Optimum Power Control for Fading CDMA with Deterministic Sequences*

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Abstract

We characterize the optimum power allocation policy that maximizes the information theoretic sum capacity of a code division multiple access (CDMA) system where the users are assigned arbitrary signature sequences in a frequency flat fading environment. We provide an iterative waterfilling algorithm to obtain the powers of all users at all channel fade levels, and prove its convergence to the optimum solution. Under certain mild conditions on the signature sequences, the optimum power allocation dictates that more than one user transmit simultaneously in some non-zero probability region of the space of all channel states. We identify these conditions, and provide an upper bound on the maximum number of users that can transmit simultaneously at any given time.

1 Introduction

Fading may be an important limiting factor in wireless communication networks unless appropriate resource allocation is applied to exploit the variations in the channel gains to the advantage of the network capacity. The resources that we concentrate on allocating optimally in this paper are the transmit powers of the users. The quality-of-service based power control approaches assign transmit powers to the users so that all users satisfy their signal-to-interference-ratio (SIR) requirements while transmitting with the least amount of power. The SIR-based power control assigns powers to the users with the aim of *compensating* for the variations in the channel; it assigns more power to the users with bad channel states, and less power to the users with good channel states [1–4].

For a single-user fading channel, [5] shows that the optimum power allocation policy, in the sense of maximizing the ergodic channel capacity, is a waterfilling of power in time. The optimum power allocation policy allocates more power to the stronger channel states, and less power to the weaker channel states; it allocates zero power to the channel states below a threshold level which is determined by the fading statistics.

The capacity of a multiple access channel is expressed as a region of achievable rates [6], and *sum capacity*, the maximum achievable sum of rates, is often used as a measure of the overall network capacity. For a multiuser scalar channel, [7] finds the optimum power allocation policy which maximizes the ergodic sum capacity of the network. The multiple access scheme in [7] is *scalar* in the sense that all users transmit with the same waveform. For this system, it was shown that the optimum power allocation policy is

*This work was supported by NSF Grant ANI 02-05330.

one where each user compares its channel state (normalized by a factor depending on the statistical characterization of the fading) to those of the other users, and transmits with non-zero power only if its normalized channel state is better than or equal to the normalized channel states of all other users. More than one user transmits simultaneously only if the normalized channel states of multiple users are the same. Since the channel gains are continuous random variables, this occurs only with zero probability. Therefore, this power control policy implies that at most one user transmits (if at all) at any given time with probability one.

For a multiple access channel with multiple antennas, [8] solves for the optimal power allocation at all transmit antennas and gives a relationship between the maximum number of active transmit and receive antennas. The problem of maximizing the sum capacity as a function of the transmit powers in a *vector* multiple access channel, such as a CDMA or multiple transmit antenna system, in fading channels, is studied for the case of large systems and random transmit vectors in [9] where a simple single-user waterfilling strategy is proposed and shown to be asymptotically optimal.

In this paper, we focus on the power control problem for a CDMA system in a fading channel where the number of users and the processing gain are finite and arbitrary, and the users are assigned arbitrary deterministic signature sequences. Our problem reduces to K independent Goldsmith-Varaiya problems [5] when the signature sequences are chosen to be orthogonal, and to a Knopp-Humblet problem [7] when the signature sequences are chosen to be identical. We show that the optimum power allocation policy is a *simultaneous waterfilling* policy that requires the solution of a set of highly nonlinear equations. We develop an iterative power allocation policy, where, at each step, only one user allocates its power optimally over all channel states of all users when the power allocations of all other users are fixed. The power allocation of each user in this iterative process is a waterfilling where the *base level of the water tank* is determined by the inverse of the SIR the user would obtain at the output of a minimum mean squared error (MMSE) receiver if it transmitted with unit power. When the signature sequences are orthogonal, this "base level" becomes the inverse of the SNR, and when the signature sequences are identical, it becomes the inverse of the SIR found at the output of a matched filter (MF), since in this case, the MMSE receivers reduce to MFs.

We prove the convergence of our algorithm to an optimum solution, and provide conditions for the uniqueness of the solution. One of the questions of interest, for an arbitrary set of signature sequences, is whether there exists a set of channel states having a non-zero probability where all users transmit simultaneously. In the case of orthogonal signature sequences, for instance, all users transmit simultaneously in an orthant of the space of all channel states where the channel states of all users exceed their corresponding thresholds; and, clearly, this region has a non-zero probability. In the case of identical signature sequences however, users transmit simultaneously only on a half-line in the space of all channel states; and, this region has a zero probability [7]. In the most general case, the existence of a region of channel states with non-zero probability where all users transmit simultaneously depends on the number of users, the dimensionality of the signal space (processing gain), and the particular set of signature sequences being used. We show that under certain mild conditions on the signature sequences, such a non-zero probability region of channel states exists. This is a result of the fact that CDMA scheme with non-identical signature sequences provides users with multiple degrees of freedom; therefore, the users do not have to *avoid* each other completely in the space of all channel states, that is, multiple users can *share* some of the channel states that are favorable to all of them.

2 System Model

We consider a CDMA system with processing gain N where all K users transmit to a single receiver. In the presence of fading and AWGN, the received signal is given by,

$$r(t) = \sum_{i=1}^{K} \sqrt{p_i h_i} b_i s_i(t) + n(t) \tag{1}$$

where, for user i, b_i denotes the information symbol with $\mathrm{E}[b_i^2] = 1$, $s_i(t)$ denotes the unit energy signature waveform, $\sqrt{h_i}$ denotes the random channel gain, and p_i denotes the transmit power, respectively, and $n(t)$ denotes the AWGN with zero-mean and power spectral density σ^2. The signature waveforms can be represented by N orthonormal basis waveforms $\{\psi_j\}_{j=1}^{N}$, such that $s_i(t) = \sum_{j=1}^{N} s_{ij}\psi_j(t)$, where $s_{ij} = \langle s_i(t), \psi_j(t)\rangle$. Projecting the received signal onto the basis waveforms, i.e., $r_j = \langle r(t), \psi_j(t)\rangle$, we obtain the sufficient statistics $\{r_j\}_{j=1}^{N}$. Therefore, the continuous channel in (1) can be represented in an equivalent vector form as [10],

$$\mathbf{r} = \sum_{i=1}^{K} \sqrt{p_i h_i} b_i \mathbf{s}_i + \mathbf{n} \tag{2}$$

where $\mathbf{s}_i = [s_{i1}, \cdots, s_{iN}]^\top$ is the signature sequence of user i, and $\mathbf{n}$ is a zero-mean Gaussian random vector with covariance $\sigma^2 \mathbf{I}_N$. We assume that the receiver and all of the transmitters have perfect knowledge of the channel states of all users represented as a vector $\mathbf{h} = [h_1, \cdots, h_K]^\top$. We further assume that although the fading is slow enough to ensure constant channel gain in a symbol interval, it is fast enough so that within the transmission time of a block of symbols the long term ergodic properties of the fading process can be observed [11].

3 Problem Definition

For a given set of signature sequences and a fixed set of channel gains, $\mathbf{h}$, the sum capacity $C_{\text{sum}}(\mathbf{h})$ is [6]

$$C_{\text{sum}}(\mathbf{h}) = \frac{1}{2} \log \left[\det \left(\mathbf{I}_N + \sigma^{-2} \sum_{i=1}^{K} h_i \bar{p}_i \mathbf{s}_i \mathbf{s}_i^\top \right) \right] \tag{3}$$

where $\bar{p}_i$ is the average power of user i. When the channel state is modeled as a random vector, the quantity $C_{\text{sum}}(\mathbf{h})$ is random as well. If a constant (channel-independent or non-adaptive) power policy is applied, the ergodic sum capacity is found as the expected value of $C_{\text{sum}}(\mathbf{h})$ over all channel states [11],

$$C_{\text{sum}} = \frac{1}{2} \int \log \left[\det \left(\mathbf{I}_N + \sigma^{-2} \sum_{i=1}^{K} h_i \bar{p}_i \mathbf{s}_i \mathbf{s}_i^\top \right) \right] f(\mathbf{h}) d\mathbf{h} \tag{4}$$

where $f(\mathbf{h})$ denotes the probability density function of the channel state vector. In (4), the transmit power of user i is fixed to $\bar{p}_i$, its average power constraint. Our aim is to choose the transmit powers of the users as a function of the channel state $p_i(\mathbf{h})$, $i = 1, \cdots, K$, with the aim of maximizing the ergodic sum capacity of the system subject to average transmit power constraints for all users. We formulate the problem as,

$$\max_{\{p_i(\mathbf{h})\}} \int \log\left[\det\left(\mathbf{I}_N + \sigma^{-2}\sum_{i=1}^{K} h_i p_i(\mathbf{h})\mathbf{s}_i\mathbf{s}_i^\top\right)\right] f(\mathbf{h})d\mathbf{h}$$

$$\text{s.t.} \int p_i(\mathbf{h})f(\mathbf{h})d\mathbf{h} = \bar{p}_i, \qquad p_i(\mathbf{h}) \geq 0, \qquad i = 1, \cdots, K \tag{5}$$

For arbitrary signature sequences, no closed form solution for this problem is known. It is interesting to note that, (5) reduces to the Knopp-Humblet problem [7] if $\mathbf{s}_i = \mathbf{s}$ for all i, and it reduces to K separable Goldsmith-Varaiya [5] problems, if the signatures are orthogonal, i.e., $\mathbf{s}_i^\top \mathbf{s}_j = 0$ for $i \neq j$, in which case each problem can be solved independently of the others. Our aim is to find the optimal power allocation for the most general case where the signature sequences are arbitrarily correlated, i.e., $\mathbf{s}_i^\top \mathbf{s}_j$ is not restricted to be zero or one.

4 Optimal Power Control via Iterative Waterfilling

We can express the ergodic sum capacity, the objective of (5), as

$$C_{\text{sum}} = C_k + \overline{C}_k \tag{6}$$

where

$$C_k = \frac{1}{2}\int \log\left(1 + h_k p_k(\mathbf{h})\mathbf{s}_k^\top \mathbf{A}_k^{-1}\mathbf{s}_k\right) f(\mathbf{h})d\mathbf{h} \tag{7}$$

represents the contribution of the kth user to the sum capacity when the transmit powers of all other users at all channel states are fixed, and

$$\overline{C}_k = \frac{1}{2}\int \log\left[\det\left(\mathbf{I}_N + \sigma^{-2}\sum_{i\neq k} h_i p_i(\mathbf{h})\mathbf{s}_i\mathbf{s}_i^\top\right)\right] f(\mathbf{h})d\mathbf{h} \tag{8}$$

represents the sum capacity of the remaining users when the kth user is removed from the system. In (7) and (8) $\mathbf{A}_k$ is defined as

$$\mathbf{A}_k = \sigma^2\mathbf{I}_N + \sum_{i\neq k} h_i p_i(\mathbf{h})\mathbf{s}_i\mathbf{s}_i^\top \tag{9}$$

It is worth noting that C_{sum}, the objective function in (5), is a concave function of the powers, and moreover, provided that the matrices $\{\mathbf{s}_i\mathbf{s}_i^\top\}_{i=1}^K$ are linearly independent, it is a strictly concave function of the powers [9]. Also, the constraint set in (5) is convex. Therefore, the optimization problem in (5) has a unique global optimum when $\{\mathbf{s}_i\mathbf{s}_i^\top\}_{i=1}^K$ are linearly independent; and all local optimums yield the same objective function value, otherwise. Lagrange optimization technique can be used to find the global optimum solution. Let us associate the Lagrange multipliers λ_i's with the equality constraints and μ_i's with the inequality constraints. The optimum power allocation policy satisfies the extended Karush-Kuhn-Tucker (KKT) conditions with mixed constraints [12, Chap. 13], which, after taking the derivatives and employing the complementary slackness conditions $p_i\mu_i = 0$, simplify to

$$\frac{h_k\mathbf{s}_k^\top \mathbf{A}_k^{-1}\mathbf{s}_k}{1 + h_k p_k(\mathbf{h})\mathbf{s}_k^\top \mathbf{A}_k^{-1}\mathbf{s}_k} \leq \lambda_k, \qquad k = 1, \cdots, K, \qquad \forall\ \mathbf{h} \in R^K \tag{10}$$

which is satisfied with equality if and only if $p_k > 0$. Using the fact that $p_i \geq 0$ for all i, (10) implies that the capacity maximizing power allocation policy satisfies

$$p_k(\mathbf{h}) = \left(\frac{1}{\lambda_k} - \frac{1}{h_k \mathbf{s}_k^\top \mathbf{A}_k^{-1} \mathbf{s}_k}\right)^+, \qquad k = 1, \cdots, K \tag{11}$$

for any realization of the channel $\mathbf{h}$. Here λ_i's are determined by inserting (11) into the average power constraints in (5). The value of λ_i depends on the statistical characterization of the channel and the choice of signature sequences.

For arbitrary signature sequences, the set of equations (11) is highly nonlinear. Although it is possible to solve for the optimum powers and transmit regions in a simple system with few users, it seems intractable for systems with large numbers of users. It is worth noting that $h_k \mathbf{s}_k^\top \mathbf{A}_k^{-1} \mathbf{s}_k$ is the SIR of user k at at the output of an MMSE receiver if it transmitted with $p_k = 1$. Therefore, all users should *simultaneously* waterfill on the "base levels" of the inverse of the SIRs they would obtain if they transmitted with unit powers. Since solving for the simultaneous waterfilling solution for all users seems intractable, we devise an iterative algorithm. Consider optimizing for the power of *only* user k over all channel states, given the powers of all other users at all channel states,

$$\begin{aligned} p_k^{n+1} &= \arg\max_{p_k} \; C_{\text{sum}}\left(p_1^{n+1}, \cdots, p_{k-1}^{n+1}, p_k, p_{k+1}^{n} \cdots, p_K^{t}\right) \\ &= \arg\max_{p_k} \; C_k\left(p_k, \mathbf{A}_k\right) \end{aligned} \tag{12}$$

where $\mathbf{A}_k$ captures the interference effects of all other users on user k. Note that $\mathbf{A}_k$ is a function of the transmit powers, signature sequences and channel states of all users except user k. We have already noted that the objective function C_{sum} is a concave function of powers, and also that C_k given by (7) is a strictly concave function of p_k. The constraint set for powers over which the maximization is to be performed is convex, and has a Cartesian product structure among the users. The solution of (12) can be found as a single-user waterfilling over all channel states of the system,

$$p_k(\mathbf{h}) = \left(\frac{1}{\tilde{\lambda}_k} - \frac{1}{h_k \mathbf{s}_k^\top \mathbf{A}_k^{-1} \mathbf{s}_k}\right)^+ \tag{13}$$

If we let only one user allocate its power over all channel states using (13), and iterate over all users sequentially, this iterative *one-user-at-a-time algorithm* is guaranteed to converge to the global optimum solution of (5) [13, Prop. 3.9].

At any given iteration, a user waterfills over the inverse of the SIRs it would obtain if it transmitted with unit power, given the current power allocations of all other users at all channel states: the user puts more power into the channel states where its expected SIR with unit transmit power is larger. For orthogonal signature sequences, the iteration in (13) becomes

$$p_k(\mathbf{h}) = \left(\frac{1}{\tilde{\lambda}_k} - \frac{\sigma^2}{h_k}\right)^+ \tag{14}$$

and converges to the optimum solution found in [5] in one step. For identical signature sequences, the iteration in (13) becomes

$$p_k(\mathbf{h}) = \left(\frac{1}{\tilde{\lambda}_k} - \frac{\sigma^2 + \sum_{i \neq k} h_i p_i(\mathbf{h})}{h_k}\right)^+ \tag{15}$$

and converges to the solution found in [7]. Finally, we note that, the iterative implementation of the "simultaneous waterfilling in time" presented in this paper is analogous to the iterative implementation of the "simultaneous waterfilling over parallel channels" in [14].

5 Properties of the Optimal Power Allocation

Let us now consider the inverse problem of finding the channel state of the system for a given non-zero transmit power vector. Since all components of the power vector are non-zero, this means that, all users transmit simultaneously at this particular channel state, and (10) should be satisfied with equality for all k. Therefore, given any arbitrary power vector $\mathbf{p}$ with $0 < p_i < 1/\lambda_i$, the channel state where this power vector is used can be found by solving

$$\mathbf{h} = \mathbf{f}(\mathbf{h}) \tag{16}$$

where the vector function $\mathbf{f}(\mathbf{h})$ is defined as

$$f_k(\mathbf{h}) = \frac{\lambda_k p_k}{(1-\lambda_k p_k)} \frac{1}{p_k \mathbf{s}_k^\top \mathbf{A}_k^{-1} \mathbf{s}_k}, \qquad k = 1, \cdots, K \tag{17}$$

Note that, for $0 < p_i < 1/\lambda_i$, $\mathbf{f}(\mathbf{h})$ is a standard function as defined in [3]. This means that, if there is a solution for (16), it is unique. In fact, one can devise an iterative algorithm to find this solution,

$$\mathbf{h}(n+1) = \mathbf{f}(\mathbf{h}(n)) \tag{18}$$

It is interesting to note that the problem in (16) with the definition of $\mathbf{f}(\mathbf{h})$ in (17) is very similar to the joint power control and receiver design problem studied in [15]. In [15], the problem is to solve for the powers (and receiver filters) when the SIR targets and channel gains of the users are given. In (16), the problem is to solve for the channel gains when the powers are given. The role played by the channel gains in [15] is the same as the role played by the powers in (16). Also, here, the quantity that plays the role of the SIR target in [15], denoted by β_k for user k, is

$$\beta_k = \frac{\lambda_k p_k}{(1-\lambda_k p_k)} \tag{19}$$

which depends on the power of user k.

Therefore, once the powers of the users are fixed, assuming that the SIR targets produced by the powers through (19) are feasible, in the sense that (16) has a solution, we can find that solution, and therefore, we can obtain a unique set of channel gains where the given power vector is used by the system as the transmit power vector. Therefore, corresponding to a set of feasible power values, there always exists a set of channel gains where all the users in the system transmit with non-zero powers. This set however can have zero probability as in [7]. To determine the set of feasible powers, it is sufficient to determine the set of feasible SIR targets in [15]. The SIR targets $\beta_1, \cdots, \beta_k$ in a joint power control and receiver design problem are feasible if and only if [16, Theorem 10]

$$\sum_{k \in U} \frac{\beta_k}{1+\beta_k} < \text{rank}(\mathbf{S}(U)), \qquad \forall\, U \subset \{1, \cdots, K\} \tag{20}$$

where $\mathbf{S}(U)$ is the matrix containing the signature sequences of the users in the subset U. Inserting (19) into (20), for our problem, a power vector $\mathbf{p}$ is feasible, if and only if it satisfies

$$\sum_{k \in U} \lambda_k p_k < \text{rank}(\mathbf{S}(U)), \qquad \forall\, U \subset \{1, \cdots, K\} \tag{21}$$

The significance of (21) for our purposes is that the set of feasible power vectors is a volume in K dimensional space. For the set of feasible power vectors satisfying (21), and having strictly positive components, if the set of corresponding channel states found by solving (16) have a non-zero measure, then we can conclude that all users transmit simultaneously with a positive probability.

Theorem 1 *There exists a non-zero probability region of fading states* $\mathbf{h}$ *where all* K *users transmit simultaneously, if and only if* $\{\mathbf{s}_i\mathbf{s}_i^\top\}_{i=1}^K$ *are linearly independent.*

Proof: It is clear that the set of feasible powers as given by (21) constitutes a volume V in R^K. Let us then pick any point in this set, and compute the channel state which corresponds to this particular solution of powers. By the feasibility of $\mathbf{p}_0$, the resulting channel state $\mathbf{h}_0$ is unique, and the original vector $\mathbf{p}_0$ satisfies the KKT conditions at $\mathbf{h}_0$. Given $\{\mathbf{s}_i\mathbf{s}_i^\top\}_{i=1}^K$ are linearly independent, we know that there exists a unique global maximum for C_{sum}. Therefore, the waterfilling solution we get at the fading state $\mathbf{h}_0$ should be equal to $\mathbf{p}_0$, as it is a possible solution to the problem, and the problem has a unique global optimum. Hence, we obtain a unique fading state for a power level, and a unique power for a fading state, for a set of powers satisfying (21). This implies that there exists a one-to-one mapping from the space of feasible non-zero powers to the space of fading states. This one-to-one mapping maps the volume $V \in R^K$ of feasible powers to a volume of fading states $\tilde{V} \in R^K$ implying that the resulting set of fading states where K users transmit simultaneously has non-zero probability. This completes the proof of the if part of the theorem.

For the only if part, consider the case where $\{\mathbf{s}_i\mathbf{s}_i^\top\}_{i=1}^K$ are linearly dependent. For all K users to transmit simultaneously with non-zero powers, (10) must be satisfied with equality for all k. By applying matrix inversion lemma, and defining $\mathbf{A} = \sigma^2\mathbf{I}_N + \mathbf{SPS}^\top$, which contains all users' powers and signatures, (10) can be written alternatively as

$$h_k \mathbf{s}_k^\top \mathbf{A}^{-1} \mathbf{s}_k = \lambda_k, \qquad k = 1, \cdots, K \tag{22}$$

Each of these equations can also be rewritten as,

$$h_k \text{tr}\left(\mathbf{A}^{-1}\mathbf{s}_k\mathbf{s}_k^\top\right) = \lambda_k, \qquad k = 1, \cdots, K \tag{23}$$

If $\{\mathbf{s}_i\mathbf{s}_i^\top\}_{i=1}^K$ are linearly dependent, then any one of the elements of this set, say $\mathbf{s}_k\mathbf{s}_k^\top$, can be written as a linear combination of the others, say, with coefficients α_i, not all equal to zero. Thus,

$$h_k \text{tr}\left(\mathbf{A}^{-1} \sum_{i \neq k} \alpha_i \mathbf{s}_i\mathbf{s}_i^\top\right) = h_k \sum_{i \neq k} \alpha_i \mathbf{s}_i^\top \mathbf{A}^{-1} \mathbf{s}_i = \lambda_k \tag{24}$$

and using (22) in (24), we get

$$\sum_{i \neq k} \alpha_i \frac{\lambda_i}{h_i} = \frac{\lambda_k}{h_k} \tag{25}$$

This means that, regardless of the power levels, for all users to transmit simultaneously, the channel states should satisfy (25). Since the channel states are continuous random variables, this event has zero probability. Therefore, given that $\{\mathbf{s}_i\mathbf{s}_i^\top\}_{i=1}^K$ are linearly dependent, all K users transmit simultaneously only with zero probability. □

Corollary 1 *When $K \leq N$, for a set of K linearly independent signature sequences, there always exists a non-zero probability region of channel states where all K users transmit simultaneously.*

Proof: The result follows from Theorem 1, and the fact that if $\{\mathbf{s}_i\}_{i=1}^K$ are linearly independent then $\{\mathbf{s}_i\mathbf{s}_i^\top\}_{i=1}^K$ are linearly independent. □

Corollary 2 *For a set of K signature sequences and processing gain N, the number of users that can transmit simultaneously cannot be larger than $N(N+1)/2$.*

Proof: The dimensionality of the space of $N \times N$ symmetric matrices is $N(N+1)/2$, therefore if $K > N(N+1)/2$, $\{\mathbf{s}_i\mathbf{s}_i^\top\}_{i=1}^K$ are guaranteed to be linearly dependent, and the result follows from Theorem 1. □

6 Numerical Examples

In this section, we give some simple numerical examples to support our analysis. Figure 1 gives an example for the two user case where the signature sequences are correlated with $\mathbf{s}_1^\top\mathbf{s}_2 = 0.86$, in which case two users may transmit at the same time, as labeled on the figure. This case corresponds to the setting in Corollary 1. The fading is assumed to be i.i.d., and uniform in $(0, 1)$ for both users. Figure 2 gives the power of user 1 for each fading level. In this figure, the transmit power of user 1 is represented by gray levels, lighter colors corresponding to more power. Note that, user 1 performs a single user waterfilling wherever user 2 does not transmit. In this region, the transmit power of user 1 for a fixed h_1 is constant (independent of h_2). However, once user 2 starts transmitting, the "base level of the water tank" is increased, decreasing the power level of user 1 with increasing h_2. Figure 3 illustrates the convergence of the iterative waterfilling algorithm to the sum capacity of the system; the convergence is very fast as suggested by the plot.

Another significance of Theorem 1 is that we can have multiple users transmit simultaneously with non-zero probability, even when the signature sequences are linearly dependent, as long as we can maintain the independence of $\{\mathbf{s}_i\mathbf{s}_i^\top\}_{i=1}^K$. Figure 4 shows the region where all users transmit simultaneously for $K = 3$ and $N = 2$.

7 Conclusion

We proposed an algorithm to compute the optimum transmit powers of the users that maximize the sum capacity of a CDMA system with arbitrary signature sequences in a fading channel. The algorithm is an iterative waterfilling of powers of all users over all fading states treating at each step all other users' signals as additional colored noise. We showed that this iterative strategy converges to a globally optimum solution, and that the global optimum is unique if the signature sequence set is such that $\{\mathbf{s}_i\mathbf{s}_i^\top\}_{i=1}^K$ are linearly independent.

We also showed that, the optimum power allocation scheme in the vector multiple access channel of interest dictates more than one user to transmit simultaneously at some channel states, and the set of such channel states has a non-zero probability. In fact, all

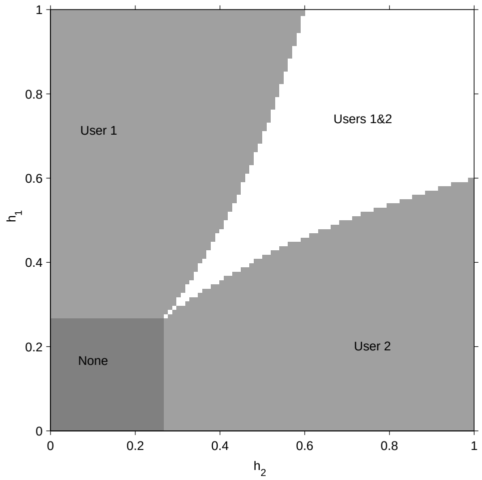

Figure 1: Transmit regions.

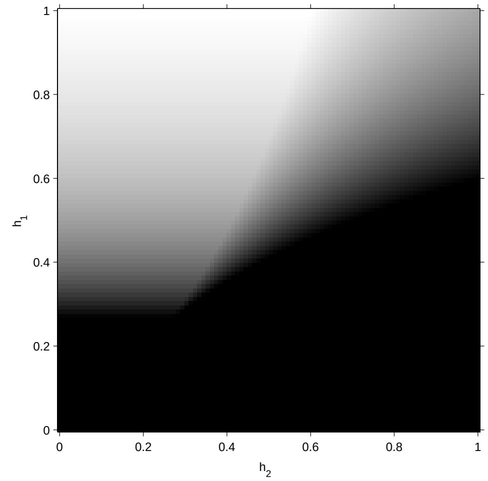

Figure 2: Power distribution of user 1.

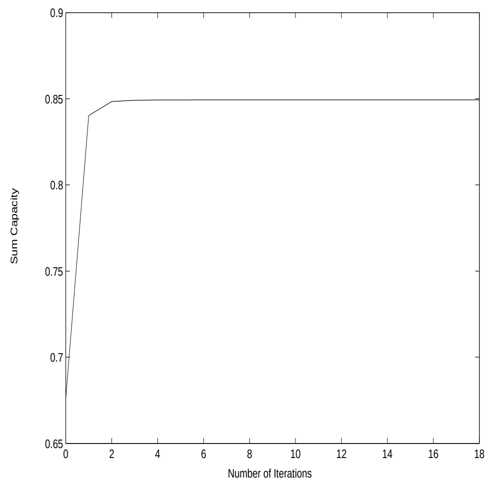

Figure 3: Sum capacity versus number of iterations.

K users in the system are shown to transmit simultaneously with non-zero probability, if and only if $\{\mathbf{s}_i \mathbf{s}_i^\top\}_{i=1}^K$ are linearly independent. An immediate implication of this is that, if the signature sequences $\{\mathbf{s}_i\}_{i=1}^K$ are linearly independent, then all users transmit simultaneously in a non-zero probability region of the channel states.

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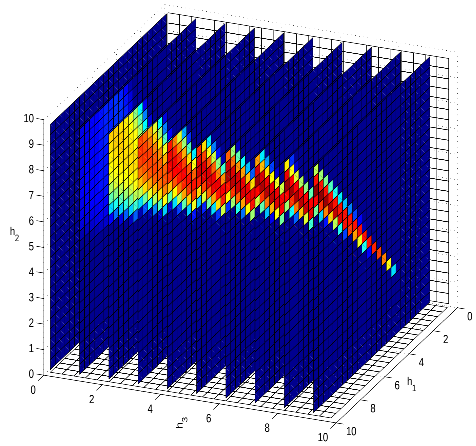

Figure 4: Transmit region for all three users when $K = 3$ and $N = 2$.

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