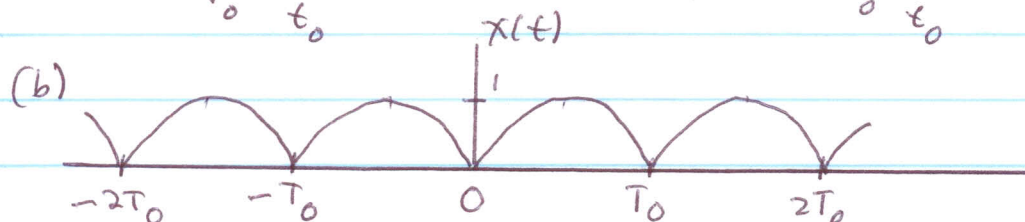


Solutions to ENEE 322 EXAM 2, Fall 2013

1. (a) $x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$; $\omega_0 = 2\pi/T_0$

$$a_n = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} x(t) \cos n\omega_0 t dt, \quad b_n = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} x(t) \sin n\omega_0 t dt$$



$$\omega_0 = 2\omega_1, \quad T_0 = 2\pi/\omega_0 = \pi/\omega_1$$

(c) $x(t)$ is even, so $b_n = 0 \quad \forall n$

$$a_n = \frac{2}{\pi/\omega_1} \int_0^{\pi/\omega_1} \sin \omega_1 t \cos 2n\omega_1 t dt$$

$$= \frac{\omega_1}{\pi} \int_0^{\pi/\omega_1} \sin (1+2n)\omega_1 t + \sin (1-2n)\omega_1 t dt$$

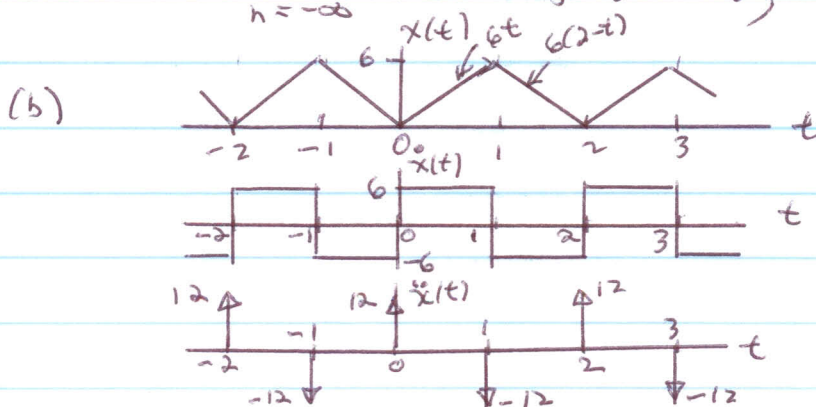
$$= \frac{\omega_1}{\pi} \left(-\frac{\cos(1+2n)\omega_1 t}{(1+2n)\omega_1} - \frac{\cos(1-2n)\omega_1 t}{(1-2n)\omega_1} \right) \Big|_{t=0}^{\pi/\omega_1}$$

$$= \frac{1}{\pi} \left[-\frac{\cos(1+2n)\omega_1 \pi/\omega_1}{1+2n} - \frac{\cos(1-2n)\omega_1 \pi/\omega_1}{1-2n} \right]$$

$$+ \frac{1}{1+2n} + \frac{1}{1-2n}$$

$$= \frac{2}{\pi} \left(\frac{1}{1+2n} + \frac{1}{1-2n} \right) = \frac{4}{\pi(1-4n^2)}$$

2. (a) $x(t) = \sum_{n=-\infty}^{\infty} x_n e^{jn\omega_0 t}$; $\omega_0 = 2\pi/T_0$, $x_n = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} x(t) e^{-jn\omega_0 t} dt$



$$\omega_0 = \frac{2\pi}{2} = \pi$$

$$(c) \hat{x}(t) = 12 \sum_{n=-\infty}^{\infty} \delta(t-2n) - 12 \sum_{n=-\infty}^{\infty} \delta(t-2n-1)$$

$$\hat{x}_n = \frac{12}{2} - \frac{12}{2} e^{-jn\omega_0} = 6 - 6e^{-jn\pi} = 6[1 - (-1)^n]$$

$$x_n = \hat{x}_n / (jn\omega_0)^2 = \frac{6[1 - (-1)^n]}{-n^2\pi^2} \quad \text{for } n \neq 0$$

$$x_0 = \frac{1}{2} \int_0^2 x(t) dt = 2 \times \frac{1}{2} \int_0^1 6t dt = 3$$

$$(d) \sum_{n=-\infty}^{\infty} |x_n|^2 = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt = \frac{2}{T_0} \int_0^1 |x(t)|^2 dt$$

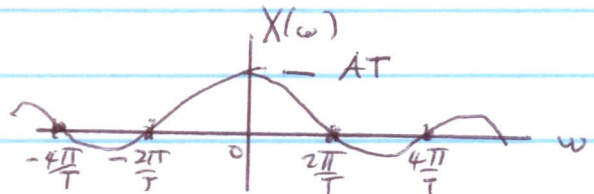
$$= \frac{2}{2} \int_0^1 (6t)^2 dt = 36 \left. \frac{t^3}{3} \right|_0^1 = 12$$

$$3. (a) X(\omega) = \int_{-T/2}^{T/2} A e^{-j\omega t} dt = A \left. \frac{e^{-j\omega t}}{-j\omega} \right|_{-T/2}^{T/2} = A \frac{e^{j\omega T/2} - e^{-j\omega T/2}}{2j\omega}$$

$$= 2A \frac{\sin \omega T/2}{\omega}$$

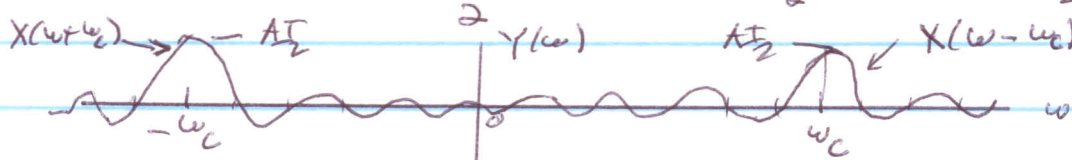
(b)

$$X(0) = \int_{-T/2}^{T/2} A dt = AT$$



Zeros at $\omega T/2 = k\pi$ or $\omega = k \cdot 2\pi/T$ for $k \neq 0$

$$(c) y(t) = x(t) e^{j\omega_c t} + e^{-j\omega_c t} \xleftrightarrow{F} \frac{1}{2} X(\omega - \omega_c) + \frac{1}{2} X(\omega + \omega_c)$$



$$(d) \frac{dx(t)}{dt} \xleftrightarrow{F} j\omega X(\omega) = j2A \sin \omega T/2$$

$$(e) \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-T/2}^{T/2} A^2 dt = A^2 T$$

$$4. (a) \quad j\omega Y(\omega) + 3Y(\omega) = X(\omega)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{j\omega + 3} = \frac{1}{\sqrt{\omega^2 + 9}} e^{-j \tan^{-1} \frac{\omega}{3}}$$

$$(b) \quad h(t) = \mathcal{F}^{-1}\{H(\omega)\} = e^{-3t} u(t)$$

$$(c) \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} [\pi \delta(\omega - \omega_0) + \pi \delta(\omega + \omega_0)] e^{j\omega t} d\omega$$

$$= \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} = \cos \omega_0 t$$

$$(d) \quad y(\omega) = X(\omega) H(\omega) = [\pi \delta(\omega - \omega_0) + \pi \delta(\omega + \omega_0)] H(\omega)$$

$$= \pi H(\omega_0) \delta(\omega - \omega_0) + \pi H(-\omega_0) \delta(\omega + \omega_0)$$

$$= \frac{\pi}{\sqrt{\omega_0^2 + 9}} \left[e^{-j \tan^{-1} \omega_0/3} \delta(\omega - \omega_0) + e^{j \tan^{-1} \omega_0/3} \delta(\omega + \omega_0) \right]$$

$$y(t) = \frac{\pi}{\sqrt{\omega_0^2 + 9}} \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(\omega - \omega_0) e^{j(\omega t - \tan^{-1} \omega_0/3)} d\omega \right.$$

$$\left. + \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(\omega + \omega_0) e^{j(\omega t + \tan^{-1} \omega_0/3)} d\omega \right]$$

$$= \frac{1}{\sqrt{\omega_0^2 + 9}} \frac{e^{j(\omega_0 t - \tan^{-1} \omega_0/3)} + e^{-j(\omega_0 t - \tan^{-1} \omega_0/3)}}{2}$$

$$= \frac{1}{\sqrt{\omega_0^2 + 9}} \cos(\omega_0 t - \tan^{-1} \omega_0/3)$$