

Solutions to Exam 1, Spring 2009

1. (a) $s(t) = (1 + \cos 20\pi t) \cos 400\pi t$

$$= \cos 400\pi t + \frac{1}{2} \cos 420\pi t + \frac{1}{2} \cos 380\pi t$$

(b) $y(t) = H(400\pi) \cos 400\pi t + \frac{1}{2} H(420\pi) \cos 420\pi t$
 $+ \frac{1}{2} H(380\pi) \cos 380\pi t$

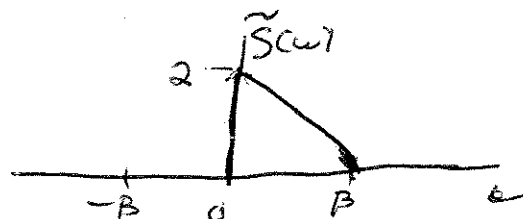
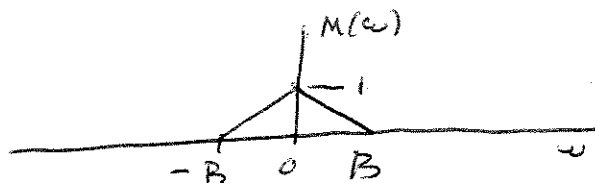
$$= \cos 400\pi t + \frac{1}{4} \cos 420\pi t + \frac{1}{4} \cos 380\pi t$$

$$= (1 + \frac{1}{2} \cos 20\pi t) \cos 400\pi t$$

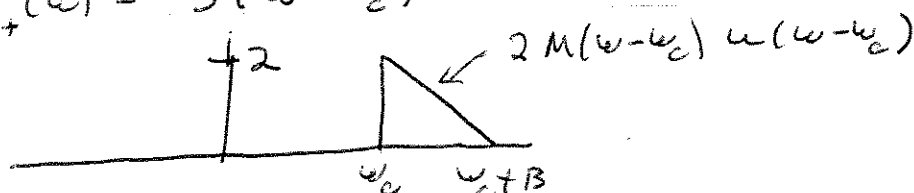
$$\mu = 1/2 = 50\%$$

2. (a) $\hat{M}(\omega) = (-j \operatorname{sign} \omega)^M = \begin{cases} -j(1 - \frac{\omega}{B}) & \text{for } 0 \leq \omega < B \\ j(1 + \frac{\omega}{B}) & \text{for } -B < \omega < 0 \\ 0 & \text{elsewhere} \end{cases}$

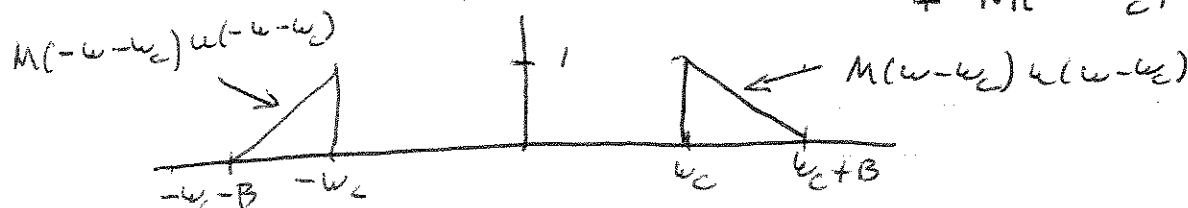
(b) $\tilde{S}(\omega) = 2 M(\omega) u(\omega)$



(c) $S_+(\omega) = \tilde{S}(\omega - \omega_c)$



(d) $s(t) = \frac{s_+(t) + \overline{s_+(t)}}{2} \xleftrightarrow{\mathcal{F}} \frac{1}{2} S_+(\omega) + \frac{1}{2} \overline{S_+(\omega)}$
 $= M(\omega - \omega_c) u(\omega - \omega_c) + M(-\omega - \omega_c) u(-\omega - \omega_c)$



$$\begin{aligned}
 (e) \quad s(t) &= \operatorname{Re} \{ [m(t) + j \hat{m}(t)] [\cos \omega_c t + j \sin \omega_c t] \} \\
 &= m(t) \cos \omega_c t - \hat{m}(t) \sin \omega_c t \\
 &\text{upper sideband SSB}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad (a) \quad s(t) &= A_c \cos \left(\omega_c t + 2\pi k_f \int_0^t A_m \cos \omega_m \tau d\tau \right) \\
 &= A_c \cos \left(\omega_c t + \frac{2\pi k_f A_m}{\omega_m} \sin \omega_m t \right) \\
 &= A_c \cos (\omega_c t + \beta \sin \omega_m t) \quad \text{where } \beta = \frac{2\pi k_f A_m}{\omega_m}
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad s(t) &= \operatorname{Re} \left\{ A_c e^{-j\omega_c t} e^{j\beta \sin \omega_m t} \right\} \\
 e^{j\beta \sin \omega_m t} &= \sum_{n=-\infty}^{\infty} c_n e^{-jn\omega_m t}
 \end{aligned}$$

$$\text{where } c_n = \left(\frac{2\pi}{\omega_m} \right)^{-1} \int_{-\pi/\omega_m}^{\pi/\omega_m} e^{j\beta \sin \omega_m t} e^{-jn\omega_m t} dt$$

$$\text{Let } x = \omega_m t, \quad \text{so } dx = \omega_m dt$$

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j[\beta \sin x - nx]} dx = J_n(\beta)$$

$$\text{so } s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos (\omega_c + n\omega_m)t$$

$$4. \quad s(t) = A_c \cos \left(\omega_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right)$$

$$x(t) = \frac{d}{dt} s(t) = -A_c \sin \left(\omega_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right) (\omega_c + 2\pi k_f m(t))$$

$$\text{envelope} = e(t) = A_c \left[\omega_c + 2\pi k_f m(t) \right] \quad \text{Yes, an FM demod.}$$