - \mu_0) \left(\frac{1}{\frac{h_1}{\lambda_1}} - \frac{1}{\frac{h_1}{\lambda_1} + \frac{h_2}{\lambda_2}} \right)^{-1}, & \text{if } \left(\frac{h_1}{\lambda_1}, \frac{h_2}{\lambda_2} \right) \in \mathcal{T}_5 \\ \mu_0 \left(\frac{h_1}{\lambda_1} + \frac{h_2}{\lambda_2} \right) - (\mu_2 - \mu_0) \left(\frac{1}{\frac{h_2}{\lambda_2}} - \frac{1}{\frac{h_1}{\lambda_1} + \frac{h_2}{\lambda_2}} \right)^{-1}, & \text{if } \left(\frac{h_1}{\lambda_1}, \frac{h_2}{\lambda_2} \right) \in \mathcal{T}_6 \bigcup \mathcal{T}_8 \\ 0, & \text{otherwise} \end{cases} \tag{54}$$

$$p_1(\mathbf{h}) = \begin{cases} \mu_1 \frac{h_1}{\lambda_1} - 1, & \text{if } \left(\frac{h_1}{\lambda_1}, \frac{h_2}{\lambda_2} \right) \in \mathcal{T}_3 \\ (\mu_1 - \mu_0) \left(\frac{1}{\frac{h_1}{\lambda_1}} - \frac{1}{\frac{h_1}{\lambda_1} + \frac{h_2}{\lambda_2}} \right)^{-1} - 1, & \text{if } \left(\frac{h_1}{\lambda_1}, \frac{h_2}{\lambda_2} \right) \in \mathcal{T}_5 \\ (\mu_1 - \mu_2) \left(\frac{1}{\frac{h_1}{\lambda_1}} - \frac{1}{\frac{h_2}{\lambda_2}} \right)^{-1} - 1, & \text{if } \left(\frac{h_1}{\lambda_1}, \frac{h_2}{\lambda_2} \right) \in \mathcal{T}_7 \bigcup \mathcal{T}_8 \\ 0, & \text{otherwise} \end{cases} \tag{55}$$

$$p_2(\mathbf{h}) = \begin{cases} \mu_2 \frac{h_2}{\lambda_2} - 1, & \text{if } \left(\frac{h_1}{\lambda_1}, \frac{h_2}{\lambda_2} \right) \in \mathcal{T}_4 \\ (\mu_2 - \mu_0) \left(\frac{1}{\frac{h_2}{\lambda_2}} - \frac{1}{\frac{h_1}{\lambda_1} + \frac{h_2}{\lambda_2}} \right)^{-1} - 1, & \text{if } \left(\frac{h_1}{\lambda_1}, \frac{h_2}{\lambda_2} \right) \in \mathcal{T}_6 \\ \mu_2 \frac{h_2}{\lambda_2} - (\mu_1 - \mu_2) \left(\frac{1}{\frac{h_1}{\lambda_1}} - \frac{1}{\frac{h_2}{\lambda_2}} \right)^{-1}, & \text{if } \left(\frac{h_1}{\lambda_1}, \frac{h_2}{\lambda_2} \right) \in \mathcal{T}_7 \\ (\mu_2 - \mu_0) \left(\frac{1}{\frac{h_2}{\lambda_2}} - \frac{1}{\frac{h_1}{\lambda_1} + \frac{h_2}{\lambda_2}} \right)^{-1} - (\mu_1 - \mu_2) \left(\frac{1}{\frac{h_1}{\lambda_1}} - \frac{1}{\frac{h_2}{\lambda_2}} \right)^{-1}, & \text{if } \left(\frac{h_1}{\lambda_1}, \frac{h_2}{\lambda_2} \right) \in \mathcal{T}_8 \\ 0, & \text{otherwise} \end{cases} \tag{56}$$

As in the previous two cases, the transmit powers are found by dividing these with the the corresponding channel gains. There are two subcases in the case of $\mu_1 \geq \mu_2 \geq \mu_0$. When $\frac{1}{\mu_1} + \frac{1}{\mu_2} < \frac{1}{\mu_0}$, i.e., μ_0 is very small, common message never gets transmitted due to its small weight. When both channels are poor, no one transmits. When channel of the first transmitter is much better than that of the second transmitter, individual message W_1 is transmitted only. When channel of the second transmitter is much better than that of the first transmitter, individual message W_2 is transmitted only. When both channels



(a) $\mu_1 \geq \mu_2 \geq \mu_0$ and $\frac{1}{\mu_1} + \frac{1}{\mu_2} < \frac{1}{\mu_0}$.

(b) $\mu_1 \geq \mu_2 \geq \mu_0$ and $\frac{1}{\mu_1} + \frac{1}{\mu_2} > \frac{1}{\mu_0}$.

Figure 3: Power control policies.

are more or less equally good, both individual messages are transmitted. These regions are shown in Figure 3(a).

In the other subcase of $\frac{1}{\mu_1}+\frac{1}{\mu_2} > \frac{1}{\mu_0}$, all three messages get a chance to be transmitted. These regions are shown in Figure 3(b).

Thus far, we have solved the optimization in (20) in terms of the lagrange multipliers λ. Next, we need to solve λ. Since there is no duality gap, we will solve λ by solving the dual problem, i.e., we will find λ that maximizes the dual function, $g(\lambda)$. It can be shown that λ thus solved enables the power policies to satisfy the power constraints with equalities. We will solve the dual problem by using the subgradient method [12]. For our problem,

$$u(\lambda) \triangleq \begin{bmatrix} E[\frac{\rho^{*2}}{h_1}p_0 + \frac{p_1}{h_1}] - \bar{P}_1 \\ E[\frac{(1-\rho^*)^2}{h_2}p_0 + \frac{p_2}{h_2}] - \bar{P}_2 \end{bmatrix} \tag{57}$$

is a subgradient of the dual function and the set $\{\lambda : \lambda \geq 0, g(\lambda) > -\infty\} = \{\lambda : \lambda > 0\}$. We start from an arbitrary point $\lambda(0) \in \{\lambda : \lambda > 0\}$. At iteration k, we have available $\lambda(k-1)$ from the previous iteration, and we compute (p_0, p_1, p_2, ρ) by setting $\lambda = \lambda(k-1)$. Then, using the (p_0, p_1, p_2, ρ) we obtained, we compute the subgradient vector $u(\lambda(k-1))$ by equation (57) and update λ using

$$\lambda(k) = \max[\lambda(k-1) + s(k)u(\lambda(k-1)), \epsilon] \tag{58}$$

where $s(k)$ is a positive scalar stepsize at step k and $\epsilon = [\epsilon_1, \epsilon_2]^T$ is a positive vector very close to zero so that $\lambda(k)$ stays in $\{\lambda : \lambda > 0\}$. We stop when both components of vector $u(\lambda(k))$ are small enough. In [12], it is proved that for small enough step sizes, this algorithm converges.

Finally, we state here without proof that the boundary rate triplet that solves (17) is unique for all μ vectors except for the following three singular cases: $\mu_0 = \mu_1 = \mu_2 = 0$, $\mu_1 > \mu_0 = \mu_2 = 0$ and $\mu_2 > \mu_0 = \mu_1 = 0$. Thus, by varying the μ vector over all possible values, and expressing the rates in limiting expressions for the singular cases, we obtain the entire boundary surface of the capacity region.

5 Conclusion

In this paper, we study the Gaussian MAC with common data in fading. We characterize the ergodic capacity region, as well as the power control policies that achieve the rate tuples on the boundary of the capacity region. As expected, the common message enjoys a beamforming gain. Hence, if all three rates are weighted equally, i.e., we are interested in the sum capacity, then we would always transmit only the common message using beamforming.

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