

$$1. (a) e^{5t} \left[\frac{d}{dt} y(t) + 5y(t) \right] = \frac{d}{dt} \left[e^{5t} y(t) \right] = e^{5t} 3x(t)$$

$$\int_{t_0}^t \frac{d}{d\tau} \left[e^{5\tau} y(\tau) \right] d\tau = 3 \int_{t_0}^t e^{5\tau} x(\tau) d\tau$$

$$e^{5t} y(t) - e^{5t_0} y(t_0) = 3 \int_{t_0}^t e^{5\tau} x(\tau) d\tau$$

$$y(t) = e^{-5(t-t_0)} y(t_0) + 3 \int_{t_0}^t x(\tau) e^{-5(t-\tau)} d\tau$$

A memoryless system uses only the present input.

This system is not memoryless because it uses

$x(\tau)$ for $t_0 < \tau < t$.

(b) A system is linear if $x_1(t) \rightarrow y_1(t)$ and

$x_2(t) \rightarrow y_2(t)$. Then $c_1 x_1(t) + c_2 x_2(t) \rightarrow c_1 y_1(t) + c_2 y_2(t)$ for all $c_1, c_2, x_1(t)$, and $x_2(t)$.

Suppose $x_1(t) = A$ with $A > 0$. Then $y_1(t) = A$.

Let $x_2(t) = -x_1(t) = -A$. Then $y_2(t) = 0 \neq -A$

so not linear because scaling does not work.

(c) A system is BIBO stable iff every bounded input gives a bounded output.

Let $x(t) = u(t)$. Then $y(t) = t u(t)$ which

becomes unbounded. System not BIBO stable

(d) A system is time-invariant if for every

$x(t) \rightarrow y(t)$, $x(t-T) \rightarrow y(t-T)$ for every T .

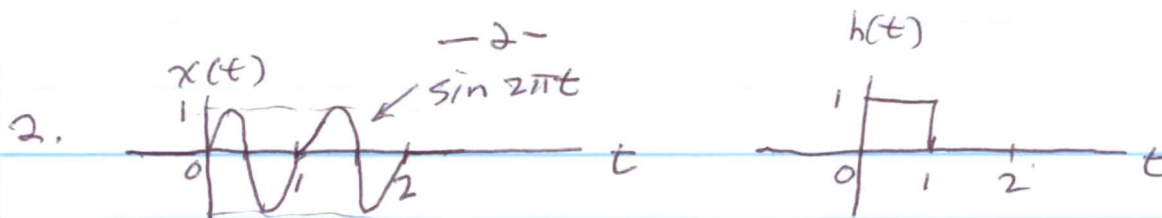
$$\text{Let } y_1(t) = \int_{-\infty}^t x_1(\tau) e^{-2(t-\tau)} d\tau$$

Let $x_2(t) = x_1(t-T)$. Then

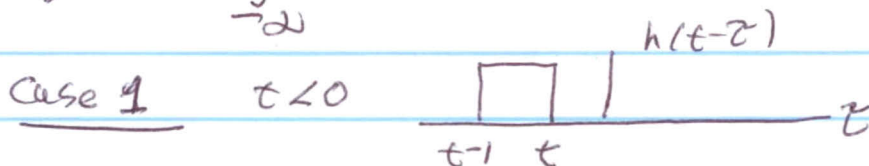
$$y_2(t) = \int_{-\infty}^t x_1(\tau-T) e^{-2(t-\tau)} d\tau$$

$$\text{Let } u = \tau - T \text{ so } du = d\tau. \text{ Then } y_2(t) = \int_{-\infty}^{t-T} x_1(u) e^{-2(t-T-u)} du$$

$= y_1(t-T)$. So time-invariant

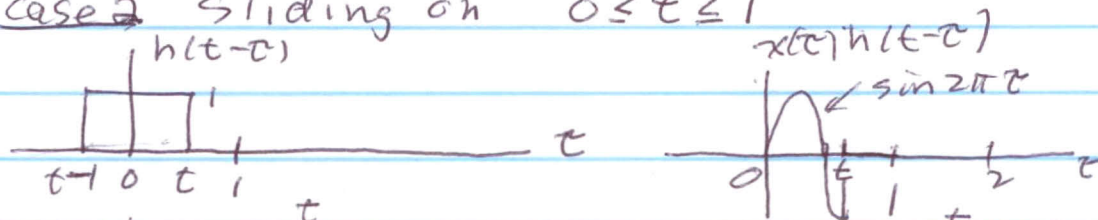


$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$



$$x(\tau) h(t-\tau) = 0 \quad \forall \tau \Rightarrow y(t) = 0$$

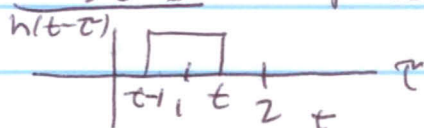
Case 2 sliding on $0 \leq t \leq 1$



$$y(t) = \int_0^t \sin 2\pi \tau d\tau = -\frac{1}{2\pi} \cos 2\pi \tau \Big|_0^t = \frac{1}{2\pi} [1 - \cos 2\pi t]$$

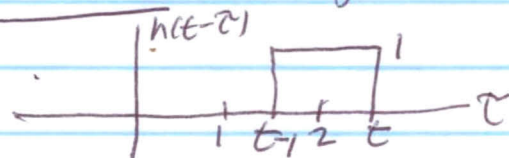
$$= \frac{1}{\pi} \sin^2 \pi t$$

Case 3 completely under $1 \leq t \leq 2$



$$y(t) = \int_{t-1}^t \sin 2\pi \tau d\tau = 0 = \text{area under one cycle}$$

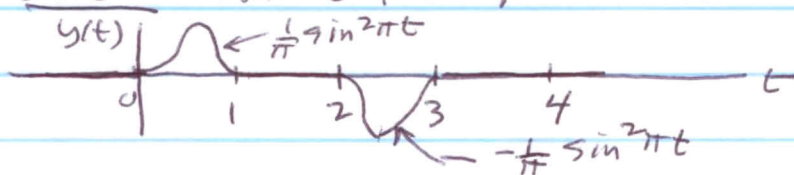
Case 4 sliding off $2 \leq t \leq 3$



$$y(t) = \int_{t-1}^2 \sin 2\pi \tau d\tau = -\frac{1}{2\pi} \cos 2\pi \tau \Big|_{t-1}^2 = \frac{1}{2\pi} [\cos 2\pi(t-1) - 1]$$

$$= \frac{1}{2\pi} [\cos(2\pi t) - 1] = -\frac{1}{\pi} \sin^2 \pi t$$

Case 5 off to the right $t > 3$ $y(t) = 0$



3. (a) Characteristic equation

$$z^2 - 1.1z + 0.1 = (z-1)(z-0.1) = 0 \text{ so } z_1=1, z_2=0.1$$

Homogeneous solution for $n > 0$

$$y[n] = h[n] = c_1 \times 1^n + c_2 (0.1)^n$$

Initial conditions when $x[n] = \delta[n]$

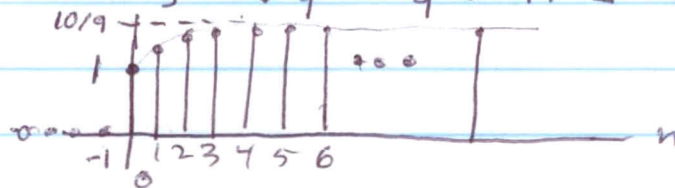
$$y[0] = \underbrace{x[0]}_1 + 1.1 \underbrace{y[-1]}^0 - 0.1 \underbrace{y[-2]}^0 = 1$$

$$y[1] = \cancel{\delta[1]}^0 + 1.1 \underbrace{y[0]}_1 - 0.1 \underbrace{y[-1]}^0 = 1.1$$

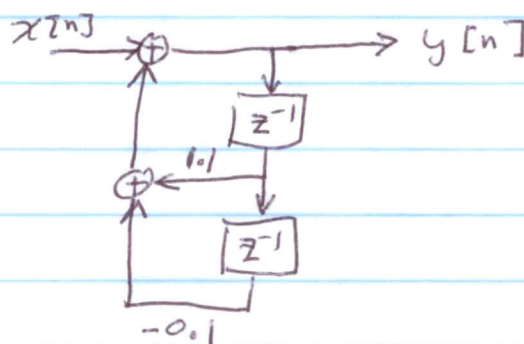
$$h[0] = y[0] = 1 = c_1 + c_2 \Rightarrow c_1 = \frac{10}{9}, c_2 = -\frac{1}{9}$$

$$h[1] = y[1] = 1.1 = c_1 + 0.1 c_2$$

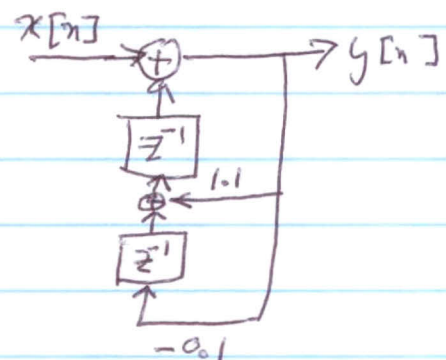
$$\text{So } h[n] = \left[\frac{10}{9} - \frac{1}{9} (0.1)^n \right] u[n]$$



(b)



Direct Form II



Direct Form III

4. (a) Characteristic equation

$$m^2 + 3m + 2 = (m+1)(m+2) = 0 \Rightarrow m_1 = -1, m_2 = -2$$

Homogeneous solution for $t > 0$ with $x(t) = \delta(t)$

$$h(t) = c_1 e^{-t} + c_2 e^{-2t}$$

$$\text{Then } h'(t) = -c_1 e^{-t} - 2c_2 e^{-2t}$$

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Initial conditions

$$h(0) = 0 = c_1 + c_2$$

$$h'(0) = \frac{1}{e} = 1 = -c_1 - 2c_2$$

$$\text{So } c_2 = -1, c_1 = -c_2 = 1$$

$$h(t) = (e^{-t} - e^{-2t}) u(t)$$

(b) Let $h_1(t)$ be the new impulse response. Then

$$h_1(t) = 2h(t) + 10 \frac{d^2}{dt^2} h(t)$$

Next two lines not required

$$= 2(e^{-t} - e^{-2t}) u(t) + 10 [\delta'(t) + (e^{-t} - 4e^{-2t}) u(t)]$$

$$= 10 \delta'(t) + [12e^{-t} - 42e^{-2t}] u(t)$$

(c)

