

Homework Problem for Implementing an IIR Filter

In this problem you will investigate what happens when sine waves are passed through a discrete-time LTI system. Consider a system where the input signal is passed through the cascade of two second-order IIR sections. First the input $x[n]$ is passed through a system S_1 to get the output

$$y_1[n] = b_{0,1}x[n] + b_{1,1}x[n-1] + b_{2,1}x[n-2] - a_{1,1}y_1[n-1] - a_{2,1}y_1[n-2]$$

The output of S_1 is passed through a second system S_2 giving the output

$$y_2[n] = b_{0,2}y_1[n] + b_{1,2}y_1[n-1] + b_{2,2}y_1[n-2] - a_{1,2}y_2[n-1] - a_{2,2}y_2[n-2]$$

The numerical values of the coefficients are:

$$b_{0,1} = 0.292893, b_{1,1} = 0.585786, b_{2,1} = 0.292893$$

$$a_{1,1} = 0.91018, a_{2,1} = 0.414213$$

$$b_{0,2} = 1, b_{1,2} = -2, b_{2,2} = 1$$

$$a_{1,2} = -0.91018, a_{2,2} = 0.414214$$

Each system is implemented as a Direct Form II. The sampling rate for the system is 8,000 Hz, so the sampling period is $T = 1/8000 = 125\mu s$.

Suppose the input $x[n] = x_c[n] = \cos(\omega nT)$ gives the overall output $y_2[n] = y_c[n]$ and the input $x[n] = x_s[n] = \sin(\omega nT)$ gives the output $y_2[n] = y_s[n]$. Then the input $x[n] = e^{j\omega nT} = \cos(\omega nT) + j\sin(\omega nT)$ results in the output $y[n] = y_c[n] + jy_s[n]$.

1. Sketch the block diagram for the system as a cascade of two Direct Form II systems. Label the state variables on your diagram.
2. Let the input be $x[n] = x_c[n] = \cos(\omega nT)$. Let $\omega = 2\pi \times 1600$. Write a MATLAB program to compute the output $y_2[n] = y_c[n]$. DO NOT use the MATLAB filter functions. Write your own program to implement the Direct Form II equations we discussed in class.

Start the system in the zero state and let n increase until a steady state is reached. You should see a sampled sine-wave of 1600 Hz at the output with close to unity amplitude. You can use the MATLAB plotting capability to display and print your results. When you are confident your program is working properly, go on to the next item.
3. Repeat the previous item for the 101 frequencies $\omega_K = 2\pi K \times 40$ for $K = 0, \dots, 100$. The output may be very small for some frequencies. Just save the last value of the output for each frequency.
4. Repeat the previous item with input $x[n] = x_s[n] = \sin(\omega nT)$.

5. After steady state is reached in each of the 101 cases, compute and plot $A(f_K) = |y[n]|/|x[n]| = \sqrt{y_c^2[n] + y_s^2[n]}$ vs. frequency $f_K = \omega_K/(2\pi) = K \times 40$ for $K = 0, \dots, 100$.

6. The theoretical amplitude response is

$$A(f) = \left| \frac{b_{0,1} + b_{1,1}e^{-j2\pi f/f_s} + b_{2,1}e^{-j4\pi f/f_s}}{1 + a_{1,1}e^{-j2\pi f/f_s} + a_{2,1}e^{-j4\pi f/f_s}} \right| \times \left| \frac{b_{0,2} + b_{1,2}e^{-j2\pi f/f_s} + b_{2,2}e^{-j4\pi f/f_s}}{1 + a_{1,2}e^{-j2\pi f/f_s} + a_{2,2}e^{-j4\pi f/f_s}} \right|$$

Plot the theoretical amplitude response for $0 \leq f \leq f_s/2 = 4000$ and compare it with your experimental plot.