Diffusion

The mathematics of diffusion can get involved, but if we limit ourselves to one-dimension it's pretty straightforward. This is not much of a limitation because when we consider diffusion in a semiconductor device the one-dimensional approximation is good.

The simplest conceptual picture of diffusion is probably the motion of ink molecules in water. Imagine a small drop of ink placed carefully into a glass of still water (a glass of water at uniform temperature that has been sitting untouched for a few days, so there are no currents in it). From experience we know that the ink drop will slowly spread out and that, if we wait a long time, the ink molecules will eventually be distributed uniformly throughout the glass. What is happening is that the molecules are moving from regions of high ink density to regions of low ink density. Why do they do this? Here is a simple analogy. Imagine a basketball court aligned along a north-south line with the north half having one person every square foot and the south half having one person every ten square feet. If the people move randomly but only towards one basket or the other, then at any given time half the people will be moving towards the south basket and half towards the north basket. Near the center court there will be, on the average, ten times as many people moving south as moving north and so, after a while the densities of people will start to equilibrate.

Now consider the following situation: particles are arranged in three dimensions but can move only along the direction of the x-axis. The particles have an average speed v_x and their density ρ is a function of x: $\rho = \rho(x)$. Consider a plane perpendicular to the x-axis at location x and another plane perpendicular to the x-axis but located at x + dx. The density of particles at the first plane is $\rho(x)$ and the density at the second plane is $\rho(x+dx)$. The average number of particles cm^{-2} sec^{-1} moving from the plane at x towards the plane at x + dx will be ½ $\rho(x)\, v_x$ and the average number of particles cm^{-2} sec^{-1} moving towards the plane at x from the plane at x + dx will be ½ $\rho(x+dx)\, v_x$. The net number of particles per cm^2 sec^1 crossing the plane at x will then be $-\frac{1}{2}\, v_x \left(\rho(x+dx) - \rho(x)\right) = -\frac{1}{2}\, v_x \frac{d\rho}{dx}\, dx$ so the flux F of particles is proportional to the derivative of the density: $F = -\text{constant} \cdot \frac{d\rho}{dx}$. The constant has units of cm^2 sec^{-1} and is called the diffusion constant. The minus sign means the flow is *away* from the high density region.