

1. (a) $R_{xx}(\tau) = \frac{N_0}{2} \delta(\tau)$

(b) $H(\omega) = \frac{1}{j\omega + 1}$

$$S_{yy}(\omega) = |H(\omega)|^2 S_{xx}(\omega) = \frac{1}{\omega^2 + 1} \frac{N_0}{2}$$

(c) $S_{yy}(\omega) = \frac{N_0}{4} \frac{2}{\omega^2 + 1}$, let $a=1$. Then by hint

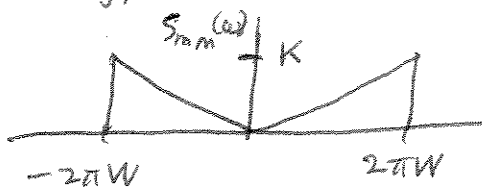
$$R_{yy}(\tau) = \frac{N_0}{4} e^{-|\tau|}$$

$$E\{y^2(t)\} = R_{yy}(0) = N_0/4$$

(d) $S_{yx}(\omega) = H(\omega) S_{xx}(\omega) = \frac{N_0}{2} \frac{1}{j\omega + 1}$

$$R_{yx}(\tau) = \mathcal{F}^{-1}\left\{\frac{N_0}{2} \frac{1}{j\omega + 1}\right\} = \frac{N_0}{2} e^{-\tau} u(\tau)$$

2. (a)



$$P = E\{m^2(t)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{mm}(\omega) d\omega$$

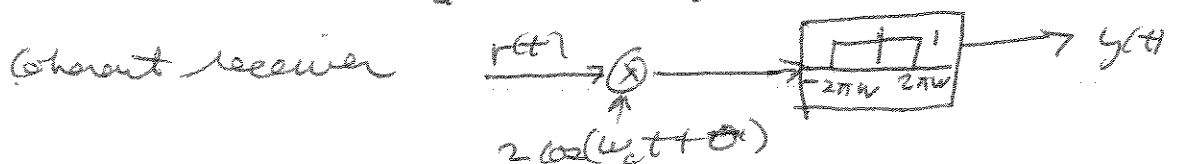
$$= 2 \times \frac{1}{2\pi} \times \frac{1}{2} \times 2\pi W K = WK$$

(b) $P_s = E\{s^2(t)\} = E\{A_c^2 m^2(t) \cos^2(\omega_c t + \theta)\}$
 $= A_c^2 E\{m^2(t)\} E\left\{\frac{1}{2} + \frac{1}{2} \cos(2\omega_c t + 2\theta)\right\} = \frac{A_c^2}{2} P$

(c) Noise power in Band $[-2\pi W, 2\pi W] = \frac{N_0}{2} \times 4\pi W \times \frac{1}{2\pi} = N_0 W$

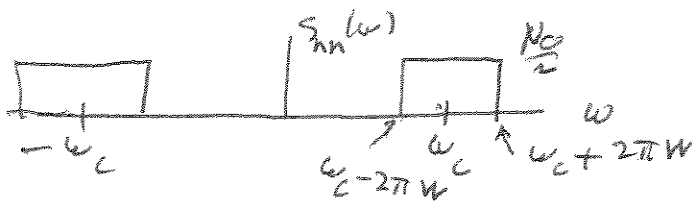
$$(SNR)_c = \frac{P_s}{N_0 W} = \frac{A_c^2 P}{2 N_0 W} = \frac{A_c^2 WK}{2 N_0 W} = \frac{A_c^2 K}{2 N_0}$$

(d) $r(t) = A_c m(t) \cos(\omega_c t + \theta) + n_I(t) \cos(\omega_c t + \theta) - n_Q(t) \sin(\omega_c t + \theta)$



$$y(t) = A_c m(t) + n_I(t)$$

$$(SNR)_o = \frac{A_c^2 P}{E\{n^2(t)\}} = \frac{A_c^2 kW}{2N_0 W} = \frac{A_c^2 K}{2N_0}$$



$$E\{n^2(t)\} = \frac{1}{2\pi} \times 2 \times \frac{N_0}{2} \times 4\pi W = 2N_0 W$$

(c) Figure of merit = $\frac{(SNR)_o}{(SNR)_c} = 1$

3. (a) $P_s = E\{A_c^2 [1 + k m(t)]^2 \cos^2(\omega_c t + \theta)\}$
 $= \frac{A_c^2}{2} E\{1 + 2k m(t) + k^2 m^2(t)\} = \frac{A_c^2}{2} [1 + k^2 P]$
 where $P = E\{m^2(t)\}$

(b) $r(t) = A_c [1 + k m(t)] \cos(\omega_c t + \theta) + n_I(t) \cos(\omega_c t + \theta) - n_Q(t) \sin(\omega_c t + \theta)$

envelope $e(t) = \{(A_c [1 + k m(t)] + n_I(t))^2 + n_Q^2(t)\}^{1/2}$
 $\cong A_c [1 + k m(t)] + n_I(t), E\{n_I^2\} = 2N_0 W$

$$(SNR)_o = \frac{k^2 A_c^2 P}{2N_0 W}$$

(c) Figure of merit = $\frac{(SNR)_o}{(SNR)_c} = \frac{\frac{k^2 A_c^2 P}{2N_0 W}}{\frac{A_c^2 (1 + k^2 P)}{2N_0 W}} = \frac{k^2 P}{1 + k^2 P}$

4. (a) $c_n = \frac{1}{T} \int_{-T/2}^{T/2} p(t) e^{-jn\omega_s t} dt = \frac{1}{T}$

(b) $x^*(t) = x(t) \sum_{n=-\infty}^{\infty} \frac{1}{T} e^{jn\omega_s t} = \frac{1}{T} \sum_{n=-\infty}^{\infty} x(t) e^{jn\omega_s t}$

$$X^*(\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$$

