Maxwell-Boltzmann distribution

Using classical statistical mechanics based on classical probability theory it is possible to derive a relationship between the *temperature* of an ensemble of particles such as atoms or electrons, which is a measure of the average energy of the particles, and the kinetic energy of each particle. The result is a distribution function giving the probability for finding a certain number of particles with energy between U and U + dU. The Maxwell-Boltzmann (M-B) distribution function is $f(U) = \frac{2}{\sqrt{\pi}} (kT)^{-3/2} \sqrt{U}\, e^{-\frac{U}{kT}}$ where the kinetic energy is $U = p^2/2m$ (p is the momentum). $\int_0^\infty f(U)\, dU = 1$. The shape of this distribution (f(U) plotted vs. U) is shown in Figure 1.

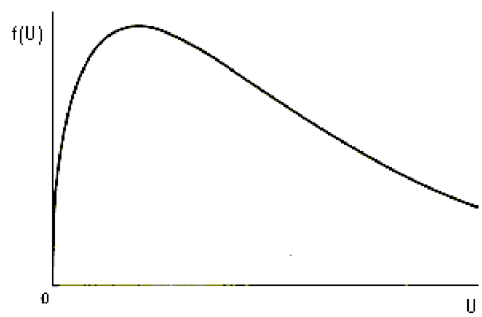

Figure 1. The distribution function f(U) plotted vs. the energy U.

For an example of the use of the M-B distribution, the average kinetic energy <U> of a particle is found by integrating the distribution multiplied by the energy of a particle:

$\overline{U} = \frac{2}{\sqrt{\pi}} (kT)^{-3/2} \int_0^\infty U \sqrt{U}\, e^{-\frac{U}{kT}} = \frac{3}{2} kT$, thus relating average energy and temperature. The exponential factor $e^{-\frac{Energy}{kT}}$ from the M-B distribution comes up all the time in physics; it is not exactly correct because the M-B distribution was derived before the discovery of quantum mechanics; classical probability theory isn't correct at the atomic level, but for many purposes the difference between the exact statistics (called Fermi-Dirac statistics for electrons and holes) and the M-B distribution is small.